June 2021

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MATHEMATICS TEST

60 Minutes—60 Questions

DIRECTIONS: Solve each problem, choose the correct answer, and then fill in the corresponding oval on your answer document.

Do not linger over problems that take too much time. Solve as many as you can; then return to the others in the time you have left for this test.

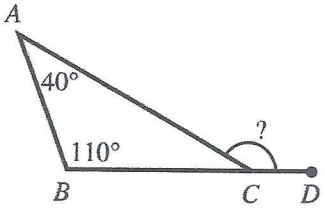
You are permitted to use a calculator on this test. You may use your calculator for any problems you choose, but some of the problems may best be done without using a calculator.

Note: Unless otherwise stated, all of the following should be assumed.

1. Illustrative figures are NOT necessarily drawn to scale.
2. Geometric figures lie in a plane.
3. The word *line* indicates a straight line.
4. The word *average* indicates arithmetic mean.

DO YOUR FIGURING HERE.

1. In the figure below, C is on $\overline{BD}$, $\angle BAC$ measures 40°, and $\angle ABC$ measures 110°. What is the measure of $\angle ACD$?



A. 110°
B. 120°
C. 130°
D. 140°
E. 150°

2. For what value of a is the equation $\frac{1}{2}a + 10 = 6$ true?

F. −32
G. −8
H. −2
J. 8
K. 32

3. What is the least common denominator of the fractions $\frac{4}{15}$, $\frac{1}{20}$, and $\frac{3}{8}$?

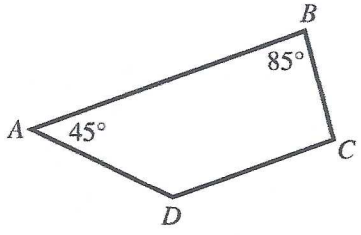
A. 24
B. 120
C. 300
D. 480
E. 2,400

4. $|5 - 3| - |1 - 6| = ?$

F. −7
G. −3
H. 3
J. 7
K. 15

DO YOUR FIGURING HERE.

5. In the trapezoid below, $\overline{AB}$ is parallel to $\overline{DC}$. What is the measure of $\angle C$?



A. 50°
B. 95°
C. 115°
D. 130°
E. 135°

6. Gao earns his regular pay of $12 per hour for up to 40 hours of work per week. For each hour over 40 hours of work per week, Gao is paid $1\frac{1}{2}$ times his regular pay. How much does Gao earn in a week in which he works 56 hours?

F. $ 672
G. $ 756
H. $ 768
J. $1,008
K. $1,344

7. On the first day of school, Ms. Dubacek gave her third-grade students 6 new spelling words to learn. On each day of school after that, she gave the students 3 new spelling words. How many new spelling words had she given the students by the end of the 21st day of school?

A. 60
B. 63
C. 66
D. 69
E. 72

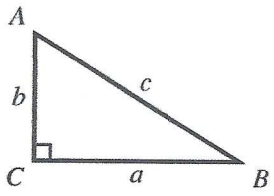
8. What is the value of the expression $\frac{8!}{(4!)^2}$?

(Note: 3! = 3(2)(1) and 6! = 6(5)(4)(3)(2)(1))

F. 0
G. $\frac{1}{2}$
H. 1
J. 70
K. 420

DO YOUR FIGURING HERE.

9. Right triangle $\triangle ABC$ and its side lengths given in inches are shown below. What is sin B ?



A. $\frac{a}{b}$

B. $\frac{a}{c}$

C. $\frac{b}{a}$

D. $\frac{b}{c}$

E. $\frac{c}{a}$

10. $(6a^3 - 5ac^2 + 12c) - (4c - 3a^3 - 2ac^2)$ is equivalent to:

F. $2a^3 - 2ac^2 + 14c$
G. $3a^3 - 7ac^2 + 16c$
H. $9a^3 - 3ac^2 + 8c$
J. $3a^6 - 7a^2c^4 + 16c^2$
K. $9a^6 - 3a^2c^4 + 8c^2$

11. Which of the following (x,y) pairs is the solution for the system of equations $x + 2y = 2$ and $-x + y = 7$?

A. (−4,3)
B. (−1,1.5)
C. (1,0.5)
D. (0,1)
E. (2,0)

12. Tim's flight was originally scheduled to depart at 4:51 p.m., but it was delayed 563 minutes. What time did Tim's flight eventually depart?

