Mark Scheme

Q1.

Question Number	Scheme			Marks
	$\frac{dN}{dt} = \frac{(kt-1)(5000-N)}{t}, \quad t>0,\ 0<N<5000$			
(a)	$\int \frac{1}{5000-N}\,dN = \int \frac{(kt-1)}{t}\,dt \quad \left\{\text{or } = \int\left(k-\frac{1}{t}\right)dt\right\}$ See notes			B1
	$-\ln(5000-N) = kt - \ln t; +c$ See notes			M1 A1; A1
	then eg either...	*or...*	*or...*	
	$-kt+c = \ln(5000-N) - \ln t$	$kt+c = \ln t - \ln(5000-N)$	$\ln(5000-N) = -kt + \ln t + c$	
	$-kt+c = \ln\left(\frac{5000-N}{t}\right)$	$kt+c = \ln\left(\frac{t}{5000-N}\right)$	$5000-N = e^{-kt+\ln t+c}$	
	$e^{-kt+c} = \frac{5000-N}{t}$	$e^{kt+c} = \frac{t}{5000-N}$	$5000-N = te^{-kt+c}$	
	leading to $N = 5000 - Ate^{-kt}$ with no incorrect working/statements. See notes			A1 * cso [5]
(b)	$\{t=1,\ N=1200 \Rightarrow\}\ 1200 = 5000 - Ae^{-k}$ $\{t=2,\ N=1800 \Rightarrow\}\ 1800 = 5000 - 2Ae^{-2k}$ — At least one correct statement written down using the boundary conditions			B1
	So $Ae^{-k} = 3800$ and $2Ae^{-2k} = 3200$ or $Ae^{-2k} = 1600$			
	Eg. $\frac{e^{-k}}{2e^{-2k}} = \frac{3800}{3200}$ or $\frac{2e^{-2k}}{e^{-k}} = \frac{3200}{3800}$ So $\frac{1}{2}e^{k} = \frac{3800}{3200}$ or $2e^{-k} = \frac{3200}{3800}$ — An attempt to eliminate A by producing an equation in only k.			M1
	$k = \ln\left(\frac{7600}{3200}\right)$ or equivalent $\left\{\text{eg } k = \ln\left(\frac{19}{8}\right)\right\}$ — At least one of $A = 9025$ cao or $k = \ln\left(\frac{7600}{3200}\right)$ or exact equivalent			A1
	$\left\{A = 3800(e^{k}) = 3800\left(\frac{19}{8}\right) \Rightarrow\right\}\ A = 9025$ — Both $A = 9025$ cao or $k = \ln\left(\frac{7600}{3200}\right)$ or exact equivalent			A1 [4]
	Alternative Method for the M1 mark in (b) $e^{-k} = \frac{3800}{A}$ $2A\left(\frac{3800}{A}\right)^2 = 3200$ — An attempt to eliminate k by producing an equation in only A			M1
(c)	$\left\{t=5,\ N = 5000 - 9025(5)e^{-5\ln\left(\frac{19}{8}\right)}\right\}$ $N = 4402.828401... = 4400$ (fish) (nearest 100) — anything that rounds to 4400			B1 [1] 10

