Reference Information. The following information is for your reference in answering some of the questions in this test.

Volume of a right circular cone with radius r and height h: $V = \frac{1}{3}\pi r^2 h$

Volume of a sphere with radius r: $V = \frac{4}{3}\pi r^3$

Volume of a pyramid with base area B and height h: $V = \frac{1}{3}Bh$

Surface Area of a sphere with radius r: $S = 4\pi r^2$

Mathematics Level 1

1. A band wants to distribute its music on compact discs (CDs). The equipment to produce the CDs costs $250, and blank CDs cost $5.90 for a package of 10. Which of the following represents the total cost, in dollars, to produce n CDs, where n is a multiple of 10?

   (A) $(250 + 0.59)n$ (B) $250 + 0.59n$
   (C) $(250 + 5.90)n$ (D) $250 + 5.90n$
         (E) $250n + 5.90$
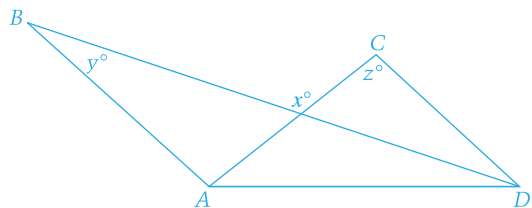
2. In the figure above, $\overline{AB}$ and $\overline{CD}$ are parallel. What is x in terms of y and z?
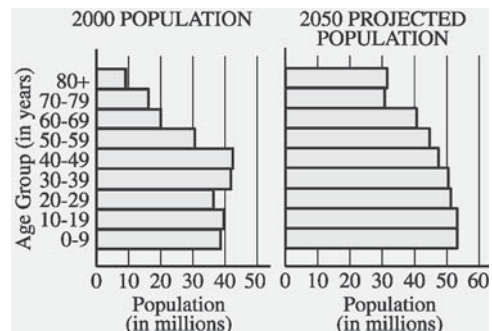
   (A) $y + z$
   (B) $2y + z$
   (C) $2y - z$
   (D) $180 - y - z$
   (E) $180 + y - z$

3. A number n is increased by 8. If the cube root of that result equals -0.5, what is the value of n?

   (A) -15.625
   (B) -8.794
   (C) -8.125
   (D) -7.875
   (E) 421.875

4. If a and b are real numbers, $i^2 = -1$, and $(a + b) + 5i = 9 + ai$, what is the value of b?

   (A) 4   (B) 5   (C) 9
     (D) $4 + 5i$ (E) $5 + 4i$

5. What are all values of x for which $4 - x^2 \geq x - 2$?

   (A) $x \geq -3$ (B) $-5 \leq x \leq 0$
   (C) $-3 \leq x \leq 2$ (D) $x \leq -3$ or $x \geq 2$
      (E) $-2 \leq x \leq 3$

6. The graphs above show United States Census Bureau population figures for the year 2000 for various age groups, together with projections for the year 2050. Of the following age groups, for which is the projected percent increase in population from 2000 to 2050 greatest?

   (A) 30–39   (B) 40–49   (C) 50–59
     (D) 60–69   (E) 70–79

7. If $\log_c a = x$, which of the following must be true?

   (A) $a^c = x$ (B) $a^x = c$ (C) $c^a = x$
       (D) $c^x = a$ (E) $x^c = a$

8. If $f(x) = x + 3$ and $g(x) = \frac{x^2 - 9}{x - 3}$, which of the following statements are true about the graphs of f and g in the xy-plane?

   I. The graphs are exactly the same.
   II. The graphs are the same except when $x = 3$.
   III. The graphs have an infinite number of points in common.

   (A) I only   (B) II only   (C) III only
       (D) I and III   (E) II and III

9. If line ℓ is the perpendicular bisector of the line segment with endpoints $(2, 0)$ and $(0, -2)$, what is the slope of line ℓ?

(A) 2   (B) 1   (C) 0
(D) −1   (E) −2

10. Twenty students have each sampled one or more of three kinds of candy bars that a school store sells. If 3 students have sampled all three kinds, and 5 have sampled exactly two kinds, how many of these students have sampled only one kind?

(A) 8   (B) 12   (C) 15
(D) 17   (E) 18
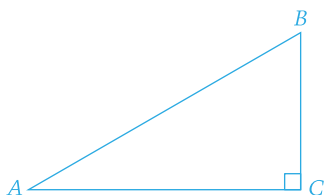
Note: Figure not drawn to scale.

11. In the figure above, $\triangle ABC$ has a right angle at C. If the length of side $\overline{AC}$ is 10 and the measure of $\angle BAC$ is 22°, what is the length of side $\overline{BC}$?