F. 1:12 a.m.
G. 1:28 a.m.
H. 2:14 a.m.
J. 10:14 p.m.
K. 10:54 p.m.

13. A circle with the equation $x^2 + y^2 = 144$ is graphed in the standard (x,y) coordinate plane. At what points does the circle intersect the x-axis?

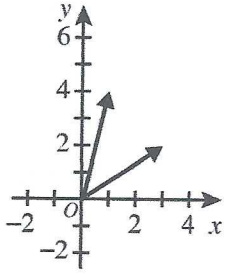
A. (−6,0) and (6,0)
B. (−12,0) and (12,0)
C. (−24,0) and (24,0)
D. (−72,0) and (72,0)
E. (−144,0) and (144,0)

14. Given that $x^2 - 5x - 36$ factors into 2 binomial factors with integer coefficients, which of the following binomials is 1 of those factors?

F. $x - 12$
G. $x - 9$
H. $x - 4$
J. $x + 6$
K. $x + 12$

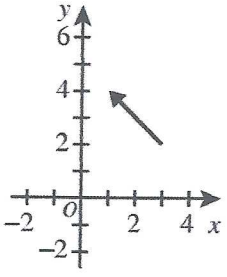
DO YOUR FIGURING HERE.

15. Two vectors are shown in the standard (x,y) coordinate plane below.

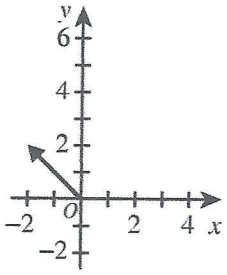


One of the following vectors in the standard (x,y) coordinate plane is the sum of these 2 vectors. Which one?

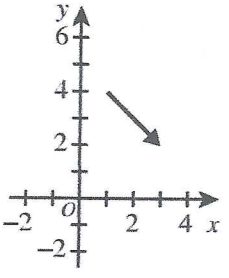
A.



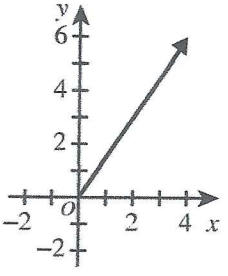
D.



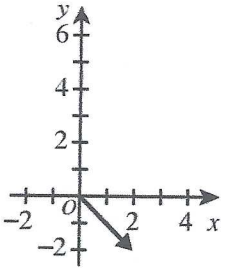
B.



E.



C.



16. A square and a rectangle have the same area. The length of the rectangle is 196 centimeters, and the width of the rectangle is 4 centimeters. What is the length, in centimeters, of a side of the square?

F. 20
G. 28
H. 100
J. 400
K. 784

DO YOUR FIGURING HERE.

17. T-shirts are on sale for D dollars each, including tax. Valentina has N dollars with which to purchase T-shirts. After she purchases the *maximum* number she can, Q T-shirts, she has R dollars left. For all possible choices of D and N, which of the following equations models a correct relationship between D, N, Q, and R, as defined?

A. $N = Q + R$
B. $N = Q + RD$
C. $N = QD + RD$
D. $N = QD + R$
E. $N = QR + D$

18. At a sandwich shop, customers can order either a meat or a vegetarian sandwich on either white or wheat bread. Out of a total of 50 customers, 20 ordered a sandwich on white bread, 28 ordered a meat sandwich, and 12 ordered a meat sandwich on white bread. The given information is summarized in the table below.

Type of bread	Meat	Vegetarian	Total
White	12	?	20
Wheat	?	?	?
Total	28	?	50

How many customers ordered a vegetarian sandwich on wheat bread?

