

Exam Questions – Normal Distribution

Q1.

In a large school, 20% of students own a touch screen laptop. A random sample of n students is chosen from the school. Using a normal approximation, the probability that more than 55 of these n students own a touch screen laptop is 0.0401 correct to 3 significant figures.

Find the value of n .

(8)

(Total for question = 8 marks)

Q2.

A packing plant fills bags with cement. The weight X kg of a bag of cement can be modelled by a normal distribution with mean 50 kg and standard deviation 2 kg.

(a) Find $P(X > 53)$.

(3)

(b) Find the weight that is exceeded by 99% of the bags.

(5)

Three bags are selected at random.

(c) Find the probability that two weigh more than 53 kg and one weighs less than 53 kg.

(4)

(Total 12 marks)

Q3.

(a) State the conditions under which the normal distribution may be used as an approximation to the binomial distribution.

(2)

A company sells seeds and claims that 55% of its pea seeds germinate.

(b) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce.

(1)

To test the company's claim, a random sample of 220 pea seeds was planted.

(c) State the hypotheses for a two-tailed test of the company's claim.

(1)

Given that 135 of the 220 pea seeds germinated,

(d) use a normal approximation to test, at the 5% level of significance, whether or not the company's claim is justified.

(7)

(Total 11 marks)

Q4.

A machine puts liquid into bottles of perfume. The amount of liquid put into each bottle, D ml, follows a normal distribution with mean 25 ml

Given that 15% of bottles contain less than 24.63 ml

- (a) find, to 2 decimal places, the value of k such that $P(24.63 < D < k) = 0.45$

(5)

A random sample of 200 bottles is taken.

- (b) Using a normal approximation, find the probability that fewer than half of these bottles contain between 24.63 ml and k ml

(3)

The machine is adjusted so that the standard deviation of the liquid put in the bottles is now 0.16 ml

Following the adjustments, Hannah believes that the mean amount of liquid put in each bottle is less than 25 ml

She takes a random sample of 20 bottles and finds the mean amount of liquid to be 24.94 ml

- (c) Test Hannah's belief at the 5% level of significance.

You should state your hypotheses clearly.

(5)

(Total for question = 13 marks)

Q5.

A machine cuts strips of metal to length L cm, where L is normally distributed with standard deviation 0.5 cm.

Strips with length either less than 49 cm or greater than 50.75 cm **cannot** be used.

Given that 2.5% of the cut lengths exceed 50.98 cm,

- (a) find the probability that a randomly chosen strip of metal **can** be used.

(5)

Ten strips of metal are selected at random.

- (b) Find the probability fewer than 4 of these strips **cannot** be used.

(2)

A second machine cuts strips of metal of length X cm, where X is normally distributed with standard deviation 0.6 cm

A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm

- (c) Stating your hypotheses clearly and using a 1% level of significance, test whether or not the mean length of all the strips, cut by the second machine, is greater than 50.1 cm

(5)

(Total for question = 12 marks)

Q6.

A study was made of adult men from region *A* of a country. It was found that their heights were normally distributed with a mean of 175.4 cm and standard deviation 6.8 cm.

- (a) Find the proportion of these men that are taller than 180 cm.

(1)

A student claimed that the mean height of adult men from region *B* of this country was different from the mean height of adult men from region *A*.

A random sample of 52 adult men from region *B* had a mean height of 177.2 cm

The student assumed that the standard deviation of heights of adult men was 6.8 cm both for region *A* and region *B*.

- (b) Use a suitable test to assess the student's claim.

You should

- state your hypotheses clearly
- use a 5% level of significance

(4)

- (c) Find the *p*-value for the test in part (b)

(1)

(Total for question = 6 marks)

Q7.

The heights of an adult female population are normally distributed with mean 162 cm and standard deviation 7.5 cm.

- (a) Find the probability that a randomly chosen adult female is taller than 150 cm.

(3)

Sarah is a young girl. She visits her doctor and is told that she is at the 60th percentile for height.

- (b) Assuming that Sarah remains at the 60th percentile, estimate her height as an adult.

(3)

The heights of an adult male population are normally distributed with standard deviation 9.0 cm.

Given that 90% of adult males are taller than the mean height of adult females,

- (c) find the mean height of an adult male.

(4)

(Total 10 marks)

Q8.

The time, in minutes, taken to fly from London to Malaga has a normal distribution with mean 150 minutes and standard deviation 10 minutes.

- (a) Find the probability that the next flight from London to Malaga takes less than 145 minutes.

(3)

The time taken to fly from London to Berlin has a normal distribution with mean 100 minutes and standard deviation d minutes.

Given that 15% of the flights from London to Berlin take longer than 115 minutes,

- (b) find the value of the standard deviation d .

(4)

The time, X minutes, taken to fly from London to another city has a normal distribution with mean μ minutes.

Given that $P(X < \mu - 15) = 0.35$

- (c) find $P(X > \mu + 15 \mid X > \mu - 15)$.

(3)

(Total 10 marks)

Q9.

A manufacturer uses a machine to make metal rods.

The length of a metal rod, L cm, is normally distributed with

- a mean of 8 cm
- a standard deviation of x cm

Given that the proportion of metal rods less than 7.902 cm in length is 2.5%

(a) show that $x = 0.05$ to 2 decimal places.

(2)

(b) Calculate the proportion of metal rods that are between 7.94 cm and 8.09 cm in length.

(1)

The **cost** of producing a single metal rod is 20p

A metal rod

- where $L < 7.94$ is **sold** for scrap for 5p
- where $7.94 \leq L \leq 8.09$ is **sold** for 50p
- where $L > 8.09$ is shortened for an extra **cost** of 10p and then **sold** for 50p

(c) Calculate the expected profit per 500 of the metal rods.
Give your answer to the nearest pound.

(5)

The same manufacturer makes metal hinges in large batches.

The hinges each have a probability of 0.015 of having a fault.

A random sample of 200 hinges is taken from each batch and the batch is accepted if fewer than 6 hinges are faulty.

The manufacturer's aim is for 95% of batches to be accepted.

(d) Explain whether the manufacturer is likely to achieve its aim.

(4)

(Total for question = 12 marks)