Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(i)	$x^2-6x+10=(x-3)^2+1$	M1	2.1
	Deduces "always true" as $(x-3)^2\geqslant 0\Rightarrow\ (x-3)^2+1\geqslant 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true. This could be if $a<0$ then $ax>b\Rightarrow x<\frac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a>0$ then $ax>b\Rightarrow x>\frac{b}{a}$; if $a<0$ then $ax>b\Rightarrow x<\frac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $=(n+1)^2-n^2=2n+1$	M1	3.1a
	Deduces "Always true" as $2n+1$ = (even +1) = odd	A1	2.2a
		(2)	
		(6 marks)	

Notes:

(i)

M1: Attempts to complete the square or any other valid reason. Allow for a graph of $y=x^2-6x+10$ or an attempt to find the minimum by differentiation

A1: States always true with a valid reason for their method

(ii)

M1: For an explanation that it need not be true (sometimes). This could be if $a<0$ then $ax>b\Rightarrow x<\frac{b}{a}$ or simply $-3x>6\Rightarrow x<-2$

A1: Correct statement (sometimes true) and explanation

(iii)

M1: Sets up the proof algebraically.
For example by attempting $(n+1)^2-n^2=2n+1$ or $m^2-n^2=(m-n)(m+n)$ with $m=n+1$

A1: States always true with reason and proof
Accept a proof written in words. For example
If integers are consecutive, one is odd and one is even
When squared $\text{odd}\times\text{odd}=\text{odd}$ and $\text{even}\times\text{even}=\text{even}$
The difference between odd and even is always odd, hence always true
Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.

(**Q06 9MA0/02, Specimen papers**)

Q2.

Question	Scheme	Marks	AOs
(i)	The statement is **not true** because e.g. when $x = -4$, $x^2 = 16$ (which is > 9 but $x < 3$)	B1	2.3
		(1)	
(ii)	$n^3 + 3n^2 + 2n = n\left(n^2 + 3n + 2\right) = n\left(n+1\right)\left(n+2\right)$	M1	2.1
	$n(n + 1)(n + 2)$ is the product of 3 consecutive integers	A1	2.2a
	As $n(n + 1)(n + 2)$ is a multiple of 2 **and** a multiple of 3 it must be a multiple of 6 and so $n^3 + 3n^2 + 2n$ is divisible by 6 for all integers n	A1	2.4
		(3)	
			(4 marks)

Notes

(i)

B1: Identifies the error in the statement by giving
- a counter example and a reason eg $x = -4$ with $x^2 = 16$ eg $x = -4$ with $(-4)^2 > 9$
- concludes **not true**

There should be no errors seen including the use of brackets. The conclusion could be a preamble. Do not accept "sometimes true" or equivalent.

Alternatively, explains why the statement is **not true**

Eg. It is not true as when $x < -3$ then $x^2 > 9$ so x does not have to be greater than 3.

Eg. $x^2 > 9 \Rightarrow x < -3$ or $x > 3$ so not true

(ii)

M1: Takes out a factor of n and attempts to factorise the resulting quadratic.

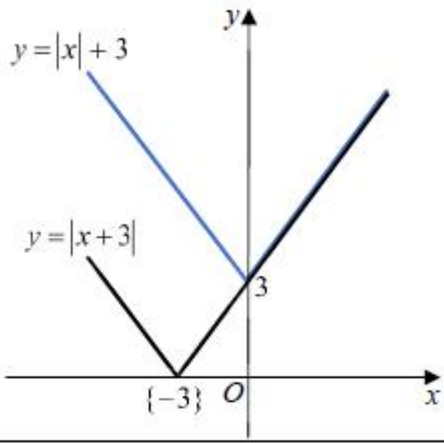
A1: Deduces that the expression is the product of 3 consecutive integers

A1: Explains that as the expression is a multiple of 3 **and** 2, it must be a multiple of 6 and so is divisible by 6

If you see any method which appears to be credit worthy but is not covered by the scheme then send to review

(**Q14 8MA0/01, June 2022**)

Q3.

Question	Scheme	Marks	AOs
	Statement: "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."		
(a)	E.g. $m = \sqrt{3}, n = \sqrt{12}$	M1	1.1b
	$\{mn =\}\ (\sqrt{3})(\sqrt{12}) = 6$ $\Rightarrow$ statement untrue **or** 6 is not irrational or 6 is rational	A1	2.4
		(2)	
(b)(i), (ii) Way 1	 V shaped graph {reasonably} symmetrical about the y-axis with vertical interpret $(0, 3)$ or 3 stated or marked on the positive y-axis	B1	1.1b
	Superimposes the graph of $y = \|x+3\|$ on top of the graph of $y = \|x\| + 3$	M1	3.1a
	the graph of $y = \|x\| + 3$ is either the same or above the graph of $y = \|x+3\|$ {for corresponding values of x} **or** when $x \geq 0$, both graphs are equal (or the same) when $x < 0$, the graph of $y = \|x\| + 3$ is above the graph of $y = \|x+3\|$	A1	2.4
		(3)	
(b)(ii) Way 2	**<u>Reason 1</u>** When $x \geq 0$, $\|x\| + 3 = \|x+3\|$ — Any one of Reason 1 or Reason 2	M1	3.1a
	<u>Reason 2</u> When $x < 0$, $\|x\| + 3 > \|x+3\|$ — Both Reason 1 and Reason 2	A1	2.4
		(5 marks)	

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Notes for Question			
(a)			
M1:	States or uses any pair of ***different*** numbers that will disprove the statement. E.g. $\sqrt{3}, \sqrt{12}$; $\sqrt{2}, \sqrt{8}$; $\sqrt{5}, -\sqrt{5}$; $\frac{1}{\pi}, 2\pi$; $3e, \frac{4}{5e}$;		
A1:	Uses correct reasoning to disprove the given statement, with a correct conclusion		
Note:	Writing $(3e)\left(\frac{4}{5e}\right) = \frac{12}{5} \Rightarrow$ untrue is sufficient for M1A1		
(b)(i)			
B1:	See scheme		
(b)(ii)			
M1:	For constructing a method of comparing $\|x\|+3$ with $\|x+3\|$. See scheme.		
A1:	Explains fully why $\|x\|+3 \geq \|x+3\|$. See scheme.		
Note:	Do not allow either $x > 0,\ \|x\|+3 \geq \|x+3\|$ or $x \geq 0,\ \|x\|+3 \geq \|x+3\|$ as a valid reason		
Note	$x = 0$ (or where necessary $x = -3$) need to be considered in their solutions for A1		
Note:	Do not allow an incorrect statement such as $x \leq 0,\ \|x\|+3 > \|x+3\|$ for A1		
(b)(ii)			
Note:	Allow M1A1 for $x > 0,\ \|x\|+3 = \|x+3\|$ and for $x \leq 0,\ \|x\|+3 \geq \|x+3\| \geqslant$		
Note:	Allow M1 for any of • x is positive, $\|x\|+3 = \|x+3\|$ • x is negative, $\|x\|+3 > \|x+3\|$ • $x > 0,\ \|x\|+3 = \|x+3\|$ • $x \leq 0,\ \|x\|+3 \geq \|x+3\|$ • $x > 0,\ \|x\|+3$ and $\|x+3\|$ are equal • $x \geq 0,\ \|x\|+3$ and $\|x+3\|$ are equal • when $x \geq 0$, both graphs are equal • for positive values $\|x\|+3$ and $\|x+3\|$ are the same Condone for M1 • $x \leq 0,\ \|x\|+3 > \|x+3\|$ • $x < 0,\ \|x\|+3 \geq \|x+3\|$		
(b)(ii) Way 3	• For $x > 0,\ \|x\|+3 = \|x+3\|$ • For $-3 < x < 0$, as $\|x\|+3 > 3$ and $\{0 <\}\ \|x+3\| < 3$, then $\|x\|+3 > \|x+3\|$ • For $x \leq -3$, as $\|x\|+3 = -x+3$ and $\|x+3\| = -x-3$, then $\|x\|+3 > \|x+3\|$	M1 A1	3.1a 2.4

(**Q03 9MA0/02, June 2018**)

Q4.