F. 2
G. 8
H. 10
J. 14
K. 16

19. A team of biologists tagged and released 90 deer in a forest. From the same forest 2 weeks later, the biologists collected a random sample of 30 deer, 5 of which were tagged. Let p be the proportion of deer in this forest that are tagged. What is $\hat{p}$, the sample proportion, for this sample?

A. $\frac{1}{24}$
B. $\frac{1}{18}$
C. $\frac{1}{6}$
D. $\frac{1}{5}$
E. $\frac{1}{3}$

20. $\sqrt{2} + \sqrt{8} + \sqrt{18} = ?$

F. $2\sqrt{7}$
G. $6\sqrt{2}$
H. $12\sqrt{2}$
J. $14\sqrt{2}$
K. 14

DO YOUR FIGURING HERE.

21. Which of the following inequalities is equivalent to $3 - 2x > 7 - x$?

A. $x < -\frac{4}{3}$

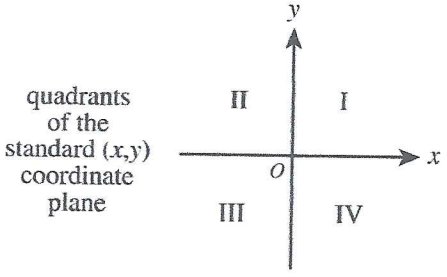
B. $x > -\frac{4}{3}$

C. $x < -4$

D. $x > -4$

E. $x > 6$

22. Let a be positive and b be negative. If it can be determined, in which quadrant of the standard (x,y) coordinate plane is the point $(-a,b^2)$ located?



F. I
G. II
H. III
J. IV
K. Cannot be determined from the given information

23. The mass of a certain type of bacteria grows exponentially, doubling every 20 minutes. What was the mass, in milligrams, of the bacteria exactly 2 hours after the mass first reached 10 milligrams?

A. 70
B. 200
C. 320
D. 640
E. 4,000

24. One day will be randomly selected from the 7 days in a week. Then 1 month will be randomly selected from the 12 months in a year. What is the probability that the selected day will be Tuesday and the selected month will be January?

F. $\frac{1}{84}$

G. $\frac{1}{42}$

H. $\frac{1}{19}$

J. $\frac{2}{19}$

K. $\frac{19}{84}$

DO YOUR FIGURING HERE.

25. The average weight of Juan, Jim, and Malik is exactly 160 pounds. The average weight of Juan, Jim, Malik, and Harry is exactly 150 pounds. How many pounds does Harry weigh?

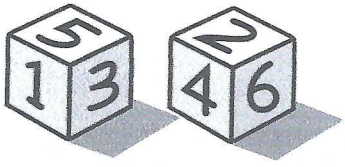
A. 100
B. 120
C. 130
D. 155
E. 190

26. What is the value of the expression $(\log_6(36))(\log_3(9))$?

F. 2
G. 3
H. 4
J. 5
K. 18

27. Each of 2 identical number cubes, shown below, has a different integer, 1 through 6, on each face. Consider the sample space determined by rolling these number cubes and adding the 2 integers on the faces that land on top. What is the positive difference between the greatest sum and the least sum in this sample space?

A. 5
B. 10
C. 11
D. 12
E. 34



28. What angle measure, in radians, is equal to 30° ?

F. $\frac{\pi}{6}$
G. $\frac{\pi}{5}$
H. $\frac{\pi}{3}$
J. $\frac{\pi}{2}$
K. $\frac{5\pi}{6}$

29. For one school week, Hannah recorded the following temperatures, in degrees Fahrenheit, so she could investigate the difference between the high and low temperature each day.

Day	Low	High
Monday	−3°	26°
Tuesday	−5°	32°
Wednesday	−7°	22°
Thursday	3°	40°
Friday	2°	40°

To the nearest degree, what was the mean of the differences in daily high and low temperatures for these 5 days?

A. 28°
B. 29°
C. 30°
D. 32°
E. 34°

DO YOUR FIGURING HERE.

30. A family's budgeted items are expressed as a fraction of their weekly income in the chart below.

Expense	Fraction of income
fixed	$\frac{3}{8}$
food	$\frac{1}{4}$
utilities	$\frac{1}{8}$
transportation	$\frac{1}{12}$
personal	$\frac{1}{16}$
entertainment	$\frac{1}{24}$

What fractional part of their weekly income is left for unbudgeted items?

