# Power Output Optimization of a 

## Variable Speed Wind Turbine

Dr Paul Oram<br>CEO, Ortomation Ltd<br>www.ortomation.io<br>paul@ortomation.io<br>Revised $12^{\text {th }}$ November 2021


#### Abstract

The capture of wind energy is playing a critical role in the establishment of an environmentally sustainable low carbon economy. To meet $\mathrm{CO}_{2}$ emission reduction targets, countries around the world are replacing fossil fuel power generation with wind turbines. Wind turbine size and numbers are growing rapidly, both onshore and offshore. This paper proposes the use of ORTO agents, a novel proprietary real-time optimization technology, to maximize the power generated by a variable speed wind turbine. Dynamic simulation is used to show that power generation may be increased by over $3 \%$, through the addition of an ORTO agent optimization scheme, placed above an existing regulatory control layer.


Keywords: Wind Turbine; Power Generation; Optimization; ORTO Agents

## 1. Introduction

### 1.1. Wind Turbine Overview

A variable speed wind turbine converts kinetic energy of air flow into mechanical energy, via a rotor, and then into electricity, via a gearbox and generator. There are three variables, manipulated by the turbine's control system, which determine energy conversion efficiency:
(1) Pitch angle, or angle of attack. This sets the amount of rotor blade surface area available upon which the incoming wind can apply force.
(2) Rotational speed. This is set in ratio to wind velocity, creating what is known as the tip speed ratio, which directly affects turbine efficiency.
(3) Yaw angle: the angle of the entire wind turbine in the horizontal axis. For maximum
 efficiency the turbine must be kept perpendicular to the wind direction.

### 1.2. ORTO Overview

ORTO (Oram Real-time Optimization) is a proprietary novel model-free real-time optimization approach, marketed by Ortomation Ltd. ORTO offers distinct advantages over traditional RTO approaches. These advantages include greater flexibility, a more intuitive design procedure and straightforward implementation. ORTO also automatically tracks non-stationary (time dependent) optima, e.g., caused by changes in operating philosophy, discrete system modifications, or changes in process dynamics over time. An ORTO scheme is built using independent ORTO agents, each manipulating one system variable. Being model free, ORTO agents have no prior knowledge of the optimization n-dimensional plane. The 'ORTO agent principle' ensures agents learn from each other and work together, to move to and then track the desired optimum value.

## 2. Variable Speed Wind Turbine Operation

There are distinct regions of wind turbine operation, as depicted in Fig. 1 (Apata \& Oyedokun, 2020). In region 1, below a cut-in speed, there is no power generation. In this region, wind speeds are too light, and the wind turbine is in an idle mode. Rotor rotation only begins at the point where the wind speed surpasses the cut-in wind speed. In region 2, the wind turbine can generate power within a range of wind speeds but below a maximum rated power. In this region, the maximization of power generation is the primary focus, using yaw angle, pitch angle and rotor speed. In region 3, the wind speed is between the rated and cut-out speed. Rotor rotation is controlled to a nominal speed, until region 4 is reached upon which power generation is cut to zero, to protect against machinery damage during periods of severe weather. This paper concerns maximization of power output in region 2, within which a turbine is principally designed to operate.


Fig. 1. Wind Turbine Regions of Operation
Wind turbine regulatory feedback control structures are relatively simple. As shown in Fig. 2, three dedicated PID feedback controllers are used, to control each variable. Setpoints are fixed, using settings calculated to maximize power output, across region 2.


Fig. 2. Wind Turbine Controls

## 3. Variable Speed Wind Turbine Dynamic Model

To analyze possible improvements in turbine efficiency through the use of real-time optimization a standard model is used. The steady-state power characteristics of a wind turbine can be described as follows (Siegfried, 1998).

