Exam Questions – Parametric integration and differentiation

Q1.

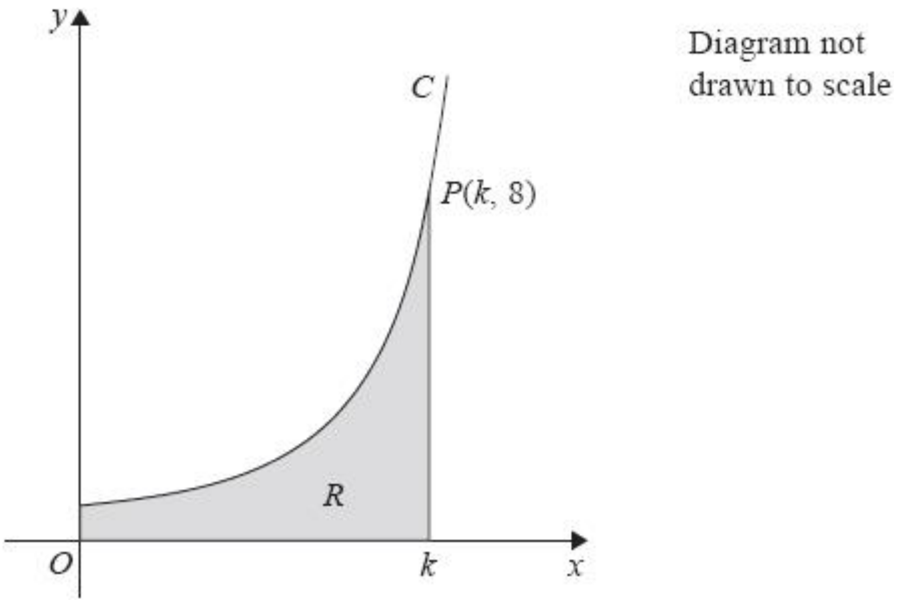

Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$x = 3\theta\sin\theta, \quad y = \sec^3\theta, \quad 0 \leq \theta < \frac{\pi}{2}$

The point $P(k, 8)$ lies on C, where k is a constant.

(a) Find the exact value of k.

(2)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the y-axis, the x-axis and the line with equation $x = k$.

(b) Show that the area of R can be expressed in the form

$$\lambda\int_{\alpha}^{\beta}\left(\theta\sec^2\theta + \tan\theta\sec^2\theta\right)\mathrm{d}\theta$$

where λ, α and β are constants to be determined.

(4)

(c) Hence use integration to find the exact value of the area of R.

(6)

(Total for question = 12 marks)

Q2.

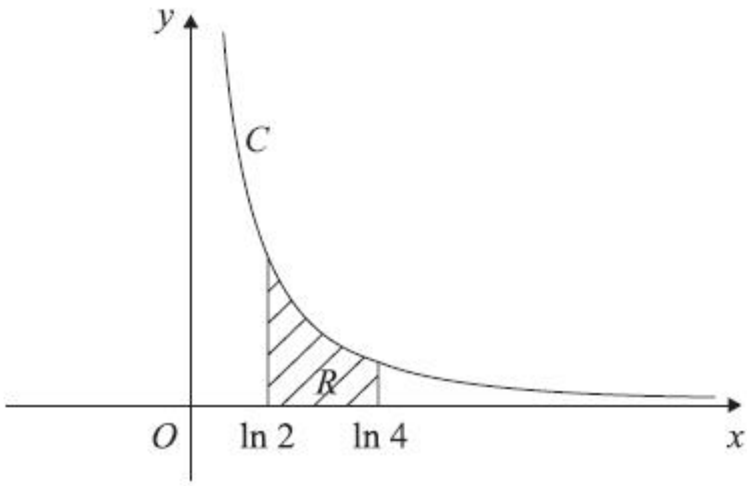

Figure 3

The curve C has parametric equations

$$x = \ln(t+2), \qquad y = \frac{1}{(t+1)}, \qquad t > -1.$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = l_n\ 2$ and $x = l_n\ 4$, is shown shaded in Figure 3.

(a) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)}\,\mathrm{d}t.$$

(4)

(b) Hence find an exact value for this area.

