Questions

Q1.

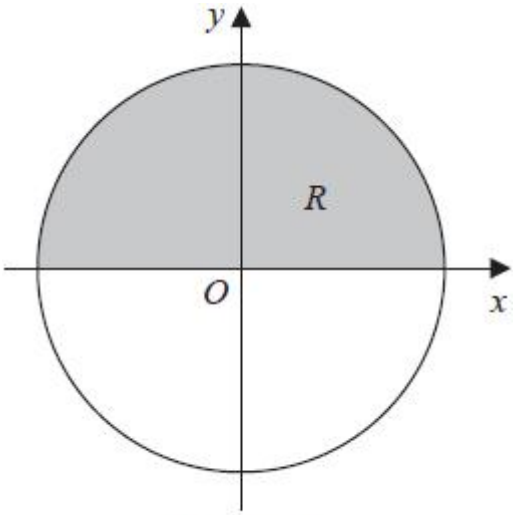

Figure 1

Figure 1 shows a circle with radius *r* and centre at the origin.
The region *R*, shown shaded in Figure 1, is bounded by the *x*-axis and the part of the circle for which $y > 0$
The region R is rotated through 360° about the *x*-axis to create a sphere with volume *V*

Use integration to show that $V = \frac{4}{3}\pi r^3$

(Total for question = 5 marks)

Q2.

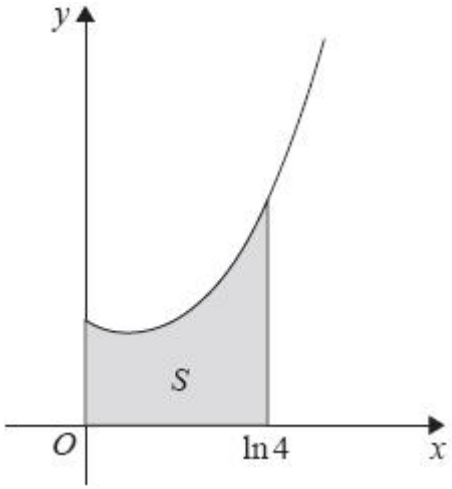

Diagram not drawn to scale

Figure 2

The finite region S, shown shaded in Figure 2, is bounded by the y-axis, the x-axis, the line with equation $x = \ln 4$ and the curve with equation

$$y = e^{x} + 2e^{-x}, \qquad x \geq 0$$

The region S is rotated through 2π radians about the x-axis.

Use integration to find the exact value of the volume of the solid generated.
Give your answer in its simplest form.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(7)

(Total for question = 7 marks)

Q3.

$$f(x) = 2x^{\frac{1}{3}} + x^{-\frac{2}{3}} \qquad x > 0$$

The finite region bounded by the curve $y = f(x)$, the line $x = \frac{1}{8}$, the x-axis and the line $x = 8$ is rotated through θ radians about the x-axis to form a solid of revolution.

Given that the volume of the solid formed is $\frac{461}{2}$ units cubed, use algebraic integration to find the angle θ through which the region is rotated.

(Total for question = 8 marks)

Q4.

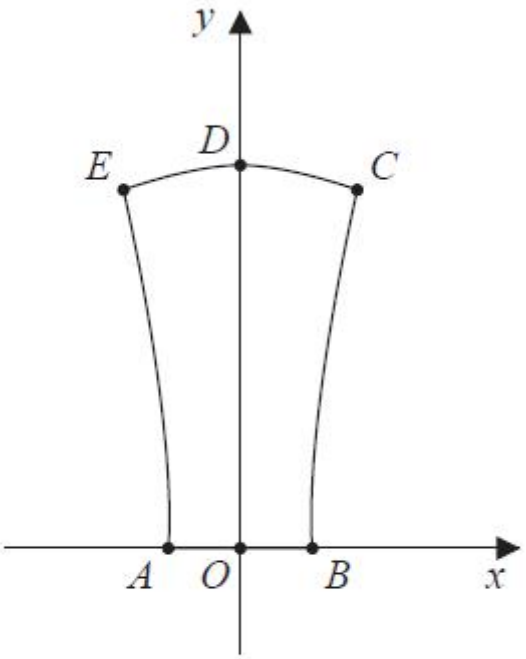

Figure 2

Figure 2 shows the vertical cross-section, *AOBCDE*, through the centre of a wax candle.

