Exam Questions – Chapter 5 and 7 Friction and Moments

Q1.

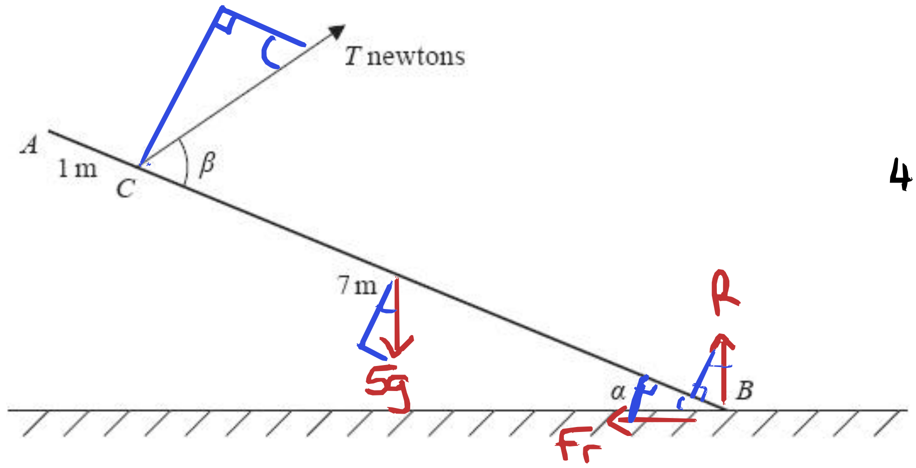
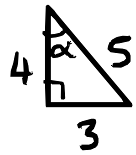


Figure 3



$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

A uniform rod *AB*, of mass 5 kg and length 8 m, has its end *B* resting on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$, by a rope attached to the rod at *C*. The distance *AC* = 1 m. The rope is in the same vertical plane as the rod. The angle between the rope and the rod is β and the tension in the rope is *T* newtons, as shown in Figure 3. The coefficient of friction between the rod and the ground is $\frac{2}{3}$. The vertical component of the force exerted on the rod at *B* by the ground is *R* newtons.

(a) Find the value of *R*.

$Fr = \frac{2}{3}R$

Moments about C

$3 \times 5g\cos\alpha + 7 \times Fr\sin\alpha = 7\cos\alpha \times R$

$3 \times 5 \times g \times \frac{4}{5} + 7 \times \frac{2}{3}R \times \frac{3}{5} = 7 \times \frac{4}{5} \times R$

$12g + \frac{14}{5}R = \frac{28}{5} \times R$

$12g = \frac{14}{5}R$

$R = 42\text{N}$

(6)

(b) Find the size of angle β.

Moments about B

$4 \times 5g \times \cos\theta = T\sin\beta \times 7$

$\frac{16g}{7} = T\sin\beta$

$T\cos\beta + 5g\sin\alpha = Fr\cos\alpha + R\sin\alpha$

$T\cos\beta + 5g \times \frac{3}{5} = \frac{2}{3} \times 42 \times \frac{4}{5} + 42 \times \frac{3}{5}$

$T\cos\beta = 18.2$

$\tan\beta = \frac{\frac{16g}{7}}{18.2}$

$\beta = 50.9°$

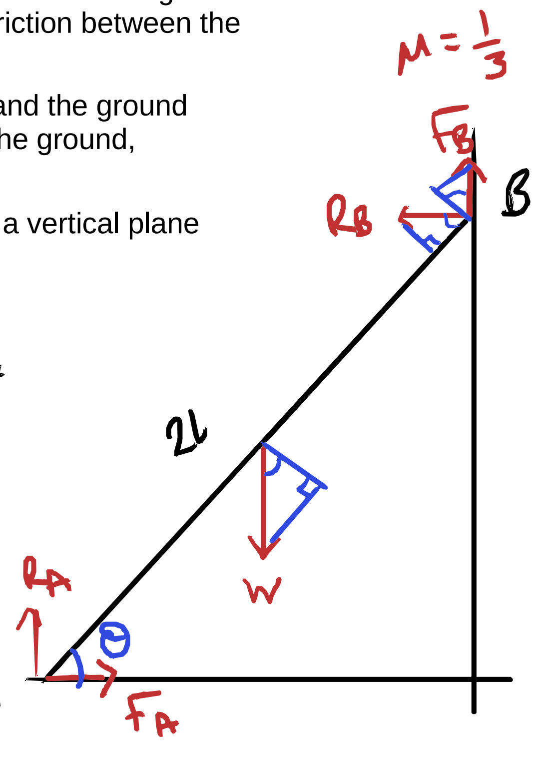
(5)

(Total for question = 11 marks)

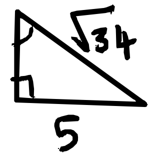
Q2.