F. $\frac{5}{48}$

G. $\frac{1}{48}$

H. $\frac{1}{24}$

J. $\frac{1}{16}$

K. $\frac{1}{8}$

31. What is the 322nd digit after the decimal point in the repeating decimal $0.\overline{1357}$?

A. 0
B. 1
C. 3
D. 5
E. 7

32. You and a friend each have a can full of water. You start pouring the water from your can into an empty bucket at a constant rate of 4 ounces per second. While you are still pouring water, 3 seconds after you started, your friend starts pouring the water from her can into the same bucket at a constant rate of 2 ounces per second. How many seconds after you first started pouring the water into the bucket will it contain 24 ounces of water?

F. 4
G. 5
H. 6
J. 8
K. 12

DO YOUR FIGURING HERE.

33. A package of candy contains pieces each of which is 1 of 6 possible colors: brown, red, green, yellow, orange, and blue. In each package, $\frac{1}{3}$ of the pieces are brown and the remaining pieces have an even distribution of the other 5 colors. What is the probability that a piece drawn randomly from the package is red?

A. $\frac{1}{15}$

B. $\frac{2}{15}$

C. $\frac{1}{6}$

D. $\frac{1}{5}$

E. $\frac{2}{3}$

34. Which of the following intervals is the range of the function $f(x) = -(x - 3)^2 + 4$?

F. $(-\infty, 3]$

G. $(-\infty, 4]$

H. $[3, 4]$

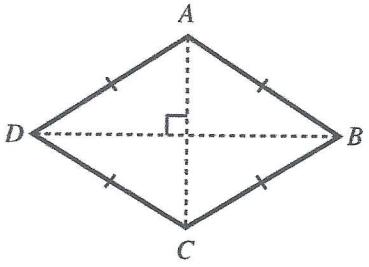
J. $[3, \infty)$

K. $[4, \infty)$

35. Anoki made a scale drawing of his rectangular classroom. The classroom is 7.5 meters by 9.0 meters. In his scale drawing, Anoki made the length of the shorter side of the classroom 9.0 centimeters. What is the length, in centimeters, of the longer side of the classroom in Anoki's scale drawing?

A. 7.5
B. 10.5
C. 10.8
D. 15.0
E. 16.5

36. In rhombus $ABCD$ shown below, $AC = 5$ feet and $BD = 6$ feet. What is the area of $ABCD$, in square feet?



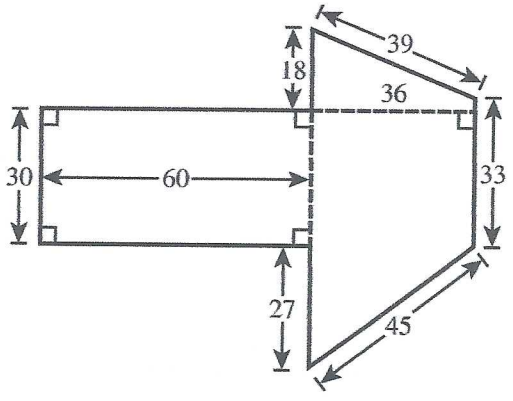
F. 5.5
G. 7.5
H. 11
J. 15
K. 30

DO YOUR FIGURING HERE.

37. One number is 25% of a second number, and the second number is 70% of a third number. The first number is what percent of the third number?

A. 17.5%
B. 42.5%
C. 45%
D. 87.5%
E. 95%

38. The monthly rent charged for a store at Center Street Mall is $2 per square foot of floor area. The floor plan of a store at Center Street Mall is shown in the figure below, with right angles as indicated and all distances given in feet. How much monthly rent is charged for this store?



F. $1,656
G. $1,872
H. $6,624
J. $7,380
K. $7,488

39. Mrs. Neeson, a science teacher, told her students that 30.0% of their final semester grades will come from their homework averages, and the remaining 70.0% will come from their test averages. She also said that the final exam will count for 20.0% of the test average. What percent of the science final semester grade is the final exam grade?