In a model, the candle is formed by rotating the region bounded by the *y*-axis, the line *OB*, the curve *BC*, and the curve *CD* through 360° about the *y*-axis.

The point *B* has coordinates (3, 0) and the point *C* has coordinates (5, 15).

The units are in centimetres.

The curve *BC* is represented by the equation

$$y = \frac{\sqrt{225x^2 - 2025}}{a} \qquad 3 \leqslant x < 5$$

where *a* is a constant.

(a) Determine the value of *a* according to this model.

(2)

The curve *CD* is represented by the equation

$$y = 16 - 0.04x^2 \qquad 0 \leqslant x < 5$$

(b) Using algebraic integration, determine, according to the model, the exact volume of wax that would be required to make the candle.

(9)

(c) State a limitation of the model.

(1)

When the candle was manufactured, 700 cm^3 of wax were required.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning.

(1)

(Total for question = 13 marks)

Q5.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

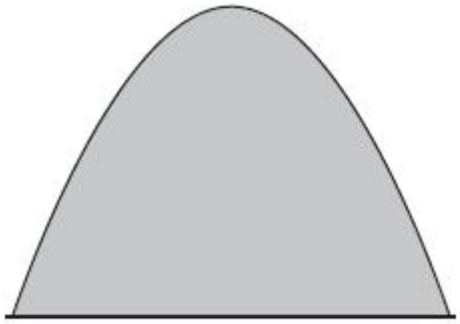

Figure 1

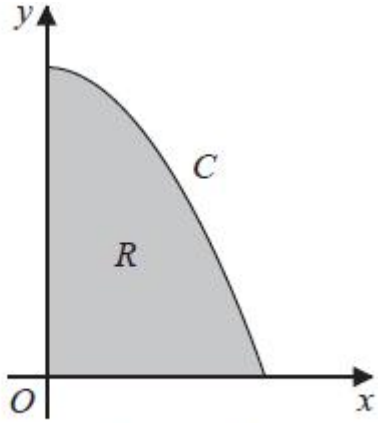

Figure 2

A large pile of concrete waste is created on a building site.

Figure 1 shows a central vertical cross-section of the concrete waste.

The curve *C*, shown in Figure 2, has equation

$$y + x^2 = 2 \qquad 0 \leqslant x \leqslant \sqrt{2}$$

The region *R*, shown shaded in Figure 2, is bounded by the *y*-axis, the *x*-axis and the curve *C*.

The volume of concrete waste is modelled by the volume of revolution formed when *R* is rotated through 360° about the *y*-axis. The units are metres.

The density of the concrete waste is 900 kgm^{-3}

(a) Use the model to estimate the mass of the concrete waste. Give your answer to 2 significant figures.

(6)

(b) Give a limitation of the model.

(1)

The mass of the concrete waste is approximately 5500 kg.

(c) Use this information and your answer to part (a) to evaluate the model, giving a reason for your answer.

(1)

(Total for question = 8 marks)

Q6.