A ladder *AB*, of weight *W* and length 2*l*, has one end *A* resting on rough horizontal ground. The other end *B* rests against a rough vertical wall. The coefficient of friction between the ladder and the wall is $\frac{1}{3}$. The coefficient of friction between the ladder and the ground is μ. Friction is limiting at both *A* and *B*. The ladder is at an angle θ to the ground, where $\tan\theta = \frac{5}{3}$. The ladder is modelled as a uniform rod which lies in a vertical plane perpendicular to the wall.

Find the value of μ.



$\tan\theta = \frac{5}{3}$



$\cos\theta = \frac{3}{\sqrt{34}}$

$\sin\theta = \frac{5}{\sqrt{34}}$

$\updownarrow \quad R_A + F_B = W$

$\leftrightarrow \quad F_A = R_B$

$F_A = \mu R_A \qquad F_B = \frac{1}{3} R_B$

① $R_A + \frac{1}{3} R_B = W$ ② $\mu R_A = R_B$

$R_A = \frac{R_B}{\mu}$

Moments at A

$$L \times W\cos\theta = R_B \times 2L \times \sin\theta + F_B \times 2L \times \cos\theta$$

$$W \times \frac{3}{\sqrt{34}} = R_B \times 2 \times \frac{5}{\sqrt{34}} + F_B \times 2 \times \frac{3}{\sqrt{34}}$$

$$\left(\frac{R_B}{\mu} + \frac{1}{3}R_B\right) \times \frac{3}{\sqrt{34}} = R_B \times \frac{10}{\sqrt{34}} + \frac{1}{3} R_B \times \frac{6}{\sqrt{34}}$$

$$R_B\left(\frac{1}{\mu} + \frac{1}{3}\right)\frac{3}{\sqrt{34}} = R_B \times \frac{10}{\sqrt{34}} + R_B \times \frac{2}{\sqrt{34}}$$

$$\left(\frac{1}{\mu} + \frac{1}{3}\right) \times 3 = 10 + 2$$

$$\frac{1}{\mu} + \frac{1}{3} = 4$$

$$\frac{1}{\mu} = \frac{11}{3}$$

$$\mu = \frac{3}{11}$$

(9)

(Total for question = 9 marks)

Q3.

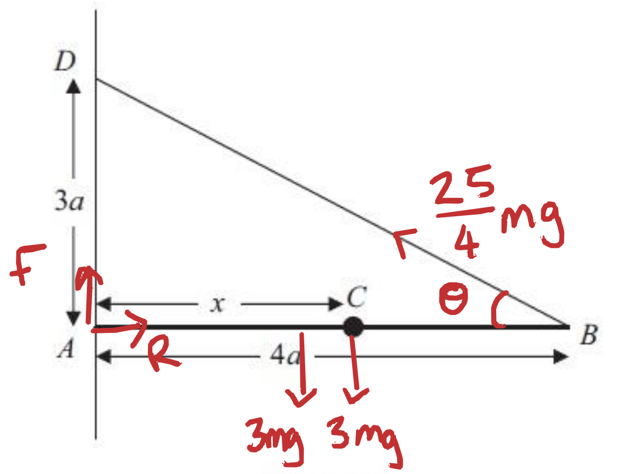
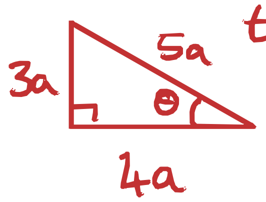


Figure 3



$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

A uniform rod AB, of mass $3m$ and length $4a$, is held in a horizontal position with the end A against a rough vertical wall. One end of a light inextensible string BD is attached to the rod at B and the other end of the string is attached to the wall at the point D vertically above A, where $AD = 3a$. A particle of mass $3m$ is attached to the rod at C, where $AC = x$. The rod is in equilibrium in a vertical plane perpendicular to the wall as shown in Figure 3. The tension in the string is $\frac{25}{4}mg$.