A. 6.0%
B. 10.5%
C. 14.0%
D. 20.0%
E. 28.6%

40. A rectangle with an area of 30 square inches has length and width, in inches, that are both integers. Which of the following CANNOT be the perimeter, in inches, of the rectangle?

F. 22
G. 26
H. 34
J. 60
K. 62

DO YOUR FIGURING HERE.

Use the following information to answer questions 41–43.

A couple is deciding between 2 condos to purchase. Some information about each condo is given below.

Property details	List price	Area in ft^2	Price per ft^2	Annual property tax
Condo X	$210,000	2,274	$92	$3,824
Condo Y	$189,900	1,726	?	$3,524

The couple assumes that the market value of either condo will increase exponentially at a rate of 4% per year.

41. What is the positive difference, in dollars, of the 2 list prices?

A. 2.01×10^2
B. 2.01×10^3
C. 2.01×10^4
D. 2.10×10^4
E. 2.10×10^5

42. The couple will consider the price per square foot of each condo. Let x and y be the price per square foot, rounded to the nearest $1, of Condo X and Condo Y, respectively. One of the following comparisons is true. Which one?

F. x is $ 3 greater than y.
G. x is $ 6 less than y.
H. x is $ 6 greater than y.
J. x is $18 less than y.
K. x is $18 greater than y.

43. The annual property tax for Condo X is 2% of its assessed value. What is the assessed value of Condo X ?

A. $ 19,120
B. $ 42,000
C. $186,200
D. $191,200
E. $205,900

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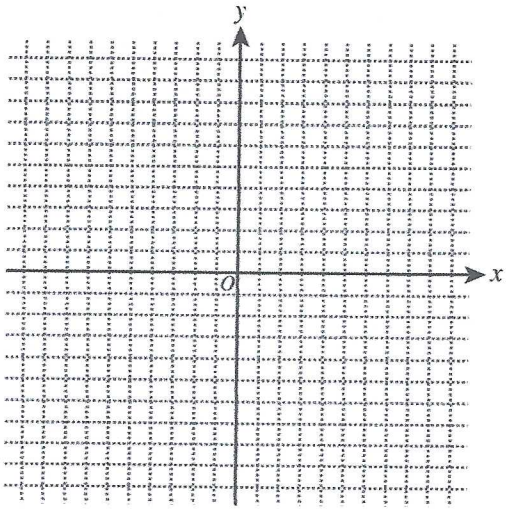
44. In the complex plane, consider the segment whose endpoints are the points corresponding to $-6 + 3i$ and $2 - 7i$. The midpoint of this segment corresponds to which of the following complex numbers?

F. $-4 - 4i$
G. $-4 + 5i$
H. $-2 - 2i$
J. $-2 + 2i$
K. $4 + 5i$

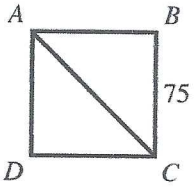
DO YOUR FIGURING HERE.

45. In the standard (x,y) coordinate plane, the 3 lines with equations $y = \frac{3}{5}x - 3$, $y = -\frac{2}{5}x + 2$, and $x = 0$ bound a triangular region. What is the area, in square coordinate units, of that triangular region? (A blank grid has been provided for your use.)



A. 2.5
B. 5.0
C. 7.5
D. 12.5
E. 62.5

46. In square $ABCD$ shown below, $\overline{AC}$ is a diagonal and the length of $\overline{BC}$ is 75 feet. Which of the following quantities is NOT a rational number?