Online Maths Teaching

		Question Notes
(a)		
	B1	Separates variables as shown. dN and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	M1	**Either** $\pm\lambda\ln(5000-N)$ or $\pm\lambda\ln(N-5000)$ **or** $kt-\ln t$ where $\lambda \neq 0$ is a constant.
	A1	For $-\ln(5000-N)=kt-\ln t$ or $\ln(5000-N)=-kt+\ln t$ or $-\frac{1}{k}\ln(5000-N)=t-\frac{1}{k}\ln t$ oe
	A1	**which is dependent on the 1st M1 mark being awarded.** For applying a constant of integration, eg. $+c$ or $+\ln e^c$ or $+\ln c$ or A to their integrated equation
	Note	$+c$ can be on either side of their equation for the 2nd A1 mark.
	A1	Uses a constant of integration eg. "c" or "$\ln e^c$" "$\ln c$" or and applies a fully correct method to prove the result $N=5000-Ate^{-kt}$ with no incorrect working seen. (Correct solution only.)
	NOTE	**IMPORTANT** There needs to be an intermediate stage of justifying the A **and** the e^{-kt} in Ate^{-kt} by for example • **either** $5000-N=e^{\ln t-kt+c}$ • **or** $5000-N=te^{-kt+c}$ • **or** $5000-N=te^{-kt}e^c$ or equivalent needs to be stated before achieving $N=5000-Ate^{-kt}$
(b)	B1	At least one of **either** $1200=5000-Ae^{-k}$ (or equivalent) **or** $1800=5000-2Ae^{-2k}$ (or equivalent)
	M1	• **Either** an attempt to eliminate A by producing an equation in only k. • or an attempt to eliminate k by producing an equation in only A
	A1	At least one of $A=9025$ cao or $k=\ln\left(\frac{7600}{3200}\right)$ or equivalent
	A1	Both $A=9025$ cao or $k=\ln\left(\frac{7600}{3200}\right)$ or equivalent
	Note	Alternative correct values for k are $k=\ln\left(\frac{19}{8}\right)$ or $k=-\ln\left(\frac{8}{19}\right)$ or $k=\ln 7600-\ln 3200$ or $k=-\ln\left(\frac{3800}{9025}\right)$ or equivalent.
	Note	$k=0.8649...$ without a correct exact equivalent is A0.
(c)	B1	anything that rounds to 4400

Q2.

Question	Scheme		Marks	AOs
(a)	$\frac{dr}{dt} \propto \pm\frac{1}{r^2}$ or $\frac{dr}{dt} = \pm\frac{k}{r^2}$ (for k or a numerical k)		M1	3.3
	$\int r^2\,dr = \int \pm k\,dt \Rightarrow ...$ (for k or a numerical k)		M1	2.1
	$\frac{1}{3}r^3 = \pm kt\ \{+c\}$		A1	1.1b
	$t=0, r=5$ and $t=4, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$,	$t=0, r=5$ and $t=240, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$,	M1	3.1a
	where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth	where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth	A1	1.1b
			(5)	
(b)	$r=0 \Rightarrow 0 = -\frac{49}{6}t + \frac{125}{3} \Rightarrow\ 0 = -49t + 250\ \Rightarrow t = ...$		M1	3.4
	time = 5 minutes 6 seconds		A1	1.1b
			(2)	
(c)	Suggests a suitable limitation of the model. E.g. • Model does not consider how the mint is sucked • Model does not consider whether the mint is bitten • Model is limited for times up to 5 minutes 6 seconds, o.e. • Not valid for times greater than 5 minutes 6 seconds, o.e. • Mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked • The model indicates that the radius of the mint is negative after it dissolves • Model does not consider the temperature in the mouth • Model does not consider rate of saliva production • Mint could be swallowed before it dissolves in the mouth		B1	3.5b
			(1)	
			(8 marks)	

Notes for Question	
(a)	
M1:	Translates the description of the model into mathematics. See scheme.
M1:	Separates the variables of their differential equation which is in the form $\frac{dr}{dt} = f(r)$ and some attempt at integration. (e.g. attempts to integrate at least one side). e.g. $\int r^2\,dr = \int \pm k\,dt$ and some attempt at integration. Condone the lack of integral signs
Note:	You can imply the M1 mark for $r^2 dr = -k\,dt \Rightarrow \frac{1}{3}r^3 = -kt$
Note:	A numerical value of k (e.g. $k = \pm 1$) is allowed for the first two M marks
A1:	Correct integration to give $\frac{1}{3}r^3 = \pm kt$ with or without a constant of integration, c
M1:	For a complete process of using the boundary conditions to find both their unknown constants and finds an equation linking r and t So applies either • $t = 0,\ r = 5$ and $t = 4,\ r = 3$, or • $t = 0,\ r = 5$ and $t = 240,\ r = 3$, *on their integrated equation* to find their constants k and c *and* obtains an equation linking r and t
A1:	Correct equation, with variables r and t fully defined including correct reference to units. • $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$, {or an equivalent equation,} where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth • $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$, {or an equivalent equation,} where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth
Note:	Allow correct equations such as • in minutes, $r = \sqrt[3]{\frac{250-49t}{2}}$, $r^3 = -\frac{49}{2}t + 125$ or $t = \frac{250-2r^3}{49}$ • in seconds, $r = \sqrt[3]{\frac{15000-49t}{120}}$, $r^3 = -\frac{49}{120}t + 125$ or $t = \frac{15000-120r^3}{49}$
Note:	t defined as "the time from the start" is not sufficient for the final A1