Score as below so M0 A0 M1 A1 or M1 A0 M1 A1 are not possible

Generally the marks are awarded for
M1: Suitable approach to answer the question for n being even **OR** odd
A1: Acceptable proof for n being even **OR** odd
M1: Suitable approach to answer the question for n being even **AND** odd
A1: Acceptable proof for n being even **AND** odd **WITH** concluding statement.

There is no merit in a

- student taking values, or multiple values, of n and then drawing conclusions. So $n = 5 \Rightarrow n^3 + 2 = 127$ which is not a multiple of 8 scores no marks.
- student using divided when they mean divisible. Eg. "Odd numbers cannot be divided by 8" is incorrect. We need to see either "odd numbers are not divisible by 8" or "odd numbers cannot be divided by 8 **exactly**"
- stating $\frac{n^3+2}{8} = \frac{1}{8}n^3 + \frac{1}{4}$ which is not a whole number
- stating $\frac{(n+1)^3+2}{8} = \frac{1}{8}n^3 + \frac{3}{8}n^2 + \frac{3}{8}n + \frac{3}{8}$ which is not a whole number

There must be an attempt to generalise either logic or algebra.

Example of a logical approach

Logical approach	States that if n is odd, n^3 is odd	M1	2.1
	so $n^3 + 2$ is odd and therefore cannot be divisible by 8	A1	2.2a
	States that if n is even, n^3 is a multiple of 8	M1	2.1
	so $n^3 + 2$ cannot be a multiple of 8 So (Given $n \in \mathbb{N}$), $n^3 + 2$ is not divisible by 8	A1	2.2a
		(4)	
			4 marks

First M1: States the result of cubing an odd or an even number
First A1: Followed by the result of adding two and gives a valid reason why it is not divisible by 8.
So for odd numbers accept for example

"odd number + 2 is still odd and odd numbers are not divisible by 8"

"$n^3 + 2$ is odd and cannot be divided by 8 **exactly**"

and for even numbers accept

"a multiple of 8 add 2 is not a multiple of 8, so $n^3 + 2$ is not divisible by 8"

"if n^3 is a multiple of 8 then $n^3 + 2$ cannot be divisible by 8"

Second M1: States the result of cubing an odd and an even number
Second A1: Both valid reasons must be given followed by a concluding statement.

Example of algebraic approaches

Question	Scheme	Marks	AOs
Algebraic approach	(If n is even,) $n = 2k$ and $n^3 + 2 = (2k)^3 + 2 = 8k^3 + 2$	M1	2.1
	Eg.'This is 2 more than a multiple of 8, hence not divisible by 8' Or 'as $8k^3$ is divisible by 8, $8k^3 + 2$ isn't'	A1	2.2a
	(If n is odd,) $n = 2k+1$ and $n^3 + 2 = (2k+1)^3 + 2$	M1	2.1
	$= \underline{\underline{8k^3 + 12k^2 + 6k + 3}}$ which is an even number add 3, therefore odd. Hence it is not divisible by 8. So (given $n \in \mathbb{N}$,) $n^3 + 2$ is not divisible by 8	A1	2.2a
		(4)	
Alt algebraic approach	(If n is even,) $n = 2k$ and $\frac{n^3+2}{8} = \frac{(2k)^3+2}{8} = \frac{8k^3+2}{8}$	M1	2.1
	$= k^3 + \frac{1}{4}$ oe which is not a whole number and hence not divisible by 8	A1	2.2a
	(If n is odd,) $n = 2k+1$ and $\frac{n^3+2}{8} = \frac{(2k+1)^3+2}{8}$	M1	2.1
	$= \frac{8k^3+12k^2+6k+3}{8}$ ** The numerator is odd as $\underline{\underline{8k^3 + 12k^2 + 6k + 3}}$ is an even number +3 hence not divisible by 8. So (Given $n \in \mathbb{N}$,) $n^3 + 2$ is not divisible by 8	A1	2.2a
		(4)	

Notes
Correct expressions are required for the M's. There is no need to state "**If *n* is even,**" $n=2k$ and "**If *n* is odd,** $n=2k+1$" for the two M's as the expressions encompass all numbers. However the concluding statement must attempt to show that it has been proven for all $n \in \mathbb{N}$ Some students will use $2k-1$ for odd numbers There is no requirement to change the variable. They may use $2n$ and $2n \pm 1$ Reasons must be correct. Don't accept $8k^3+2$ cannot be divided by 8 for example. (It can!) Also $**" = \frac{8k^3+12k^2+6k+3}{8} = k^3+\frac{3}{2}k^2+\frac{3}{4}k+\frac{3}{8}$ which is not whole number" is too vague so A0

(**Q15 8MA0/01, June 2019**)

Q5.

Question	Scheme	Marks	AOs
(a)	States $(2a-b)^2 .. 0$	M1	2.1
	$4a^2 + b^2 .. 4ab$	A1	1.1b
	(As $a > 0, b > 0$) $\frac{4a^2}{ab} + \frac{b^2}{ab} .. \frac{4ab}{ab}$	M1	2.2a
	Hence $\frac{4a}{b} + \frac{b}{a} ... 4$ * CSO	A1*	1.1b
		(4)	
(b)	$a = 5, b = -1 \Rightarrow \frac{4a}{b} + \frac{b}{a} = -20 - \frac{1}{5}$ which is less than 4	B1	2.4
		(1)	
			(5 marks)

Notes

(a) (condone the use of $>$ for the first three marks)

M1: For the key step in stating that $(2a-b)^2 .. 0$

A1: Reaches $4a^2 + b^2 ... 4ab$

M1: Divides each term by $ab \Rightarrow \frac{4a^2}{ab} + \frac{b^2}{ab} .. \frac{4ab}{ab}$

A1*: Fully correct proof with steps in the correct order and gives the reasons why this is true:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a > 0$ and $b > 0$

(b)

B1: Provides a counter example and shows it is not true.
This requires values, a calculation or embedded values(see scheme) and a conclusion. The conclusion must be in words eg the result does not hold or not true
Allow 0 to be used as long as they explain or show that it is undefined so the statement is not true.

Proof by contradiction: Scores all marks

M1: Assume that there exists an $a, b > 0$ such that $\dfrac{4a}{b} + \dfrac{b}{a} < 4$

A1: $4a^2 + b^2 < 4ab \Rightarrow 4a^2 + b^2 - 4ab < 0$

M1: $(2a - b)^2 < 0$

A1*: States that this is not true, hence we have a contradiction so $\dfrac{4a}{b} + \dfrac{b}{a} \ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a > 0$ and $b > 0$

Attempt starting with the left-hand side

M1: $(\text{lhs} =)\dfrac{4a}{b} + \dfrac{b}{a} - 4 = \dfrac{4a^2 + b^2 - 4ab}{ab}$

A1: $= \dfrac{(2a - b)^2}{ab}$

M1: $= \dfrac{(2a - b)^2}{ab} \ldots 0$

A1*: Hence $\dfrac{4a}{b} + \dfrac{b}{a} - 4 \ldots 0 \Rightarrow \dfrac{4a}{b} + \dfrac{b}{a} \ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- ab is positive as $a > 0$ and $b > 0$

Attempt using given result: For 3 out of 4

$\dfrac{4a}{b} + \dfrac{b}{a} .. 4$ M1 $\Rightarrow 4a^2 + b^2 .. 4ab \Rightarrow 4a^2 + b^2 - 4ab \ldots 0$

A1 $\Rightarrow (2a - b)^2 \ldots 0$ oe

M1 gives both reasons why this is true

- "square numbers are greater than or equal to 0"
- "multiplying by *ab* does not change the sign of the inequality because *a* and *b* are positive"

(**Q13 8MA0/01, Oct 2020**)

Q6.