F. The perimeter of $ABCD$, in feet
G. The area of $ABCD$, in square feet
H. The length of $\overline{AB}$, in feet
J. The length of $\overline{AC}$, in feet
K. The measure of $\angle CAD$, in degrees

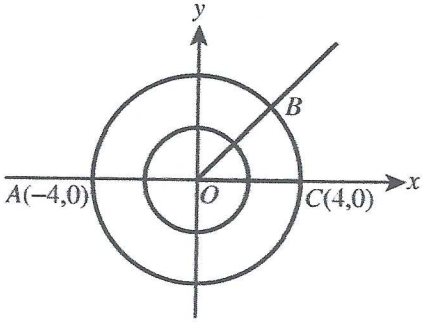
47. The volume of a solid object is equal to the volume of water it displaces when completely submerged in water. A solid object will be placed in a rectangular tank that has a base of 35 cm by 30 cm and is filled with water to a uniform depth of 13 cm. When the object is completely submerged, the new depth of the water in the tank is 15 cm. What is the volume, in cubic centimeters, of the object?

A. 135
B. 525
C. 780
D. 1,212
E. 2,100

DO YOUR FIGURING HERE.

Use the following information to answer questions 48–51.

The 2 circles graphed in the standard (x,y) coordinate plane below are centered at the origin, O. In coordinate units, the radius of the smaller circle is 2, and the radius of the larger circle is 4. Points $A(-4,0)$, B, and $C(4,0)$ are on the larger circle. The measure of $\angle BOC$ is 45°.



(Note: Both axes have the same scale.)

48. What is the x-coordinate of B ?

F. $\frac{4}{\sqrt{3}}$

G. $\frac{4}{\sqrt{2}}$

H. 4

J. $4\sqrt{2}$

K. $4\sqrt{3}$

49. A 3rd circle, not shown, is the image resulting from applying the 1st transformation listed below to the *smaller* circle and then applying the 2nd transformation listed below to the result of the 1st transformation.

1st: A dilation with center O and scale factor 2
2nd: A translation of 8 coordinate units to the right

The 3rd circle has how many points in common with the *larger* circle?

A. 0
B. 1
C. 2
D. 4
E. Infinitely many

50. What is the area, in square coordinate units, of the region that is outside the smaller circle and inside the larger circle?

F. 4π
G. 12π
H. 20π
J. 48π
K. 80π

51. Which of the following is an equation of $\overleftrightarrow{OB}$?

A. $y = -4x$
B. $y = -x$
C. $y = x$
D. $y = 2x$
E. $y = 4x$

DO YOUR FIGURING HERE.

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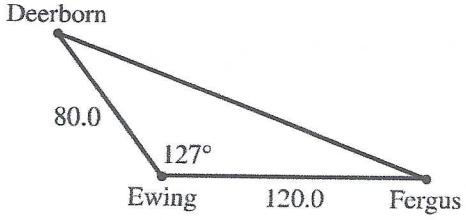
52. A sequence is given by $s_1 = 4$ and $s_{n+1} = 2s_n - 3$ for $n \geq 1$. What is s_5 ?

F. 5
G. 7
H. 11
J. 19
K. 35

53. In the figure below, the distances between 2 pairs of cities are shown, as well as the angle formed at Ewing, which has a measure of 127°. Which of the following values is closest to the distance, in miles, from Deerborn to Fergus?



(Note: $\cos 127° \approx -0.6$; $\sin 127° \approx 0.8$)

A. 100
B. 140
C. 160
D. 180
E. 200

54. Which of the following expressions is equivalent to $\frac{1}{x-a} - \frac{1}{x+a}$?

F. $\frac{2a}{x^2 - a^2}$

G. $\frac{a^2}{x^2 - a^2}$

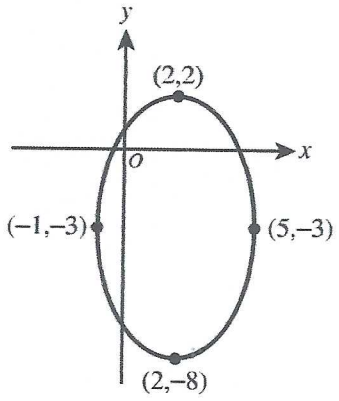
H. $\frac{2x}{x^2 - a^2}$

J. $\frac{1}{x^2 - a^2}$

K. $-\frac{1}{2a}$

DO YOUR FIGURING HERE.