(b)	
M1:	Sets $r = 0$ in their part (a) equation which links r with t and rearranges to make $t = \ldots$
A1:	5 minutes 6 seconds cao (**Note:** 306 seconds with no reference to 5 minutes 6 seconds is A0)
Note:	Give M0 if their equation would solve to give a negative time or a negative time is found
Note:	You can mark part (a) and part (b) together
(c)	
B1:	See scheme
Note:	Do not accept by itself • mint may not dissolve at a constant rate • rate of decrease of mint must be constant • $0 \le t < \frac{250}{49}$, $r \ge 0$; without any written explanation • reference to a mint having $r > 5$

Q3.

Question Number	Scheme	Notes	Marks
	$\frac{dh}{dt} = k\sqrt{(h-9)},\ \ 9 < h \leq 200;\ \ \ h = 130,\ \frac{dh}{dt} = -1.1$		
(a)	$-1.1 = k\sqrt{(130-9)} \Rightarrow k = ...$	Substitutes $h = 130$ and either $\frac{dh}{dt} = -1.1$ or $\frac{dh}{dt} = 1.1$ into the printed equation and rearranges to give $k = ...$	M1
	so, $k = -\frac{1}{10}$ or -0.1	$k = -\frac{1}{10}$ or -0.1	A1
			[2]
(b) **Way 1**	$\int \frac{dh}{\sqrt{(h-9)}} = \int k\,dt$	Separates the variables correctly. dh and dt should not be in the wrong positions, although this mark can be implied by later working. Ignore the integral signs.	B1
	$\int (h-9)^{-\frac{1}{2}}\,dh = \int k\,dt$		
	$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt\,(+c)$	Integrates $\frac{\pm\lambda}{\sqrt{(h-9)}}$ to give $\pm\mu\sqrt{(h-9)}$; $\lambda, \mu \neq 0$	M1
		$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt$ or $\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} =$ (their k)t, with/without $+\,c$, or equivalent, which can be un-simplified or simplified.	A1
	$\{t = 0,\ h = 200 \Rightarrow\}\ \ 2\sqrt{(200-9)} = k(0) + c$	Some evidence of applying both $t = 0$ and $h = 200$ to changed equation containing a constant of integration, e.g. c or A	M1
	$\Rightarrow c = 2\sqrt{191} \Rightarrow 2(h-9)^{\frac{1}{2}} = -0.1t + 2\sqrt{191}$ $\{h = 50 \Rightarrow\}\ \ 2\sqrt{(50-9)} = -0.1t + 2\sqrt{191}$ $t = ...$	**dependent on the previous M mark** Applies $h = 50$ and their value of c to their changed equation and rearranges to find the value of $t = ...$	dM1
	$t = 20\sqrt{191} - 20\sqrt{41}$ or $t = 148.3430145... = 148$ (minutes) (nearest minute)	$t = 20\sqrt{191} - 20\sqrt{41}$ isw or awrt 148	A1 cso
			[6]

(b) **Way 2**	$\int_{200}^{50} \frac{dh}{\sqrt{(h-9)}} = \int_0^T k \, dt$	Separates the variables correctly. dh and dt should not be in the wrong positions, although this mark can be implied by later working. Integral signs and limits not necessary.	B1
	$\int_{200}^{50} (h-9)^{-\frac{1}{2}} dh = \int_0^T k \, dt$		
	$\left[\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_{200}^{50} = [kt]_0^T$	Integrates $\frac{\pm\lambda}{\sqrt{(h-9)}}$ to give $\pm\mu\sqrt{(h-9)}$; $\lambda, \mu \neq 0$	M1
		$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt$ or $\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} =$ (their k)t, with/without limits, or equivalent, which can be un-simplified or simplified.	A1
	$2\sqrt{41} - 2\sqrt{191} = kt$ or kT	Attempts to apply limits of $h = 200, h = 50$ and (can be implied) $t = 0$ to their changed equation	M1
	$t = \frac{2\sqrt{41} - 2\sqrt{191}}{-0.1}$	**dependent on the previous M mark** Then rearranges to find the value of $t = \ldots$	dM1
	$t = 20\sqrt{191} - 20\sqrt{41}$ or $t = 148.3430145\ldots = 148$ (minutes) (nearest minute)	$t = 20\sqrt{191} - 20\sqrt{41}$ **or** awrt 148 **or** 2 hours and awrt 28 minutes	A1 cso
			[6]
			8