Question	Scheme	Marks	AOs
	$(\sin x-\cos x)^2 < 1 \Rightarrow \sin^2 x-2\sin x\cos x+\cos^2 x \ (<1)$ o.e.	**M1**	1.1b
	Examples: $1-2\sin x\cos x<1,\ 1-\sin 2x<1,\ -2\sin x\cos x<0,\ -\sin 2x<0$	**A1**	2.2a
	As x is obtuse then $-2\sin x\cos x$ is positive because $\sin x>0$ and $\cos x<0$ so we have a contradiction. Therefore $\sin x-\cos x \ldots 1$ *	**A1***	2.4
			(3 marks)
	Notes		

Condone poor notation e.g. $\sin x^2$ **or e.g.** $-2\sin\theta\cos x<1$ **for the first two marks only.**

M1: Expands $(\sin x-\cos x)^2$ to obtain $\sin^2 x \pm k\sin x\cos x+\cos^2 x$ where $k=1$ or 2 o.e. May be implied.

A1: Uses a correct identity $\sin^2 x+\cos^2 x=1$ or e.g. $-\sin^2 x-\cos^2 x=-1$ to obtain a correct inequality in any form that does not include the $\sin^2 x$ and $\cos^2 x$ terms. Condone e.g. $-2\sin\ \cos x<0$

A1*: Fully correct work which includes

- a convincing argument that explains why their inequality is not true
- a statement that indicates there is a contradiction
- a conclusion that $\sin x-\cos x \ldots 1$ (there is no need to repeat "when x is obtuse")
- no contradictory statements
- no mixed/missed variables, e.g., $-2\sin\theta\cos x<1$ or $1-\sin 2<1$

Examples:

From $-2\sin x\cos x<0$:

In the second quadrant $-2\sin x\cos x$ is $-\times+\times-=+$

"(this is a) contradiction" or equivalent (therefore) $\sin x-\cos x \ldots 1$

or

As x is obtuse, $\sin x>0$, $\cos x<0$ so $-2\sin x\cos x>0$

"(this is a) contradiction" or equivalent (therefore) $\sin x-\cos x \ldots 1$

From $-\sin 2x<0$:

As x is obtuse, $2x$ is reflex o.e. (i.e. $\pi<2x<2\pi$) so $-\sin 2x>0$

"(this is) wrong" or equivalent (therefore) $\sin x-\cos x \ldots 1$

From $1-\sin 2x<1$:

As x is obtuse, $2x$ is reflex o.e. (i.e. $180<2x<360$) so $\sin 2x<0$ so $1-\sin 2x>1$

"(this is a) contradiction" or equivalent (therefore) $\sin x-\cos x \ldots 1$

From $\sin 2x>0$:

As x is obtuse, $2x$ is reflex o.e. (i.e. $180<2x<360$) so $\sin 2x<0$

"(this is) incorrect" or equivalent (therefore) $\sin x-\cos x \ldots 1$

Note that you may condone the absence of a statement referring to the fact that $(\sin x-\cos x)^2<1$ is only valid since $\sin x-\cos x>0$ when x is obtuse.

Q7.

Question	Scheme	Marks	AOs
	When n is even: $(2k+1)^3-(2k)^3=8k^3+12k^2+6k+1-8k^3=6k(2k+1)+1$ $\Rightarrow$ which is odd	M1	3.1a
	or		
	When n is odd: $(2k+2)^3-(2k+1)^3=8(k^3+3k^2+3k+1)-(8k^3+12k^2+6k+1)=6k(2k+3)+7$ $\Rightarrow$ which is odd	A1	2.2a
	When n is even: $(2k+1)^3-(2k)^3=8k^3+12k^2+6k+1-8k^3=6k(2k+1)+1$ $\Rightarrow$ which is odd **and** When n is odd: $(2k+2)^3-(2k+1)^3=8(k^3+3k^2+3k+1)-(8k^3+12k^2+6k+1)=6k(2k+3)+7$ $\Rightarrow$ which is odd	dM1	2.1
	Hence odd for all $n\,(\in \mathbb{N})$ *	A1*	2.4
			(4 marks)

Notes
General guidance **It is likely that you will see a mixture of methods and approaches within some solutions.** **Mark the approach which scores the highest number of marks.** There should be no errors in the algebra but allow e.g. invisible brackets to be "recovered". Withhold the final mark if n is used instead of k or reference to $n \in \square$ but $n \in \square^{+}$ is acceptable. **Main scheme algebraic method using e.g.** $n = 2k$ **and** $n = 2k \pm 1$ **You will need to look at both cases and mark the one which is fully correct first.** Allow a different variable to k and may be different letters for odd and even M1: For the key step attempting to find $(n+1)^3 - n^3$ when $n = 2k$ **or** $n = 2k \pm 1$ and attempting to multiply out and simplify to achieve a three term quadratic (allow equivalent representation of odd or even e.g. $n = 2k + 2$ **or** $2n \pm 5$) Condone arithmetical slips and condone the use of e.g. $n = 2n$ and $n = 2n \pm 1$ A1: Complete argument for $n = 2k$ **or** $n = 2k + 1$ (or e.g. $n = 2k - 1$) showing the result is odd. Requires: • Correct simplified quadratic expression e.g. $12k^2 + 6k + 1$ (when $n = 2k$), $12k^2 + 18k + 7$ (when $n = 2k + 1$), $12k^2 - 6k + 1$ (when $n = 2k - 1$) (may be factorised) • A reason why the expression is odd e.g. $2k(6k + 3) + 1$ or may use a divisibility argument e.g. $\frac{12k^2 + 6k + 1}{2} = 6k^2 + 3k + \frac{1}{2}$ • Concludes "odd" o.e. (may be within their final conclusion) There should be no errors in the algebra but allow e.g. invisible brackets if they are "recovered" Condone the use of e.g. $n = 2n$ and $n = 2n \pm 1$ dM1: Attempts to find $(n+1)^3 - n^3$ when $n = 2k$ **and** $n = 2k \pm 1$ and attempts to multiply out and simplify to achieve a three term quadratic (allow equivalent representation of odd or even e.g. $n = 2k + 2$, $2n \pm 5$ Condone arithmetical slips and condone the use of e.g. $n = 2n$ and $n = 2n \pm 1$

A1*: Complete argument for **both** $n = 2k$ **and** $n = 2k+1$ (or e.g. $n = 2k-1$) showing the result is odd for all $n\left(\in \square\right)$

Requires for both cases:

- Correct simplified expressions for both odd and even (which may be factorised)
- A reason why both of the expressions are odd
- Minimal conclusion (may be within their final conclusion)

An overall conclusion is also required. "Hence odd for all $n\left(\in \square\right)$" Accept "hence proven", "statement proved", "QED"

The conclusion for when $n = 2k$ and $n = 2k+1$ may be within the final conclusion rather than separate which is acceptable e.g. "when $n = 2k$ and when $n = 2k+1$ the expression is odd, hence proven" (following correct simplified expressions and reasons)

	$(n+1)^3$	n^3	$(n+1)^3 - n^3$
$n = 2k-1$	$8k^3$	$8k^3 - 12k^2 + 6k - 1$	$12k^2 - 6k + 1$
$n = 2k$	$8k^3 + 12k^2 + 6k + 1$	$8k^3$	$12k^2 + 6k + 1$
$n = 2k+1$	$8k^3 + 24k^2 + 24k + 8$	$8k^3 + 12k^2 + 6k + 1$	$12k^2 + 18k + 7$

Alternative methods:

Algebraic with logic example

M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic. Condone arithmetical slips.

A1: Correct quadratic expression $3n^2 + 3n + 1$

dM1: Attempts to factorise their quadratic such that $n^2 + n \to n(n+1)$ within their expression e.g. $3n(n+1)+1$

A1*: Explains that e.g. $n(n+1)$ is always even as it is the product of two consecutive numbers so $3n(n+1)$ is odd $\times$ even $=$ even so $3n(n+1)+1$ is odd hence odd for all $n\left(\in \square\right)$

Proof by contradiction example

M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic.

