

Mark Scheme

Q1.

Question	Scheme								Marks	AOs
(a)		A	B	C	D	E	F	G	B2, 1, 0	1.1b 1.1b
	A	-	9	∞	25	35	∞	11		
	B	∞	-	40	8	∞	35	∞		
	C	∞	40	-	17	∞	∞	47		
	D	∞	8	17	-	11	31	∞		
	E	35	∞	∞	11	-	14	∞		
	F	∞	35	∞	31	14	-	12		
	G	11	∞	47	∞	24	12	-		
								(2)		
(b)	Fourth iteration:								M1	2.1
		A	B	C	D	E	F	G		
	A	-	9	34	17	28	44	11		
	B	∞	-	25	8	19	35	72		
	C	∞	25	-	17	28	48	47		
	D	∞	8	17	-	11	31	64		
	E	35	19	28	11	-	14	46		
	F	∞	35	48	31	14	-	12		
	G	11	20	45	28	24	12	-		
	Fifth iteration:								M1	2.1
		A	B	C	D	E	F	G		
	A	-	9	34	17	28	42	11		
	B	54	-	25	8	19	33	65		
	C	63	25	-	17	28	42	47		
D	46	8	17	-	11	25	57			
E	35	19	28	11	-	14	46			
F	49	33	42	25	14	-	12			
G	11	20	45	28	24	12	-			
								(5)		
(c)(i)	NNA starting at B: B – D – E – F – G – A – C – B								B1	2.2a
(ii)	Time taken: $8 + 11 + 14 + 12 + 11 + 34 + 25 = 115$ (minutes)								B1	1.1b
(iii)	Actual villages visited: BDEFGABDCDB								B1	2.2a
								(3)		
(10 marks)										

Notes:

a1B1: At least three rows and three columns correct (condone at most 2 empty cells but accept consistent blanks for the leading diagonal)

a2B1: CAO

b1M1: No change in the fourth row and fourth column with at least two values reduced correctly (condone at most 2 empty cells but accept consistent blanks for the leading diagonal)

b1A1: CAO for the fourth iteration (changes in bold and blue shading)

b2M1: No change in the fifth row and fifth column with at least two values reduced correctly (follow through from their previous iteration) (condone at most 2 empty cells but accept consistent blanks for the leading diagonal)

b2A1: At least eight values reduced correctly (from the values shown in bold and orange shading)

b3A1: CSO for the fourth and fifth iterations

(if correct changes for the fifth iteration are shown in bold and orange shading)

ci1B1: CAO (Nearest neighbour cycle starting at B – must include return to B) (condone the use of arcs BD DE EF FG GA AC CB – must be written in this manner and not AG or BC)

cii2B1: Correct time

ciii3B1: Correct route – must be stated as nodes

(Q03 9FM0/03D, June 2025)

(e)	NNA starting at D: D – E – H – J – C – B – G – F – A – D $24 + 30 + 33 + 45 + 28 + 45 + 47 + 40 + 26 = 318$ (miles)	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(f)	The best upper bound is the one found in (e) as 318 is less than both 352 and 482	dB1ft	2.2a
		(1)	
(g)	$(241 - 26) + 26 + 40 = 281$ (miles)	M1 A1	3.1b 2.2a
		(2)	
(h)	The best lower bound is the one found in (g) as 281 is greater than 274	dB1ft	2.2a
		(1)	
(13 marks)			

(a)

M1: First three arcs correctly chosen in order {DE, AD, EH, ...} or first four nodes correctly chosen in order {D, E, A, H, ...}. If any rejections seen at any point then **M1** (max) only. Order of nodes may be seen at the top of the matrix {3, -, -, 1, 2, -, -, 4, -} so please check the top of the matrix carefully. If they start at any other vertex **M1** only for the first three arcs in order (Starting at A AD DE EH).

A1: First six arcs correctly chosen in order {DE, AD, EH, EJ, CD, BC, ...} or all nine nodes correctly chosen in order {D, E, A, H, J, C, B, G, F}. Order of nodes may be seen at the top of the matrix so for the first two marks accept {3, 7, 6, 1, 2, 9, 8, 4, 5} (do not condone any missing numbers e.g. the number 9 must be above F)

A1: CSO – all arcs correctly stated and chosen in the correct order. Candidates must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)

(b)

B1: CAO (241) – no units required (in this or any of the subsequent parts)

(c)

B1: Correct MST (If multiple attempts not rejected check all and award mark if any one is correct)

(d)

B1ft: Follow through double their answer to (b)

(e)

M1: Nearest neighbour route starting at D – must have at least D – E – H – J – C – B – ... allow if stated in terms of arcs (DE EH HJ JC CB) but not for just numbering across the matrix

A1: CAO route (must return to D and can be stated as arcs DE EH HJ JC CB BG GF FA AD)

A1: CAO length (318)

(f)

dB1ft: Follow through their value from (e) (must include reason e.g. 318 is the lowest of the values found, but they do not need to mention the other values) **Must score at least M1 in (e)**

(g)

M1: (weight of their MST from (b) or 241 only) – 26 + 26(AD) + 40(AF) (oe so may not see the – 26 + 26). A correct answer of 281 can imply this (and the next) mark. Alternatively may delete A and find RMST (215) and add AD (26) and AF (40)

A1: 281

(h)

dB1ft: Follow through their “281” with 274 and makes conclusion (e.g. 281 is the bigger value, but they do not need to mention 274). **Must be a smaller value than their answer to (f) and must score at least M1 in (g)**

(Q02 9FM0/03D, June 2024)

Q3.

