

**DISCLAIMER: THERE IS NO GUARANTEE
THESE TOPICS WILL COME UP ON
PAPER 2.**

These are common topics not appeared on
P1 (or topics that regularly appear twice).

REVISE THOROUGHLY!!

**NOTE: I've compiled this paper using very
challenging questions!!**

Q1.

In a large theatre there are n rows of seats, where n is a constant.

The number of seats in the first row is a , where a is a constant.

In each subsequent row there are 4 more seats than in the previous row so that

- in the 2nd row there are $(a + 4)$ seats
- in the 3rd row there are $(a + 8)$ seats
- the number of seats in each row form an **arithmetic** sequence

Given that the **total** number of seats in the first 10 rows is 360

(a) find the value of a .

(2)

Given also that the total number of seats in the n rows is 2146

(b) show that

$$n^2 + 8n - 1073 = 0$$

(2)

(c) Hence

- state the number of rows of seats in the theatre,
- find the maximum number of seats in any one row.

(3)

(Total for question = 7 marks)

Q2.

Given $\log_a b = k$, find, in simplest form in terms of k ,

(i) $\log_a \left(\frac{\sqrt{a}}{b} \right)$

(2)

(ii) $\frac{\log_a a^2 b}{\log_a b^3}$

(2)

(iii) $\sum_{n=1}^{50} (k + \log_a b^n)$

(3)

(Total for question = 7 marks)

Q3.

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

(i) Solve

$$2\log_2(4-x) = 3 + \log_2\left(\frac{x+11}{2}\right)$$

(5)

(ii) The curves C_1 and C_2 with equations

$$y = 3^{2x+1} \quad \text{and} \quad y = 6 \times 3^x$$

meet at the point P .

Find the exact coordinates of P , writing your answer in the form $(\log_3 a, b)$ where a and b are integers.

(5)

(Total for question = 10 marks)

Q4.

(a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\left(2 - \frac{1}{4}x\right)^6$$

(4)

(b) Given that x is small, so terms in x^4 and higher powers of x may be ignored, show

$$\left(2 - \frac{1}{4}x\right)^6 + \left(2 + \frac{1}{4}x\right)^6 = a + bx^2$$

where a and b are constants to be found.

(3)

(Total for question = 7 marks)

Q5.

The circle C has centre $X(3, 5)$ and radius r

The line l has equation $y = 2x + k$, where k is a constant.

(a) Show that l and C intersect when

$$5x^2 + (4k - 26)x + k^2 - 10k + 34 - r^2 = 0$$

(3)

Given that l is a tangent to C ,

(b) show that $5r^2 = (k + p)^2$, where p is a constant to be found.

(3)

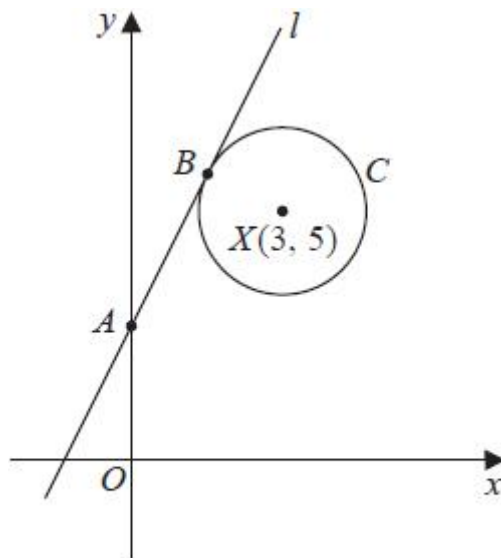


Figure 2

The line l

- cuts the y -axis at the point A
- touches the circle C at the point B

as shown in Figure 2.

Given that $AB = 2r$

(c) find the value of k

(6)

(Total for question = 12 marks)

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Q6.

The circle C_1 has equation

$$x^2 + y^2 - 6x + 5y - 41 = 0$$

(a) Find

- (i) the coordinates of the centre of C_1
- (ii) the radius of C_1

(3)

The circle C_2 has

- centre $(-k, 0)$ where k is a positive constant
- radius 5

Given that circles C_1 and C_2 touch

(b) find the exact value of k .

(3)

(Total for question = 6 marks)

Q7.

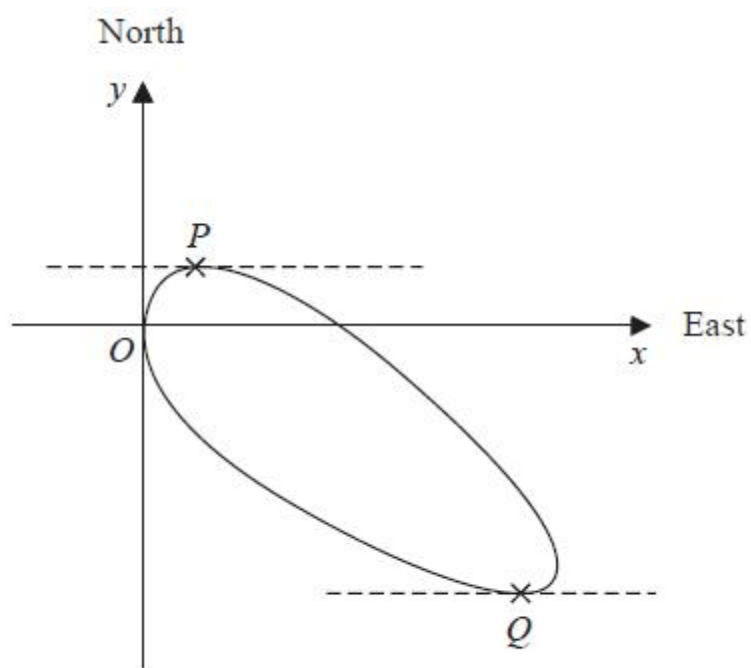


Figure 4

Figure 4 shows a sketch of the closed curve with equation

$$(x + y)^3 + 10y^2 = 108x$$

(a) Show that

$$\frac{dy}{dx} = \frac{108 - 3(x + y)^2}{20y + 3(x + y)^2}$$

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest north and furthest south of the origin O , as shown in Figure 4.

Using the result given in part (a),

(b) find how far the point Q is south of O . Give your answer to the nearest 100 m .

(4)

(Total for question = 9 marks)

Q8.

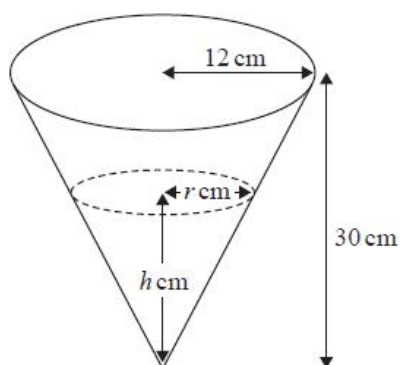


Figure 3

Figure 3 shows a container in the shape of a hollow, inverted, right circular cone.

The height of the container is 30 cm and the radius is 12 cm, as shown in Figure 3.

The container is initially empty when water starts flowing into it.

When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm³

(a) Show that

$$V = \frac{4\pi h^3}{75}$$

[The volume V of a right circular cone with vertical height h and base radius r is given by the formula

$$V = \frac{1}{3}\pi r^2 h$$

(2)

Given that water flows into the container at a constant rate of 2π cm³ s⁻¹

(b) find, in cm s⁻¹, the rate at which h is changing, exactly 1.5 **minutes** after water starts flowing into the container.

(4)

(Total for question = 6 marks)

Q9.

In a controlled experiment, the number of microbes, N , present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b \quad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b .

(2)

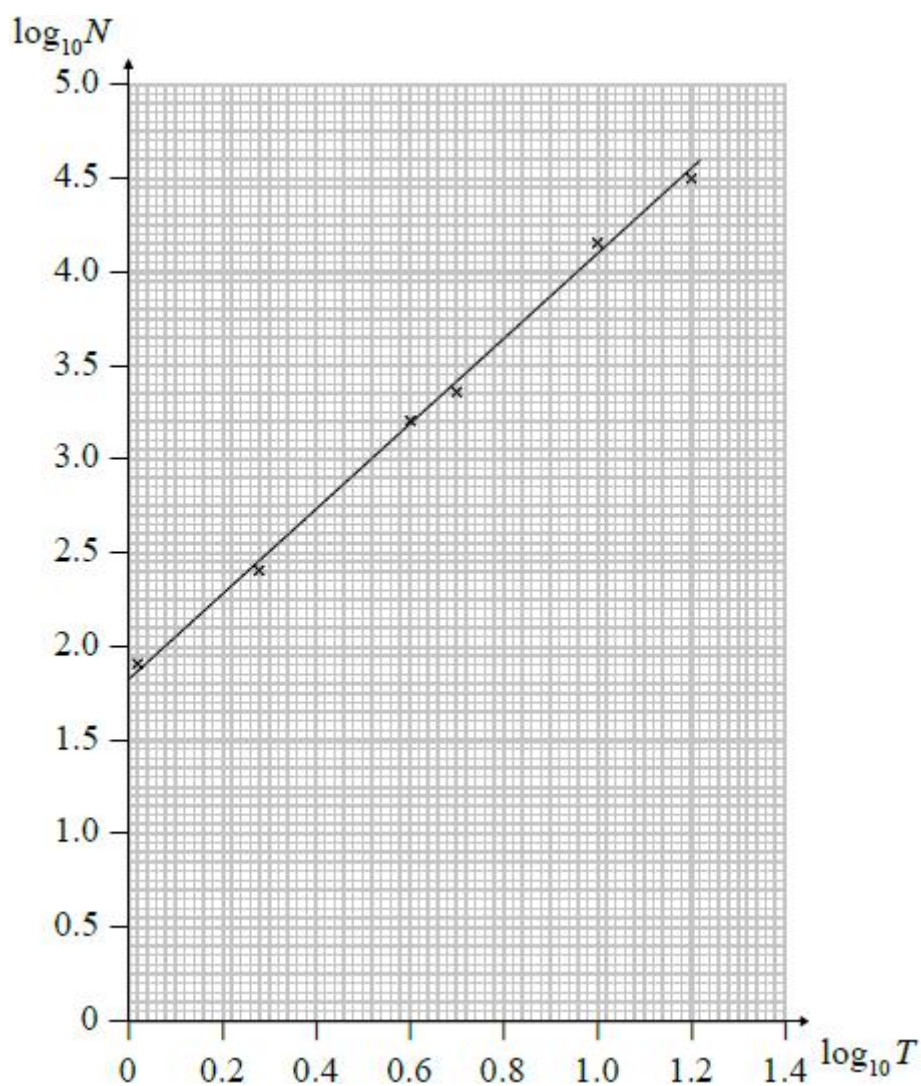


Figure 2

Figure 2 shows the line of best fit for values of $\log_{10}N$ plotted against values of $\log_{10}T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) With reference to the model, interpret the value of the constant a .

(1)

(Total for question = 7 marks)

Q10.

(a) Given that

$$3 \log_3(2x - 1) = 2 + \log_3(14x - 25)$$

show that

$$2x^3 - 3x^2 - 30x + 56 = 0$$

(4)

(b) Show that -4 is a root of this cubic equation.

(2)

(c) Hence, using algebra and showing each step of your working, solve

$$3 \log_3(2x - 1) = 2 + \log_3(14x - 25)$$

(4)

(Total for question = 10 marks)

Q11.

$$f(x) = x^3 + (p + 3)x^2 - x + q$$

where p and q are constants and $p > 0$

Given that $(x - 3)$ is a factor of $f(x)$

(a) show that

$$9p + q = -51$$

(2)

Given also that when $f(x)$ is divided by $(x + p)$ the remainder is 9

(b) show that

$$3p^2 + p + q - 9 = 0$$

(2)

(c) Hence find the value of p and the value of q .