(6)

(c) Find a cartesian equation of the curve C, in the form $y = \mathrm{f}(x)$.

(4)

(d) State the domain of values for x for this curve.

(1)

(Total 15 marks)

Q3.

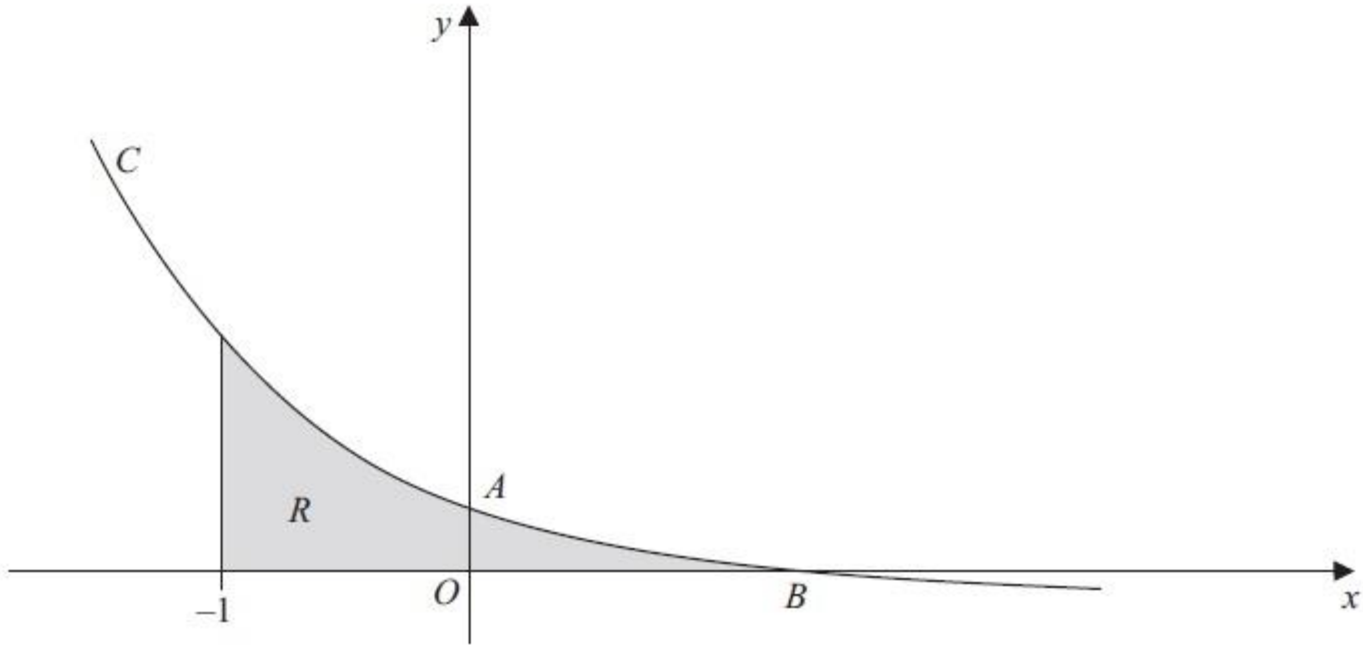

Figure 2

Figure 2 shows a sketch of part of the curve *C* with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the *y*-axis at the point *A* and crosses the *x*-axis at the point *B*.

(a) Show that *A* has coordinates (0, 3).

(2)

(b) Find the *x* coordinate of the point *B*.

(2)

(c) Find an equation of the normal to *C* at the point *A*.

(5)

The region *R*, as shown shaded in Figure 2, is bounded by the curve *C*, the line $x = -1$ and the *x*-axis.

(d) Use integration to find the exact area of *R*.

(6)

(Total 15 marks)

Q4.

A curve C has parametric equations

$$x = 4t + 3, \quad y = 4t + 8 + \frac{5}{2t}, \quad t \neq 0$$

(a) Find the value of $\frac{dy}{dx}$ at the point on C where $t = 2$, giving your answer as a fraction in its simplest form.

(3)

(b) Show that the cartesian equation of the curve C can be written in the form

$$y = \frac{x^2 + ax + b}{x - 3}, \quad x \neq 3$$

where a and b are integers to be determined.