Show that

(a) $x = 3a$,

Moments at A

$$3mg \times 2a + 3mg \times x = \frac{25}{4}mg \times 4a \times \sin\theta$$

$$6a + 3x = \frac{25}{4} \times 4a \times \frac{3}{5}$$

$$6a + 3x = 15a$$

$$3x = 9a$$

$$x = 3a$$

(5)

(b) the horizontal component of the force exerted by the wall on the rod has magnitude $5mg$.

$$R = \frac{25}{4}mg \times \cos\theta$$

$$R = \frac{25}{4}mg \times \frac{4}{5}$$

$$R = 5mg$$

(3)

The coefficient of friction between the wall and the rod is μ. Given that the rod is about to slip,

(c) find the value of μ.

$\updownarrow \quad F + \frac{25}{4}mg\sin\theta = 6mg$

$\mu \times R + \frac{25}{4}mg \times \frac{3}{5} = 6mg$

$\mu \times 5mg + \frac{15}{4}mg = 6mg$

$5\mu + \frac{15}{4} = 6$

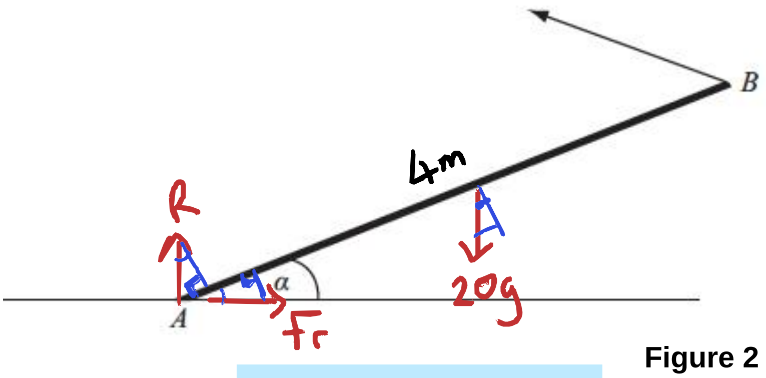
$5\mu = \frac{9}{4}$

$\mu = 0.45 \text{ or } \frac{9}{20}$ //

(5)

(Total 13 marks)

Q4.



$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

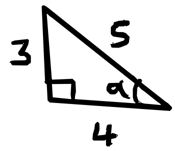


Figure 2

A uniform rod *AB*, of mass 20 kg and length 4 m, rests with one end *A* on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$, by a force acting at *B*, as shown in Figure 2. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5. Find the magnitude of the normal reaction of the ground on the rod at *A*.

Moments at B

$$2 \times 20g \times \cos\alpha + 4 \times Fr \times \sin\alpha = 4 \times R\cos\alpha$$

$$40g \times \frac{4}{5} + 4 \times 0\cdot5R \times \frac{3}{5} = 4 \times R \times \frac{4}{5}$$

$$32g + \frac{6}{5}R = \frac{16}{5}R$$

$$32g = \frac{10}{5}R$$

$$R = 156\cdot8$$

$$R = 157\text{N}$$

(7)

(Total 7 marks)

Q5.

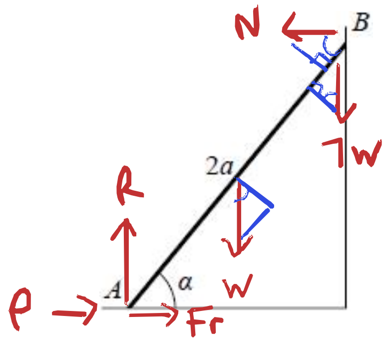
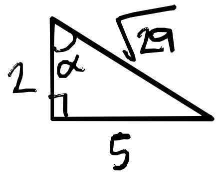


Figure 1



$\tan\alpha = \frac{5}{2}$

$\cos\alpha = \frac{2}{\sqrt{29}}$

$\sin\alpha = \frac{5}{\sqrt{29}}$

A uniform ladder *AB*, of length 2*a* and weight *W*, has its end *A* on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end *B* of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight 7*W* stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude *P* to the ladder at *A*, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle *α* with the horizontal ground, where $\tan\alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at *B* has magnitude 3*W*.