A1: Correct quadratic expression $3n^2 + 3n + 1$

dM1: Sets $3n^2 + 3n + 1 = 2k$ (for some integer k) $\Rightarrow 3n(n+1) = 2k - 1$

A1*: Explains that $n(n+1)$ is always even as it is the product of two consecutive numbers so $3n(n+1)$ is odd $\times$ even $=$ even but $2k-1$ is odd hence we have a contradiction so $(n+1)^3 - n^3$ is odd (for all $n\left(\in \square\right)$). There must have been a correct opening statement setting up the contradiction e.g. "assume that there exists a value for n for which $(n+1)^3 - n^3$ is even"

Solutions via just logic (no algebraic manipulation)
e.g.
If n is odd, then $(n+1)^3 - n^3$ is $\text{even}^3 - \text{odd}^3 = \text{even} - \text{odd} = \text{odd}$

If n is even, then $(n+1)^3 - n^3$ is $\text{odd}^3 - \text{even}^3 = \text{odd} - \text{even} = \text{odd}$

Both cases must be considered to score any marks and scores SC 1010 if fully correct

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Further Maths method (proof by induction) – you may see these but please send to review for TLs or above to mark

M1: Assumes true for $n = k$, substitutes $n = k+1$ into $(n+1)^3 - n^3$, multiplies out the brackets and attempts to simplify to a three term quadratic e.g. $3k^2 + 9k + 7$ Condone arithmetical slips

A1: $\left(\text{f}(k+1) = 3k^2 + 3k + 1 + 6(k+1) =\right)$ $(k+1)^3 - k^3 + 6(k+1) = \text{f}(k) + 6(k+1)$
which is odd + even = odd

dM1: Attempts to substitute $n = 1 \Rightarrow (1+1)^3 - 1^3 = 7$ (which is true) (Condone arithmetical slips evaluating)

A1*: Explains that
- it is true when $n = 1$
- if it is true for $n = k$ then it is true for $n = k+1$
- therefore it is true for all $n\left(\in \mathbb{N}\right)$

(**Q14 9MA0/01, June 2023**)

Q8.

Question	Scheme	Marks	AOs
(a)	e.g. The last line should start $25k^2+20k+4$	B1	2.3
		(1)	
(b)	Considers one of the missing calculations $m=5k+3$ and attempts $m^2=(5k+3)^2=...$ **or** $m=5k+4$ and attempts $m^2=(5k+4)^2=...$	M1	2.1
	Achieves one correct statement $m^2=(5k+3)^2=25k^2+30k+9=5(5k^2+6k+2)-1$ **or** $m^2=(5k+4)^2=25k^2+40k+16=5(5k^2+8k+3)+1$	A1	1.1b
	Considers **both** of the missing calculations	dM1	1.1b
	Achieves both correct statements with final concluding remark (see notes)	A1	2.1
		(4)	
			(5 marks)

Notes:

(a)

B1: Corrects the error for the case when $m = 5k + 2$. The correction may be on the proof in the box or may be described in the main body of the text. May just see the $10k$ crossed out and replaced with $20k$ in the box or described in the main body of the work.

Sight of the quadratic $\left(25k^2\right) + 20k\ (+4)$ scores B1 and isw e.g. if they make other errors in the expansion.

If B1 is not scored in (a) then allow to score if seen correct in (b) e.g. they may attempt the case $m = 5k + 2$ as part of their proof in (b).

(b) **Main scheme method uses $m = 5k + 3$ and $m = 5k + 4$**

You will need to look at both cases and mark the one which is fully correct first.

Allow a different variable to k and may be different letters for the two cases.

If a candidate attempts repeated cases e.g. $m = 5k + 4$ and $m = 5k - 1$ then mark both and award the higher mark of the two.

Condone use of m as a variable for the first three marks.

There should be no errors in the algebra for the A marks including invisible brackets but do not be concerned with any re-attempt at doing the case $m = 5k + 2$

Note that there are other allowable valid pairs of combinations covering the final two distinct cases e.g. $m = 5k - 2$ and $m = 5k + 4$, or $m = 5k - 1$ and $m = 5k + 3$ but NOT e.g. $m = 5k + 3$ and $m = 5k - 2$

Note that we are not expecting candidates to state what set of numbers k belongs to but we will condone pairs such as $m = -5k - 1$ and $m = -5k - 2$

Typically candidates will show the algebraic steps as in the main scheme but for this particular pair they may justify equivalence using the results in the box for $m = 5k + 1$ **and** $m = 5k + 2$ without requiring calculations which is acceptable.

M1: Considers one valid case e.g. $m = 5k + 3$ and attempts $m^2 = (5k + 3)^2$ or $m = 5k + 4$ and attempts $m^2 = (5k + 4)^2$

Look for expanding out the brackets and simplifying to a 3TQ. Condone slips.

A1: Achieves one correct statement which includes the case, the quadratic multiplied out and written in the required form

e.g. $m^2 = (5k + 3)^2 = 25k^2 + 30k + 9 = 5\left(5k^2 + 6k + 2\right) - 1$ or

e.g. $m^2 = (5k + 4)^2 = 25k^2 + 40k + 16 = 5\left(5k^2 + 8k + 3\right) + 1$

dM1: Considers **both cases** for a valid pair (see first M1 for guidance). It is dependent on the previous method mark. Condone slips.

A1: Full proof with correct statements for both cases for a valid pair. Each must include the case, the quadratic multiplied out and written in the form which is not in terms of m (we do not need the "where $n = ...$" at the end of the statements - you can ignore these)

Requires a minimal overall conclusion eg. Proven, QED, tick

Condone recovery of interchanging of variables.

m	m^2	$5n \pm 1$
$5k+3$	$25k^2+30k+9$	$5\left(5k^2+6k+2\right)-1$
$5k+4$	$25k^2+40k+16$	$5\left(5k^2+8k+3\right)+1$
$5k-1$	$25k^2-10k+1$	$5\left(5k^2-2k\right)+1$
$5k-2$	$25k^2-20k+4$	$5\left(5k^2-4k+1\right)-1$

Ignore any additional cases that are not required to complete the proof

(and ignore replications of the ones given in the box in the question)

(**Q08 9MA0/01, June 2025**)

Q9.

Question	Scheme	Marks	AOs
(a)	Attempts $\pm\left((2p+3)^2-(2p+1)^2\right)$ or equivalent	M1	3.1a
	$= 8p+8$ o.e.	A1	1.1b
	Requires • correct expression • minimal conclusion "hence multiple of 8", "proven", "true" etc.	A1	2.1
		(3)	
(b)	Provides a counter example. E.g $4^2-2^2=12$ which is not a multiple of 8	B1	2.4
		(1)	
			(4 marks)

Notes:

(a)

M1: Awarded for the key step in writing down an expression of the form

$$\pm\left((2p+3)^2-(2p+1)^2\right) \text{ oe e.g. } \pm\left((2p+1)^2-(2p-1)^2\right)$$

A1: Correct simplified expression for their $\pm\left((2p+3)^2-(2p+1)^2\right)$

Note that $\pm\left((2p+1)^2-(2p-1)^2\right)=\pm 8p$

A1: Rigorous proof showing all key steps with explanations

(b)

B1: Shows that the statement is not true for consecutive even numbers

Note that a proof such as $(2p+2)^2-(2p)^2=8p+4$ would need fully correct work and an explanation why $8p+4$ is not a multiple of 8.

(**Q09 9MA0/02/M, June 2025**)

Q10.

Question	Scheme	Marks	AOs
(a)	n^3+4n		
	Attempts n^3+4n for any 2 natural numbers.	M1	1.1b
	$1^3+4\times1=5$ prime and e.g. $2^3+4\times2=16$ not prime $\therefore$ sometimes true.	A1	2.4
		(2)	

Condone the use of *n* for e.g. *k* for both marks.

All methods require attempting n^3+4n when $n=1$ for full marks.

Way 1:

M1: Attempts n^3+4n for any 2 natural numbers.

A1: Requires:

- obtains $n^3+4n=5$ when $n=1$ and states "prime" or "true" or ✓
- correct evaluation for any other natural number and states "not prime" or "composite" or "not true" or ×
- states "sometimes true"

You can ignore any other incorrectly evaluated examples or e.g. use of negative numbers as long as these conditions are met.