The output power of the turbine is given by:

$$
\begin{equation*}
P_{m}=c_{p}(\lambda, \beta) \frac{\pi l^{2} \rho}{2} \mathrm{v}_{\text {wind }}^{3} \tag{1}
\end{equation*}
$$

where:

| $P_{m}$ | Mechanical output power of the turbine $(\mathrm{W})$ |
| :--- | :--- |
| $c_{p}$ | Performance coefficient of the turbine |
| $\rho$ | Air density $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ |
| $l$ | Turbine blade length $(\mathrm{m})$ |
| $v_{\text {wind }}$ | Wind speed (m/s) |
| $\lambda$ | Tip speed ratio: rotor blade tip speed, $\varpi(\mathrm{deg} / \mathrm{s})$ to wind speed ratio |
| $\beta$ | Blade pitch angle (deg) |

A generic equation can be used to model $c_{p}(\lambda, \beta)$, based on turbine design:

$$
\begin{equation*}
c_{p}(\lambda, \beta)=0.5176\left(\frac{116}{\lambda_{i}}-0.4 \beta-5\right) \mathrm{e}^{-\frac{21}{\lambda_{i}}}+0.0068 \lambda_{i} \tag{2}
\end{equation*}
$$

with:

$$
\begin{equation*}
\lambda=\frac{\varpi}{v_{\text {wind }}} \tag{3}
\end{equation*}
$$

and:

$$
\begin{equation*}
\frac{1}{\lambda_{i}}=\frac{1}{\lambda+0.08 \beta}-\frac{0.035}{\beta^{3}+1} \tag{4}
\end{equation*}
$$

The above steady state model produces a three-dimensional relationship between pitch angle ( $\beta$ ), generator speed $(\varpi)$ and power generated $\left(P_{m}\right)$. This relationship describes the optimization plane which any real-time optimization scheme must navigate to maximize power generated. The shape of this plane is however affected by the two disturbance variables, wind speed ( $v_{\text {wind }}$ ) and air density ( $\rho$ ). Any optimization scheme must therefore accommodate how these disturbance variables change over time. The three-dimensional plane produced when $\mathrm{v}_{\text {wind }}=8 \mathrm{~m} / \mathrm{s}$ and $\rho=$ $1.225 \mathrm{~kg} / \mathrm{m}^{3}$, for a wind turbine having a blade length of 50 m , is shown in Fig. 3. One noticeable attribute is the sharp drop-off in power output beyond a 'critical pitch angle', which is by design $<0^{\circ}$ and varies slightly with generator speed.


Fig. 3. Optimization Plane (with wind speed and air density fixed at $8 \mathrm{~m} / \mathrm{s}$ and $1.225 \mathrm{~kg} / \mathrm{m}^{3}$ respectively)

The above model assumes that the wind direction is constant and wind turbine is always perpendicular to the wind direction. During periods of misalignment, where yaw control lags, power output is reduced. Wind speed needs to be modified by a factor $\cos (\gamma)$, where $\gamma$ is the yaw angle error to the perpendicular, to give an effective wind speed. Thus, from [1], some power output is lost (Wan et al, 2015).

Any real-time optimization scheme also needs to accommodate process dynamics. The following first order plus time delay Laplace transfer functions are used as approximations and assumed to hold true throughout the defined optimization search space.

Pitch angle closed loop control:

$$
\begin{gather*}
\frac{\beta_{p v}(s)}{\beta_{s p}(s)}=\frac{e^{-20 s}}{70 s+1}  \tag{5}\\
\frac{\lambda_{p v}(s)}{\lambda_{s p}(s)}=\frac{e^{-20 s}}{250 s+1}  \tag{6}\\
\frac{P_{m}(s)}{v_{\text {wind }}(s)}=\frac{1}{20 s+1} \tag{7}
\end{gather*}
$$

Generated power response to a change in wind speed (dynamics only):

Faster dynamics facilitate faster optimization. Conversely, slower dynamics will slow the rate of optimization. Suitable low pass filters can be used to counteract measurement noise. Dynamics associated with changes in air density to power generated are assumed negligible, as too are the measurement dynamics of wind speed, air density and power output.

## 4. Wind Turbine Optimization

The optimization objective is to maximize power output from the wind turbine, above the cut-in speed and below the maximum rated power (region 2 in Fig.1), through the continuous adjustment of pitch angle and tip speed ratio. For the purposes of optimization analysis, it is assumed that yaw control is perfect and instantaneous i.e., during simulations the wind direction is constant and wind turbine is always perpendicular to the wind direction.