Moments at A

$$a \times W\cos\alpha + 2a \times 7W\cos\alpha = 2a \times N\sin\alpha$$

$$W \times \frac{2}{\sqrt{29}} + 14W \times \frac{2}{\sqrt{29}} = 2 \times N \times \frac{5}{\sqrt{29}}$$

$$2W + 28W = 10N$$

$$\frac{30W}{10} = N$$

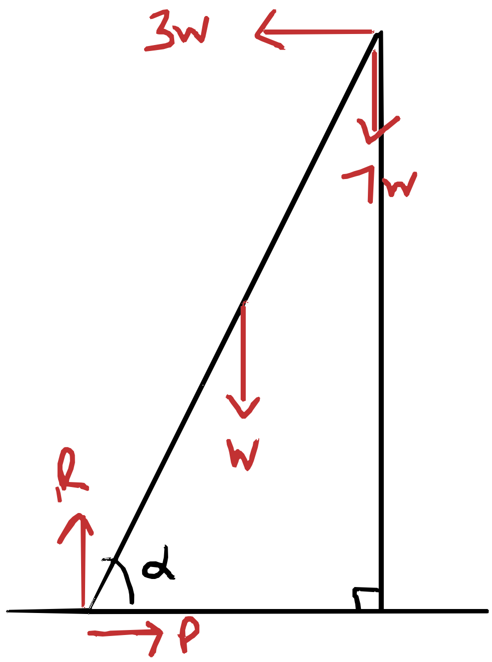
$$N = 3W$$

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

$\updownarrow \quad 8W = R$

$\mu = \frac{1}{4}$



Friction away from wall

$3W + F_r = P$

$3W + \frac{1}{4} \times 8W = P$

$3W + 2W = P$

$P = 5W$

Friction towards wall

$P + F_r = 3W$

$P + \frac{1}{4} \times 8W = 3W$

$P + 2W = 3W$

$P = W$

$W \leq P \leq 5W$

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

Moments at A would mean reaction at B unchanged.

Resolving vertically will mean R increases.

Increasing R will also increase F_{max}.

(3)

(Total for question = 13 marks)

Q6.

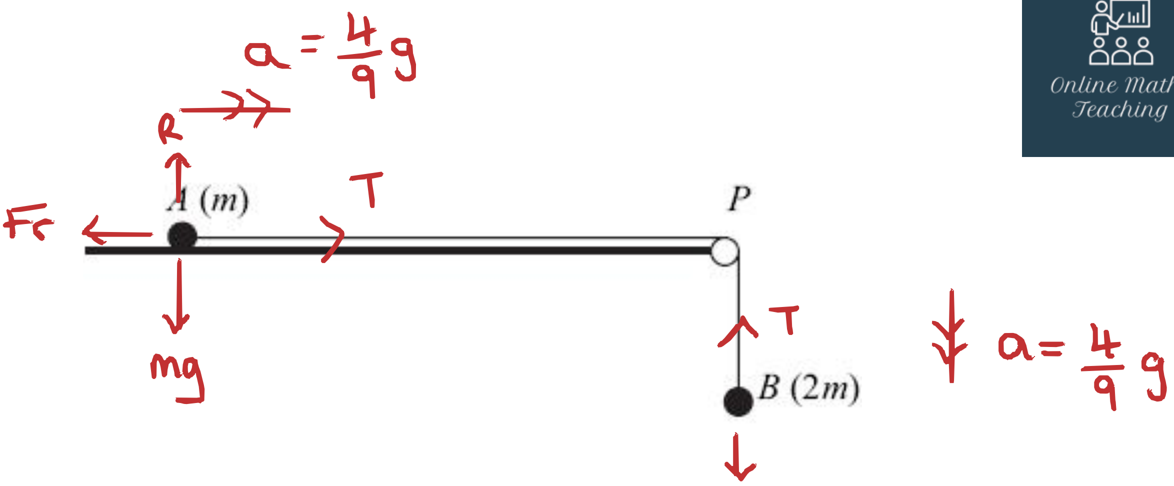


Figure 3 2mg

Two particles *A* and *B*, of mass *m* and 2*m* respectively, are attached to the ends of a light inextensible string. The particle *A* lies on a rough horizontal table. The string passes over a small smooth pulley *P* fixed on the edge of the table. The particle *B* hangs freely below the pulley, as shown in Figure 3. The coefficient of friction between *A* and the table is *μ*. The particles are released from rest with the string taut.