Diagrams not drawn to scale

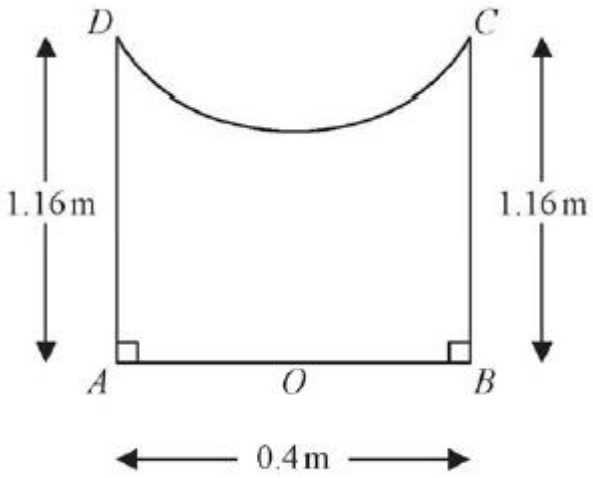

Figure 1

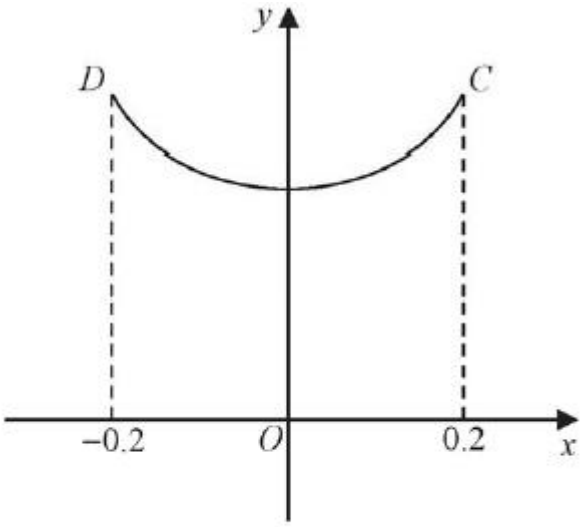

Figure 2

Figure 1 shows the central cross-section *AOBCD* of a circular bird bath, which is made of concrete. Measurements of the height and diameter of the bird bath, and the depth of the bowl of the bird bath have been taken in order to estimate the amount of concrete that was required to make this bird bath.

Using these measurements, the cross-sectional curve *CD*, shown in Figure 2, is modelled as a curve with equation

$$y = 1 + kx^2 \qquad -0.2 \leqslant x \leqslant 0.2$$

where k is a constant and where O is the fixed origin.

The height of the bird bath measured 1.16 m and the diameter, *AB*, of the base of the bird bath measured 0.40 m, as shown in Figure 1.

(a) Suggest the maximum depth of the bird bath.

(1)

(b) Find the value of k.

(2)

(c) Hence find the volume of concrete that was required to make the bird bath according to this model. Give your answer, in m^3, correct to 3 significant figures.

(7)

(d) State a limitation of the model.

(1)

It was later discovered that the volume of concrete used to make the bird bath was 0.127 m^3 correct to 3 significant figures.

(e) Using this information and the answer to part (c), evaluate the model, explaining your reasoning.

(1)

(Total for question = 12 marks)

Q7.