(3)

(Total for question = 6 marks)

Q5.

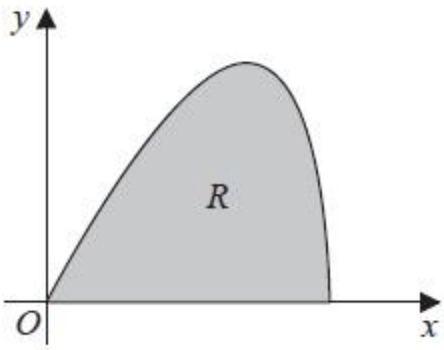

Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6\sin t \qquad y = 5\sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

(a) (i) Show that the area of R is given by $\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t \, dt$

(3)

(ii) Hence show, by algebraic integration, that the area of R is exactly 20

(3)

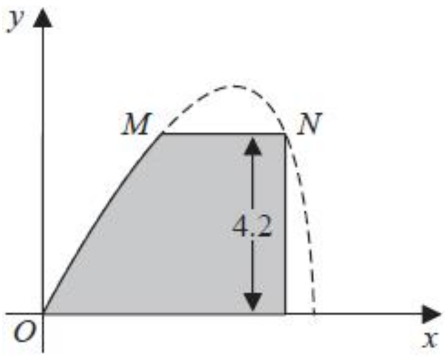

Figure 4

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4.
Using the model and given that

- *x* and *y* are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width *MN* along the top of the dam

(b) calculate the width of the walkway.

(5)

(Total for question = 11 marks)

Q6.

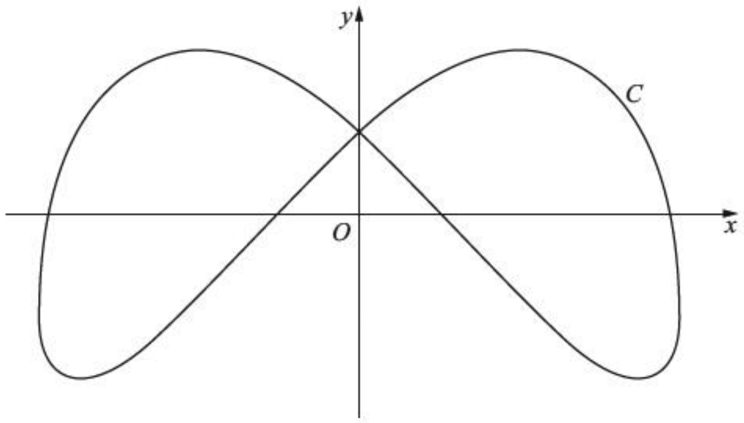

Figure 2

Figure 2 shows a sketch of the curve *C* with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \qquad y = 3\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of *t*.

(3)

Find the coordinates of all the points on *C* where $\frac{dy}{dx} = 0$

(5)

(Total 8 marks)

Q7.

Q7.

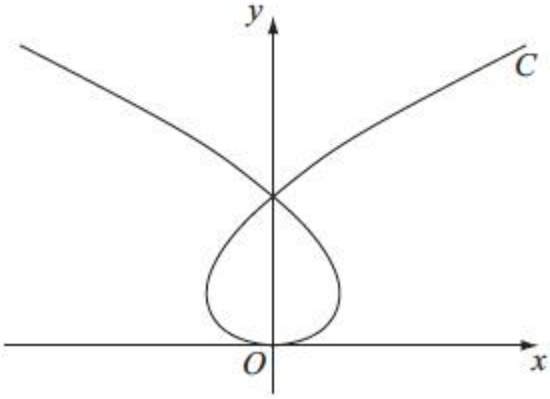

Figure 3

The curve *C* shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \; y = t^2$$

where *t* is a parameter. Given that the point *A* has parameter $t = -1$,

(a) find the coordinates of *A*.

(1)

The line l is the tangent to *C* at *A*.

(b) Show that an equation for l is $2x - 5y - 9 = 0$.

(5)

The line l also intersects the curve at the point *B*.

(c) Find the coordinates of *B*.

(6)

(Total 12 marks)