		Question Notes
(b)	**Note**	Allow first B1 for writing $\frac{dt}{dh} = \frac{1}{k\sqrt{(h-9)}}$ or $\frac{dt}{dh} = \frac{1}{(\text{their } k)\sqrt{(h-9)}}$ or equivalent
	Note	$\frac{dt}{dh} = \frac{1}{k\sqrt{(h-9)}}$ leading to $t = \frac{2}{k}\sqrt{(h-9)}\ (+c)$ with/without $+c$ is B1M1A1
	Note	After finding $k = 0.1$ in part (a), it is only possible to gain full marks in part (b) by **initially writing** $\frac{dh}{dt} = -k\sqrt{(h-9)}$ **or** $\int \frac{dh}{\sqrt{(h-9)}} = \int -k\,dt$ **or** $\frac{dh}{dt} = -0.1\sqrt{(h-9)}$ **or** $\int \frac{dh}{\sqrt{(h-9)}} = \int -0.1\,dt$ Otherwise, those candidates who find $k = 0.1$ in part (a), should lose at least the final A1 mark in part (b).

Online Maths Teaching

Q4.

Question Number	Scheme		Marks
(a)	$\frac{2}{P(P-2)} = \frac{A}{P} + \frac{B}{(P-2)}$		
	$2 \equiv A(P-2) + BP$	Can be implied.	M1
	$A = -1,\ B = 1$	Either one.	A1
	giving $\frac{1}{(P-2)} - \frac{1}{P}$	See notes. **cao, aef**	A1
			[3]
(b)	$\frac{dP}{dt} = \frac{1}{2}P(P-2)\cos 2t$		
	$\int \frac{2}{P(P-2)}\, dP = \int \cos 2t\, dt$	can be implied by later working	B1 oe
	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t \quad (+c)$	$\pm\lambda\ln(P-2) \pm \mu\ln P$, $\lambda \neq 0,\ \mu \neq 0$	M1
		$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t$	A1
	$\{t = 0,\ P = 3 \Rightarrow\}\quad \ln 1 - \ln 3 = 0 + c \quad \{\Rightarrow\ c = -\ln 3 \text{ or } \ln(\frac{1}{3})\}$	See notes	M1
	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t - \ln 3$ $\ln\left(\frac{3(P-2)}{P}\right) = \frac{1}{2}\sin 2t$		
	$\frac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	M1
	$3(P-2) = Pe^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = Pe^{\frac{1}{2}\sin 2t}$ gives $3P - Pe^{\frac{1}{2}\sin 2t} = 6 \Rightarrow P(3 - e^{\frac{1}{2}\sin 2t}) = 6$	A complete method of rearranging to make P the subject. **Must have a constant of integration that need not be evaluated (see note)**	dM1
	$P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	A1 * cso
			[7]
(c)	$\{\text{population} = 4000 \Rightarrow\}\ P = 4$	States $P = 4$ or applies $P = 4$	M1
	$\frac{1}{2}\sin 2t = \ln\left(\frac{3(4-2)}{4}\right) \left\{= \ln\left(\frac{3}{2}\right)\right\}$	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, $\lambda \neq 0,\ k > 0$ where λ and k are numerical values and λ can be 1	M1
	$t = 0.4728700467...$	anything that rounds to 0.473 Do not apply isw here	A1
			[3] **13**