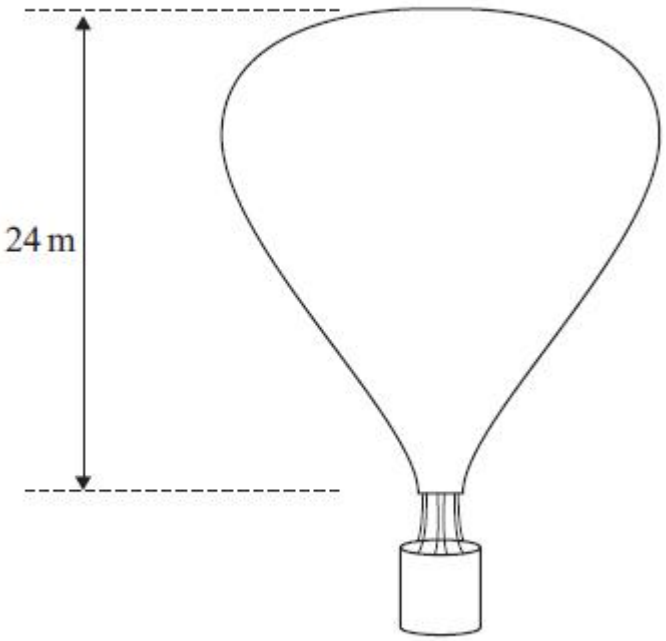

Figure 1

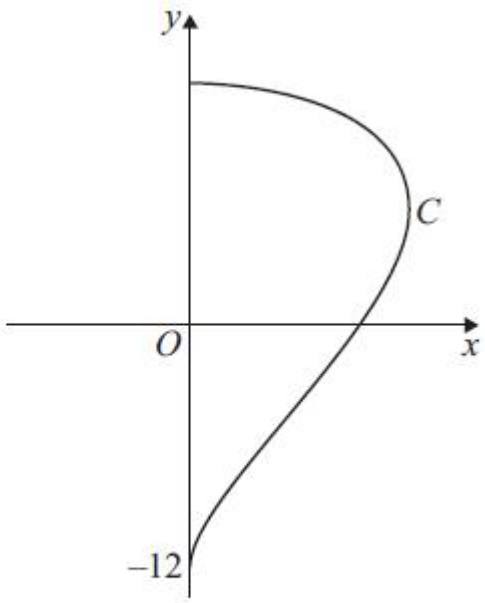

Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of a hot air balloon.

When filled with air the balloon has a height of 24 metres.

Figure 2 shows a sketch of the curve C with equation

$$350x^2 = (12 + y)^2 (A - y^2) \qquad x \geq 0$$

where A is a constant.

The balloon is modelled by rotating C through 360° about the y-axis.

Given that one y intercept of C is −12

(a) show that $A = 144$

(1)

(b) Use algebraic integration to determine the volume of air needed to fill the balloon, according to the model, giving the answer to 2 significant figures.

(5)

(c) Modify the equation $350x^2 = (12 + y)^2 (144 - y^2)$ to model a mathematically similar balloon with a height of 26 metres.

(1)

(d) State one limitation of the models.

(1)

(Total for question = 8 marks)

Q8.

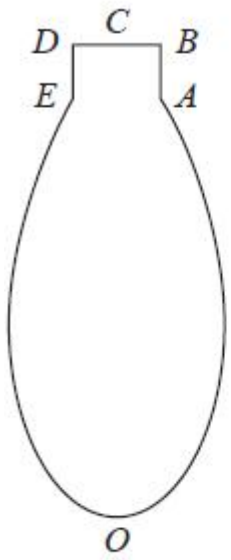

Figure 1

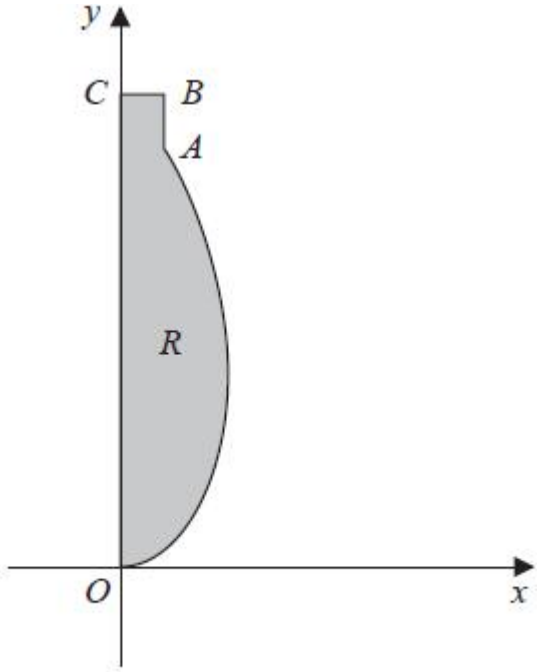

Figure 2

Figure 1 shows the central vertical cross-section, *OABCDEO*, of the design for a solid glass ornament.

Figure 2 shows the finite region, *R*, which is bounded by the *y*-axis, the horizontal line *CB*, the vertical line *BA*, and the curve *AO*.