Qu	Scheme	Marks	AOs																																																																									
(a)	<p style="text-align: center;">Distance table</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> </tr> </thead> <tbody> <tr> <th>A</th> <td>-</td> <td>15</td> <td>7</td> <td>18</td> <td>3</td> </tr> <tr> <th>B</th> <td>15</td> <td>-</td> <td>∞</td> <td>∞</td> <td>∞</td> </tr> <tr> <th>C</th> <td>7</td> <td>5</td> <td>-</td> <td>4</td> <td>9</td> </tr> <tr> <th>D</th> <td>18</td> <td>∞</td> <td>4</td> <td>-</td> <td>3</td> </tr> <tr> <th>E</th> <td>∞</td> <td>∞</td> <td>9</td> <td>3</td> <td>-</td> </tr> </tbody> </table>		A	B	C	D	E	A	-	15	7	18	3	B	15	-	∞	∞	∞	C	7	5	-	4	9	D	18	∞	4	-	3	E	∞	∞	9	3	-	<p style="text-align: center;">Route table</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> </tr> </thead> <tbody> <tr> <th>A</th> <td>A</td> <td>B</td> <td>C</td> <td>D</td> <td>E</td> </tr> <tr> <th>B</th> <td>A</td> <td>B</td> <td>C</td> <td>D</td> <td>E</td> </tr> <tr> <th>C</th> <td>A</td> <td>B</td> <td>C</td> <td>D</td> <td>E</td> </tr> <tr> <th>D</th> <td>A</td> <td>B</td> <td>C</td> <td>D</td> <td>E</td> </tr> <tr> <th>E</th> <td>A</td> <td>B</td> <td>C</td> <td>D</td> <td>E</td> </tr> </tbody> </table>		A	B	C	D	E	A	A	B	C	D	E	B	A	B	C	D	E	C	A	B	C	D	E	D	A	B	C	D	E	E	A	B	C	D	E	B1	1.1b
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(c)(i)	Start at E(5 th row) and read across to the A (1 st column), there is a D there so the route from E to A is via D	B1	2.4
	Now consider both E to D and D to A – for E reading across to the D (4 th column), there is a D indicating that the shortest path from E to D is ED. For D reading across to the A (1 st column), there is a C indicating that the shortest path from D to A is via C	dB1	2.4
(ii)	EDCA	B1	2.2a
		(3)	
(d)(i)	NNA: D – E – C – B – A – D	B1	1.1b
(ii)	$3 + 7 + 5 + 15 + 6 = 36$ miles	B1	1.1b
(iii)	D – E – D – C – B – A – E – D	B1	3.2a
(iv)	e.g. the cycle A – E – D – C – B – A has a length of 30 miles < 36 miles so Mabintou's route is not optimal	B1	2.4
		(4)	
(14 marks)			

Notes for Question

IN THE DISTANCE AND ROUTE TABLE FOR PARTS (a) and (b) IGNORE WHATEVER IS WRITTEN IN THE LEAD DIAGONAL (TOP LEFT TO BOTTOM RIGHT)

(a) B1: Correct distance table (condone dashes, crosses, etc. for infinity but do not condone a 'large' number in these cells)

B1: Correct route table

(b) M1: No change in the first row and first column of both tables with at least two values in the distance table correctly reduced and two letters in the route table correctly changed – all cells complete

A1: CAO (condone dashes, etc. in cells EA and EB)

A1ft: No change from candidate's first iteration to second iteration for either table or ft from candidate's first iteration

M1: No change in the third row and third column of both tables with at least two values in the distance table correctly reduced from their second iteration and two values in the route table correctly changed

A1: CAO for third iteration (**note that the entry in row B column D for the route table could be an A**)

(c) B1: Row E column A is D so the route is E to A via D (or implies that the order of the nodes in the route is EDA) **or** D implied from general argument **or** Row E column A is D therefore the route begins ED (in all cases must clearly imply **row E** and **column A**)

dB1: Row D column A is C therefore the route goes via C (before A) **or** complete general argument that allows the route from D to A to be found **or** allow those who say that row D column A is C so the route is EDC and then row C column A is A

B1: CAO (EDCA)

(d)(i) B1: CAO (D – E – C – B – A – D)

(ii) B1: CAO (36 – no units required)

(iii) B1: CAO (D – E – D – C – B – A – E – D) **or** mentions that the cycle would visit E twice and D three times (or visit D before the end of the cycle – if D visited once stated and it is not clear that this isn't the start or finish then B0) **or** mention of E to C via D and A to D via E

(iv) B1: A correct cycle stated (e.g. a cyclic permutation of A – E – D – C – B – A) with corresponding correct length – **dependent on second B mark in this part** (so must have had 36 in **(d)(ii)**)

(Q03 9FM0/03D, June 2019)

Q4.

Question	Scheme	Marks	AOs
(a)	e.g. in the practical problem each vertex must be visited at least once. In the classical problem each vertex must be visited just once	B2,1,0	2.4 2.4
		(2)	
(b)	Prim's algorithm on reduced network starting at A: AD, AF, AE, CE, CG	M1	1.1b
	Lower bound = $107 + 17 + 25 = 149$ (km)	M1 A1	1.1b 1.1b
		(3)	
(c)	NNA from A: $A - D - F - E - C - G - B - A = 126 + x$ NNA from C: $C - E - A - D - F - B - G - C = 139 + x$	M1 A1 A1	1.1b 1.1b 1.1b
	$(126 + x) + (139 + x) = 331 \Rightarrow x = 33$	A1	1.1b
		(4)	
(d)	$149 < \text{optimal} \leq 159$	M1 A1	2.2b 1.1b
		(2)	
			(11 marks)