(A) 3.7   (B) 4.0   (C) 5.8
(D) 6.8   (E) 9.3

12. The function h given by $h(t) = -16t^2 + 46t + 5$ represents the height of a ball, in feet, t seconds after it is thrown. To the nearest foot, what is the maximum height the ball reaches?

(A) 5   (B) 23   (C) 35
(D) 38   (E) 46

13. The front, side, and bottom faces of a rectangular solid have areas of 24 square centimeters, 8 square centimeters, and 3 square centimeters, respectively. What is the volume of the solid, in cubic centimeters?

(A) 24   (B) 96   (C) 192
(D) 288   (E) 576
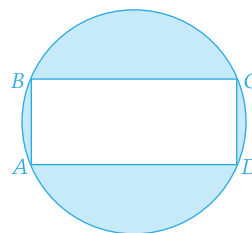
14. Rectangle $ABCD$ is inscribed in the circle shown above. If the length of side $\overline{AB}$ is 5 and the length of side $\overline{BC}$ is 12, what is the area of the shaded region?

(A) 40.8   (B) 53.1   (C) 72.7
(D) 78.5   (E) 81.7

15. If $f(x) = x^4 - 3x^3 - 9x^2 + 4$, for how many real numbers k does $f(k) = 2$?

(A) None   (B) One   (C) Two
(D) Three   (E) Four

16. If the measure of one angle of a rhombus is 60°, then the ratio of the length of its longer diagonal to the length of its shorter diagonal is

(A) 2   (B) $\sqrt{3}$ (C) $\sqrt{2}$
(D) $\dfrac{\sqrt{3}}{2}$ (E) $\dfrac{\sqrt{2}}{2}$

| Time t (years)	0	1	2	5
Value $v(t)$ (dollars)	15,000	13,000	10,900	3,000

17. When purchased, an automobile is valued at $15,000. Its value depreciates at the rate shown in the table above. Based on a least-squares linear regression, what is the value, to the nearest hundred dollars, of the automobile when $t = 4$?

(A) $5,400   (B) $5,500   (C) $5,600
(D) $6,400   (E) $7,000

Mathematics Level 2

18. What is the distance in space between the points with coordinates $(-3, 6, 7)$ and $(2, -1, 4)$?

    (A) 4.36   (B) 5.92   (C) 7.91
       (D) 9.11   (E) 22.25

19. If $f(x) = \dfrac{3x + 12}{2x - 12}$, what value does $f(x)$ approach as x gets infinitely larger?

    (A) -6 (B) $-\dfrac{3}{2}$ (C) -1
       (D) $\dfrac{2}{3}$ (E) $\dfrac{3}{2}$

20. In January 1990 the world's population was 5.3 billion. Assuming a growth rate of 2 percent per year, the world's population, in billions, for t years after 1990 can be modeled by the equation $P = 5.3(1.02)^t$. According to the model, the population growth from January 1995 to January 1996 was

    (A) 106,000,000
    (B) 114,700,000
    (C) 117,000,000
    (D) 445,600,000
    (E) 562,700,000

21. What is the measure of one of the larger angles of a parallelogram in the xy-plane that has vertices with coordinates $(2, 1)$, $(5, 1)$, $(3, 5)$, and $(6, 5)$?

    (A) 93.4°   (B) 96.8°   (C) 104.0°
       (D) 108.3°   (E) 119.0°

22. For some real number t, the first three terms of an arithmetic sequence are $2t, 5t - 1$, and $6t + 2$. What is the numerical value of the fourth term?

    (A) 4   (B) 8   (C) 10
       (D) 16   (E) 19

23. The diameter and height of a right circular cylinder are equal. If the volume of the cylinder is 2, what is the height of the cylinder?

    (A) 1.37   (B) 1.08   (C) 0.86
       (D) 0.80   (E) 0.68

24. If $\sin \theta = 0.57$, then $\sin(\pi - \theta) =$

    (A) -0.57 (B) -0.43 (C) 0
       (D) 0.43   (E) 0.57

25. In a group of 10 people, 60 percent have brown eyes. Two people are to be selected at random from the group. What is the probability that <u>neither</u> person selected will have brown eyes?

    (A) 0.13   (B) 0.16   (C) 0.25
       (D) 0.36   (E) 0.64
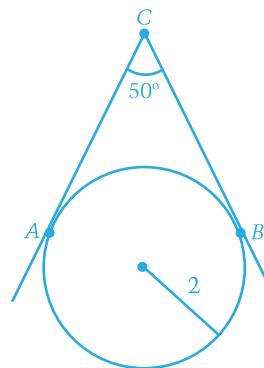
26. In the figure above, two lines are tangent to a circle of radius 2 at points A and B. What is the length of segment AB (not shown)?