The ornament is formed by rotating the region *R* through 360° about the *y*-axis.

The curve *AO* is modelled by the equation

$$x = ky^2 + \sqrt{y} \qquad 0 \leqslant y \leqslant 4$$

where *k* is a constant.

The point *A* has coordinates (0.4, 4) and the point *B* has coordinates (0.4, 4.5)

The units are centimetres.

(a) Determine the value of *k* according to this model.

(2)

(b) Use algebraic integration to determine the exact volume of glass that would be required to make the ornament, according to the model.

(7)

(c) State a limitation of the model.

(1)

When the ornament was manufactured, 9 cm^3 of glass was required.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning.

(1)

(Total for question = 11 marks)

Q9.

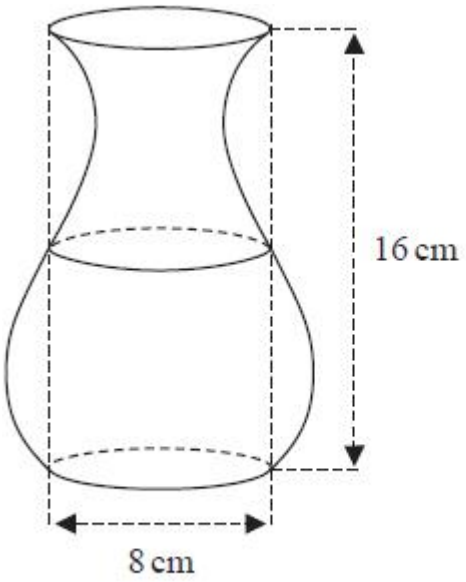

Figure 1

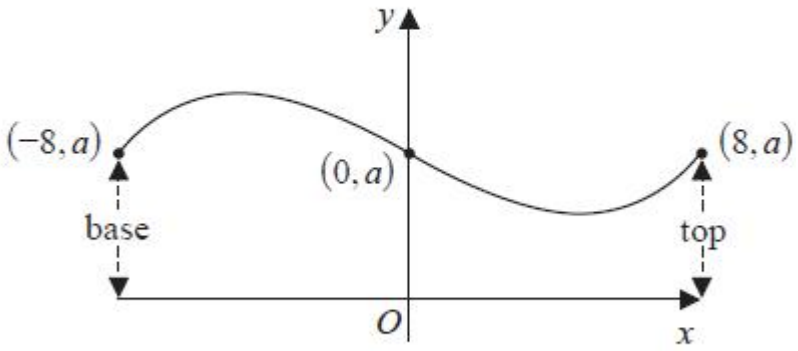

Figure 2

Figure 1 shows a sketch of a 16 cm tall vase which has a flat circular base with diameter 8 cm and a circular opening of diameter 8 cm at the top.

A student measures the circular cross-section halfway up the vase to be 8 cm in diameter.

The student models the shape of the vase by rotating a curve, shown in Figure 2, through 360° about the *x*-axis.

(a) State the value of *a* that should be used when setting up the model.

(1)

Two possible equations are suggested for the curve in the model.

Model A $\quad y = a - 2\sin\left(\frac{45}{2}x\right)^{\circ}$

Model B $\quad y = a + \frac{x(x-8)(x+8)}{100}$

For each model,

(b) (i) find the distance from the base at which the widest part of the vase occurs,

(ii) find the diameter of the vase at this widest point.

(7)

The widest part of the vase has diameter 12 cm and is just over 3 cm from the base.

(c) Using this information and making your reasoning clear, suggest which model is more appropriate.

(1)

(d) Using algebraic integration, find the volume for the vase predicted by Model B.
You must make your method clear.

(5)

The student pours water from a full one litre jug into the vase and finds that there is 100 ml left in the jug when the vase is full.

(e) Comment on the suitability of Model B in light of this information.

(1)

(Total for question = 15 marks)