Notes:	
(a)	
B1:	Understands the difference is connected to the number of times each vertex may be visited (but maybe incorrectly attributed). Must be an attempt at a difference (so must refer to both the classical and practical problems explicitly). Technical language (vertex/node) must be correct. Need not imply each/every/all (oe) vertices for this first mark
B1:	Correctly reasons which is classical and which is practical and correctly states the difference. Must imply that each/every/all (oe) vertices are visited, so for example, 'the practical problem visits a vertex at least once while the classical visits a vertex only once' is B1B0 (note that B0B1 is not possible in (a))
(b)	
M1:	Correctly applying Prim's algorithm from node A for the first four arcs (or five nodes)
M1:	Candidates weight of their RMST + 17 + 25 (the two smallest arcs incident to B)
A1:	CAO (condone lack of units)
(c)	
M1:	Either one route, must return to A
A1:	One correct route, must return to A and corresponding length correct (do not is in part (c) if correct lengths seen but are then doubled)
A1:	Both routes correct and their corresponding lengths correct
A1:	CAO for x
Notes: (continued)	
(d)	
M1:	Their numbers correctly used, accept any inequalities or any indication of interval from their 149 to their 159 (so 149 – 159 can score this mark). This mark is dependent on two routes seen in (c), however, neither of the two totals need to be correct. Please note that $UB > LB$ for this mark
A1:	CAO (no follow through on their values) including correct inequalities or equivalent set notation (but condone $149 \leq \text{optimal} \leq 159$)

(Q03 9FM0/3D-4D, Specimen papers)

Q5.

Question Number	Scheme	Marks
(a)(i)	Prim's starting from A: AE, AC, BE, AD; EF $2 \times 391 = 782$	M1 A1 B1
(ii)	Nearest neighbour: A – E – B – C – D – F – A $69 \ 77 \ 94 \ 97 \ 125 \ 97 = 559$	M1 A1 (5)
(b)	$500 \leq \text{length} \leq 559$ (accept $500 < \text{length} \leq 559$)	B2, 1, 0 (2)
		7 marks

Notes for Question

a1M1: First four arcs (or first five nodes: A, E, C, B, D or equivalent numbers across the top of the table {1, 4, 3, 5, 2, -}) selected correctly. Award M1 only for a correct tree with no working or for a correct tree starting at a different node

a1A1: CAO (order of arcs correct or all six nodes correct: A, E, C, B, D, F – but not just the numbers across the top of the table)

a1B1: CAO (782) – must follow from the correct MST (so dependent on at least the M mark in (a)(i)) – do not isw if attempt at short cuts reduces this value

a2M1: Nearest neighbour A – E – B – C – D – F – (condone lack of return to start) **or** correct route length of 559. Accept AE, EB, BC, CD, DF but do not accept weights only

a2A1: CAO both route (either in terms of vertices or arcs but not weights) **and** length correct

b1B1: Any indication of an interval from 500 to their 559 (their $559 > 500$ and is the smallest value from either the MST method or NN method – must have stated two values in (a) but ignore how these values were derived)

b2B1: $500 \leq \text{length} \leq 559$ **or** $500 < \text{length} \leq 559$ (no ft on this mark) – accept set notation e.g. [500,559] or (500, 559]

(Q07 6690/01, June 2017)

Q6.

Question Number	Scheme	Marks
(a)	e.g. in the practical problem each vertex must be visited at least once. In the classical problem each vertex must be visited just once	B2, 1, 0 (2)
(b)	A – D – C – F – B – E – G – A 12+16+19+25+14+41+22 = 149	M1
	A – D – C – F – G – E – B – A 12+16+19+25+41+14+31 = 158	A1 A1 (3)
(c)	RMST weight = 86 (miles) 86 + 12 + 15 = 113 (miles)	B1 M1 A1 (3)
(d)	113 ≤ optimal distance ≤ 149	B2, 1, 0 (2) 10 marks
Notes for Question		
<p>a1B1: Understands the difference is connected to the number of times each vertex may be visited (but maybe incorrectly attributed). Must be an attempt at a difference (so must refer to both the classical and practical problems explicitly). Technical language (vertex/node) must be correct. Need not imply each/every/all (oe) vertices for this first mark</p> <p>a2B1: Correctly identifies which is classical and which is practical and correctly states the difference. Must imply that each/every/all (oe) vertices are visited, so for example, 'the practical problem visits a vertex at least once while the classical visits a vertex only once' is B1B0 (note that B0B1 is not possible in (a))</p> <p>b1M1: Either one correct route, must return to A, or one correct length stated (do not isw in part (b) if correct lengths seen but are then doubled)</p> <p>b1A1: One correct route, must return to A and corresponding length correct</p> <p>b2A1: Both routes correct and their corresponding lengths correct</p> <p>c1B1: CAO for RMST weight (either 86 or 20 + 16 + 14 + 19 + 17) – maybe implied by later working</p> <p>c1M1: Adding 12 + 15 (the two least weighted arcs) to their RMST length (the length of their RMST must be in the interval $66 \leq \text{RMST} \leq 106$) – this mark maybe implied by the correct value for the lower bound</p> <p>c1A1: CAO - if 113 seen without working then award all 3 marks in (c)</p> <p>d1B1: Their numbers correctly used, accept any inequalities or any indication of interval from their 113 to their 149 (so 113 – 149 can score this mark). This mark is dependent on two totals seen in (b), however, neither of the two totals need to be correct. Please note that $UB > LB$ for this mark</p> <p>d2B1: CAO (no follow through on their values) including correct inequalities or equivalent set notation (but condone $113 < \text{optimal distance} \leq 149$)</p>		

(Q08 6690/01, June 2016)

Q7.