Question Number	Scheme		Marks
	Method 2 for Q7(b)		
(b)	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t \ \ (+c)$	As before for…	**B1M1A1**
	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$		
	$\frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t + c}$ or $\frac{(P-2)}{P} = Ae^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	**3rd M1**
	$(P-2) = APe^{\frac{1}{2}\sin 2t} \Rightarrow P - APe^{\frac{1}{2}\sin 2t} = 2$ $\Rightarrow P(1 - Ae^{\frac{1}{2}\sin 2t}) = 2 \Rightarrow P = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2t})}$	A complete method of rearranging to make P the subject. Condone sign slips or constant errors. **Must have a constant of integration that need not be evaluated (see note)**	**4th dM1**
	$\{t = 0, P = 3 \Rightarrow\}\ \ 3 = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2(0)})}$	See notes **(Allocate this mark as the 2nd M1 mark on ePEN).**	**2nd M1**
	$\left\{\Rightarrow 3 = \frac{2}{(1-A)} \Rightarrow A = \frac{1}{3}\right\}$		
	$\Rightarrow P = \frac{2}{\left(1 - \frac{1}{3}e^{\frac{1}{2}\sin 2t}\right)} \Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	**A1 * cso**

Question Notes

(a)	**M1**	Forming a correct identity. For example, $2 \equiv A(P-2) + BP$ from $\frac{2}{P(P-2)} \equiv \frac{A}{P} + \frac{B}{(P-2)}$
	Note	A and B are not referred to in question.
	A1	Either one of $A = -1$ or $B = 1$.
	A1	$\frac{1}{(P-2)} - \frac{1}{P}$ or any equivalent form. This answer *cannot* be recovered from part (b).
	Note	M1A1A1 can also be given for a candidate who finds both $A = -1$ and $B = 1$ and $\frac{A}{P} + \frac{B}{(P-2)}$ is seen in their working.
	Note	Candidates can use 'cover-up' rule to write down $\frac{1}{(P-2)} - \frac{1}{P}$, so as to gain all three marks.
	Note	Equating coefficients from $2 \equiv A(P-2) + BP$ gives $A + B = 2, -2A = 2 \Rightarrow A = -1, B = 1$

(b)	B1	Separates variables as shown on the Mark Scheme. $\mathrm{d}P$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Eg:** $\int \frac{2}{P^2 - 2P}\,\mathrm{d}P = \int \cos 2t\,\mathrm{d}t$ or $\int \frac{1}{P(P-2)}\,\mathrm{d}P = \frac{1}{2}\int \cos 2t\,\mathrm{d}t$ o.e. are also fine for B1.
	1st M1	$\pm\lambda\ln(P-2) \pm \mu\ln P,\ \lambda \neq 0,\ \mu \neq 0$. Also allow $\pm\lambda\ln(M(P-2)) \pm \mu\ln NP$; M, N can be 1.
	Note	Condone $2\ln(P-2) + 2\ln P$ or $2\ln(P(P-2))$ or $2\ln(P^2 - 2P)$ or $\ln(P^2 - 2P)$
	1st A1	Correct result of $\ln(P-2) - \ln P = \frac{1}{2}\sin 2t$ or $2\ln(P-2) - 2\ln P = \sin 2t$ o.e. with or without $+c$
	2nd M1	Some evidence of using both $t = 0$ and $P = 3$ in an integrated equation containing a constant of integration. Eg: c or A, etc.
	3rd M1	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c,\ \lambda,\mu,\beta,K,\delta \neq 0$, applies a fully correct method to eliminate their logarithms.
	4th M1	**dependent on the third method mark being awarded.** A complete method of rearranging to make P the subject. Condone sign slips or constant errors.
	Note	For the 3rd M1 and 4th M1 marks, a candidate needs to have included a constant of integration, in their working. eg. c, A, $\ln A$ or an evaluated constant of integration.
	2nd A1	Correct proof of $P = \frac{6}{(3 - \mathrm{e}^{\frac{1}{2}\sin 2t})}$. **Note:** This answer is given in the question.
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$ followed by $\frac{(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} + \mathrm{e}^{c}$ is 3rd M0, 4th M0, 2nd A0.
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c \rightarrow \frac{(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t + c} \rightarrow \frac{(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} + \mathrm{e}^{c}$ is final M1M0A0
	4th M1 for making P the subject	**Note there are three type of manipulations here which are considered acceptable for making P the subject.** (1) M1 for $\frac{3(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3(P-2) = P\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = P\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow P(3 - \mathrm{e}^{\frac{1}{2}\sin 2t}) = 6 \Rightarrow P = \frac{6}{(3 - \mathrm{e}^{\frac{1}{2}\sin 2t})}$ (2) M1 for $\frac{3(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3 - \frac{6}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3 - \mathrm{e}^{\frac{1}{2}\sin 2t} = \frac{6}{P} \Rightarrow\Rightarrow P = \frac{6}{(3 - \mathrm{e}^{\frac{1}{2}\sin 2t})}$ (3) M1 for $\left\{\ln(P-2) + \ln P = \frac{1}{2}\sin 2t + \ln 3 \Rightarrow\right\} P(P-2) = 3\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow P^2 - 2P = 3\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow (P-1)^2 - 1 = 3\mathrm{e}^{\frac{1}{2}\sin 2t}$ leading to $P = ..$
(c)	M1	States $P = 4$ or applies $P = 4$
	M1	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, where λ and k are numerical values and λ can be 1
	A1	anything that rounds to 0.473. *(Do not apply isw here)*
	Note	*Do not apply ignore subsequent working for A1.* (Eg: 0.473 followed by 473 years is A0.)
	Note	Use of $P = 4000$: Without the mention of $P = 4$, $\frac{1}{2}\sin 2t = \ln 2.9985$ or $\sin 2t = 2\ln 2.9985$ or $\sin 2t = 2.1912...$ will usually imply M0M1A0
	Note	Use of Degrees: $t =$ awrt 27.1 will usually imply M1M1A0