Two ORTO agents are now added to the control structure; ORTO agent 1 (OA1) writing to the setpoint of the tip speed ratio PID controller and ORTO agent 2 (OA2) writing to the setpoint of the pitch angle PID controller. Each ORTO agent acts to maximize the defined optimization variable (OV) by adjusting its respective manipulated variable (MV). Power output is to be maximized; however, we note that both wind speed and air density are wild 'feed-through' disturbance variables i.e., they affect power output directly, irrespective of the tip speed ratio or pitch angle. Now, we know from [1], ignoring effects of [3], that:

$$
\begin{equation*}
P_{m} \propto v_{\text {wind }}^{3} \tag{8}
\end{equation*}
$$

And:

$$
\begin{equation*}
P_{m} \propto \rho \tag{9}
\end{equation*}
$$

Thus, a superior OV is the ratio of power output to wind speed cubed multiplied by air density:

$$
\begin{equation*}
\mathrm{OV}=\frac{P_{m}}{\rho v_{\text {wind }}^{3}} \tag{10}
\end{equation*}
$$

Maximizing this ratio will inherently maximize power output, for any given wind speed or air density value. To improve performance, a feedforward block can also be added if power to wind dynamics [7] are estimated. Fig. 4 depicts the required ORTO structure, cascaded to the regulatory PID controls. Appendix A describes the general methodology for designing an ORTO optimization scheme.

The set-up parameters of each ORTO agent are detailed in Table 1. Sample times have been selected to accommodate associated system dynamics, [5], [6] and [7]. Maximum move sizes, set in engineering units (EU), have been specified based on sensitivity. For example, power output is very sensitive to pitch angle, thus the ORTO agent 2 maximum move size has been set relatively low. An average wind speed of $8 \mathrm{~m} / \mathrm{s}$ is assumed within region 2 , and this figure is used to compensate respective OV and MV ranges and maximum move sizes. MV constraint bounds are set to define an appropriate optimization search space, again assuming an average wind speed of $8 \mathrm{~m} / \mathrm{s}$. For each
scenario, initial push directions are set to 'best guesses' of where the optimum value may reside. Appendix B provides a deeper overview of how ORTO agent configuration parameters should be set.


Fig. 4. Control Structure with ORTO Agents

| ORTO Agent 1 <br> (MV: Tip Speed Ratio) | Optimization Objective | Maximize |
| :---: | :---: | :---: |
|  | OV Engineering Range | [4000000/(8^3) 0] |
|  | MV Engineering Range | [150/8 0] |
|  | Sample Time (Secs) | 1500 |
|  | Max. Move Size | 1/8 |
|  | MV Upper and Lower Constraint Bounds | [70/8 50/8] |
|  | Initial Push Direction? | +ve |
| ORTO Agent 2 <br> (MV: Pitch Angle) | Optimization Objective | Maximize |
|  | OV Engineering Range | [4000000/(8^3) 0] |
|  | MV Engineering Range | [900] |
|  | Sample Time (Secs) | 425 |
|  | Max. Move Size | 0.1 |
|  | MV Upper and Lower Constraint Bounds | [5-1] |
|  | Initial Push Direction? | -ve |

Table 1. ORTO Agent Configuration

## 5. Simulation Analysis

For the purposes of analyzing optimization performance, the wind turbine model and associated control and optimization scheme have been built in Matlab / Simulink. To reiterate, ORTO agents have no prior knowledge of the model. They must work together to navigate and find the optimum. Four case studies are now presented.

## Case1: Initial Conditions Far From Optimum

To illustrate ORTO agent performance, the initial conditions are set, unrealistically, some distance from the optimum. The wind speed and air density disturbance variables are fixed at $\mathrm{v}_{\text {wind }}=8 \mathrm{~m} / \mathrm{s}$ and $\rho=1.225 \mathrm{~kg} / \mathrm{m}^{3}$ respectively

As shown in Fig. 5, the ORTO agents work together to navigate the optimization plane to achieve the maximum possible power output. Through the action of ORTO agents, power generation is increased by $\sim 80 \%$, over a period of $\sim 24$ hours. The optimization speed and path taken is a function of ORTO agent move sizes and sample times.