Question Number	Scheme	Marks
(a)	Prim: AF, EF, BE, BC, CD, DG	M1 A1 (2)
(b)	$2 \times 136 = 272$ (km)	B1 (1)
(c)	A F E B C D G A 21 20 19 27 24 25 30 = 166 (km)	B1 B1 (2)
(d)	Starting at F route length is $153 + x$ With $x > 21$, $153 + x$ is greater than 166 so the better upper bound is the one starting at A	B1 DB1 (2)
(e)	Length of RMST = 115 $115 + 21 + x = 159 \therefore x = 23$ (km)	B1 M1 A1 (3)
(f)	$159 \leq \text{optimal} \leq 166$ [accept $159 < \text{optimal} \leq 166$]	B2,1,0 (2) 12 marks

Notes for Question

a1M1: Must be using Prim's algorithm not NNA – if any arc creates a cycle then M0. First four arcs (or all 7 nodes / or numbers across the top of the matrix) selected correctly. Award M1 only for a correct tree with no working. Award M1 only for the first four arcs (oe) selected correctly if starting at a different node than A

a1A1: CAO (order of arc selection clear)

b1B1: CAO (272)

c1B1: CAO – must be either in terms of nodes or arcs (not weights)

c2B1: CAO (166)

d1B1: Either $153 + x$ or states a value in the interval $174 < \text{value} < 180$ or considers one of the intervals $174 < \text{value} < 180$ or $175 \leq \text{value} \leq 179$

d2DB1: Correct argument that A gives the better upper bound. Must be considering either $x > 21$ or $x \geq 22$ with 153 (so expect to see as a minimum the mention of > 174 or ≥ 175) – must be clear that the upper bound starting at A is the better upper bound. This mark is dependent on the previous B mark in (d)

e1B1: CAO (length of RMST) – the length (115 or $19 + 20 + 27 + 24 + 25$) must be either explicitly stated or seen in their working (not just implied by their working)

e1M1: Adding the correct two least values (21 and x) to their RMST length (their RMST may be incorrect but must contain only 5 arcs) and equating to 159. Accept, for example, $136 + x = 159$ or $136 + 23 = 159$ or $115 + 21 + 23 = 159$ or equivalent calculations using the length of their RMST

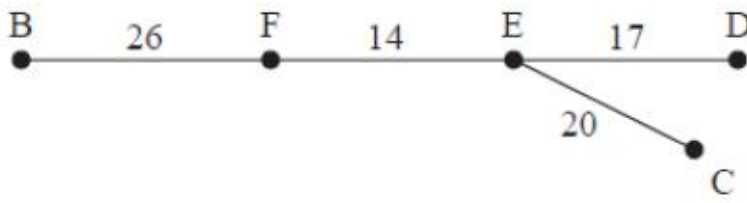
e1A1: CAO (must be clear that $(x =) 23$ not just embedded in a calculation)

f1B1: Any indication of an interval containing 159 (as a lower bound) and their stated better upper bound from (d)

f2B1: CAO either $159 \leq \text{optimal (oe)} \leq 166$ or $159 < \text{optimal (oe)} \leq 166$

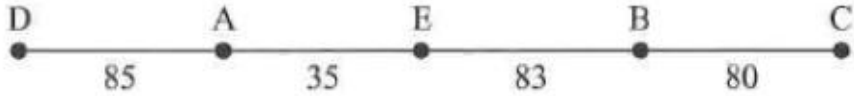
(Q07 6690/01, June 2015)

Q8.

Question Number	Scheme	Marks
(a)	In the practical problem each vertex must be visited at least once. In the classical problem each vertex must be visited just once.	B2, 1, 0 (2)
(b)	A D E F B C A $15 + 17 + 14 + 26 + 50 + 48 = 170$	M1 A1 A1 (3)
(c)	 <p>RMST weight = $26 + 14 + 17 + 20 = 77$ (km) Lower bound = $77 + 15 + 30 = 122$ (km)</p>	M1 A1 A1 (3)
(d)	$122 \leq \text{length} \leq 170$	B2,1,0 (2)
Notes for Question		
<p>a1B1: Understands the difference is connected to the number of times each vertex may be visited. a2DB1: Correctly identifies which is classical and which is practical and correctly states the difference. b1M1: Nearest neighbour A – D – E – F – B – C – or accept 145623 across top of table (condone lack of return to start). b1A1: Route correctly stated, must return to A, accept link back to A. b2A1: Length correctly stated. Do not ISW if candidates then go on to double the route length. c1M1: Finding RST (maybe implicit) and using the correct two least lengths. Their RST must have only four arcs of which none are incident to A. c1A1: RMST correct or list of arcs or 77 or $26 + 14 + 17 + 20$ seen. c2A1: CAO 122 d1B1ft: Their correct numbers correctly used (their upper bound must be a cycle and their lower bound must have scored M1 in (c)), accept any inequalities or any indication of interval from their 122 to their 170. d2B2: CAO including correct inequalities (but condone $122 < \text{length} \leq 170$).</p>		

(Q08 6690/01/R, June 2014)

Q9.

Question Number	Scheme	Marks
(a)	A E F B C D A and A E F D B C A $35+75+88+80+108+85 = 471$ $35+75+88+100+80+130 = 508$	M1 A1 A1 A1 (4)
(b)	 RMST weight = $85 + 35 + 83 + 80 = 283$ (seconds) Lower bound = $283 + 75 + 88 = 446$ (seconds)	M1 A1 A1 (3)
(c)	$446 \leq \text{time} \leq 471$ [accept $446 < \text{time} \leq 471$]	B3,2,1,0 (3)
Notes for Question		
a1M1: Nearest neighbour either A – E – F – B – C – D – or A – E – F – D – B – C –, condone lack of return to start. Accept 145623 or 156423 across top of table (numbers must be from NN not Prim). a1A1: One route correctly stated, must return to A, accept link back to A. a2A1: One route length correctly stated. Do not ISW if candidates then go on to double the route length in (a). a3A1: Second route and its length correctly stated. Do not ISW if candidates then go on to double the route length in (a). b1M1: Finding RST (maybe implicit) and using the correct two least lengths. Their RST must have only four arcs none of which are incident to F. b1A1: RMST correct or list of arcs or 283 or $85 + 35 + 83 + 80$ seen. b2A1: CAO 446 c1B1ft: their 471 (must be a cycle) as an upper bound – allow recovery in this part. c2B1ft: any indication of interval from their 446 (must come from six arcs) to their 471. c3B1: $446 \leq \text{time} \leq 471$ or $446 < \text{time} \leq 471$		