55. One of the following equations represents the ellipse shown below in the standard (x,y) coordinate plane. Which one?



A. $\frac{(x-2)^2}{3} + \frac{(y+3)^2}{5} = 1$

B. $\frac{(x+2)^2}{3} + \frac{(y-3)^2}{5} = 1$

C. $\frac{(x+2)^2}{5} + \frac{(y-3)^2}{3} = 1$

D. $\frac{(x-2)^2}{9} + \frac{(y+3)^2}{25} = 1$

E. $\frac{(x+2)^2}{9} + \frac{(y-3)^2}{25} = 1$

56. One of the following equations is that of a parabola with x-intercepts -5 and $\frac{3}{4}$ in the standard (x,y) coordinate plane. Which equation?

F. $y = 3x^2 - 11x - 20$

G. $y = 3x^2 + 11x - 20$

H. $y = 4x^2 - 17x - 15$

J. $y = 4x^2 + 17x - 15$

K. $y = 15x^2 - 17x - 4$

57. There are 100 fractions in the following set.

$$\left\{\frac{1}{4}, \frac{4}{7}, \frac{7}{10}, \frac{10}{13}, \ldots, \frac{292}{295}, \frac{295}{298}, \frac{298}{301}\right\}$$

Each fraction after the first is found by adding 3 to the preceding fraction's numerator *and* denominator. What is the product of these 100 fractions?

A. 1

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{100}$

E. $\frac{1}{301}$

58. If $2^x = 7$ and $2^y = 14$, then $x - y = ?$

F. -14
G. -7
H. -1
J. 1
K. 49

DO YOUR FIGURING HERE.

59. The table indicates the grade (10 or 11) and high school (North or South) of the 270 students enrolled in Algebra II in the Green City School District.

	North	South
Grade 10	47	93
Grade 11	73	57

Suppose 2 of these students will be chosen at random to represent the Algebra II classes at a local STEM (Science, Technology, Engineering, and Mathematics) event. Which of the following expressions gives the probability that both chosen students will be from the same grade and the same high school?

A. $\frac{47(46)}{270(269)} + \frac{93(92)}{270(269)} + \frac{73(72)}{270(269)} + \frac{57(56)}{270(269)}$

B. $\frac{1}{4}\left(\frac{47}{270} + \frac{93}{270} + \frac{73}{270} + \frac{56}{270}\right)$

C. $\frac{47(73)}{270(269)} + \frac{93(57)}{270(269)}$

D. $\frac{47(93)}{270(269)} + \frac{73(57)}{270(269)}$

E. $\frac{1}{4}\left(\frac{1}{4}\right)$

60. A certain company has 120 employees, 85 of whom have business degrees. Of the employees with business degrees, 75 are certified public accountants (CPAs). There are 14 employees who are not CPAs and also do not hold a business degree. One employee of the company will be selected at random to be interviewed for a television news program. What is the probability that the selected employee will be a CPA ?

(Note: A business degree is NOT required to be a CPA.)

F. $\frac{75}{120}$

G. $\frac{85}{120}$

H. $\frac{89}{120}$

J. $\frac{96}{120}$

K. $\frac{99}{120}$

END OF TEST 2

STOP! DO NOT TURN THE PAGE UNTIL TOLD TO DO SO.

DO NOT RETURN TO THE PREVIOUS TEST.

JUNE 2021 – MATH ANSWERS

1)	E
2)	G
3)	B
4)	G
5)	B
6)	H
7)	C
8)	J
9)	D
10)	H
11)	A
12)	H
13)	B
14)	G
15)	E
16)	G
17)	D
18)	J
19)	C
20)	G
21)	C
22)	G
23)	D
24)	F
25)	B
26)	H
27)	B
28)	F
29)	E
30)	J

31)	C
32)	G
33)	B
34)	G
35)	C
36)	J
37)	A
38)	K
39)	C
40)	J
41)	C
42)	J
43)	D
44)	H
45)	D
46)	J
47)	E
48)	G
49)	B
50)	G
51)	C
52)	J
53)	D
54)	F
55)	D
56)	J
57)	E
58)	H
59)	A
60)	J