Fig. 5. Variable Trends Over 48 Hours and Optimization Path (Initial Conditions: Pitch Angle $4^{\circ}$, Speed $50 \mathrm{deg} / \mathrm{s}$ )

### 5.1. Case 2: Initial conditions at Wind Turbine Manufacturer's Default Settings

Wind turbines are usually designed to provide maximum power at a designated $0^{\circ}$ pitch angle. Dissecting the 3-D plane along the $0^{\circ}$ pitch line on the 3-D plane, shown in Fig. 3, a rotor speed of approximately $64 \mathrm{deg} / \mathrm{s}$ then delivers the maximum power. These manufacturer's default settings are now used as the initial conditions for case 2 .


Fig. 6. Variable Trends Over 48 Hours and Optimization Path (Initial Conditions: Pitch Angle $0^{\circ}$, Speed $64 \mathrm{deg} / \mathrm{s}$ )
Fig. 6 shows how the true optimum is obtained using the ORTO agents. Power output is improved by $\sim 3.1 \%$, when compared with manufacturer's default settings. Most of the gain is achieved by adjusting the pitch angle within the
first three hours and allowing the turbine to run closer to, but still comfortably some distance from, the sharp decline in power output i.e., closer to the 'critical pitch angle'.

### 5.2. Case 3: Initial conditions at Turbine Manufacturer's Default Settings, Real Wind Speed and Air Density Source Data

Case 2 is now repeated but with varying wind speed and air density, using historic real-time values sourced from UK met office datasets (Dyce weather station, https://www.metoffice.gov.uk/research/climate/maps-anddata/data/index). Sampling rates are 10 minutes and 3 hours respectively, with data linearly interpolated between samples. For the purposes of calculating the optimization variable [10], measurement errors are assumed to be zero. A cut-in wind power of 0.1 MW (ref. Fig. 1, boundary between regions $1 \& 2$ ) and a maximum power rating of 4.2 MW (ref. Fig. 1, boundary between regions $2 \& 3$ ) are assumed.


Fig. 7. Variable Trends Over 5 Days (Initial Conditions: Pitch Angle 0 ${ }^{\circ}$, Speed $64 \mathrm{deg} / \mathrm{s}$, Wind Speed and Air Density Varying)
Fig. 7 shows that pitch angle and generator speed are again adjusted, mostly within the first $\sim 12$ hours of operation, to maximize power output. Under ORTO agent optimization, power output is improved by $\sim 2.2 \%$, when compared with manufacturer's default settings. This is less that that achieved in case 2 because of the breaching of the minimum and maximum power rating thresholds throughout the test period i.e., additional power is only achieved whilst operating in region 2.

### 5.3. Case 4: Initial conditions at Turbine Manufacturer's Default Settings, with Wind Speed and Air Density Measurement Error

Case 3 is now repeated, but with a $+/-0.5 \%$ random error superimposed on both the wind speed and air density measurements. This results in imprecise compensation of the wild variables within the OV calculation [10] and noise being superimposed on the OV signal. Incremental improvements in the OV made by each ORTO agent, through respective MV changes, now need to be distinguished from that caused by the noise. Because of the cubing effect and wind speed variability, wind speed measurement noise has a much larger impact on optimization performance, than that of air density measurement noise. Furthermore, the impact of the OV noise is greater on OA1 performance, because the contribution made by changing the tip speed ratio is relatively small compared to pitch angle changes i.e., given the initial conditions, OA1 changes are more easily masked by the noise. To counter the adverse effects, low pass filters are placed on the OV signals to each OA. Time constants of 500 s for OA1 and 50 s for OA2 are chosen.

As can be seen from Fig. 8, the ORTO agents move their respective MVs towards their optimum values as before, but paths taken are not as smooth. Comparing accumulated power generated using ORTO and manufacturer's default settings, ORTO delivers a $\sim 2.2 \%$ improvement, as achieved in case 3 . To summarize, wild variable measurement noise can be tolerated but appropriate OV filtering is required.