Q5.

Question Number	Scheme	Marks
	(a) $\frac{dV}{dt} = 0.48\pi - 0.6\pi h$	M1 A1
	$V = 9\pi h \quad \Rightarrow \frac{dV}{dt} = 9\pi \frac{dh}{dt}$	B1
	$9\pi \frac{dh}{dt} = 0.48\pi - 0.6\pi h$	M1
	Leading to $75\frac{dh}{dt} = 4 - 5h$ * cso	A1 **(5)**
	(b) $\int \frac{75}{4-5h} dh = \int 1 dt$ separating variables	M1
	$-15\ln(4-5h) = t \ (+C)$	M1 A1
	$-15\ln(4-5h) = t + C$	
	When $t = 0$, $h = 0.2$	
	$-15\ln 3 = C$	M1
	$t = 15\ln 3 - 15\ln(4-5h)$	
	When $h = 0.5$	
	$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$ awrt 10.4	M1 A1
	Alternative for last 3 marks	
	$t = \left[-15\ln(4-5h)\right]_{0.2}^{0.5}$	
	$= -15\ln 1.5 + 15\ln 3$	M1 M1
	$= 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$ awrt 10.4	A1 **(6)**

Q6.

Question Number	Scheme		Marks
(a)	$\frac{dV}{dt} = 1600 - c\sqrt{h}$ or $\frac{dV}{dt} = 1600 - k\sqrt{h}$,	Either of these statements	M1
	$(V = 4000h \Rightarrow)\ \frac{dV}{dh} = 4000$	$\frac{dV}{dh} = 4000$ or $\frac{dh}{dV} = \frac{1}{4000}$	M1
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}}$		
	Either, $\frac{dh}{dt} = \frac{1600 - c\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{c\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$ or $\frac{dh}{dt} = \frac{1600 - k\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{k\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$	Convincing proof of $\frac{dh}{dt}$	A1 **AG**
			[3]
(b)	When $h = 25$ water ***leaks out such that*** $\frac{dV}{dt} = 400$		
	$400 = c\sqrt{h} \Rightarrow 400 = c\sqrt{25} \Rightarrow 400 = c(5) \Rightarrow c = 80$		
	From above; $k = \frac{c}{4000} = \frac{80}{4000} = 0.02$ as required	Proof that $k = 0.02$	B1 **AG**
			[1]
Aliter (b) **Way 2**	$400 = 4000k\sqrt{h}$		
	$\Rightarrow 400 = 4000k\sqrt{25}$		
	$\Rightarrow 400 = k(20000) \Rightarrow k = \frac{400}{20000} = 0.02$	Using 400, 4000 and $h = 25$ or $\sqrt{h} = 5$. Proof that $k = 0.02$	B1 **AG**
			[1]
(c)	$\frac{dh}{dt} = 0.4 - k\sqrt{h} \Rightarrow \int \frac{dh}{0.4 - k\sqrt{h}} = \int dt$	***Separates the variables*** with $\int \frac{dh}{0.4 - k\sqrt{h}}$ and $\int dt$ on either side with integral signs not necessary.	M1 oe
	$\therefore$ time required $= \int_0^{100} \frac{1}{0.4 - 0.02\sqrt{h}}\, dh \quad \frac{\div 0.02}{\div 0.02}$		
	time required $= \int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh$	Correct proof	A1 **AG**
			[2]