    (A) 1.37   (B) 1.69   (C) 3.06
       (D) 3.63   (E) 4

27. If $x - 2$ is a factor of $x^3 + kx^2 + 12x - 8$, then $k =$

    (A) -6 (B) -3 (C) 2
       (D) 3   (E) 6

28. If $f(x) = \sqrt[3]{x^3 + 1}$, what is $f^{-1}(1.5)$?

    (A) 3.4   (B) 2.4   (C) 1.6
       (D) 1.5   (E) 1.3

x	-9.8	-0.9	5.2	8.8
y	0.12	2.43	18.46	68.4

29. Which of the following equations best models the data in the table above?
    (A) $y = -3.3(1.4)^x$
    (B) $y = -1.4(3.3)^x$
    (C) $y = 1.4(3.3)^x$
    (D) $y = 3.3(1.4)^x$
    (E) $y = 1.4x^{3.3}$

$$C = -1.02F + 93.63$$

30. The linear regression model above is based on an analysis of nutritional data from 14 varieties of cereal bars to relate the percent of calories from fat (F) to the percent of calories from carbohydrates (C). Based on this model, which of the following statements must be true?

   I. There is a positive correlation between C and F.
   II. When 20 percent of calories are from fat, the predicted percent of calories from carbohydrates is approximately 73.
   III. The slope indicates that as F increases by 1, C decreases by 1.02.

  (A) II only   (B) I and II only
  (C) I and III only   (D) II and III only
        (E) I, II, and III

31. A line has parametric equations $x = 5 + t$ and $y = 7 + t$, where t is the parameter. The slope of the line is

  (A) $\dfrac{5}{7}$ (B) 1   (C) $\dfrac{7+t}{5+t}$
     (D) $\dfrac{7}{5}$ (E) 7

32. What is the range of the function defined by $f(x) = \dfrac{1}{x} + 2$?

  (A) All real numbers
  (B) All real numbers except $-\dfrac{1}{2}$
  (C) All real numbers except 0
  (D) All real numbers except 2
  (E) All real numbers between 2 and 3

33. The number of hours of daylight, d, in Hartsville can be modeled by $d = \dfrac{35}{3} + \dfrac{7}{3}\sin\left(\dfrac{2\pi}{365}t\right)$, where t is the number of days after March 21. The day with the greatest number of hours of daylight has how many more daylight hours than May 1? (March and May have 31 days each. April and June have 30 days each.)

  (A) 0.8 hr   (B) 1.5 hr   (C) 2.3 hr
     (D) 3.0 hr   (E) 4.7 hr

	Day 1	Day 2	Day 3
Model X	20	18	3
Model Y	16	5	8
Model Z	19	11	10

34. The table above shows the number of digital cameras that were sold during a three-day sale. The prices of models X, Y, and Z were $99, $199, and $299, respectively. Which of the following matrix representations gives the total income, in dollars, received from the sale of the cameras for each of the three days?

(A) $\begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix}\begin{bmatrix} 99 & 199 & 299 \end{bmatrix}$

(B) $\begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix}\begin{bmatrix} 99 \\ 199 \\ 299 \end{bmatrix}$

(C) $\begin{bmatrix} 99 & 199 & 299 \end{bmatrix}\begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix}$

(D) $\begin{bmatrix} 99 \\ 199 \\ 299 \end{bmatrix}\begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix}$

(E) $99\begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix} + 199\begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix} + 299\begin{bmatrix} 20 & 18 & 3 \\ 16 & 5 & 8 \\ 19 & 11 & 10 \end{bmatrix}$

ANSWERS

The estimated difficulty level, on a scale of 1 to 5, with 1 the easiest and 5 the most difficult, is in parentheses.

Mathematics Level 1

1. B (2)   5. C (3)   9. D (4)   13. A (4)   17. C (5)
2. A (2)   6. D (4)   10. B (3)   14. C (4)
3. C (2)   7. D (3)   11. B (3)   15. E (3)
4. A (3)   8. E (3)   12. D (4)   16. B (5)

Mathematics Level 2

18. D (2)   22. E (4)   26. D (4)   30. D (4)   34. C (3)
19. E (2)   23. A (3)   27. A (2)   31. B (3)
20. C (4)   24. E (3)   28. E (4)   32. D (3)
21. C (4)   25. A (4)   29. D (4)   33. A (4)