Fig. 8. Variable Trends Over 5 Days (Initial Conditions: Pitch Angle $0^{\circ}$, Speed $64 \mathrm{deg} / \mathrm{s}$, Wind Speed and Air Density Varying with Noise)

## 6. Discussion

As demonstrated, ORTO increases power output beyond that achieved by using manufacturer's default operational settings. Moreover, using these settings may in fact cause the wind turbine to be operated further away from the true optimum than first assumed and indicated by the dynamic model used. Reasons include:

1. Inaccuracies within manufacturer's model used to determine optimum tip speed ratio and pitch angle settings.
2. Inherent inaccuracies in wind speed and air density measurements.
3. Small variations in optimal tip speed ratio and pitch angle, through the operational range of wind velocities.
4. Degradation of mechanical parts over time.
5. Non-optimal operation during changes in wind direction due to transitory yaw angle error.

It is therefore reasonable to assume that the manufacturer's default pitch angle and tip speed ratio settings have some additional inherent suboptimality, which may grow over time. The application of ORTO finds the tip speed ratio and pitch angle to deliver the true optimum, without being impacted by any of the above sources of inaccuracy. Benefits, before the turbine's rated power is reached, may therefore be bigger than the $+3 \%$ demonstrated.

## 7. Conclusions and Further Work

This work has shown that there is clear scope for optimizing wind turbine performance. The application of ORTO agents, cascading to traditional pitch angle and tip speed ratio regulatory controls, could deliver more than a $3 \%$ improvement in power output, before full rated power is reached, and maintain optimal operation over time.

Further work is required to investigate the benefit of adding a third ORTO agent to adjust the yaw control setpoint to maximize power generation during periods of yaw angle error. The use of ORTO agents to maximize power across multiple wind turbines within a wind turbine farm should also be examined.

## References

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## Appendix A. ORTO Scheme General Design Methodology

The procedure to design and build an ORTO scheme, for any given process or system, is as follows:
Step 1: Review business objectives. Identify the key commercial, environmental or safety variable to be optimized (OV).
Step 2: Identify all manipulated variables (MVs) which have an influence on the optimization objective and define the range within which an ORTO agent can safely move them within. The number of MVs identified equals the number of ORTO agents required.
Step 3: Identify any wild 'feed-through' variables (WVs) which directly impact the optimization variable, irrespective of MV action. Modify the OV to take account of the WVs accordingly.
Step 4: Identify any external constraint variables (CVs) that the optimization must respect e.g., product quality variables.
Step 5: Implement each ORTO agent, with outputs cascaded to each respective MV and with OV, MV readback and CV logic inputs configured.
Step 6: Configure each ORTO agent by setting the parameters, as listed and described in Appendix B.
Step 7: Switch each ORTO agent on, in turn, monitoring OV improvements. Adjust maximum move sizes and search space, as confidence grows in optimization performance. Remove any ORTO agents making a negligible contribution. Add further ORTO agents as additional MVs are identified.

## Appendix B. ORTO Agent Configuration Parameters

| Optimization Objective | Set to 'Maximize' or 'Minimize'. All ORTO <br> agents must have the same optimization <br> objective within an optimization scheme. |
| :--- | :--- |
| OV Engineering Range (EU) | Knowing the OV and MV ranges helps ORTO <br> to calibrate its MV moves. |
| MV Engineering Range (EU) | Set as a function of system dynamics. <br> Combined with maximum move size, sets the <br> speed of optimization. |
| Sample Time (Secs) | The maximum acceptable net move across the <br> defined sample time. Setting maximum move <br> size too low will slow down optimization. <br> Setting it too high may cause excessive <br> movement of MV at around the optimum <br> value. |
| Max. Move Size (EU) | The bounds within which the MV can be <br> moved. |
| MV Hi-Lo Constraint Bounds (EU) | Set to 'Positive', 'Negative' or 'Not Sure'. <br> Direction is a function of the initial condition <br> i.e., starting point. |
| Initial Push Direction | (Eit |