(3)

(d) Hence find a quadratic expression $g(x)$ such that

$$f(x) = (x - 3)g(x)$$

(2)

(Total for question = 9 marks)

Q12.

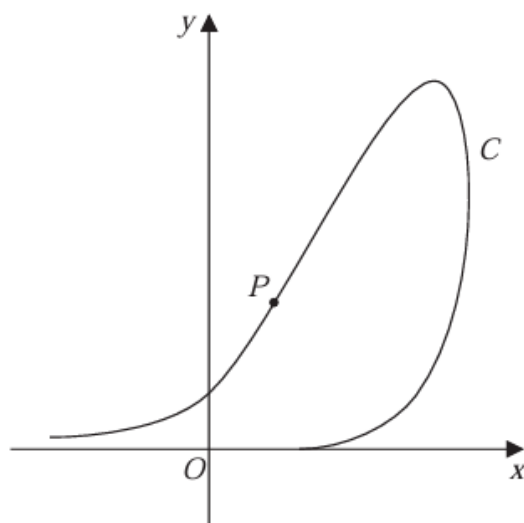


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$2^x - 4xy + y^2 = 13 \quad y \geq 0$$

The point P lies on C and has x coordinate 2

(a) Find the y coordinate of P .

(2)

(b) Find $\frac{dy}{dx}$ in terms of x and y .

(5)

The tangent to C at P crosses the x -axis at the point Q .

$$\frac{a \ln 2 + b}{c \ln 2 + d}$$

(c) Find the x coordinate of Q , giving your answer in the form $\frac{a \ln 2 + b}{c \ln 2 + d}$ where a , b , c and d are integers to be found.

(3)

(Total for question = 10 marks)

Q13.

In this question you should show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Using the substitution $u = 3 + \sqrt{2x - 1}$ find the exact value of

$$\int_1^{13} \frac{4}{3 + \sqrt{2x - 1}} dx$$

giving your answer in the form $p + q \ln 2$, where p and q are integers to be found.

(8)

(Total for question = 8 marks)

Q14.

In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve

$$2\log_3(4x + 5) - \log_3(x + 3) = 2$$

(5)

(ii) Given that $a > 0$, $b > 0$ and

$$\log_{10}a + \log_{10}b = \log_{10}(a + b)$$

(a) prove that $a = \frac{b}{b-1}$

(3)

(b) Hence write down the full restriction on the value of b , giving a reason for your answer.

(2)

(Total for question = 10 marks)

Q15.

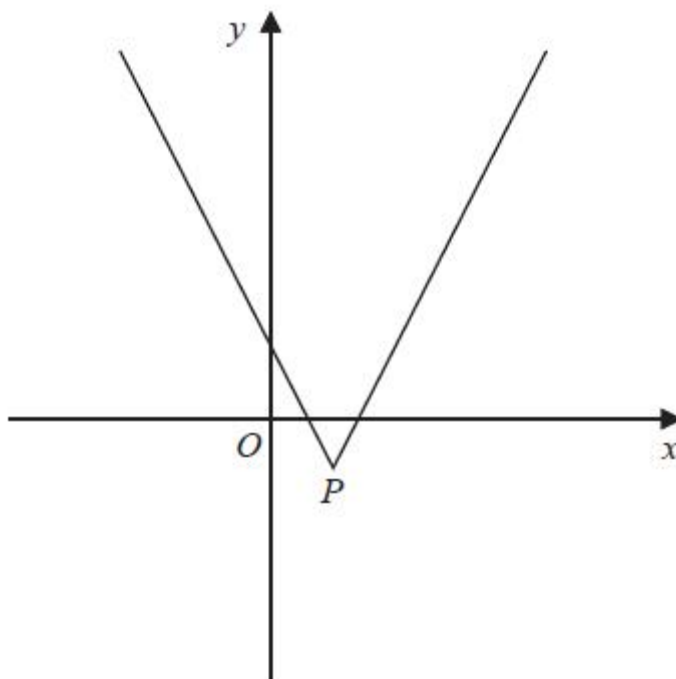


Figure 1

Figure 1 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = |3x - 11| - 4$$

The vertex of the graph is at the point P , as shown in Figure 1.

(a) State the coordinates of P .

(2)

(b) Solve the equation $f(x) = 8$

(3)

The line l has equation $y = mx$, where m is a constant.

Given that l intersects the graph of $y = f(x)$ at exactly one point,

(c) find the possible values of m .

(3)

The graph with equation $y = f(x)$ is transformed onto the graph with equation $y = a f(x - b)$, where a and b are constants.

Given that the vertex of the graph with equation $y = a f(x - b)$ is $(5, 16)$

(d) find the value of a and the value of b .

(2)

(Total for question = 10 marks)

Q16.

(a) Sketch the curve with equation

$$y = a^x + 4$$

where a is a positive constant greater than 1

On your sketch, show

- the coordinates of the point of intersection of the curve with the y -axis
- the equation of the asymptote of the curve

(3)

x	2	2.3	2.6	2.9	3.2	3.5
y	0	0.3246	0.8629	1.6643	2.7896	4.3137

The table shows corresponding values of x and y for

$$y = 2^x - 2x$$

with the values of y given to 4 decimal places as appropriate.

Using the trapezium rule with all the values of y in the given table,

- (b) obtain an estimate for $\int_2^{3.5} (2^x - 2x) dx$, giving your answer to 2 decimal places.

(3)

(c) Using your answer to part (b) and making your method clear, estimate

(i) $\int_2^{3.5} (2^x + 2x) dx$

(ii) $\int_2^{3.5} (2^{x+1} - 4x) dx$

(3)

(Total for question = 9 marks)

Q17.

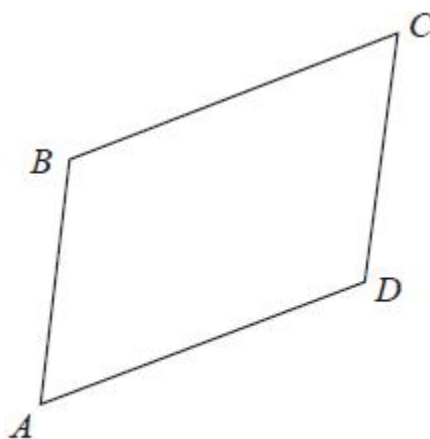


Figure 1

Figure 1 shows a sketch of parallelogram $ABCD$.

Given that $\vec{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\vec{BC} = 2\mathbf{i} + 5\mathbf{j} + 8\mathbf{k}$

(a) find the size of angle ABC , giving your answer in degrees, to 2 decimal places.

(3)

(b) Find the area of parallelogram $ABCD$, giving your answer to one decimal place.

(2)

(Total for question = 5 marks)

Q18.

A tree was planted in the ground.

Exactly 2 years after it was planted, the height of the tree was 1.85 m.

Exactly 7 years after it was planted, the height of the tree was 3.45 m.

Given that the height, H metres, of the tree, t years after it was planted in the ground, can be modelled by the equation

$$H = at + b$$

where a and b are constants,

(a) find the value of a and the value of b .

(4)

(b) State, according to the model, the height of the tree when it was planted.

(1)

(Total for question = 5 marks)

Q19.

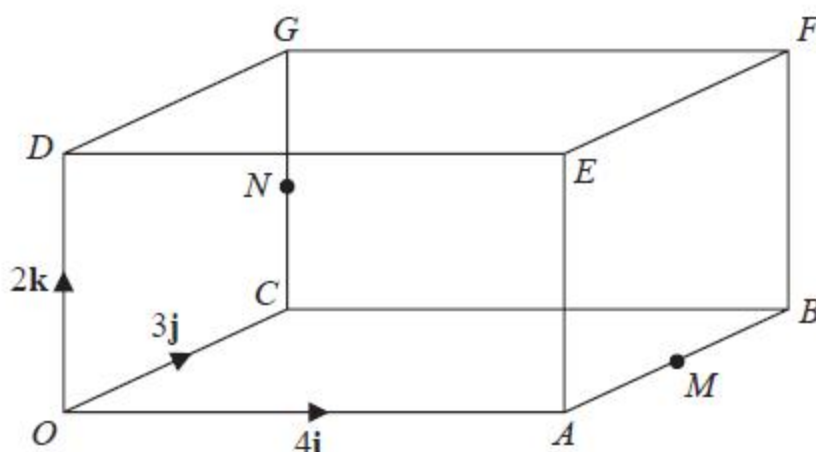


Figure 5

Figure 5 shows a sketch of the cuboid $OABCGDEF$

Unit vectors \mathbf{i} , \mathbf{j} and \mathbf{k} lie on the lines OA , OC and OD respectively where O is the origin.

Given that

- $\vec{OA} = 4\mathbf{i}$ and $\vec{OC} = 3\mathbf{j}$ and $\vec{OD} = 2\mathbf{k}$
- the points M and N are the midpoints of AB and CG respectively
- the point P lies on MN such that $\vec{MP} = 3\vec{PN}$

(a) show that

$$\vec{OP} = \mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}$$

(4)

The straight line through O and P meets the face $BFGC$ at the point Q

(b) Find the coordinates of Q

(2)

(Total for question = 6 marks)

Q20.

Given that θ is small and measured in radians, use the small angle approximations to show that

$$4 \sin \frac{\theta}{2} + 3 \cos^2 \theta \approx a + b\theta + c\theta^2$$

where a , b and c are integers to be found.

(Total for question = 3 marks)

Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$\text{Uses } S_{10} = 360 \Rightarrow \frac{10}{2} \{2 \times a + 9 \times 4\} = 360$ $\Rightarrow 2a + 36 = 72 \Rightarrow a = 18$	M1 A1 (2)
(b)	$\text{Uses } S_n = 2146 \Rightarrow \frac{n}{2} \{2 \times "18" + (n-1) \times 4\} = 2146$ $\Rightarrow n \{16 + 2n\} = 2146$ $\Rightarrow n^2 + 8n - 1073 = 0 \quad *$	M1 A1* (2)
(c)	(i) States 29 (ii) Attempts $"18" + ("29" - 1) \times 4 = 130$	B1 M1, A1 (3)
		(7 marks)

(a)

M1: Uses $S_{10} = 360$ with a correct formula to set up a linear equation in "a". E.g. $\frac{10}{2} \{2 \times a + 9 \times 4\} = 360$

Alternatively uses the sum of all 10 terms

$$a + (a+4) + (a+8) + (a+12) + (a+16) + (a+20) + (a+24) + (a+28) + (a+32) + (a+36) = 360$$

A1: $a = 18$ following a correct equation

(b)

M1: Attempts to use $S_n = 2146 \Rightarrow \frac{n}{2} \{2 \times "18" + (n-1) \times 4\} = 2146$ following through on their value of "a"

A1*: Proceeds to $n^2 + 8n - 1073 = 0$ showing sufficient working, which in most circumstances should be regarded as one correct simplified intermediate line. (See main scheme)

(c)(i)

B1: States 29.

The -37 if written down must be deleted/crossed out, or the 29 chosen by being "ringed" or underlined

(c)(ii)

M1: Attempts $a + (n-1) \times 4$ with their values of a and n. E.g. $"18" + ("29" - 1) \times 4$

$$\text{Alternatively uses } 2146 \Rightarrow \frac{29}{2} \{ "18" + l \} \Rightarrow l = \dots$$

A1: Maximum number of seats = 130 which may be labelled $l = 130$

This alone scores both marks (but only if they had $a = 18$ and $n = 29$ earlier)

(Q08 WMA12/01, Oct 2023)



Q2.