Way 2 (factorisation):

M1: Attempts to factorise $n^3+4n=n(n^2+4)$

Allow for $n^3+4n=n(n^2+...)$ or $n^3+4n=n(...+4)$

A1: Requires:

- uses $n=1$ and obtains $n^3+4n=5$ and states "prime" or "true"
- correct factorisation and states $n^3+4n=n(n^2+4)$ is "composite" or "not prime" or "not true" (when $n>1$)
- states "sometimes true"

Way 3 (odd/even):

M1: Attempts to substitute $n=2k$ oe **or** $n=2k+1$ oe

A1: Requires:

- uses $n=1$ and obtains $n^3+4n=5$. Here the value for $n=1$ might be found using $k=1$ for $n=2k-1$ or $k=0$ for $n=2k+1$ and states "prime" or "true"
- correctly factorises any one correct form for odd or even e.g. $8k^3+8k=8k(k^2+1)$ and concludes "not prime" or "composite" or "not true" (only one form needed here)
- states "sometimes true"

(b)	n^3+5n		
	$n^3+5n=n(n^2+5)$	M1	3.1a
	Since $1^3+5\times1=6$, n^3+5n is not prime for $n=1$ For all other n, $n^3+5n=n(n^2+5)$ is not prime as it is the product of two other numbers not equal to 1. Hence never true.	A1	2.4
		(2)	
		(4 marks)	

Notes:

Condone the use of *n* for e.g. *k* for both marks.

Way 1 (Factorisation):

M1: Attempts to factorise $n^3+5n=n(n^2+5)$

Allow $n^3+5n=n(n^2+...)$ or $n^3+5n=n(...+5)$

A1: Requires:

- Correct factorisation $n^3+5n=n(n^2+5)$
- Substitution of $n=1$: $1^3+5\times1=6$
  or states $n^3+5n\neq2$ oe e.g. $n^3+5n>2$ oe
- "Never true"

Way 2 (Odd/Even):

M1: Attempts to substitute $n=2k$ **and** either $n=2k+1$ or $n=2k-1$ oe

A1: Requires:

- Correctly factorising both even and odd forms
  e.g. $8k^3+10k=2k(4k^2+5)$ **and**
  e.g. $(2k+1)^3+5(2k+1)=(2k+1)((2k+1)^2+5)$ or e.g. $2(2k+1)(2k^2+2k+3)$
  (in this part both cases are needed)
- Substitution of $n=1$: $1^3+5\times1=6$, or e.g. $k=1$ for $n=2k-1$ or $k=0$ for $n=2k+1$
  or states $n^3+5n\neq2$ oe e.g. $n^3+5n>2$ oe
  e.g. $2k(4k^2+5)$ and $2(2k+1)(2k^2+2k+3)$ are >2 or $\neq2$
- "Never true"

Way 3 (Odd/Even via logic):

M1: Considers n^3+5n with "odds" and "evens" e.g.

If n is **odd** then n^3+5n = odd + odd = even

or e.g. $n^3+5n=n(n^2+5)$ = odd(odd + odd) = odd×even = even

If n is **even** then n^3+5n = even + even = even

or e.g. $n^3+5n=n(n^2+5)$ = even(even + odd) = even×odd = even

A1: Requires:

- Fully correct argument for both odds and evens
- A full justification of any assumed results e.g. n odd $\Rightarrow n^3$ is odd via algebra or e.g. n odd means n^3 = odd×odd×odd = odd
- When $n=1$, $n^3+5n=6$ so $n^3+5n\neq2$
- "Never true"

(**Q15 8MA0/01, June 2025**)

Q11.

General points for marking question (i):

- Students who just try random numbers in part (i) are not going to score any marks.
- Students can mix and match methods. Eg you may see odd numbers via logic and even via algebra
- Students who state $4m^2+2$ **cannot be divided** by (instead of is not divisible by) cannot be awarded credit for the accuracy/explanation marks, unless they state correctly that $4m^2+2$ **cannot be divided by 4 to give an integer.**
- Students who write $n^2+2=4k \Rightarrow k=\frac{1}{4}n^2+\frac{1}{2}$ which is not a whole number gains no credit unless they then start to look at odd and even numbers for instance
- Proofs via induction usually tend to go nowhere unless they proceed as in the main scheme
- Watch for unusual methods that are worthy of credit (See below)
- If the final conclusion is $n \in \mathbb{R}$ then the final mark is withheld. $n \in \mathbb{Z}^+$ is correct

Watch for methods that may not be in the scheme that you feel may deserve credit.

If you are uncertain of a method please refer these up to your team leader.

Eg 1. Solving part (i) by modulo arithmetic.

All $n \in \mathbb{N}$ mod 4	0	1	2	3
All $n^2 \in \mathbb{N}$ mod 4	0	1	0	1
All $n^2+2 \in \mathbb{N}$ mod 4	2	3	2	3

Hence for all n, n^2+2 is not divisible by 4.

Question (i)	Scheme	Marks	AOs

Notes: Note that M0 A0 M1 A1 and M0 A0 M1 A0 are not possible due to the way the scheme is set up

(i)

M1: Awarded for setting up the proof for either the even or odd numbers.

A1: Concludes correctly with a reason why n^2+2 cannot be divisible by 4 for either n odd or even.

dM1: Awarded for setting up the proof for both even and odd numbers

A1: Fully correct proof with valid explanation and conclusion for all n

Example of an algebraic proof

For $n = 2m$, $n^2 + 2 = 4m^2 + 2$	M1	2.1
Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1	1.1b
For $n = 2m+1$, $n^2 + 2 = (2m+1)^2 + 2 = \dots$ FYI $(4m^2 + 4m + 3)$	dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4(m^2 + m) + 3$ ....AND states ......hence true for all	A1*	2.4
	(4)	

Example of a very similar algebraic proof

For $n = 2m$, $\frac{4m^2 + 2}{4} = m^2 + \frac{1}{2}$	M1	2.1
Concludes that this is not divisible by 4 due to the $\frac{1}{2}$ (A suitable reason is required)	A1	1.1b
For $n = 2m+1$, $\frac{n^2 + 2}{4} = \frac{4m^2 + 4m + 3}{4} = m^2 + m + \frac{3}{4}$	dM1	2.1
Concludes that this is not divisible by 4 due to the $\frac{3}{4}$ ...AND states ...... hence for all n, $n^2 + 2$ is not divisible by 4	A1*	2.4
	(4)	

Example of a proof via logic

When n is odd, "odd $\times$ odd" = odd	M1	2.1
so $n^2 + 2$ is odd, so (when n is odd) $n^2 + 2$ cannot be divisible by 4	A1	1.1b
When n is even, it is a multiple of 2, so "even $\times$ even" is a multiple of 4	dM1	2.1
Concludes that when n is even $n^2 + 2$ cannot be divisible by 4 because n^2 is divisible by 4.....AND STATES ....trues for all n.	A1*	2.4
	(4)	

Example of proof via contradiction

Sets up the contradiction 'Assume that n^2+2 is divisible by 4 $\Rightarrow n^2+2=4k$'	M1	2.1
$\Rightarrow n^2=4k-2=2(2k-1)$ and concludes even Note that the M mark (for setting up the contradiction must have been awarded)	A1	1.1b
States that n^2 is even, then n is even and hence n^2 is a multiple of 4	dM1	2.1
Explains that if n^2 is a multiple of 4 then n^2+2 cannot be a multiple of 4 and hence divisible by 4 Hence there is a contradiction and concludes Hence true for all n.	A1*	2.4
	(4)	

A similar proof exists via contradiction where

A1: $n^2=2(2k-1)\Rightarrow n=\sqrt{2}\times\sqrt{2k-1}$

dM1: States that $2k-1$ is odd, so does not have a factor of 2, meaning that n is irrational

Question (ii)	Scheme	Marks	AOs

(ii)

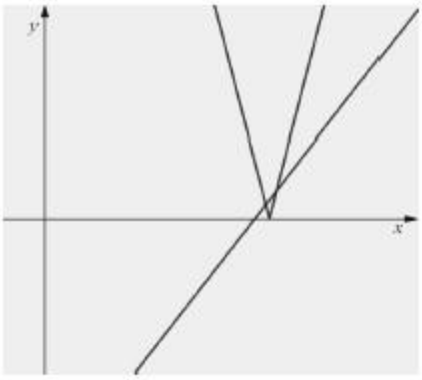
M1: States or implies 'sometimes true' or 'not always true' and gives an example where it is not true.