Question Number	Scheme		Marks
(d)	$\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h$ with substitution $h=(20-x)^2$		
	$\frac{\mathrm{d}h}{\mathrm{d}x}=2(20-x)(-1)$ or $\frac{\mathrm{d}h}{\mathrm{d}x}=-2(20-x)$	Correct $\frac{\mathrm{d}h}{\mathrm{d}x}$	B1 aef
	$h=(20-x)^2 \Rightarrow \sqrt{h}=20-x \Rightarrow x=20-\sqrt{h}$		
	$\int \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \int \frac{50}{x}.-2(20-x)\,\mathrm{d}x$	$\pm\lambda\int \frac{20-x}{x}\,\mathrm{d}x$ or $\pm\lambda\int \frac{20-x}{20-(20-x)}\,\mathrm{d}x$ where λ is a constant	M1
	$=100\int \frac{x-20}{x}\,\mathrm{d}x$		
	$=100\int \left(1-\frac{20}{x}\right)\mathrm{d}x$		
	$=100(x-20\ln x)\ (+c)$	$\pm\alpha x \pm \beta \ln x\ ;\ \alpha, \beta \neq 0$ $100x-2000\ln x$	M1 A1
	change limits: when $h=0$ then $x=20$ and when $h=100$ then $x=10$		
	$\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = [100x-2000\ln x]_{20}^{10}$		
	or $\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \left[100\left(20-\sqrt{h}\right)-2000\ln\left(20-\sqrt{h}\right)\right]_0^{100}$		
	$=(1000-2000\ln 10)-(2000-2000\ln 20)$	Correct use of limits, ie. putting them in the correct way round Either $x=10$ and $x=20$ or $h=100$ and $h=0$	ddM1
	$=2000\ln 20-2000\ln 10-1000$	Combining logs to give...	
	$=2000\ln 2-1000$	$2000\ln 2-1000$ or $-2000\ln\left(\frac{1}{2}\right)-1000$	A1 aef
			[6]
(e)	Time required $=2000\ln 2-1000 = 386.2943611...$ sec		
	$=386$ seconds (nearest second)		
	$=6$ minutes and 26 seconds (nearest second)	6 minutes, 26 seconds	B1
			[1]
			13 marks

Q7.

Question	Scheme	Marks	AOs
(a)	$\dfrac{dV}{dt} = 0.48 - 0.1h$	B1	3.1b
	$V = 24h \Rightarrow \dfrac{dV}{dh} = 24$ or $\dfrac{dh}{dV} = \dfrac{1}{24}$	B1	3.1b
	$\dfrac{dh}{dt} = \dfrac{dV}{dt} \times \dfrac{dh}{dV} = \dfrac{0.48 - 0.1h}{24}$ or e.g. $\dfrac{dV}{dt} = \dfrac{dV}{dh}\dfrac{dh}{dt} \Rightarrow 0.48 - 0.1h = 24\dfrac{dh}{dt}$	M1	2.1
	$1200\dfrac{dh}{dt} = 24 - 5h$ *	A1*	1.1b
		(4)	
(b)	$1200\dfrac{dh}{dt} = 24 - 5h \Rightarrow \displaystyle\int \dfrac{1200}{24-5h}dh = \int dt$ $\Rightarrow$ *e.g.* $\alpha \ln(24-5h) = t(+c)$ oe or $1200\dfrac{dh}{dt} = 24 - 5h \Rightarrow \dfrac{dt}{dh} = \dfrac{1200}{24-5h}$ $\Rightarrow$ *e.g.* $t(+c) = \alpha \ln(24-5h)$ oe	M1	3.1a
	$t = -240\ln(24-5h)(+c)$ oe	A1	1.1b
	$t = 0, h = 2 \Rightarrow 0 = -240\ln(24-10) + c \Rightarrow c = ...(240\ln 14)$	M1	3.4
	$t = 240\ln(14) - 240\ln(24-5h)$	A1	1.1b
	$t = 240\ln\dfrac{14}{24-5h} \Rightarrow \dfrac{t}{240} = \ln\dfrac{14}{24-5h} \Rightarrow e^{\frac{t}{240}} = \dfrac{14}{24-5h}$ $\Rightarrow 14e^{-\frac{t}{240}} = 24 - 5h \Rightarrow h = ...$	ddM1	2.1
	$h = 4.8 - 2.8e^{-\frac{t}{240}}$ oe e.g. $h = \dfrac{24}{5} - \dfrac{14}{5}e^{-\frac{t}{240}}$	A1	3.3
		(6)	