Question Number	Scheme	Marks
(i)	$\log_a \left(\frac{\sqrt{a}}{b} \right) = \frac{1}{2} \log_a a - \log_a b = \frac{1}{2} - k$	M1 A1 (2)
(ii)	$\frac{\log_a a^2 b}{\log_a b^3} = \frac{2 \log_a a + \log_a b}{3 \log_a b} = \frac{2+k}{3k}$	M1 A1 (2)
(iii)	$\sum_{n=1}^{50} (k + \log_a b^n) = 50k + (1k + 2k + 3k + \dots + 50k)$ or $(2k + 3k + 4k + \dots + 51k)$ Uses the sum formula an AP with $n = 50, d = k$ $S = 50k + \frac{50}{2}(2k + 49k) \qquad S = \frac{50}{2}(2k + 51k)$ $= 1325k$	M1 A1 A1 (3) (7 marks)

- (i)
- M1 Uses log laws to achieve $\frac{1}{2}\log a - \log b$ or eg $0.5\log a - \log b$
- A1 $\frac{1}{2} - k$ oe
- (ii)
- M1 Uses correct log laws on the numerator or denominator. Score for $\frac{\dots}{3\log b}$ or $\frac{\dots}{3k}$ or sight of the numerator $2\log a + \log b$ which does not have to be part of a fraction (May be implied by $2+k$)
- A1 $\frac{2+k}{3k}$ or $\frac{2}{3k} + \frac{1}{3}$ do not isw
- (iii)
- M1 **Either** uses the sum formula for an AP in terms of k on part or all of the expression with $n = 50, d = k, a = k$ or $n = 50, d = k, a = 2k$. It is sufficient to see the terms substituted in for this mark.
- $$\sum_{n=1}^{50} nk = \frac{50}{2}(2k + 49k) \text{ or } \sum_{n=1}^{50} (k + nk) = \frac{50}{2}(2k + 51k).$$
- You may see the equivalent sum $\frac{n}{2}(a + L)$ eg $\frac{50}{2}(k + 50k)$
- Or** uses the sum formula for an AP in terms of $\log_a b$ on part of the expression with $n = 50, d = \log b, a = \log b$
- $$\sum_{n=1}^{50} \log b^n = \frac{50}{2}(2\log b + 49\log b)$$
- You may see the equivalent sum $\frac{n}{2}(a + L) = \sum_{n=1}^{50} \log b^n = \frac{50}{2}(\log b + 50\log b)$
- A1 A correct unsimplified answer in terms of k .
Eg. $S = 50k + \frac{50}{2}(2k + 49k)$ or $S = \frac{50}{2}(2k + 51k)$
- A1 1325k

(Q07 WMA12/01, Oct 2019)



- (i)
- B1: Sight of one correct log law applied. See scheme. There will be other equivalent examples
e.g. $\log_2\left(\frac{x+11}{2}\right) = \log_2(x+11) - 1$. This mark can still be awarded if they have made errors before applying the law and do not withhold this mark for any subsequent errors.
- M1: Correctly combines two of the three original terms of the equation. e.g. $\log_2\left(\frac{2(4-x)^2}{x+11}\right) = 3$
May be implied by a correct equation in any form not involving logs provided it has not come from incorrect log work. e.g. $\frac{\log_2(4-x)^2}{\log_2\left(\frac{x+11}{2}\right)} = 3 \Rightarrow \frac{(4-x)^2}{\frac{x+11}{2}} = 8$ is M0
- A1: Correct equation in any form not involving logs which has not come from incorrect log work.
- dM1: Solves a quadratic equation via factorisation, the formula or completing the square (usual rules apply). They are not allowed to just state the roots. Dependent on the previous method mark.
- A1: $x = -2$ only (the 14 if found must be rejected or not included in their final answer)



- (ii) Note that if logarithms are written as decimals instead of keeping their values exact then
- if they find x first the maximum score is M1dM0A0ddM0A0
 - if they find y first the maximum score is M1dM1A1ddM0A0

M1: Sets up a correct equation in x or y . Most likely in x but it is possible to form an equation in y

$$\text{e.g. } y = 3^{2x+1} = 3 \times (3^x)^2 = 3 \times \left(\frac{y}{6}\right)^2$$

dM1: A correct method of solving the equation in x (or y) to find either

- an unsimplified expression or value for x in terms of logs, or
- an expression or value for y not involving logs

It is dependent on the previous method mark. Condone arithmetical slips but any log/index work must be correct.

There may be some alternative approaches here to solving:

e.g. taking logarithms of base 3:

$$\log_3(3^{2x+1}) = \log_3(6 \times 3^x) \Rightarrow 2x+1 = \log_3 6 + \log_3 3^x \Rightarrow 2x+1 = 1 + \log_3 2 + x \Rightarrow x = \dots$$

e.g. forming a quadratic in 3^x (or another variable)

$$3 \times 3^{2x} - 6 \times 3^x = 0 \Rightarrow 3^x(3^x - 2) = 0 \Rightarrow 3^x = 2 \Rightarrow x = \dots$$

If solving an equation in y then e.g. $y = 3 \times \left(\frac{y}{6}\right)^2 \Rightarrow y = 3 \times \frac{y^2}{36} \Rightarrow y =$

A1: Correct x coordinate ($\log_3 2$ or equivalent expressions such as $\log_3 6 - 1$ or $\frac{\log 2}{\log 3}$ o.e.) or y coordinate which must be 12

ddM1: Uses a correct method to find values for both x and y . Condone arithmetical slips but any log/index work must be correct.

It is dependent on the previous two method marks.

May be implied by a correct coordinate for their x or y . Do not allow logarithms to be written as decimals.

A1: $(\log_3 2, 12)$ Accept if written separately e.g. $x = \log_3 2$, $y = 12$. isw if they incorrectly state the values of a and b after correct coordinates are seen.
Condone missing brackets but do not allow the coordinates to be the wrong way round.

(Q06 WMA12/01, June 2025)



Q4.

Question Number	Scheme	Marks
(a)	$\left(2 - \frac{1}{4}x\right)^6 = 2^6 + {}^6C_1 2^5 \left(-\frac{1}{4}x\right)^1 + {}^6C_2 2^4 \left(-\frac{1}{4}x\right)^2 + {}^6C_3 2^3 \left(-\frac{1}{4}x\right)^3 + \dots$ $= 64 - 48x + 15x^2 - 2.5x^3$	B1, M1 A1 A1 (4)
(b)	$\left(2 - \frac{1}{4}x\right)^6 + \left(2 + \frac{1}{4}x\right)^6 = (64 - 48x + 15x^2 - 2.5x^3) + (64 + 48x + 15x^2 + 2.5x^3)$ $\approx 128 + 30x^2$	M1 B1ft A1 (3) (7 marks)

Notes

(a)

B1 For either 2^6 or 64. Award for an unsimplified ${}^6C_0 2^6 \left(-\frac{1}{4}x\right)^0$

M1 For an attempt at the binomial expansion. Score for a correct attempt at term 2, 3 or 4.

Accept sight of ${}^6C_1 2^5 \left(\pm\frac{1}{4}x\right)^1$ ${}^6C_2 2^4 \left(\pm\frac{1}{4}x\right)^2$ ${}^6C_3 2^3 \left(\pm\frac{1}{4}x\right)^3$ condoning omission of brackets.

Accept any coefficient appearing from Pascal's triangle. FYI 6, 15, 20

A1 For any two simplified terms of $-48x + 15x^2 - 2.5x^3$

A1 For $64 - 48x + 15x^2 - 2.5x^3$ ignoring terms with greater powers. This may be awarded in (b) if it is not fully simplified in (a). Allow the terms to be listed $64, -48x, 15x^2, -2.5x^3$. Isw after sight of correct values. The expression written out without any method can be awarded all 4 marks.

(b) **Note that this is now marked M1 B1 A1**

M1 For adding two sequences that must be of the correct form with the correct signs.

Look for $(A - Bx + Cx^2 - Dx^3) + (A + Bx + Cx^2 + Dx^3)$ but condone

$$(A - Bx + Cx^2) + (A + Bx + Cx^2)$$

For this to be scored there must be some negative terms in (a)

B1ft For one correct term (follow through). Usually $a = 128$ but accept either $a = 2 \times$ 'their' +ve 64 or $b = 2 \times$ 'their' +ve 15

A1 For $128 + 30x^2$. CSO so must be from $(64 - 48x + 15x^2 - 2.5x^3) + (64 + 48x + 15x^2 + 2.5x^3)$

Allow $a = 128, b = 30$ following correct work.

This is a show that question so M1 must be awarded. It must be their final answer so do not isw here.

Alternative method in (a):

$$\left(2 - \frac{1}{4}x\right)^6 = 2^6 \left(1 - \frac{1}{8}x\right)^6 = 2^6 \left(1 + 6\left(-\frac{1}{8}x\right) + \frac{6 \times 5}{2} \left(-\frac{1}{8}x\right)^2 + \frac{6 \times 5 \times 4}{3!} \left(-\frac{1}{8}x\right)^3 + \dots\right)$$

B1 For sight of factor of either 2^6 or 64

M1 For an attempt at the binomial expansion seen in at least one term within the brackets.

Score for a correct attempt at term 2, 3 or 4.

Accept sight of $6\left(\pm\frac{1}{8}x\right)^1$ $\frac{6 \times 5}{2} \left(\pm\frac{1}{8}x\right)^2$ $\frac{6 \times 5 \times 4}{3!} \left(\pm\frac{1}{8}x\right)^3$ condoning omission of brackets

A1 For any two terms of $64 - 48x + 15x^2 - 2.5x^3$

A1 For all four terms $64 - 48x + 15x^2 - 2.5x^3$ ignoring terms with greater powers

Attempts to multiply out

B1 For 64

M1 Multiplies out to form $a + bx + cx^2 + dx^3 + \dots$ and gets b, c or d correct.

A1A1 As main scheme

Q5.

Question	Scheme	Marks
(a)	Equation of circle is $(x-3)^2 + (y-5)^2 = r^2$ and line is $y = 2x + k$ So intersect when $(x-3)^2 + (2x+k-5)^2 = r^2$	M1
	$\Rightarrow x^2 - 6x + 9 + 4x^2 + 4(k-5)x + (k-5)^2 = r^2$ $\Rightarrow 5x^2 + (-6 + 4k - 20)x + 9 + k^2 - 10k + 25 - r^2 = 0$	dM1
	$\Rightarrow 5x^2 + (4k - 26)x + k^2 - 10k + 34 - r^2 = 0^*$	A1*
		(3)
(b)	Tangent to $C \Rightarrow b^2 - 4ac = 0 \Rightarrow (4k - 26)^2 - 4 \times 5 \times (k^2 - 10k + 34 - r^2) = 0$	M1
	$\Rightarrow 16k^2 - 208k + 676 - 20k^2 + 200k - 680 + 20r^2 = 0$ $\Rightarrow 5r^2 = \dots$	M1
	$\Rightarrow 5r^2 = k^2 + 2k + 1 = (k+1)^2$	A1
		(3)
(b) Way 2	Gradient of BX is $-\frac{1}{2}$ so equation of BX is $y - 5 = -\frac{1}{2}(x - 3)$ $y - 5 = -\frac{1}{2}(x - 3), y = 2x + k \Rightarrow x = \dots, y = \dots \left(\frac{13 - 2k}{5}, \frac{26 + k}{5} \right)$	M1
	$\left(\frac{13 - 2k}{5} - 3 \right)^2 + \left(\frac{26 + k}{5} - 5 \right)^2 = r^2$	dM1
	$\Rightarrow 5r^2 = k^2 + 2k + 1 = (k+1)^2$	A1
		(3)



(c)	Triangle AXB is right angled so $AB^2 + r^2 = XA^2 = (3-0)^2 + (5-k)^2$	M1
	$AB^2 = 4r^2$ so $AB^2 + r^2 = 5r^2$	M1
	$\Rightarrow 5r^2 = 9 + (5-k)^2$	A1
	$\Rightarrow k^2 + 2k + 1 = 9 + 25 - 10k + k^2$	M1
	$\Rightarrow 12k = 33 \Rightarrow k = \dots$	dM1
	$k = \frac{11}{4}$	A1
		(6)

(12 marks)

Notes:

(a)

M1: Forms equation of circle and substitutes in equation of line.