A1: and gives an example where it is true,

Proof using numerical values

SOMETIMES TRUE and chooses any number $x: 9.25 < x < 9.5$ and shows false Eg $x=9.4$ $\lvert 3x-28\rvert=0.2$ and $x-9=0.4$ ×	M1	2.3
Then chooses a number where it is true Eg $x=12$ $\lvert 3x-28\rvert=8$ $x-9=3$ ✓	A1	2.4
	(2)	

Graphical Proof

 States or implies ''sometimes true'' Sketches both graphs on the same axes. Expect shapes and relative positions to be correct. V shape on +ve x -axis Linear graph with +ve gradient intersecting twice	M1	2.3
Graphs accurate and explains that as there are points where $\lvert 3x-28 \rvert < x-9$ and points where $\lvert 3x-28 \rvert > x-9$ oe in words like 'above' and 'below' or 'dips below at one point'	A1	2.4
	(2)	

Proof via algebra

States sometimes true and attempts to solve both $3x-28<x-9$ and $-3x+28<x-9$ or one of these with the bound $9.\dot{3}$	M1	2.3
States that it is false when $9.25<x<9.5$ or $9.25<x<9.\dot{3}$ or $9.\dot{3}<x<9.5$	A1	2.4
	(2)	

Alt: It is possible to find where it is always true

States sometimes true and attempts to solve where it is just true Solves both $3x-28 \geqslant x-9$ and $-3x+28 \geqslant x-9$	M1	2.3
States that it is false when $9.25<x<9.5$ or $9.25<x<9.\dot{3}$ or $9.\dot{3}<x<9.5$	A1	2.4
	(2)	

(**Q10 9MA0/01, June 2019**)

Q12.

Question	Scheme	Marks	AOs
(a)	Provides a counter example with a reason. e.g., $6^3 - 1^3 = 215$ which is a multiple of 5	B1	2.4
		(1)	
(b)	States or uses, e.g., $2n$ and $2n+2$ **or** $2n+2$ and $2n+4$	M1	2.1
	Attempts $(2n+2)^3 - (2n)^3 = 8n^3 + 24n^2 + 24n + 8 - 8n^3$ leading to a quadratic.	dM1	1.1b
	$= 24n^2 + 24n + 8$	A1	1.1b
	$24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ So $q^3 - p^3$ is a multiple of 8	A1	2.1
		(4)	
			(5 marks)

Notes:

(a)

B1: Provides a counter example with a reason. There is no need to state "not true".

e.g., $7^3 - 2^3 = 335$ which divides by 5 {exactly}.

It is sufficient to have, e.g., $9^3 - 4^3 = 665$ and $\frac{665}{5} = 133$

Here q must be greater than p and both must be natural numbers, not 0 or negatives.
Note that any pair of positive integers n and $n+5k$ will provide a counter example, but $q^3 - p^3$ must be evaluated correctly, and if they divide by 5 this also needs to be correct.

(b)

M1: For the key step in stating the algebraic form of consecutive even numbers.
See main scheme for examples. They might be used either way round for this mark.

dM1: Attempts $(2n+2)^3 - (2n)^3 = ...$ condoning slips but must lead to a quadratic.

Alternatively, $(2n+2)^3 - (2n)^3 = 2^3\{(n+1)^3 - n^3\}$

May be subtracted the wrong way round for this mark as below.

$(2n)^3 - (2n+2)^3 = ...$ but this will score M1dM1A0A0

A1: e.g., $(2n+2)^3 - (2n)^3 = 24n^2 + 24n + 8$ **or** $(2n+4)^3 - (2n+2)^3 = 24n^2 + 72n + 56$

or $(2n+2)^3 - (2n)^3 = 8\{(n+1)^3 - n^3\}$ **or** $(2n)^3 - (2n-2)^3 = 24n^2 - 24n + 8$ etc.

Must come from correct work and the algebra will need checking carefully.

A1: For a full and rigorous proof showing all necessary steps including:

- correct quadratic expression for $q^3 - p^3$ for their even numbers, e.g., $24n^2 + 24n + 8$
- reason e.g., $24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ **or**, e.g., in $24n^2 + 24n + 8$ the coefficients are all multiples of 8
- minimal conclusion, "hence true"

Alt 1:
If the even numbers are set as n and $n + 2$ there must be sufficient work seen before marks can be awarded.
e.g.,

M1dM1: $n = 2k \Rightarrow (n+2)^3 - n^3 = ...n^2 + ...n + ... = ...(2k)^2 + ...(2k) + ...$

A1: $= 24k^2 + 24k + 8$

A1: $= 8(3k^2 + 3k + 1)$ so $q^3 - p^3$ is a multiple of 8

Alt 2:
If they just use any two even numbers, e.g., $2a$ and $2b$, **or** $2m$ and $2n + 2$ then they will score as follows:

M1: $(2a)^3 - (2b)^3$ Condone missing brackets if recovered.

dM1: $= ...a^3 - ...b^3$

A1: $= 8a^3 - 8b^3$ Note $8(a^3 - b^3)$ would imply this mark.

A1: $= 8(a^3 - b^3)$ so $q^3 - p^3$ is a multiple of 8 if q and p are {any two} even {numbers}

and hence $q^3 - p^3$ is a multiple of 8 if q and p are *consecutive* even numbers

(**Q17 8MA0/01, June 2023**)

Q13.

Question	Scheme	Marks	AOs
	Sets up the proof by exploring when $n = 2k$ or $n = 2k+1$ e.g. $(2k)^2 + 5(2k) = ...k^2 + ...k$ or $(2k+1)^2 + 5(2k+1) = ...k^2 + ...k + ...$	M1	1.1b
	e.g. $4k^2 + 10k$ or $4k^2 + 14k + 6$ and shows or gives a reason why the expression is even (see notes)	A1	2.2a
	Explores when $n = 2k$ and $n = 2k+1$ eg $(2k)^2 + 5(2k) = ...k^2 + ...k$ and $(2k+1)^2 + 5(2k+1) = ...k^2 + ...k + ...$	dM1	2.1
	e.g. $4k^2 + 10k$ and $4k^2 + 14k + 6$ and shows or gives a reason why both of the expressions are even (see notes) hence $n^2 + 5n$ is even for all $n(\in \mathbb{N})$ (or equivalent)	A1*	2.4
			(4 marks)

Notes
Main scheme algebraic method using e.g. $n = 2k$ and $n = 2k \pm 1$ **You will need to look at both cases and mark the one which is fully correct first.** **Allow a different variable to k and may be different letters for odd and even.** **Condone use of n as a variable for the first three marks.** **There should be no errors in the algebra for the A marks but allow e.g. invisible brackets to be "recovered".**

M1: Sets up the proof by exploring when n is odd or even e.g. $n = 2k$ or $n = 2k+1$ (or equivalent), and either expands and achieves a quadratic expression (which may be unsimplified) or allow to factorise e.g. $2k(2k+5)$ or e.g. $(2k+2)(2k+7)$

Condone slips. e.g. $2k(2k+5) = 2k^2 + 10k$ or slips when collecting terms.

A1: Correct quadratic expression (which may be unsimplified) for $n^2 + 5n$ for either odds or evens and shows or gives a reason why the expression is even. They must have fully multiplied out or the quadratic expression must be factorised completely.

e.g. $4k^2 + 10k = 2(2k^2 + 5k)$ (which is even)

e.g. $4k^2 + 14k + 6 = 2(2k^2 + 7k + 3)$ (which is even)

e.g. $\frac{4k^2 + 10k}{2} = 2k^2 + 5k$ (hence even)

e.g. "2 is a factor of both terms", "all divisible by 2" (so even)

If a reason is given as well as an algebraic expression it must be correct

e.g. $4k^2 + 10k = 2(2k^2 + 5k)$ so even as can be multiplied by 2 can score M1A1

but $\frac{4k^2 + 10k}{2} = 2k^2 + 5k$ so it can be divided by 2 so even is M1A0 (needs to say divisible by 2)

Do not isw if they simplify their quadratic incorrectly.