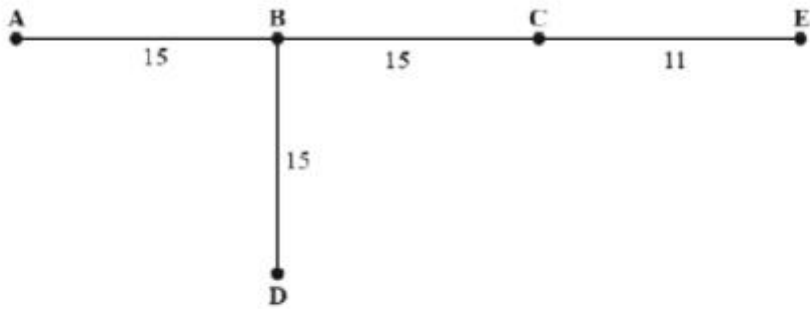
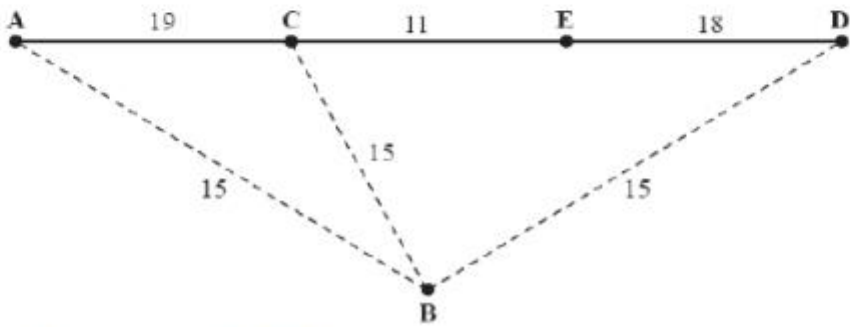
(Q08 6690/01, June 2014)

Q10.

Question Number	Scheme	Marks
(a)	E.g. If use CD as shortcut get 807 or if use CF + AD get 793	M1 A1 (2)
(b)	A F E D B C A 82 113 98 130 110 217 = 750	B1 B1 (2)
(c)	length of RMST = 439 439 + 82 + 113 = 634	B1 M1 A1 (3)
(d)	634 < optimal ≤ 750	B1ft (1)
8 marks		
Notes for Question		
<p>a1M1: Their plausible shortcut leading to a value < 810 and a length below 810 stated. a1A1: CAO – shortcut and length must be consistent.</p> <p>(Examples shortcuts: CD = 807, CF + AD = 793, CF + BD = 664, AD + EF + FC = 715, DF + FC = 785 etc.)</p> <p>b1B1: CAO b2B1: CAO c1B1: CAO c1M1: Adding two least weighted arcs to their RMST length c1A1: CAO d1B1: An interval that incorporates their lower bound from (c) and their best upper bound from either (a) or (b)</p>		

(Q07 6690/01/R, June 2013)

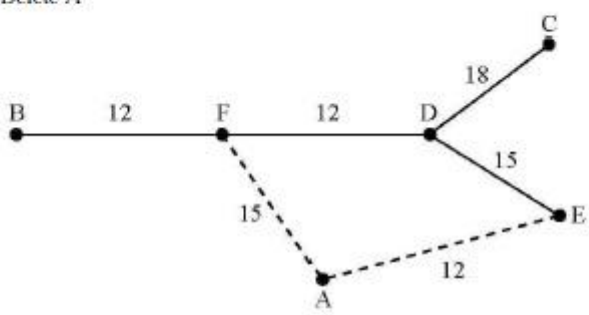
Q11.

Question Number	Scheme	Marks
(a)	 <p>e.g. starting from A: AB, BD, BC, CE or AB, BC, CE, BD</p>	M1A1 (2)
(b)	$2 \times 56 = 112$	B1 (1)
(c)	$A \ B \ C \ E \ D \ A$ and $A \ B \ D \ E \ C \ A$ $15 \ 15 \ 11 \ 18 \ 25 = 84$ $15 \ 15 \ 18 \ 11 \ 19 = 78$	M1 A1 A1 (3)
(d)	78 is the better upper bound	B1ft (1)
(e)	 <p>Lower bound = $48 + 15 + 15 = 78$</p>	1M1A1 2M1A1 (4)

Question Number	Scheme	Marks
(f)	<p>The route is ABDECA (The optimal route length is 78, since upper bound = lower bound)</p> <p>a1M1 First three arcs (or all 5 nodes / or numbers across the top of the matrix) selected correctly (may start from any node). Award M1 only for a correct tree with no working. a1A1 CAO (order of arc selection clear)</p> <p>b1B1 112 CAO</p> <p>c1M1 Nearest Neighbour either A-B-C-E-D- or A-B-D-E-C- (condone lack of return to start). Accept 12354 or 12534 across the top of the matrix. c1A1 1 route and length CAO (Do not ISW if route length is doubled) c2A1 both routes and lengths CAO (Do not ISW if route lengths are doubled)</p> <p>d1B1ft their stated shortest (must be a number)</p> <p>e1M1 Finding correct RMST (maybe implicit) 48 sufficient, or correct numbers. 3 arcs. e1A1 CAO; tree or 48 or 11 + 18 + 19 seen. e2M1 Adding 2 least arcs to B; 15 and 15 or two out of BA, BC or BD or 30 only e2A1 CAO 78</p> <p>f1B1 CAO, accept any start point for the correct tour, but must return to start. Dependent on their answer to part (d) = their answer to part (e).</p>	<p>B1 (1)</p> <p>Total 12</p>

(Q07 6690/01, June 2013)

Q12.