The circle equation must be of the form $(x \pm 3)^2 + (y \pm 5)^2 = r^2$

dM1: Expands both sets of brackets and collects terms in x^2 and x .

AI*: Reaches the answer given with no errors seen.

Note that some candidates expand the brackets first before substitution e.g.:

$$(x-3)^2 + (y-5)^2 = r^2 \Rightarrow x^2 - 6x + 9 + y^2 - 10y + 25 = r^2 \Rightarrow x^2 - 6x + 9 + (2x+k)^2 - 10(2x+k) + 25 = r^2$$

This implies the first M and then the second M will score when terms in x^2 and x are collected.

Note about poor squaring e.g. $(x-3)^2 = x^2 + 9$: The first M is available in both cases above but the second M requires at least two x^2 terms and at least two x terms from the expansions.

Note that it is acceptable to go from a completely correct full expansion to the printed answer e.g.

$$(x-3)^2 + (y-5)^2 = r^2 \Rightarrow x^2 - 6x + 9 + y^2 - 10y + 25 = r^2 \Rightarrow x^2 - 6x + 9 + (2x+k)^2 - 10(2x+k) + 25 = r^2 \\ \Rightarrow x^2 - 6x + 9 + 4x^2 + 4kx + k^2 - 20x - 10k + 25 = r^2 \Rightarrow 5x^2 + (4k-26)x + k^2 - 10k + 34 - r^2 = 0$$

Scores full marks in (a)

(b)

M1: Uses the discriminant is zero to form an equation in k and r

dM1: Expands and rearranges to make ar^2 the subject

A1: Correct answer

Way 2

M1: Attempts the equation of BX and solves simultaneously with l to find the coordinates of B

Alternatively uses $x = -\frac{b}{2a} = \frac{13-2k}{5}$ at B and uses this to find y at B

dM1: Correct use of Pythagoras for BX and sets $= r^2$

A1: Correct answer

(c)

M1: Attempts XA^2 correctly in terms of k (the k may appear as y but must be replaced by k later) and uses it in Pythagoras theorem for triangle AXB .

May be implied.

M1: Applies $AB = 2r$ to get $AB^2 + r^2$ in terms of r . Condone with $AB^2 = 2r^2$ used.

A1: Correct equation.

M1: Substitutes the result from (b) and expands brackets.

dM1: Solves a linear equation in k . **Depends on the previous M.**

A1: Correct value.

Alt (c)	$\text{At } B \quad x = -\frac{b}{2a} = \frac{13-2k}{5}, y = 2\left(\frac{13-2k}{5}\right) + k$	M1
	$AB^2 = \left(\frac{13-2k}{5} - 0\right)^2 + \left(\frac{26-4k}{5} + k - k\right)^2$	M1 A1
	$\Rightarrow 25 \times 4r^2 = (13-2k)^2 + (26-4k)^2 = \dots$	M1
	$\Rightarrow 20 \times (k+1)^2 = (13-2k)^2 + (26-4k)^2 = 20k^2 - 260k + 845$ $\Rightarrow k^2 + 2k + 1 = k^2 - 13k + \frac{169}{4} \Rightarrow k = \dots$	dM1
	$\Rightarrow k = \frac{11}{4}$	A1
		(6)

Notes:

(c)

M1: Uses the result in (a) to find the x coordinate where the line and circle meet and then finds y
 An alternative is to find the equation of BX as in (b) way 2 and solve with l to find x and y at B
 (May have already found the coordinates of B in (a) but must be re-stated or used in (c) to score this mark)

M1: Uses distance formula to find an expression in k for AB or AB^2

A1: Correct expression for AB or AB^2 . Need not be simplified.

M1: Applies $AB = 2r$ to the equation. Condone with $AB^2 = 2r^2$ used.

dM1: Substitutes the result from (b) and solves a linear equation in k . **Depends on the previous M.**

A1: Correct value.

(Q10 WMA12/01, June 2022)



Q6.

Question Number	Scheme	Marks
(a)	$x^2 + y^2 - 6x + 5y - 41 = 0$ <p>Attempts $(x-3)^2 + \left(y + \frac{5}{2}\right)^2 = \dots$</p> <p>Correct centre $\left(3, -\frac{5}{2}\right)$</p> <p>Exact radius $\frac{15}{2}$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>(3)</p>
(b)	<p>Sets up appropriate equation $(k+3)^2 + \left(\frac{5}{2}\right)^2 = \left(5 + \frac{15}{2}\right)^2$</p> $(k+3)^2 = 150 \Rightarrow k+3 = \sqrt{150}$ $\Rightarrow k = 5\sqrt{6} - 3$	<p>M1</p> <p>dM1</p> <p>A1</p> <p>(3)</p> <p>(6 marks)</p>

(a)

M1: Attempts to complete the square for both variables. Look for $(x \pm 3)^2, \left(y \pm \frac{5}{2}\right)^2 = \dots$

This mark can be implied from the coordinates of the centre $\left[3, -\frac{5}{2}\right]$ just written

down.

A1: $\left(3, -\frac{5}{2}\right)$

A1: $\frac{15}{2}$ condone $\sqrt{\frac{225}{4}}$ o.e. which may be scored following $(x-3)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{225}{4}$

(b)

M1: Uses Pythagoras' theorem to set up an equation in k

Look for $(k \pm 3)^2 + \left(\frac{5}{2}\right)^2 = \left(5 + \frac{15}{2}\right)^2$ following through on their $\left(3, -\frac{5}{2}\right)$ and $\frac{15}{2}$

Also look for $k^2 + 6k - 141 = 0$ o.e.

dM1: Uses an appropriate method to proceed to a value for k

A1: $k = 5\sqrt{6} - 3$ o.e.

(Q05 WMA12/01, Jan 2026)



Q7.

Question Number	Scheme	Marks
(a)	$(x+y)^3 + 10y^2 = 108x$ Differentiates $10y^2$ to $20y \frac{dy}{dx}$ Differentiates $(x+y)^3$ to $3(x+y)^2 \left(1 + \frac{dy}{dx}\right)$ $3(x+y)^2 \left(1 + \frac{dy}{dx}\right) + 20y \frac{dy}{dx} = 108$ $3(x+y)^2 \frac{dy}{dx} + 20y \frac{dy}{dx} = 108 - 3(x+y)^2$ $\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{20y + 3(x+y)^2} \quad *$	B1 M1 A1 dM1 A1*
(b)	Deduces that P and Q are where $108 - 3(x+y)^2 = 0 \Rightarrow (x+y)^2 = 36$ Attempts to substitute $(x+y) = \pm 6$ into $(x+y)^3 + 10y^2 = 108x$ to form an equation in y (or x) Solves $216 + 10y^2 = 108(6-y)$ and finds the negative root Awrnt 13900 metres	M1 dM1 ddM1 A1
		(5) (4) (9 marks)



(a) Allow use of y' for $\frac{dy}{dx}$. This is a proof. Look carefully for evidence of candidates working backwards.

B1: Differentiates $10y^2$ to $20y \frac{dy}{dx}$

M1: Attempts to differentiate $(x+y)^3$ Via the chain rule look for $k(x+y)^2 \left(1 + \frac{dy}{dx}\right)$ See **

Alternatively you will see attempts where $(x+y)^3$ is multiplied out. Terms may not be collected but

look for 3 of $x^3 \rightarrow \dots x^2$, $\dots x^2 y \rightarrow 2xy + x^2 \frac{dy}{dx}$, $\dots xy^2 \rightarrow y^2 + 2xy \frac{dy}{dx}$ and $y^3 \rightarrow 3y^2 \frac{dy}{dx}$

differentiated to the correct form.

A1: Correct differentiation $3(x+y)^2 \left(1 + \frac{dy}{dx}\right) + 20y \frac{dy}{dx} = 108$

$$\text{or } 3x^2 + 3 \left(2xy + x^2 \frac{dy}{dx} \right) + 3 \left(y^2 + 2xy \frac{dy}{dx} \right) + 3y^2 \frac{dy}{dx} + 20y \frac{dy}{dx} = 108$$

dM1: Collects the terms in $\frac{dy}{dx}$.

Look for either $\frac{dy}{dx}$ being factorised out of the relevant terms

or the terms in $\frac{dy}{dx}$ being set on one side of the equation and the other terms on the other side

It is dependent upon **BOTH** the B and M marks.

A1*: Proceeds to the given answer.

In the approach $3x^2 + 3 \left(2xy + x^2 \frac{dy}{dx} \right) + 3 \left(y^2 + 2xy \frac{dy}{dx} \right) + 3y^2 \frac{dy}{dx} + 20y \frac{dy}{dx} = 108$ it is

acceptable

to go from $(3x^2 + 6xy + 3y^2 + 20y) \frac{dy}{dx} = 108 - 3x^2 - 6xy - 3y^2$ straight to the given answer

Watch for candidates who use the given answer and work backwards **.

$$\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{20y + 3(x+y)^2} \Rightarrow 3(x+y)^2 \frac{dy}{dx} + 20y \frac{dy}{dx} = 108 - 3(x+y)^2$$

Therefore candidates who start with

$$3(x+y)^2 \frac{dy}{dx} + 20y \frac{dy}{dx} = 108 - 3(x+y)^2 \quad \text{or indeed} \quad 3(x+y)^2 \frac{dy}{dx} + 3(x+y)^2 + 20y \frac{dy}{dx} = 108$$

without any other working or evidence can only score B1 M0 A0 M0 A0

(b)

M1: Deduces that P and Q are where $108 - 3(x+y)^2 = 0 \Rightarrow (x+y)^2 = 36$

This is implied if there is a statement that $(x+y) = \pm 6$

dM1: Attempts to solve $(x+y) = \pm 6$ and $(x+y)^3 + 10y^2 = 108x$ simultaneously to form a quadratic equation in y or x .

ddM1: Solves $216 + 10y^2 = 108(6-y)$ and finds the negative root. Allow if both roots are found

Note that $(x+y) = 6$ must have been used.

If an equation was set up in x , then look for an attempt to find the larger root of

$216 + 10(6-x)^2 = 108x$ which should then be used to find a negative y value

A1: Awrt 13900 metres or awrt 13.9 km with the correct units. Condone answers like -13.9 km

(Q11 WMA14/01, Oct 2022)

Q8.

Question Number	Scheme	Marks
(a)	e.g. $\frac{r}{h} = \frac{12}{30} \Rightarrow r = \frac{12}{30}h \Rightarrow V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{2}{5}h\right)^2 h$	M1
	$V = \frac{4\pi h^3}{75}$ *	A1*
		(2)
(b)	$V = \frac{4\pi h^3}{75} \Rightarrow \frac{dV}{dh} = \frac{12\pi h^2}{75} \left(= \frac{4\pi h^2}{25} \right)$	B1
	$V = 1.5 \times 60 \times 2\pi$ $180\pi = \frac{4\pi h^3}{75} \Rightarrow h^3 = 3375 \Rightarrow h = \dots(15)$	M1
	$\frac{dh}{dt} = \frac{dh}{dV} \frac{dV}{dt} = \frac{75}{12\pi \times 15^2} \times 2\pi = \dots$	M1
	$\frac{1}{18}$ (cm s ⁻¹)	A1
		(4)
		Total 6

(a)

 M1: Uses the 12 and the 30 to find the ratio between h and r and substitutes into the volume formula to find V in terms of h . Condone poor bracketing / invisible brackets for this mark.

 May just state $r = \frac{2}{5}h$

 This mark cannot be for substituting in e.g. $r = \frac{2}{5}$ instead of $r = \frac{2}{5}h$

 A1*: Correct proof with no errors including brackets. There must be at least one intermediate stage of working between establishing $r = \frac{12}{30}h$ and proceeding to the given answer.