Note that they do not have to state that the expression is even if they conclude for all cases at the end.

dM1: Explores when n is odd and when n is even leading to two quadratic expressions (may be factorised) for when $n = 2k$ and $n = 2k+1$ (or equivalent) (see first M1 for guidance)

A1*: Requires

- correct quadratic expression for n^2+5n for both odds and evens
- shows or gives a reason for each why the expressions are even (see first A1 for guidance)
- makes a concluding overall statement. "Hence n^2+5n is even for all $n(\in \mathbb{N})$" (or equivalent).

Note that if they have stated for each separate case that the expression is even then allow minimal statements of "hence proven", "statement proved", "QED", tick

Do not isw this mark if they simplify their quadratic incorrectly.

	n^2+5n
$2k-3$	$4k^2-2k-6$
$2k-2$	$4k^2+2k-6$
$2k-1$	$4k^2+6k-4$
$2k$	$4k^2+10k$
$2k+1$	$4k^2+14k+6$
$2k+2$	$4k^2+18k+14$
$2k+3$	$4k^2+22k+24$

Alternative methods:

Algebraic with logic example

e.g. $n^2+5n=n(n+5)$

When n is odd then $n+5$ is even so odd x even is even

When n is even then $n+5$ is odd so even x odd is even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

Further Maths method (proof by induction) – you may see these but please send to review for TLs or above to mark

M1: Assumes true for $n=k$, substitutes $n=k+1$ into n^2+5n, multiplies out the brackets and attempts to simplify to a quadratic expression (which may be unsimplified)
e.g. k^2+7k+6 Condone arithmetical slips

A1: $\left(\mathrm{f}(k+1)=3k^2+3k+1+6(k+1)=\right)$ $k^2+5k+2k+6=\mathrm{f}(k)+2(k+3)$
which is even + even = even

dM1: Attempts to substitute $n=1 \Rightarrow 1^2+5\times1=6$ (which is true) (Condone arithmetical slips evaluating)

A1*: Explains that

- it is true when $n=1$
- if it is true for $n=k$ then it is true for $n=k+1$
- therefore it is true for all $n(\in\mathbb{N})$

Solutions via just logic (no algebraic manipulation) scores 0 marks.
e.g.
If n is odd, then n^2+5n is $odd^2+odd\times odd=odd+odd=even$
If n is even, then n^2+5n is $even^2+odd\times even=even+even=even$

(**Q14 8MA0/01, June 2024**)

Online Maths Teaching

Q14.

Question	Scheme	Marks	AOs
(a) Way 1	Since x and y are positive, their square roots are real and so $(\sqrt{x}-\sqrt{y})^2 \geq 0$ giving $x-2\sqrt{x}\sqrt{y}+y \geq 0$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x+y$ provided x and y are positive and so $\sqrt{xy} \leq \frac{x+y}{2}$ *	A1*	2.2a
		(2)	
Way 2 Longer method	Since $(x-y)^2 \geq 0$ for real values of x and y, $x^2-2xy+y^2 \geq 0$ and so $4xy \leq x^2+2xy+y^2$ i.e. $4xy \leq (x+y)^2$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x+y$ provided x and y are positive and so $\sqrt{xy} \leq \frac{x+y}{2}$ *	A1*	2.2a
		(2)	
(b)	Let $x = -3$ and $y = -5$ then LHS = $\sqrt{15}$ and RHS= -4 so as $\sqrt{15} > -4$ result does not apply	B1	2.4
		(1)	
		(3 marks)	

Notes

(a) M1 : Need two stages of the three stage argument involving the three stages, squaring, square rooting terms and rearranging.

A1*: Need all three stages making the correct deduction to achieve the printed result.

(b) B1 : Chooses two negative values and substitutes, then states conclusion

(**Q11 8MA0/01, Specimen papers**)

Q15.

Question	Scheme	Marks	AOs
	Realises that $k = 0$ will give no real roots as equation becomes $3 = 0$ (proof by contradiction)	B1	3.1a
	(For $k \neq 0$) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$	M1	2.4
	$4k(4k-3) < 0$ with attempt at solution	M1	1.1b
	So $0 < k < \frac{3}{4}$, which together with $k = 0$ gives $0 \leqslant k < \frac{3}{4}$ *	A1*	2.1
			(4 marks)

Notes

B1 : Explains why $k = 0$ gives no real roots

M1 : Considers discriminant to give quadratic inequality – does not need the $k \neq 0$ for this mark

M1 : Attempts solution of quadratic inequality

A1* : Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)

(**Q10 8MA0/01, Specimen papers**)

Q16.

Question	Scheme	Marks	AOs
	$n\left(n^2+5\right)$		
	Attempts even or odd numbers Sets $n=2k$ **or** $n=2k\pm1$ oe and attempts $n\left(n^2+5\right)$	M1	3.1a
	Achieves $2k\left(4k^2+5\right)$ (for $n=2k$) and states "even" **Or** achieves $(2k+1)\left(4k^2+4k+6\right)=2(2k+1)\left(2k^2+2k+3\right)$ (for $n=2k+1$) and states "even" Or e.g. achieves $(2k-1)\left(4k^2-4k+6\right)=2(2k-1)\left(2k^2-2k+3\right)$ (for $n=2k-1$) and states "even"	A1	2.2a
	Attempts even **and** odd numbers Sets $n=2k$ **and** $n=2k\pm1$ oe and attempts $n\left(n^2+5\right)$	dM1	2.1
	Achieves $2k\left(4k^2+5\right)$ (for $n=2k$) and states "even" **and** achieves $(2k\pm1)\left(4k^2\pm4k+6\right)=2(2k\pm1)\left(2k^2\pm2k+3\right)$ (for $n=2k\pm1$) and states "even" Correct work and states even for both WITH a final conclusion showing that true for all $n(\in\mathbb{N})$ or e.g. true for all even and odd numbers.	A1	2.4
		(4)	
			(4 marks)
Notes:			

M1: For the key step attempting to find $n\left(n^2+5\right)$ when $n=2k$ **or** $n=2k\pm1$ or equivalent

representation of odd or even e.g. $n=2k+2$ **or** $n=2k\pm7$

Condone the use of e.g. $n=2n$ and $n=2n\pm1$

A1: Achieves $2k\left(4k^2+5\right)$ or e.g. $2\left(4k^3+5k\right)$ and deduces that this is even at the appropriate time.

Or achieves $(2k\pm1)\left(4k^2\pm4k+6\right)=2(2k\pm1)\left(2k^2\pm2k+3\right)$ oe e.g. $2\left(4k^3+6k^2+8k+3\right)$ and deduces that this is even.

Note that if the bracket is expanded to e.g. $8k^3+12k^2+16k+6$ then stating "even" is insufficient – they would need to say e.g. even + even + even + even = even or equivalent

Note it is also acceptable to use a divisibility argument e.g. $\frac{8k^3+10k}{2}=4k^3+5k$ so $8k^3+10k$ must be even.

There should be no errors in the algebra but allow e.g. invisible brackets if they are "recovered".

dM1: Attempts $n\left(n^2+5\right)$ when $n=2k$ **and** $n=2k\pm1$ or equivalent representation of odd or even e.g. $n=2k+2$ **and** $n=2k\pm7$

A1: Correct work and states even for both WITH a final conclusion e.g. so true for all $n(\in\mathbb{N})$.

There should be no errors in the algebra but allow e.g. invisible brackets if they are "recovered".

A "solution" via just logic.

E.g.