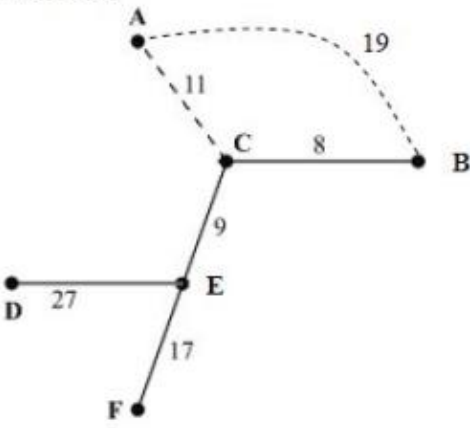
Question Number	Scheme	Marks
(a)	<p>A E D F B C A = 100 km</p> <p>12 15 12 12 24 25</p>	<p>1M1 1A1 2A1</p>
(b)	<p>Delete A</p>  <p>RMST weight = $12 + 12 + 15 + 18 = 57$ (km)</p> <p>Lower bound = $57 + 12 + 15 = 84$ (km)</p>	<p>3</p> <p>1M1</p> <p>1A1</p> <p>2M1 2A1</p> <p>Total 7</p>

Notes for question

- a1M1 NN Each vertex visited at least once, accept 156324 across top of table (condone lack of return to start).
- a1A1 Route CAO must be stated, must return to A, accept link back to A.
- a2A1 Length CAO 100. Do not ISW if candidates then go on to double the route length.
- b1M1 Finding correct RMST (maybe implicit) 57 sufficient; or 12, 12, 15 and 18. Must have 4 arcs.
- b1A1 CAO; tree **or** list of arcs **or** 57 **or** $12 + 12 + 15 + 18$ seen.
- b2M1 Adding 2 least arcs from A to 'tree'; 12 and 15 **or** AF and AE **or** 27 only. Must add these arcs distinctly.
- b2A1 CAO 84

(Q07 6690/01, June 2012)

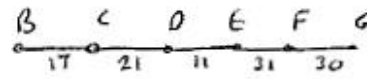
Q13.

Question Number	Scheme	Marks																																																	
(a)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td>A</td> <td>B</td> <td>C</td> <td>D</td> <td>E</td> <td>F</td> </tr> <tr> <td>A</td> <td>-</td> <td>19</td> <td>11</td> <td>23</td> <td>20</td> <td>37</td> </tr> <tr> <td>B</td> <td>19</td> <td>-</td> <td>8</td> <td>42</td> <td>17</td> <td>32</td> </tr> <tr> <td>C</td> <td>11</td> <td>8</td> <td>-</td> <td>34</td> <td>9</td> <td>26</td> </tr> <tr> <td>D</td> <td>23</td> <td>42</td> <td>34</td> <td>-</td> <td>27</td> <td>31</td> </tr> <tr> <td>E</td> <td>20</td> <td>17</td> <td>9</td> <td>27</td> <td>-</td> <td>17</td> </tr> <tr> <td>F</td> <td>37</td> <td>32</td> <td>26</td> <td>31</td> <td>17</td> <td>-</td> </tr> </table>		A	B	C	D	E	F	A	-	19	11	23	20	37	B	19	-	8	42	17	32	C	11	8	-	34	9	26	D	23	42	34	-	27	31	E	20	17	9	27	-	17	F	37	32	26	31	17	-	B3, 2, 1, 0 (3)
	A	B	C	D	E	F																																													
A	-	19	11	23	20	37																																													
B	19	-	8	42	17	32																																													
C	11	8	-	34	9	26																																													
D	23	42	34	-	27	31																																													
E	20	17	9	27	-	17																																													
F	37	32	26	31	17	-																																													
(b)	$A \ C \ B \ E \ F \ D \ A$ $11 \ 8 \ 17 \ 17 \ 31 \ 23 = 107$	M1 A1 A1 (3)																																																	
(c)	<p>Delete A</p>  <p>RMST weight = 61 Lower bound = 61 + 11 + 19 = 91 km</p>	M1 A1 M1 A1 (4) 10																																																	

Question Number	Scheme	Marks
	<u>Notes:</u>	
(a)1B1	One double entry correct	
2B1	Two double entries correct	
3B1	Three double entries correct	
(b)M1	NN route, each letter appearing once, condone lack of return vertex	
1A1	CAO	
2A1	CAO	
(c)1M1	Finding my RMST – accept 61 for both marks	
1A1	Either 8 + 9 + 17 + 27 or 61 seen	
2M1	Adding on two least arcs, accept 11 and 19 or AC and AB	
2A1	91 CAO	



Q14.