 (b) **Note that substituting $h = 1.5$ or $h = 90$ will score M0M0 in (b)**

 B1: Correct expression for $\frac{dV}{dh}$ in any form

 M1: Attempts their volume (" $2\pi \times t$ ") after 1.5 **minutes** and uses this with the given result from (a) to find h . They may find h by equating their expression for the volume to the given result from (a) before substituting $t = 90$ and rearranging to find h . Do not be concerned by the mechanics of the rearrangement. Condone slips.

 M1: Correct chain rule attempt using their h to find $\frac{dh}{dt}$. It is dependent on having used $t = 90$ (or condone $t = 1.5$) when finding h .

 A1: Correct value. Allow awrt 0.0556 (cm s⁻¹) Units not required for this mark.

Alt(b) I Solving the differential equation $\frac{dh}{dt} = \frac{75}{6h^2}$ and then using $t = 90$

B1: $\frac{dV}{dh} = \frac{12\pi h^2}{75} \left(= \frac{4\pi h^2}{25} \right)$

M1: Attempts $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} \left(= \frac{75}{6h^2} \right)$ to achieve an expression in h , attempts to solve their differential equation $\int 6h^2 dh = \int 75 dt \Rightarrow 2h^3 = 75t + c$ o.e. and uses $t = 0, h = 0 \Rightarrow c = 0$

(you may not see the working for c) to find $h = f(t)$ ($h = (37.5t)^{\frac{1}{3}}$). Do not be concerned by the mechanics of the rearrangement.

M1: Attempts $\frac{dh}{dt} \left(= \frac{1}{3} \sqrt[3]{37.5 t^{\frac{2}{3}}} \right)$ and substitutes $t = 90 \Rightarrow \frac{dh}{dt} = \dots$

A1: As above in main scheme

Alt(b) II Equating volumes, rearranging to $h = \dots$ and then using $t = 90$

B1: $V = 2\pi t$

M1: Equates their $2\pi t$ equal to $\frac{4\pi h^3}{75}$ and rearranges to $h = f(t)$ ($h = (37.5t)^{\frac{1}{3}}$). Do not be concerned by the mechanics of the rearrangement.

M1: Attempts $\frac{dh}{dt} \left(= \frac{1}{3} \sqrt[3]{37.5 t^{\frac{2}{3}}} \right)$ and substitutes $t = 90 \Rightarrow \frac{dh}{dt} = \dots$

A1: As above in main scheme

(Q05 WMA14/01, Oct 2024)

Q9.

Question	Scheme	Marks
(a)	$N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b$	M1
	$\Rightarrow \log_{10} N = \log_{10} a + b \log_{10} T$ so $m = b$ and $c = \log_{10} a$	A1
		(2)
(b)	Uses the graph to find either a or b $a = 10^{\text{intercept}}$ or $b = \text{gradient}$	M1
	Uses the graph to find both a and b $a = 10^{\text{intercept}}$ and $b = \text{gradient}$	M1
	Uses $T = 3$ in $N = aT^b$ with their a and b	M1
	Number of microbes ≈ 800	A1
		(4)
(c)	States that ' a ' is the number of microbes 1 day after the start of the experiment.	B1
		(1)
		(7 marks)

Notes:

(a)

 M1: Takes \log_{10} 's of both sides and attempts to use the addition law. Condone $\log = \log_{10}$ for this mark.

 A1: Proceeds correctly to $\log_{10} N = \log_{10} a + b \log_{10} T$ and states $m = b$ and $c = \log_{10} a$

 (b) **Way One: Main scheme**

 M1: For attempting to use the graph to find either a or b using $a = 10^{\text{intercept}}$ or $b = \text{gradient}$. This may be implied by $a = 10^{1.75 \text{ to } 1.85}$ or $b = 2.27$ to 2.33

 M1: For attempting to use the graph to find BOTH a and b (See previous M1)

 M1: Uses $T = 3$ in $N = aT^b$ with their a and b

 A1: Number of microbes ≈ 800
Way Two: Alternative using line of best fit techniques.

 M1: For $\log_{10} 3 \approx 0.48$ and using the graph to find $\log_{10} N$

 M1: For using the graph to find $\log_{10} N$ (FYI $\log_{10} N \approx 2.9$)

 M1: For $\log_{10} N = k \Rightarrow N = 10^k$

 A1: Number of microbes ≈ 800

(c)

B1: See scheme.

(Q08 WMA13/01, Specimen papers)

Q10.

Question	Scheme	Marks
(a)	$2 = \log_3 3^2$ oe seen or implied by working	B1
	$3 \log_3 (2x - 1) = \log_3 (2x - 1)^3$ or $\log_3 "3^2" + \log_3 (14x - 25) = \log_3 ("3^2"(14x - 25))$	M1
	$\Rightarrow (2x - 1)^3 = 9(14x - 25)$	dM1
	$\Rightarrow 8x^3 - 12x^2 + 6x - 1 = 126x - 225$	A1*
	$\Rightarrow 8x^3 - 12x^2 - 120x + 224 = 0 \Rightarrow 2x^3 - 3x^2 - 30x + 56 = 0^*$	
		(4)
(b)	$2(\pm 4)^3 - 3(\pm 4)^2 - 30(\pm 4) + 56 = \dots$	M1
	$2(-4)^3 - 3(-4)^2 - 30(-4) + 56 = -128 - 48 + 120 + 56 = 0$ Hence -4 is a root of the equation.	A1
		(2)
Alt (b)	$2x^3 - 3x^2 - 30x + 56 = (x \pm 4)(2x^2 + \dots \pm 14)$	M1
	$2x^3 - 3x^2 - 30x + 56 = (x + 4)(2x^2 - 11x + 14)$ so -4 is a root of the equation.	A1
		(2)
(c)	$2x^3 - 3x^2 - 30x + 56 = 0 \Rightarrow (x + 4)(2x^2 + \dots \pm 14) = 0$	M1
	$(x + 4)(2x^2 - 11x + 14) = 0$	A1
	$(x + 4)(2x - 7)(x - 2) = 0 \Rightarrow x = \dots$	dM1
	(Equation not defined for $x = -4$ so) solutions are 2 and $\frac{7}{2}$	A1
		(4)
(10 marks)		

Notes:**(a)****B1:** States or implies $2 = \log_3 3^2$ or equivalent. May be gained via $\log_3 f(x) = 2 \Rightarrow f(x) = 9$ **M1:** Uses a correct log law, either power law on $3 \log_3(2x-1) = \log_3(2x-1)^3$ or sum law on $\log_3 3^2 + \log_3(14x-25) = \log_3(3^2(14x-25))$ or equivalent work (e.g. difference law if combining the two log terms on one side first).**dM1:** Uses correct log work to remove the logs. Allow slips, but all log work must have been correct.**A1*:** Expands and simplifies to the given cubic. Must see an intermediate step between the factorised sides and final answer.**(b)****M1:** Substitutes ± 4 into the cubic and attempts to evaluate it.**A1:** Evaluates to 0 with intermediate working shown, and gives conclusion that -4 is a root.**Alt:****M1:** Attempts to take a factor of $(x \pm 4)$ out of the equation or uses long division. Look for first term $2x^2$ and last term ± 14 in the quadratic factor.**A1:** Correct factorisation with conclusion. If via long division, all work must be correct with zero remainder found.**(c)****M1:** Attempts to take a factor of $(x+4)$ out of the cubic, or attempts long division by $x+4$ **A1:** Correct quadratic factor (either by factorisation or by long division)**Note:** the first two marks may be awarded for work seen in part (b).**dM1:** Depends on first M. Solves the resulting quadratic (any means).**A1:** Correct two solutions **only** identified. Withhold if -4 is listed as a solution.

Since the question says "hence, using algebra and showing each step of your working", solutions derived from a calculator with no working shown score no marks. The intermediate quadratic must be attempted to gain marks in part (c)

(Q07 WMA12/01, June 2021)

Q11.

Question Number	Scheme	Marks
(a)	$3^3 + 3^2(p+3) - 3 + q = 0$	M1
	eg $\Rightarrow 27 + 9p + 27 - 3 + q = 0 \Rightarrow 9p + q = -51^*$	A1*
		(2)
(b)	$(-p)^3 + (p+3)(-p)^2 - (-p) + q = 9$	M1
	eg $-p^3 + p^3 + 3p^2 + p + q = 9 \Rightarrow 3p^2 + p + q - 9 = 0^*$	A1*
		(2)
(c)	$3p^2 + p + q - 9 = 0$ $\Rightarrow 3p^2 + p - 51 - 9p - 9 = 0$ $\Rightarrow 3p^2 - 8p - 60 = 0$	M1
	$p = 6$	A1
	$q = -51 - 9p = -105$	A1
		(3)
(d)	$f(x) = x^3 + 9x^2 - x - 105$ $f(x) = (x-3)(\dots x^2 + \dots x + \dots)$	M1
	$g(x) = x^2 + 12x + 35$	A1
		(2)
		Total 9



- (a)
 M1 Attempts to use the factor theorem by setting $f(\pm 3) = 0$. Score for the values embedded in the expression leading to an equation in p and q . The $= 0$ may be implied by later work for this mark. Alternatively, there may be other attempts eg to divide algebraically by $x - 3$

$$\begin{array}{r}
 x^2 + (p+6)x + 3p + 17 \\
 x-3 \overline{) x^3 + (p+3)x^2 \quad -x \quad +q} \\
 \underline{x^3 \quad -3x^2} \\
 (p+6)x^2 \\
 \underline{(p+6)x^2 + (-3p-18)x} \\
 (3p+17)x \quad +q \\
 \underline{(3p+17)x - 9p - 51} \\
 \Rightarrow -9p - 51 = q \Rightarrow 9p + q = -51
 \end{array}$$

To score the method mark they would need to proceed as far as equating q with their " $-9p - 51$ " to achieve an equation in p and q .

With alternative methods look for a correct method, condoning slips and invisible brackets leading to an equation in p and q .

- A1* Correct proof with no errors including brackets. There must be at least one intermediate stage of working and $= 0$ must be seen at some point in their solution via the factor theorem method.
 e.g. $3^3 + (p+3) \times 3^2 - 3 + q = 0 \Rightarrow 9p + q = -51$ scores M1A0 (no intermediate stage seen)
 $27 + (p+3) \times 9 - 3 + q = 0 \Rightarrow 51 + 9p + q = 0 \Rightarrow 9p + q = -51$ scores M1A1

- (b)
- M1 Attempts the remainder theorem by setting $f(\pm p) = 9$ oe Award for e.g.
 $(-p)^3 + (p+3)(-p)^2 + p + q = 9$ condoning sign slips and invisible brackets.
 Alternatively, they may attempt to divide algebraically by $x + p$ leading to a remainder in terms of p^2 , p and q which is equated to 9. Condone slips in their working.
- $$\begin{array}{r}
 x^3 + (p+3)x^2 \\
 x^3 px^2 \\
 3x^2 \\
 3x^2 + 3px \\
 (-1-3p)x \\
 (-1-3p)x + (-1-3p)p
 \end{array}$$
- With alternative methods look for a correct method, leading to a remainder in terms of p^2 , p and q which is equated to 9
- A1* Correct proof with no errors including brackets. There must be at least one intermediate stage of working before proceeding to the final answer
 e.g. $(-p)^3 + (p+3)(-p)^2 - (-p) + q = 9 \Rightarrow 3p^2 + p + q - 9 = 0$ scores M1A0 (no intermediate stage)
 $(-p)^3 + (p+3)(-p)^2 - (-p) + q = 9 \Rightarrow 3p^2 + p + q = 9 \Rightarrow 3p^2 + p + q - 9 = 0$ scores M1A1
- (c)
- M1 Attempts to use both given equations to form a 3TQ equation in p (or q). (terms do not need to be all on one side and condone the omission of $= 0$)
 Do not be too concerned by slips in their substitution/rearrangements or miscopying of the given equations. May be implied by either a correct value for p or a correct value for q
- A1 $p = 6$ (ignore any reference to $-\frac{10}{3}$)
- A1 $q = -105$ only
- (d)
- M1 Uses their values for p and q and a correct strategy (inspection or long division) to obtain the quadratic factor.
 Via inspection score for $x^2 + \dots x \pm \frac{q}{3}$
 Via long division score for proceeding as far as $x^2 \pm (p+6)x$. If they attempt algebraic division in (a) you may need to check their quotient with their value for p to see if the method mark can be scored. They may also restart which is acceptable.
 Condone the use of a negative value for p for this mark (eg even if $p = -6 \Rightarrow 0x$)
- A1 $x^2 + 12x + 35$ oe eg $(x+5)(x+7)$ Allow embedded as $(x-3)(x^2 + 12x + 35)$ and condone poor notation such as $f(x) = x^2 + 12x + 35$. Also allow this mark to be scored if seen within their long division.