If n is odd, then $n\left(n^2+5\right)$ is odd×(odd + odd) = odd × even = even

If n is even, then $n\left(n^2+5\right)$ is even×(even + odd) = even × odd = even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

OR

E.g. $n\left(n^2+5\right)=n^3+5n$

If n is odd, then n^3 is odd and $5n$ is odd, so n^3+5n is odd + odd = even

If n is even, then n^3 is even and $5n$ is even, so n^3+5n is even + even = even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

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A solution via contradiction.

M1 A1: There exists a number n such that $n\left(n^2+5\right)$ is odd, and so deduces that both n and n^2+5 are odd. Note that M1A0 is not possible via this method.

dM1: Sets $n^2+5=2k+1$ (for some integer k) $\Rightarrow n^2=2k-4=2(k-2)$ which is even
Must use algebra here for this approach and not a "logic" argument.

A1: States that "this is a contradiction as if n^2 is even, then n is even" and then concludes so "$n\left(n^2+5\right)$ is even for all n."

Attempts at proof by induction should be sent to review

(**Q11 9MA0/02, June 2022**)

Q17.

Question	Scheme	Marks	AOs
(i)	For setting up the contradiction: There exists integers p and q such that pq is even and both p and q are odd	B1	2.5
	For example, sets $p = 2m+1$ and $q = 2n+1$ and then attempts $pq = (2m+1)(2n+1) = ...$	M1	1.1b
	Obtains $pq = (2m+1)(2n+1) = 4mn + 2m + 2n + 1$ $= 2(2mn + m + n) + 1$ States that this is odd, giving a contradiction so " if pq is even, then at least one of p and q is even" *	A1*	2.1
		(3)	
(ii)			
	$(x+y)^2 < 9x^2 + y^2 \Rightarrow 2xy < 8x^2$	M1	2.2a
	States that as $x < 0 \Rightarrow 2y > 8x$ $\Rightarrow y > 4x$ *	A1*	2.1
		(2)	
			(5 marks)
Notes:			

(i)

B1: For using the "correct"/ allowable language in setting up the contradiction.
Expect to see a minimum of

- "assume" or "let" or "there is " or other similar words
- "pq is even" and "p and q are (both) odd"

M1: Uses a correct algebraic form for p and q and attempting to multiply.
Allow any correct form so $p = 2n+1$ and $q = 2m+3$ would be fine to use
Different variables must be used for p and q, so $p = 2n+1$ and $q = 2n-1$ would be M0

A1*: Full argument .
This requires (1) a correct calculation for their pq
(2) a correct reason and conclusion that it is odd
E.g. $(2m+1)(2n+1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1 = \text{odd}$
E.g. $(2m-1)(2n+1) = 4mn + 2m - 2n - 1 = \text{even} + \text{even} - \text{even} - 1 = \text{odd}$
and (3) a minimal statement implying that they have proven what was required which could be QED, proven etc

Note that B0 M1 A1 is possible

(ii)

M1: For multiplying out and cancelling terms before proceeding to a correct intermediate line such as $2xy < 8x^2$ o.e. such as $2x(4x-y) > 0$

A1*: Full and rigorous proof with reason shown as to why inequality reverses. The point at which it reverses must be correct and a correct reason given

See scheme

Alt: $2xy < 8x^2 \Rightarrow xy - 4x^2 < 0 \Rightarrow x(y-4x) < 0$

as $x < 0$, $(y-4x) > 0 \Rightarrow y > 4x$ scores M1 A1

So, the following should be scored M1 A0 as line 3 is incorrect

$2xy - 8x^2 < 0$
$\Rightarrow 2xy < 8x^2$
$\Rightarrow y < 4x$
$\Rightarrow y > 4x$ as $x < 0$

There should be no incorrect lines in their proof

(Q07 9MA0/01, June 2022)

Q18.

Question	Scheme	Marks	AOs
(i)	$x^2-8x+17=(x-4)^2-16+17$	M1	3.1a
	$=(x-4)^2+1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2\geqslant 0 \Rightarrow (x-4)^2+1\geqslant 1$ hence $x^2-8x+17>0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x=5$ $(5+3)^2=64$ whereas $(5)^2=25$ True When $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$ Not true	A1	2.4
		(2)	
			(5 marks)

Notes

(i) Method One: Completing the Square

M1: For an attempt to complete the square. Accept $(x-4)^2$ ...

A1: For $(x-4)^2+1$ with either $(x-4)^2 \geqslant 0, (x-4)^2+1 \geqslant 1$ or min at (4,1). Accept the inequality statements in words. Condone $(x-4)^2 > 0$ or a squared number is always positive for this mark.

A1: A fully written out solution, with correct statements and no incorrect statements. There must be a valid reason and a conclusion

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$x^2-8x+17$

$=(x-4)^2+1 \geqslant 1$ as $(x-4)^2 \geqslant 0$

Hence $(x-4)^2+1>0$

scores M1 A1 A1

$x^2-8x+17>0$

$(x-4)^2+1>0$

This is true because $(x-4)^2 \geqslant 0$ and when you add 1 it is going to be positive

scores M1 A1 A1

$x^2-8x+17>0$

$(x-4)^2+1>0$

which is true because a squared number is positive

scores M1 A1 A0

incorrect and incomplete

$x^2-8x+17=(x-4)^2+1$

Minimum is (4,1) so $x^2-8x+17>0$

scores M1 A1 A0

correct but not explained

$x^2-8x+17=(x-4)^2+1$

Minimum is (4,1) so as $1>0 \Rightarrow x^2-8x+17>0$

scores M1 A1 A1

correct and explained

$x^2-8x+17>0$

$(x-4)^2+1>0$

scores M1 A0 (no explanation) A0

Method Two: Use of a discriminant

M1: Attempts to find the discriminant b^2-4ac with a correct a, b and c which may be within a quadratic formula. You may condone missing brackets.

A1: Correct value of $b^2-4ac=-4$ **and** states or shows curve is U shaped (or intercept is (0,17)) or equivalent such as +ve x^2 etc

A1: Explains that as $b^2-4ac<0$, there are no roots, and curve is U shaped then $x^2-8x+17>0$

Method Three: Differentiation

M1: Attempting to differentiate and finding the turning point. This would involve attempting to find $\frac{dy}{dx}$, then setting it equal to 0 and solving to find the x value and the y value.

A1: For differentiating $\frac{dy}{dx}=2x-8 \Rightarrow (4,1)$ is the **turning point**

A1: Shows that (4,1) is the minimum point (second derivative or U shaped), hence $x^2-8x+17>0$

Method 4: Sketch graph using calculator

M1: Attempting to sketch $y = x^2 - 8x + 17$, U shape with minimum in quadrant one

A1: As above with minimum at (4,1) marked

A1: Required to state that quadratics only have one turning point and as "1" is above the x-axis then $x^2 - 8x + 17 > 0$

(ii)

Numerical approach

Do not allow any marks if the student just mentions "positive" and "negative" numbers. Specific examples should be seen calculated if a numerical approach is chosen.

M1: Attempts a value (where it is not true) and shows/implies that it is not true for that value.

For example, for -4 : $(-4+3)^2 > (-4)^2$ and indicates not true (states not true, ×)

or writing $(-4+3)^2 < (-4)^2$ is sufficient to imply that it is not true

A1: Shows/implies that it can be true for a value **AND** states sometimes true.

For example for +4 : $(4+3)^2 > 4^2$ and indicates true ✓

or writing $(4+3)^2 > 4^2$ is sufficient to imply this is true following $(-4+3)^2 < (-4)^2$

condone incorrect statements following the above such as 'it is only true for positive numbers' as long as they state "sometimes true" and show both cases.

Algebraic approach

M1: Sets the problem up algebraically Eg. $(x+3)^2 > x^2 \Rightarrow x > k$ Any inequality is fine. You may condone one error for the method mark. Accept $(x+3)^2 > x^2 \Rightarrow 6x + 9 > 0$ oe

A1: States sometimes true and states/implies true for $x > -\frac{3}{2}$ or states/implies not true for $x \leq -\frac{3}{2}$ In both cases you should expect to see the statement "sometimes true" to score the A1

(**Q02 8MA0/01, June 2018**)