Question Number	Scheme	Marks
(a)	Adds 32 to AB + BA (ACB) 47 to AE + EA (ACDE) 32 to CE + EC (CDE) 53 to DG + GD (DCG)	B1 B1 B1 B1 (4)
(b)	A C B D E F G A $15 + 17 + 38 + 11 + 31 + 30 + 23 = 165$ miles	M1 A1 A1 (3)
(c)	eg BC, CD, DE, EF, FG  weight of RST = 110 miles Lower bound = $110 + 15 + 23 = 148$ miles	M1 A1 M1 A1 (4) <div style="border: 1px solid black; width: 20px; height: 20px; display: inline-block; margin-left: 10px;"></div>
(a)	B1 Adds 32 to both AB and BA } each number pair Adds 47 to both AE and EA } correct on at least Adds 32 to both CE and EC } one diagram. Adds 53 to both DG and GD	
(b)	M1 Each vertex visited once A1 c.a.o including return to A (ACBDEFGA) A1 c.a.o (165)	
(c)	M1 Finding correct rsmst (maybe implicit) 110 sufficient A1 cao tree of 110 M1 Adding 2 least arcs to A ie 15 and 23 only A1 cao of from their rsmst values - ie 38 + their tree length (must be a tree)	

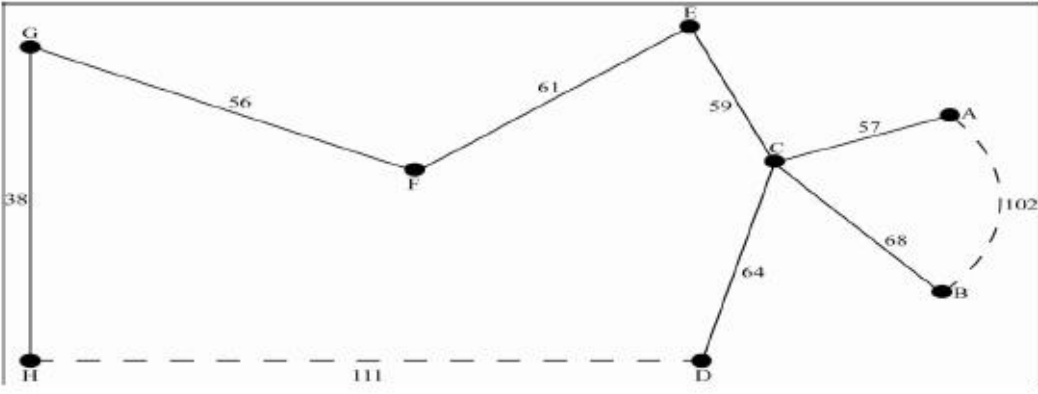
(Q06 6690/01, June 2007)

Q15.

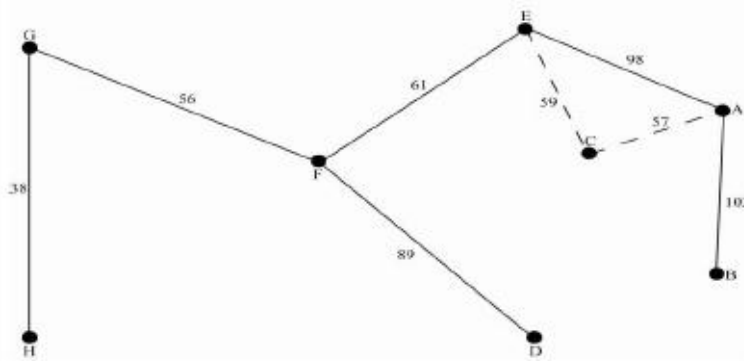
Question Number	Scheme	Marks
(a)	A walk is a finite sequence of arcs such that the end vertex of one arc is the start vertex of the next.	B2,1,0
(b)	A tour is a walk that visits every vertex, returning to its starting vertex.	B2,1,0 (4) (4 marks)

(Q07 6690/01, June 2008)

Q16.

Question Number	Scheme	Marks
(a)	GH(38) GF(56) CA(57) EC(59) FE(61) CD(64) CB(68)	M1A1 (2)
(b)	$2 \times 403 = 806$ (km)	B1 (1)
(c)	e.g. DH saves 167 AB saves 23	M1 A1
	$806 - 190 = 616$ (km)	A1
		
	eg ABC EFGH DCA	A1 (4)
(d)	B C A E F G H D B $68 + 57 + 98 + 61 + 56 + 38 + 111 + 108 = 597$ (km)	M1 A1 A1 (3)

(e) Delete C



RMST weight = 444

 Lower bound = $444 + 59 + 57 = 560$ (km)

 (f) $560 < \text{length} \leq 597$

M1

A1

M1

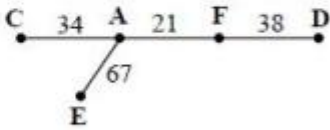
A1ft (4)

B2,1,0 (2)

(16 marks)

(Q08 6690/01, June 2008)

Q17.

Question Number	Scheme	Marks
Q		
(a)	In the classical problem each vertex must be visited only once. In the practical problem each vertex must be visited at least once.	B2, 1, 0 (2)
(b)	A F D B E C A {1 4 6 3 5 2} $21 + 38 + 58 + 36 + 70 + 34 = 257$	M1 A1 A1 (3)
(c)	257 is the better upper bound, it is lower.	B1ft (1)
(d)	R.M.S.T. 	M1 A1
	Lower bound is $160 + 36 + 58 = 254$	M1A1 (4)
(e)	Better lower bound is 254, it is higher	B1ft
(f)	$254 < \text{optimal} \leq 257$	B1 (2)
		[12]

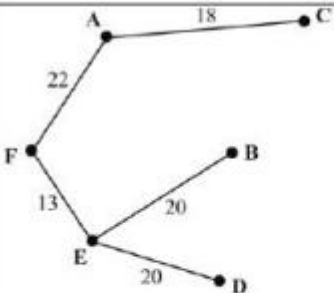
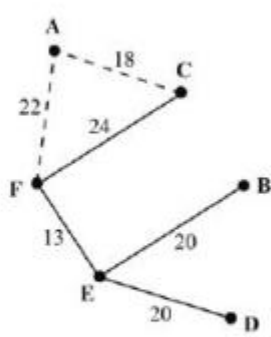


Notes:

- (a) 1B1: Generous, on the right lines bod gets B1
2B1: cao, clear answer.
- (b) 1M1: Nearest Neighbour each vertex visited once (condone lack of return to start)
1A1: Correct route cao – must return to start.
2A1: 257 cao
- (c) 1B1ft: ft their lowest.
- (d) 1M1: Finding correct RMST (maybe implicit) 160 sufficient
1A1: cao tree or 160.
2M1: Adding 2 least arcs to B, 36 and 58 only
2A1: 254
- (e) 1B1ft: ft their highest
- (f) 1B1: cao

(Q07 6690/01, June 2009)

Q18.