Q12.

Question Number	Scheme	Marks
(a)	$x = 2 \Rightarrow 4 - 8y + y^2 = 13 \Rightarrow (y^2 - 8y - 9 = 0 \Rightarrow) y = \dots$	M1
	$y = 9$	A1
		(2)
(b)	$2^x \rightarrow 2^x \ln 2$	B1
	$-4xy \rightarrow -4x \frac{dy}{dx} - 4y$ OR $y^2 \rightarrow 2y \frac{dy}{dx}$	M1
	$2^x \ln 2 - 4x \frac{dy}{dx} - 4y + 2y \frac{dy}{dx} = 0$	A1
	$\frac{dy}{dx}(2y - 4x) = 4y - 2^x \ln 2 \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{4y - 2^x \ln 2}{2y - 4x}$ or $\frac{dy}{dx} = \frac{2^x \ln 2 - 4y}{4x - 2y}$ or $\frac{dy}{dx} = \frac{\ln 2 e^{x \ln 2} - 4y}{4x - 2y}$	A1
		(5)
(c)	$(2, 9) \rightarrow \frac{dy}{dx} = \frac{4(9) - 2^2 \ln 2}{2(9) - 4(2)}$ $\Rightarrow y - "9" = \frac{36 - 4 \ln 2}{10} (x - 2)$	M1
	$y = 0 \Rightarrow 0 - "9" = \frac{36 - 4 \ln 2}{10} (x - 2) \Rightarrow x = \dots$	dM1
	$x = \frac{4 \ln 2 + 9}{2 \ln 2 - 18}$ o.e. e.g. $x = \frac{-8 \ln 2 - 18}{-4 \ln 2 + 36}$	A1
		(3)
		Total 10



(a)
 M1: Substitutes $x = 2$ into the equation for C , forms a quadratic equation in y and then solves to obtain at least one value for y
 A1: $y = 9$ only

(b) It is acceptable to use $y' \leftrightarrow \frac{dy}{dx}$ in this question

B1: Correct differentiation of 2^x . Allow also $2^x = e^{x \ln 2} \rightarrow e^{x \ln 2} \ln 2$

M1: Differentiates $-4xy \rightarrow \pm Ax \frac{dy}{dx} \pm By$ OR differentiates $y^2 \rightarrow k y \frac{dy}{dx}$

A1: Fully correct differentiation. Allow versions such as $2^x \ln 2 dx - 4x dy - 4y dx + 2y dy = 0$

M1: Attempts to make $\frac{dy}{dx}$ the subject with the 2 terms in $\frac{dy}{dx}$ coming from differentiating y^2 and $-4xy$

A1: Any correct expression for $\frac{dy}{dx}$. Note that you can isw after a correct answer. $\frac{2y - 2^{x-1} \ln 2}{y - 2x}$ is correct

(c)
 M1: Uses $x = 2$ and their y value from part (a) to find the gradient at P and attempts to form the equation of the tangent at P . Condone poor attempts at differentiation but $\frac{dy}{dx}$ must have both x and y terms.

If the form $y = mx + c$ is used they must proceed as far as $c = \dots$

dM1: Substitutes $y = 0$ into their tangent equation and rearranges to find x . Dependent upon previous M
 It is possible for a method to combine this with the previous M.

Look for methods like the following, which score M1, dM1

$-9 = "m"(x - 2) \Rightarrow x = \dots$ where ' m ' is the value of their $\frac{dy}{dx}$ at $x = 2$

or $\frac{9}{2 - x} = "m" \Rightarrow x = \dots$ where ' m ' is the value of their $\frac{dy}{dx}$ at $x = 2$

A1: Correct expression in the required form. Must come from a correct $\frac{dy}{dx}$

Do not be concerned about the order of the terms on the numerator and denominator

(Q02 WMA14/01, June 2023)



Q13.

Question Number	Scheme	Marks
	$u = 3 + \sqrt{2x-1} \Rightarrow x = \frac{(u-3)^2 + 1}{2} \Rightarrow \frac{dx}{du} = u-3$ <p style="text-align: center;">or</p> $u = 3 + \sqrt{2x-1} \Rightarrow \frac{du}{dx} = \frac{1}{2}(2x-1)^{-\frac{1}{2}} \times 2 = \frac{1}{\sqrt{2x-1}} = \frac{1}{u-3}$	M1 A1
	$\int \frac{4}{3 + \sqrt{2x-1}} dx = \int \frac{4}{u} \times (u-3) du$	M1
	$\int \frac{4}{u} \times (u-3) du = \int \left(4 - \frac{12}{u}\right) du$	dM1
	$\int \left(4 - \frac{12}{u}\right) du = 4u - 12 \ln u \quad \text{or} \quad k(4u - 12 \ln u)$	ddM1 A1ft
	$\int_1^{13} \frac{4}{3 + \sqrt{2x-1}} dx = [4u - 12 \ln u]_4^8 = (4 \times 8 - 12 \ln 8) - (4 \times 4 - 12 \ln 4)$ <p style="text-align: center;">or</p> $\int_1^{13} \frac{4}{3 + \sqrt{2x-1}} dx = \left[4(3 + \sqrt{2x-1}) - 12 \ln(3 + \sqrt{2x-1})\right]_1^{13} = (4 \times 8 - 12 \ln 8) - (4 \times 4 - 12 \ln 4)$	M1
	$= 16 - 12 \ln 2$	A1
		(8 marks)

M1: Differentiates to get $\frac{du}{dx}$ in terms of x and then obtains $\frac{dx}{du}$ in terms of u

Need to see $\frac{du}{dx} = k(2x-1)^{\frac{1}{2}} \rightarrow \frac{du}{dx} = \frac{1}{au+b}$ or $\frac{dx}{du} = au+b$

or

Attempts to change the subject of $u = 3 + \sqrt{2x-1}$ and differentiates to get $\frac{dx}{du}$ in terms of u

Need to see $x = \frac{(u \pm 3)^2 \pm 1}{2} \rightarrow \frac{dx}{du} = au+b$

A1: $\frac{dx}{du} = u-3$ or e.g. $\frac{du}{dx} = \frac{1}{u-3}$, $du = \frac{dx}{u-3}$, $dx = (u-3) du$

M1: Attempts to write the integral completely in terms of u .

Need to see $\int \frac{\dots}{u} \times \text{their } \frac{dx}{du} du$ with or without the "du" but not e.g. $\int \frac{\dots}{u} \times \frac{1}{\frac{dx}{du}} du$

dM1: Divides to reach an integral of the form $\int \left(A + B \times \frac{1}{u} \right) du$. Depends on both previous M's

dM1: Integrates to a form $Au + B \ln u$. Depends on the previous M

An alternative for the previous 2 marks is to use integration by parts:

E.g. $\int \frac{4}{u} \times (u-3) du = 4(u-3) \ln u - \int 4 \ln u du = 4u \ln u - 12 \ln u - 4u \ln u + 4u = 4u - 12 \ln u$

Score dM1 for $\int \frac{k}{u} \times (Au+B) du = k(Au+B) \ln u - \int k \ln u du$ and dM1 for integrating to a form $Au + B \ln u$.

A1ft: $4u - 12 \ln u$ or $k(4u - 12 \ln u)$ following through on $\frac{dx}{du} = k(u-3)$ only.

M1: Substitutes 8 and 4 into their $4u - 12 \ln u$ and subtracts or substitutes 13 and 1 into their $4u - 12 \ln u$ with $u = 3 + \sqrt{2x-1}$ and subtracts. This mark depends on there having been an attempt to integrate, however poor.

A1: $16 - 12 \ln 2$

(Q05 WMA14/01, Jan 2021)

Q14.

Question Number	Scheme	Marks
(i)	$2\log_3(4x+5) - \log_3(x+3) = 2 \Rightarrow \log_3 \frac{(4x+5)^2}{(x+3)} = 2$ $\Rightarrow \frac{(4x+5)^2}{(x+3)} = 9$ $\Rightarrow 16x^2 + 31x - 2 = 0$ $(16x-1)(x+2) = 0 \Rightarrow x = \frac{1}{16} \text{ only}$	M1, M1 A1 dM1, A1 (5)
(ii) (a)	States that $\log a + \log b = \log(ab)$ or else uses rule and proceeds from given equation $\log a + \log b = \log(a+b)$ to $\log ab = \log(a+b)$ Deduces $ab = a+b \Rightarrow ab - a = b \Rightarrow a(b-1) = b \Rightarrow a = \frac{b}{b-1}$ *	B1 M1, A1*
(b)	States either $b > 1$ or $b \neq 1$ as a would not be defined $b > 1$ as logs only exist for positive numbers	B1 B1 (5) (10 marks)



(i) We are now scoring this question M1, M1, A1, dM1, A1

M1: Usually scored for the power law of logs $2\log_3(4x+5) \rightarrow \log_3(4x+5)^2$ (allow without the base 3)

But may be awarded for writing 2 as $\log_3 9$

M1: For combining two terms of the original equation.

E.g. $\log_3 \frac{(4x+5)^2}{(x+3)} = 2$ (allow without the base 3) or $2\log_3(4x+5) = \log_3 9(x+3)$

A1: A correct equation not involving logs. E.g. $\frac{(4x+5)^2}{(x+3)} = 9$ or $(4x+5)^2 = 9(x+3)$

dM1: Requires all of the following

- A starting equation of the form (or equivalent to the form) $\frac{(4x+5)^2}{(x+3)} = k$, $k > 0$
- An intermediate equation that is a 3TQ
- A correct attempt to solve the 3TQ by any means condoning the use of a calculator here.

It is dependent upon scoring at least one of the previous M marks

A1: CSO $\frac{1}{16}$ oe only. The decimal equivalent is 0.0625

'Correct' solutions with missing or incorrect lines

Example 1: Incorrect statement

$$2\log_3(4x+5) - \log_3(x+3) = 2 \Rightarrow \frac{\log_3(4x+5)^2}{\log_3(x+3)} = 2 \Rightarrow \frac{(4x+5)^2}{(x+3)} = 9 \text{ leading to full and correct}$$

solution

Score Special Case M1, M0, A1, dM1, A0 for 3 out of 5

Example II: Missing lines

$$2\log_3(4x+5) - \log_3(x+3) = 2 \Rightarrow \log_3 \frac{(4x+5)^2}{(x+3)} = 2 \Rightarrow \frac{(4x+5)^2}{(x+3)} = 9$$

Or even $2\log_3(4x+5) - \log_3(x+3) = 2 \Rightarrow \frac{(4x+5)^2}{(x+3)} = 9$ leading to full and correct solution can be awarded all the marks.