(c)	**Examples:** • As $t \to \infty, e^{-\frac{t}{240}} \to 0$ • When $h > 4.8$, $\frac{dV}{dt} < 0$ • Flow in = flow out at max h so $0.1h = 4.8 \rightarrow h = 4.8$ • As $e^{-\frac{t}{240}} > 0,\ h < 4.8$ • $h = 5 \Rightarrow \frac{dV}{dt} = -0.02$ or $\frac{dh}{dt} = -\frac{1}{1200}$ • $\frac{dh}{dt} = 0 \Rightarrow h = 4.8$ • $h = 5 \Rightarrow 4.8 - 2.8e^{-\frac{t}{240}} = 5 \Rightarrow e^{-\frac{t}{240}} < 0$	M1	3.1b
	• The limit for h (according to the model) is 4.8m and the tank is 5m high so the tank will never become full • If $h = 5$ the tank would be emptying so can never be full • The equation can't be solved when $h = 5$	A1	3.2a
		(2)	
			(12 marks)

Notes
(a) B1: Identifies the correct expression for $\frac{dV}{dt}$ according to the model B1: Identifies the correct expression for $\frac{dV}{dh}$ according to the model M1: Applies $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$ or equivalent correct formula with their $\frac{dV}{dt}$ and $\frac{dV}{dh}$ which may be implied by their working A1*: Correct equation obtained with no errors Note that: $\frac{dV}{dt} = 0.48 - 0.1h \Rightarrow \frac{dh}{dt} = \frac{0.48 - 0.1h}{24} \Rightarrow 1200\frac{dh}{dt} = 24 - 5h$ * scores B1B0M0A0. There must be clear evidence where the "24" comes from and evidence of the correct chain rule being applied.

(b)

M1: Adopts a correct strategy by separating the variables correctly or rearranges to obtain $\frac{dt}{dh}$ correctly in terms of h **and** integrates to obtain $t = \alpha \ln(24-5h)(+c)$ or equivalent (condone missing brackets around the "24 – 5h") and + c not required for this mark.
A1: Correct equation in any form and + c not required. Do not condone missing brackets unless they are implied by subsequent work.
M1: Substitutes $t = 0$ and $h = 2$ to find their constant of integration (there must have been some attempt to integrate)
A1: Correct equation in any form
ddM1: Uses fully correct log work to obtain h in terms of t.
This depends on <u>both</u> previous method marks.
A1: Correct equation
Note that the marks may be earned in a different order e.g.:

$$t + c = -240\ln(24-5h) \Rightarrow -\frac{t}{240} + d = \ln(24-5h) \Rightarrow Ae^{-\frac{t}{240}} = 24-5h$$

$$t = 0, h = 2 \Rightarrow A = 14 \Rightarrow 14e^{-\frac{t}{240}} = 24-5h \Rightarrow h = 4.8 - 2.8e^{-\frac{t}{240}}$$

Score as M1 A1 as in main scheme then

M1: Correct work leading to $Ae^{\alpha t} = 24-5h$ (must have a constant "A")

A1: $Ae^{-\frac{t}{240}} = 24-5h$

ddM1: Uses t = 0, h = 2 in an expression of the form above to find A

A1: $h = 4.8 - 2.8e^{-\frac{t}{240}}$

(c)

M1: See scheme for some examples
A1: Makes a correct interpretation for their method.
There must be no incorrect working or contradictory statements.
This is not a follow through mark and if their equation in (b) is used it must be correct.