Question Number	Scheme	Marks
Q (a)		M1 A1 (2)
(b)	Minimum Spanning tree length 93, so upper bound is £186	B1ft (1)
(c)	A C F E B D A 18 24 13 20 22 28 Length 125 A C F E D B A 18 24 13 20 22 36 Length 133	M1 A1 A1 (3)
(d)	Best upper bound is £125	B1ft (1)
(e)	Delete A	
		M1 A1
	RMST weight = 77 Lower bound = 77 + 18 + 22 = £117	M1 A1 (4) [11]

Notes for question

- (a) 1M1: Spanning tree found. Allow 1x2x43 across top of table or 93
1A1: CAO must see tree or list of arcs
- (b) 1B1ft: 186 their ft93 x 2
- (c) 1M1: One Nearest Neighbour each vertex visited at least once (condone lack of return to start)
1A1: One correct route and length CAO - must return to start.
2A1: Second correct route and length CAO - must return to start.
- (d) 1B1ft: ft but only on three different values.
- (e) 1M1: Finding correct RMST (maybe implicit) 77 sufficient, or correct numbers. 4 arcs.
1A1: CAO tree or 77.
2M1: Adding 2 least arcs to A, 18 and 22 or 40 only
2A1: CAO 117

(Q07 6690/01, June 2010)

Q19.

Question	Scheme	Marks	AOs
(a)	Yes Dijkstra's algorithm can be applied to either a directed or undirected network	B1	3.5b
		(1)	
(b)	Initial tables $\begin{bmatrix} - & 5 & 11 & 8 \\ 5 & - & 3 & 2 \\ 11 & 3 & - & 4 \\ 8 & \infty & 4 & - \end{bmatrix}$ $\begin{bmatrix} A & B & C & D \\ A & B & C & D \\ A & B & C & D \\ A & B & C & D \end{bmatrix}$		
	1 st iteration $\begin{bmatrix} - & 5 & 11 & 8 \\ 5 & - & 3 & 2 \\ 11 & 3 & - & 4 \\ 8 & [13] & 4 & - \end{bmatrix}$ $\begin{bmatrix} A & B & C & D \\ A & B & C & D \\ A & B & C & D \\ A & [A] & C & D \end{bmatrix}$	M1 A1	1.1b 1.1b
	2 nd iteration $\begin{bmatrix} - & 5 & [8] & [7] \\ 5 & - & 3 & 2 \\ [8] & 3 & - & 4 \\ 8 & 13 & 4 & - \end{bmatrix}$ $\begin{bmatrix} A & B & [B] & [B] \\ A & B & C & D \\ [B] & B & C & D \\ A & A & C & D \end{bmatrix}$	M1 A1ft	1.1b 1.1b
	3 rd iteration $\begin{bmatrix} - & 5 & 8 & 7 \\ 5 & - & 3 & 2 \\ 8 & 3 & - & 4 \\ 8 & [7] & 4 & - \end{bmatrix}$ $\begin{bmatrix} A & B & B & B \\ A & B & C & D \\ B & B & C & D \\ A & [C] & C & D \end{bmatrix}$	M1 A1ft	1.1b 1.1b
	4 th iteration $\begin{bmatrix} - & 5 & 8 & 7 \\ 5 & - & 3 & 2 \\ 8 & 3 & - & 4 \\ 8 & 7 & 4 & - \end{bmatrix}$ $\begin{bmatrix} A & B & B & B \\ A & B & C & D \\ B & B & C & D \\ A & C & C & D \end{bmatrix}$ no changes therefore optimal	A1	1.1b
			(7)

(c)	Start at D (4 th) row and read across to the B (2 nd) column, there is a C there		
	so the route starts DC. Look at the C row, B column and you see B	B1	2.4
	The route is therefore DCB	B1	2.2a
		(2)	
Question	Scheme	Marks	AOs
(d)	D – C – B – A – B – D	M1	2.2a
	Length 19 (miles)	A1	1.1b
		(2)	
(e)	Dijkstra's algorithm finds the shortest distances from one vertex to all the others. Floyd's algorithm finds the shortest distance between every pair of vertices.	B1	2.5
		B1	2.5
		(2)	
(14 marks)			



Notes:
(a) B1: CAO (must include mention of 'directed' network) M1: No change in the first row and first column of both tables with at least one value in the distance table reduced and one value in the route table changed
(b) A1: CAO M1: No change in the second row and second column of both tables with at least two values in the distance table reduced and two values in the route table changed A1ft: Correct second iteration follow through from the candidate's first iteration M1: No change in the third row and third column of both tables with at least one value in the distance table reduced and one value in the route table changed A1ft: Correct third iteration follow through from the candidate's second iteration A1: CSO (no change after the fourth iteration) – all previous marks must have been awarded in this part
(c) B1: Clear indication of how the final route table can be used to get from D to B (therefore must mention the correct rows and columns in their reasoning) B1: Completely correct argument + correct route (DCB)
(d) M1: Deduce correctly their minimum route from their final distance table (dependent on all M marks in (a)) must begin and end at D A1: CAO (length of 19)
(e) B1: CAO – must use correct language 'one vertex to all other vertices' B1: CAO – must use correct language 'every pair of vertices'

(Q04 9FM0/3D-4D, Specimen papers)