(ii) You can condone the omission of the base

(a) Main method

B1: States or uses $\log a + \log b = \log(ab)$ o.e. It is implied when $\log(ab) = \log(a+b)$ is written down or $ab = a+b$ but it cannot be awarded following incorrect work. Condone $\log a + b$ for $\log(a+b)$ if the intention is clear (but the use of this would mean that the A1* mark is not scored)

Alternatives exist such as $\log(a+b) - \log b = \log\left(\frac{a+b}{b}\right)$

M1: Following correct log work, candidate removes the logs, correctly deduces $ab = a+b$, and then attempts to collect terms in a . To award this mark the two terms in a must be moved to the same side of the equation and the a factorised out. It can be awarded when the term in 'a' is isolated. E.g. $ab \pm a = b \Rightarrow a(b \pm 1) = b$

Alternatively removes the logs, deduces $a = \frac{a+b}{b}$ and then attempts to collect terms in a

Acceptable alternatives exist. E.g. $ab = a + b \Rightarrow b = 1 + \frac{b}{a} \Rightarrow \frac{b}{a} = b - 1 \Rightarrow a = \frac{b}{b-1}$

The method mark can be awarded here when there is a single term in a . See double underlined expression

A1*: Correctly proves that $a = \frac{b}{b-1}$ via $a(b-1) = b$ or $ab - a = b$.

This is a given answer and **ALL** appropriate lines must be seen and be correct.

Condone a missing trailing bracket.

The appropriate lines that must be seen (as a minimum) via the standard approach are;

$$\log ab = \log(a+b) \Rightarrow ab = a+b \Rightarrow a(b-1) = b \Rightarrow a = \frac{b}{b-1}$$

(a) Alt method

B1: Substitutes $a = \frac{b}{b-1}$ in $\log a + \log b$ and proceeds to $\log\left(\frac{b^2}{b-1}\right)$

M1: Substitutes $a = \frac{b}{b-1}$ in $\log(a + b)$ and proceeds to $\log\left(\frac{b}{b-1} + b\right)$ and attempts to write as a single fraction

A1*: Shows that $\log a + \log b = \log(ab)$ when $a = \frac{b}{b-1}$ and makes a minimal statement.

This is a given answer and all appropriate lines must be seen and be correct (including any bracketing).

(ii)(b)

B1: For either (i) giving the full restriction on b .

E.g.

$b > 1$, 'b is more than one' or 'it must be bigger than one'. No (correct) explanation required.

or (ii) stating that $b \neq 1$ with a reason such as $\frac{b}{b-1} \rightarrow \infty$ or $b-1=0$.

Allow reasons such as 'the denominator cannot be 0' 'when $b = 1$ you get a maths error'

Do not accept ambiguous statements such as $b \neq 1$ because it cannot be 0

or (iii) stating that $b \dots 1$ with a reason such as $b-1$ cannot be negative

B1: $b > 1$ or 'b must be more than one' as logs only exist for positive numbers.

Allow 'b > 1 as a must be greater than 0'

Allow variations that imply this so accept $b > 1$ as $\frac{b}{b-1}$ needs to be positive or $b-1$ needs to be positive

There really needs to be some words here and the statement and explanation must be given together and in a logical order. A minimum acceptable response could be 'as $a > 0$, $b > 1$ '

Do not accept incorrect reasons such as 'b > 1 as a must be greater than or equal to 0'

Do not award this mark if there are incorrect statements along with correct ones. If unsure use review.

(Q09 WMA12/01, Oct 2025)



Q15.

Question Number	Scheme	Marks
(a)	$\left(\frac{11}{3}, -4\right)$	B1 B1 (2)
(b)	Attempts $3x - 11 - 4 = 8$ and $-3x + 11 - 4 = 8$ $x = -\frac{1}{3}, \frac{23}{3}$	M1, dM1 A1 (3)
(c)	$m \geq 3, m = -\frac{12}{11}, m < -3$	M1, A1, A1 (3)
(d)	$a = -4, b = \frac{4}{3}$	B1ft, B1ft (2)
		(10 marks)

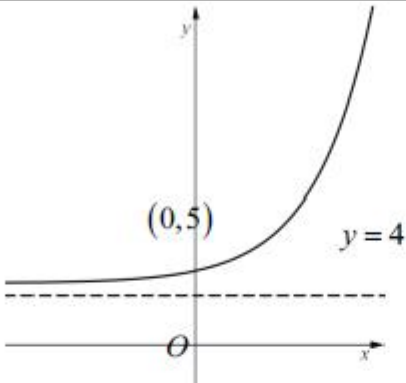


Note: Check working near to the question text or written on the diagram.

- (a)
- B1 For one correct coordinate
- B1 For $\left(\frac{11}{3}, -4\right)$. Allow missing brackets, or $x = \frac{11}{3}, y = -4$. Accept equivalent fractions for $\frac{11}{3}$
- (b)
- M1 For a full attempt at either $3x - 11 - 4 = 8$ or $-3x + 11 - 4 = 8$. Must lead to a value for x .
- dM1 Dependent on the first M1. For a full attempt at both $3x - 11 - 4 = 8$ and $-3x + 11 - 4 = 8$ leading to values for x .
- A1 $x = -\frac{1}{3}, \frac{23}{3}$ only. If one of these has clearly been selected as a final answer and the other rejected, withhold this mark.
- Alt**
- M1 Reaches $|3x - 11| = 12$, squares both sides and solves a quadratic equation. Must lead to a value for x .
- dM1 Reaches two values for x following a valid method for solving their quadratic equation.
- A1 $x = -\frac{1}{3}, \frac{23}{3}$. If one of these has clearly been selected as a final answer and the other rejected, withhold this mark.
- (c)
- M1 Finds any of the three critical values for m . This need not be presented as an inequality.
- A1 Two of $m \geq 3, m = -\frac{12}{11}, m < -3$. Do not give this mark if there are contradictions, e.g.
- $m = -\frac{12}{11}, m > -\frac{12}{11}$ unless the correct answer is clearly indicated. Accept set notation.
- A1 All three of $m \geq 3, m = -\frac{12}{11}, m < -3$ with no contradictions (see above). Accept set notation.
- (d) **Note: If $a = \dots$ and $b = \dots$ are not stated, they could be embedded in $y = af(x - b)$. The stated values take precedence.**
- B1ft Either $a = -4$ or $b = \frac{4}{3}$ but follow through on their vertex, so allow $a = \frac{16}{-4}$ or $b = 5 - \frac{11}{3}$
- B1ft Both $a = -4$ and $b = \frac{4}{3}$ but follow through on their vertex, so allow $a = \frac{16}{-4}$ and $b = 5 - \frac{11}{3}$

(Q07 WMA13/01, Jan 2026)

Q16.

Question Number	Scheme	Marks
(a)		Shape or asymptote Intercept Fully correct M1 B1 A1 (3)
(b)	$\text{Area} \approx \frac{0.3}{2} \{0 + 4.3137 + 2 \times (0.3246 + 0.8629 + 1.6643 + 2.7896)\}$ $= \text{awrt } 2.34$	$h = 0.3$ B1 M1 A1 (3)
(c) (i)	$\int_2^{3.5} (2^x + 2x) dx = \int_2^{3.5} (2^x - 2x + 4x) dx = 2.34 + [2x^2]_2^{3.5}$ $= 2.34 + 16.5 = 18.84$	M1 A1 ft
(ii)	$\int_2^{3.5} (2^{x+1} - 4x) dx = \int_2^{3.5} 2(2^x - 2x) dx = 2 \times 2.34 = 4.68$	B1 ft (3) (9 marks)

(a)

- M1: For either the correct shape, an increasing curve in any position in quadrants 1 and 2 or for the correct asymptote (labelled in some way at height 4) but must have a curve approaching as an asymptote, not just the line drawn. Be tolerant on "pen slips" approaching the asymptote.
- B1: Correct intercept (touching or crossing). Accept 5 on axis, or stated as (0,5). Condone (5,0) on the graph if it is in the right place.
- A1: Fully correct. Shape in quadrants 1 and 2 only with the y intercept at 5 and asymptote of $y = 4$ clearly drawn on the graph and labelled or stated separately. Be tolerant on "pen slips" approaching the asymptote. If there is conflicting information, what is on the graph takes precedence.

(b)

- B1: For $h = 0.3$ This is implied by sight of $\frac{0.3}{2}$ in front of the bracket.
- M1: Applies the trapezium rule with correct bracket condoning slips in copying (or even truncating) values. May use separate trapezia. Allow a missing final bracket, but otherwise bracketing must be correct or recovered by correct answer.
- A1: awrt 2.34 Note that the calculator answer for this integral is 2.30

(c) (i)

M1: For realising that $\int_2^{3.5} (2^x + 2x) dx = \int_2^{3.5} (2^x - 2x + 4x) dx = 2.34 + [kx^2]_2^{3.5}$ The 4x should be seen with an attempt to integrate (increase in power) made (though may be implied).

Could also use answer to (b) and add the area under the trapezium $\frac{1.5}{2}(4 \times 2 + 4 \times 3.5)$ - may be implied by correct answer after seeing the integral correctly split.

Another alternative is to isolate the $\int_2^{3.5} 2^x dx = [x^2]_2^{3.5} + "2.34"$ and use this to find a value for the integral required.

A1ft: "2.34" + 16.5 = 18.84 but follow through on their 2.34 and allow awrt to 3s.f.

There must be a suitable method for this mark to be awarded.

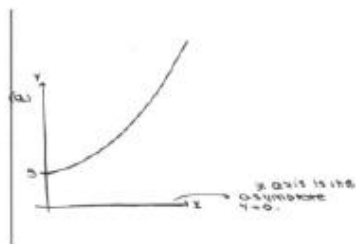
Answer via repeated trapezium rule scores M0A0

(c)(ii)

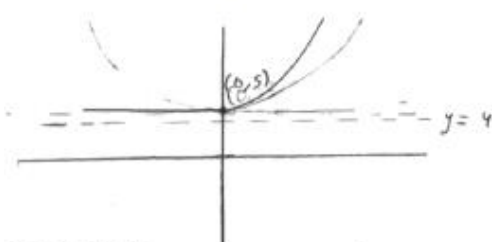
B1 ft: For $2 \times$ their "2.34" = 4.68 ((awrt to 3s.f.) must be evaluated). Follow through on their answer to (b).

Note: Answer via repeated trapezium rule is permitted to score the B1 as long the answer is twice their (b).

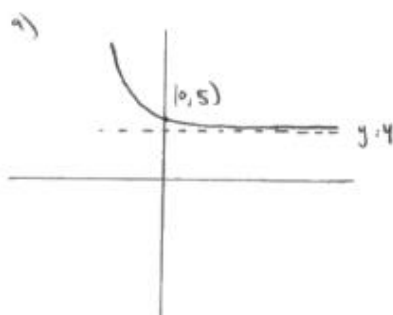
Some examples of graphs for (a):



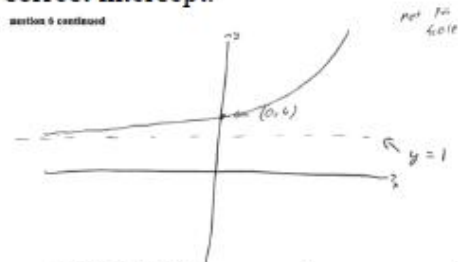
M0B1A0 Not in quadrant 2, but does have correct intercept.



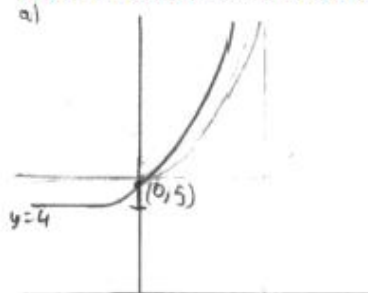
M1B1A0 Does not approach asymptote



M1B1A0 Correct asymptote and intercept



M1B0A0 Incorrect intercept and asymptote



M1B1A0 asymptote not drawn.

Q17.

Question Number	Scheme	Marks
(a)	$\overline{BA} \cdot \overline{BC} = -6 \times 2 + 2 \times 5 - 3 \times 8 = (-26)$	M1
	Uses $\overline{BA} \cdot \overline{BC} = \overline{BA} \overline{BC} \cos \theta \Rightarrow -26 = \sqrt{49} \times \sqrt{93} \cos \theta \Rightarrow \theta = \dots$	dM1
	$\theta = 112.65^\circ$	A1
		(3)
(b)	Attempts to use $ \overline{BA} \overline{BC} \sin \theta$ with their θ	M1
	Area = awrt 62.3	A1
		(2)
		(5 marks)

(a)

 M1: Attempts the scalar product of $\pm \overline{AB} \cdot \pm \overline{BC}$ condone slips as long as the intention is clear

 Or attempts the vector product $\pm \overline{AB} \times \pm \overline{BC}$ (see alternative 1)

Or attempts vector AC (see alternative 2)

 dM1: Attempts to use $\pm \overline{AB} \cdot \overline{BC} = |\overline{AB}| |\overline{BC}| \cos \theta$ AND proceeds to a value for θ

Expect to see at least one correct attempted calculation for a modulus.

 For example $\sqrt{2^2 + 5^2 + 8^2} (= \sqrt{93})$ or $\sqrt{6^2 + 2^2 + 3^2} (= 7)$

 Note that we condone poor notation such as: $\cos \theta = \frac{26}{7\sqrt{93}} = 67.35^\circ$ Depends on the first mark.

Must be an attempt to find the correct angle.

 A1: $\theta =$ awrt 112.65° Versions finishing with $\theta =$ awrt 67.35° will normally score M1 dM1 A0

 Angles given in radians also score A0 (NB $\theta = 1.9661\dots$ or acute $1.1754\dots$)

 Allow e.g. $\theta = 67.35^\circ \Rightarrow \theta = 180 - 67.35^\circ = 112.65$ and allow $\cos \theta = \frac{26}{7\sqrt{93}} \Rightarrow \theta = 112.65$

1. Alternative using the vector product:

 M1: Attempts the vector product $\pm \overline{AB} \times \pm \overline{BC} = \pm \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} \times \pm \begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix} = \pm \begin{pmatrix} -31 \\ -42 \\ 34 \end{pmatrix}$ condone slips as long as the intention is

clear

 dM1: Attempts to use $\pm \overline{AB} \times \overline{BC} = |\overline{AB}| |\overline{BC}| \sin \theta$ AND proceeds to a value for θ

Expect to see at least one correct attempted calculation for a modulus on rhs and attempt at the modulus of the vector product

 For example $\sqrt{2^2 + 5^2 + 8^2}$ or $\sqrt{6^2 + 2^2 + 3^2}$ and $\sqrt{31^2 + 42^2 + 34^2} (= \sqrt{3881})$

Note that we condone poor notation such as: $\sin \theta = \frac{\sqrt{3881}}{7\sqrt{93}} = 67.35^\circ$ Depends on the first mark.

Must be an attempt to find the correct angle.

A1: $\theta = \text{awrt } 112.65^\circ$ Versions finishing with $\theta = \text{awrt } 67.35^\circ$ will normally score M1 dM1 A0

2. Alternative using cosine rule:

M1: Attempts $\pm \overline{AC} = \pm (\overline{AB} + \overline{BC}) = \pm (8\mathbf{i} + 3\mathbf{j} + 11\mathbf{k})$ condone slips and poor notation as long as the intention is

clear e.g. allow $\begin{pmatrix} 8\mathbf{i} \\ 3\mathbf{j} \\ 11\mathbf{k} \end{pmatrix}$

dM1: Attempts to use $AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cos \theta$ AND proceeds to a value for θ

Must be an attempt to find the correct angle.

A1: $\theta = \text{awrt } 112.65^\circ$

(b)

M1: Attempts to use $|\overline{AB}| |\overline{BC}| \sin \theta$ with their θ . You may see $\frac{1}{2} |\overline{AB}| |\overline{BC}| \sin \theta$ found first before it is doubled.

or attempts the magnitude of their vector product e.g. $\sqrt{3881}$

A1: Area = awrt 62.3. If this is achieved from an angle of $\theta = \text{awrt } 67.35^\circ$ full marks can be scored

Note that there are other more convoluted methods for finding the area – score M1 for a complete and correct method using their values and send to review if necessary.

(Q02 WMA14/01, Jan 2021)

Q18.

Question Number	Scheme	Marks
(a)	$1.85 = 2a + b$ and $3.45 = 7a + b$ Solves simultaneously to get $a = 0.32, b = 1.21$ (oe)	M1 A1 dM1 A1 (4)
(b)	States 1.21 m or 121 cm (oe)	B1ft (1) (5 marks)

- (a)
- M1 For either $1.85 = 2a + b$ or $3.45 = 7a + b$
- A1 For both $1.85 = 2a + b$ and $3.45 = 7a + b$
- dM1 Solves simultaneously to get a value for a and a value for b
- A1 $a = 0.32, b = 1.21$ or equivalent fractions. May be seen in the equation.
- (b)
- B1 ft States 1.21 m or 121 cm (oe including units). Correct answer or follow through on their positive b
- Alt part (a)
- M1 Attempts $\frac{3.45 - 1.85}{7 - 2}$ Allow from attempts at use of arithmetic series, or from incorrect indexing. So
- E.g. $1.85 = 3a + b, 3.45 = 8a + b \Rightarrow a = \dots$ or $a_n = a + (n - 1)d, a_1 = 1.85, a_6 = 3.45 \Rightarrow a = \dots$ gain this mark.
- A1 $a = 0.32$ (may be called d if using AS)
- dM1 Full correct method to find b E.g substitutes their $a = 0.32$ into either correct equation (with correct indexing), or in " $a_n = a + (n - 1)d$ " finds " a " (=1.53) and expands to find $b = "a" - "d"$.
- A1 $a = 0.32, b = 1.21$

(Q02 WMA11/01, Oct 2019)

Q19.

Question	Scheme	Marks	AOs
(a)	Attempts \overline{OM} and \overline{ON} and subtracts to find $\pm\overline{MN}$ E.g. $\overline{OM} = 4\mathbf{i} + \frac{3}{2}\mathbf{j}$ $\overline{ON} = 3\mathbf{j} + \mathbf{k}$ $(\overline{MN}) = (3\mathbf{j} + \mathbf{k}) - \left(4\mathbf{i} + \frac{3}{2}\mathbf{j}\right) = -4\mathbf{i} + \frac{3}{2}\mathbf{j} + \mathbf{k}$	M1	1.1b
	Uses the given information to find either $\pm\overline{MP}$ or $\pm\overline{PN}$ Either $\overline{MP} = \frac{3}{4}\overline{MN} = \frac{3}{4}\left(-4\mathbf{i} + \frac{3}{2}\mathbf{j} + \mathbf{k}\right) = -3\mathbf{i} + \frac{9}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}$ Or $\overline{PN} = \frac{1}{4}\overline{MN} = \frac{1}{4}\left(-4\mathbf{i} + \frac{3}{2}\mathbf{j} + \mathbf{k}\right) = -\mathbf{i} + \frac{3}{8}\mathbf{j} + \frac{1}{4}\mathbf{k}$	M1 A1	3.1a 1.1b
	$\overline{OP} = \overline{OM} + \overline{MP} = \left(4\mathbf{i} + \frac{3}{2}\mathbf{j}\right) + \left(-3\mathbf{i} + \frac{9}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}\right) = \mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}^*$ or e.g. $\overline{OP} = \overline{ON} + \overline{NP} = (3\mathbf{j} + \mathbf{k}) + \left(\mathbf{i} - \frac{3}{8}\mathbf{j} - \frac{1}{4}\mathbf{k}\right) = \mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}^*$	A1*	2.1
	(4)		
(b)	$\overline{OQ} = \frac{8}{7}\left(\mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}\right) = \dots$	M1	3.1a
	Coordinates $Q = \left(\frac{8}{7}, 3, \frac{6}{7}\right)$	A1	3.2a
	(2)		
(6 marks)			

**Notes:**

(a)

M1: Attempts to find \overline{MN} (ignore labelling for this mark)**M1:** For the key step in using $\overline{MP} = 3\overline{PN}$ in a correct attempt to find $\pm\overline{MP}$ or $\pm\overline{PN}$ **A1:** Correct $\pm\overline{MP}$ or $\pm\overline{PN}$ **A1*:** Completes proof showing all key steps to obtain $\mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}$ or e.g. $\begin{pmatrix} 1 \\ \frac{21}{8} \\ \frac{3}{4} \end{pmatrix}$ but not $\begin{pmatrix} \mathbf{i} \\ \frac{21}{8}\mathbf{j} \\ \frac{3}{4}\mathbf{k} \end{pmatrix}$

(b)

M1: Attempts $\overline{OQ} = \frac{8}{7}\left(\mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}\right) = \dots$ May be implied by a correct vector or point.**A1:** Deduces that $Q = \left(\frac{8}{7}, 3, \frac{6}{7}\right)$.

Do not allow as a vector unless the correct coordinates are seen first then isw.

(Q12 9MA0/02/M, June 2025)

Q20.

Question	Scheme	Marks	AOs
	Examples: $4 \sin \frac{\theta}{2} \approx 4 \left(\frac{\theta}{2} \right), \quad 3 \cos^2 \theta \approx 3 \left(1 - \frac{\theta^2}{2} \right)^2$ $3 \cos^2 \theta = 3(1 - \sin^2 \theta) \approx 3(1 - \theta^2)$ $3 \cos^2 \theta = 3 \frac{(\cos 2\theta + 1)}{2} \approx \frac{3}{2} \left(1 - \frac{4\theta^2}{2} + 1 \right)$	M1	1.1a
	Examples: $4 \sin \frac{\theta}{2} + 3 \cos^2 \theta \approx 4 \left(\frac{\theta}{2} \right) + 3 \left(1 - \frac{\theta^2}{2} \right)^2$ $4 \sin \frac{\theta}{2} + 3 \cos^2 \theta = 4 \left(\frac{\theta}{2} \right) + 3(1 - \sin^2 \theta) \approx 2\theta + 3(1 - \theta^2)$ $4 \sin \frac{\theta}{2} + 3 \cos^2 \theta = 4 \sin \frac{\theta}{2} + 3 \frac{(\cos 2\theta + 1)}{2} \approx 4 \left(\frac{\theta}{2} \right) + \frac{3}{2} \left(1 - \frac{4\theta^2}{2} + 1 \right)$	dM1	1.1b
	$= 2\theta + 3(1 - \theta^2 + \dots) = 3 + 2\theta - 3\theta^2$	A1	2.1
		(3)	
(3 marks)			
Notes			
<p>M1: Attempts to use at least one correct approximation within the given expression.</p> <p>Either $\sin \frac{\theta}{2} \approx \frac{\theta}{2}$ or $\cos \theta \approx 1 - \frac{\theta^2}{2}$ or e.g. $\sin \theta \approx \theta$ if they write $\cos^2 \theta$ as $1 - \sin^2 \theta$ or e.g. $\cos 2\theta \approx 1 - \frac{(2\theta)^2}{2}$ (condone missing brackets) if they write $\cos^2 \theta$ as $\frac{1 + \cos 2\theta}{2}$.</p> <p>Allow sign slips only with any identities used but the appropriate approximations must be applied.</p> <p>dM1: Attempts to use correct approximations with the given expression to obtain an expression in terms of θ only. Depends on the first method mark.</p> <p>A1: Correct terms following correct work. Allow the terms in any order and ignore any extra terms if given correct or incorrect.</p>			

(Q04 9MA0/02, Oct 2021)