Immediately after release, the magnitude of the acceleration of *A* and *B* is $\frac{4}{9}g$. By writing down separate equations of motion for *A* and *B*,

(a) find the tension in the string immediately after the particles begin to move,

A $\quad F = ma$

$T - Fr = m \times \frac{4}{9}g$

B $\quad F = ma$

$2mg - T = 2m \times \frac{4}{9}g$

$2mg - T = \frac{8}{9}mg$

$2mg - \frac{8}{9}mg = T$

$T = \frac{10}{9}mg$ //

(3)

(b) show that $\mu = \frac{2}{3}$.

$\frac{10}{9}mg - \mu \times R = m \times \frac{4}{9}g$

$\frac{10}{9}mg - \mu mg = \frac{4}{9}mg$

$\frac{10}{9} - \frac{4}{9} = \mu$

$\frac{6}{9} = \mu$

$\mu = \frac{2}{3}$ //

(5)

When B has fallen a distance h, it hits the ground and does not rebound. Particle A is then a distance $\frac{1}{3}h$ from P.

(c) Find the speed of A as it reaches P.

until B hits floor

$s = h$
$u = 0$
$v = x$
$a = \frac{4}{9}g$
t

$v^2 = u^2 + 2as$

$x^2 = 2 \times \frac{4}{9}g \times h$

$x = \sqrt{\frac{8}{9}gh}$

Acceleration of A after B hits floor

$F = ma$

$-\frac{2}{3}mg = ma$

$-\frac{2}{3}g = a$

$s = \frac{1}{3}h$
$u = \sqrt{\frac{8}{9}gh}$
$v =$
$a = -\frac{2}{3}g$
t

$v^2 = u^2 + 2as$

$v^2 = \frac{8}{9}gh + 2 \times -\frac{2}{3}g \times \frac{1}{3}h$

$v^2 = \frac{8}{9}gh - \frac{4}{9}gh$

$v = \sqrt{\frac{4}{9}gh}$

$v = \frac{2}{3}\sqrt{gh}$ //

(6)

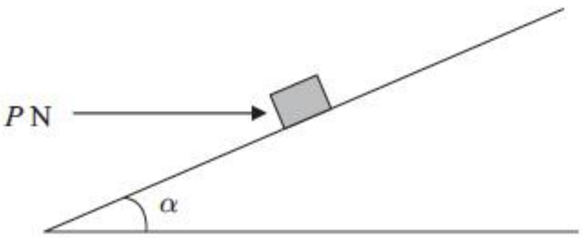
(d) State how you have used the information that the string is light.

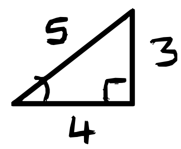
Same tension on A and B //

(1)

(Total 15 marks)

Q7.





$\tan\alpha = \frac{3}{4}$

$\cos\alpha = \frac{4}{5}$

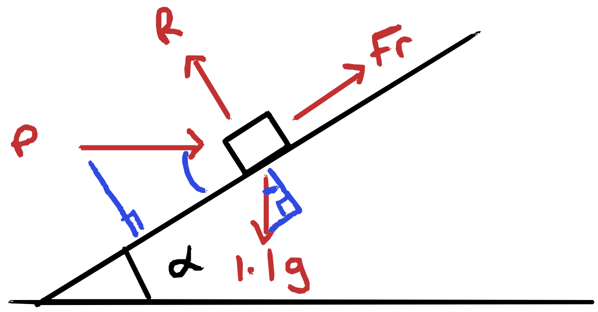
$\sin\alpha = \frac{3}{5}$

Figure 2

A small package of mass 1.1 kg is held in equilibrium on a rough plane by a horizontal force. The plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The force acts in a vertical plane containing a line of greatest slope of the plane and has magnitude P newtons, as shown in Figure 2.

The coefficient of friction between the package and the plane is 0.5 and the package is modelled as a particle. The package is in equilibrium and on the point of slipping down the plane.

(a) Draw, on Figure 2, all the forces acting on the package, showing their directions clearly.



(2)

(b) (i) Find the magnitude of the normal reaction between the package and the plane.

(ii) Find the value of P.

$\nearrow \quad Fr + P\cos\alpha = 1.1g\sin\alpha$

$0.5R + P \times \frac{4}{5} = 1.1g \times \frac{3}{5}$

$0.5R + \frac{4}{5}P = \frac{33}{50}g$

$0.5\left(\frac{3}{5}P + \frac{22}{25}g\right) + \frac{4}{5}P = \frac{33}{50}g$

$\frac{3}{10}P + \frac{11}{25}g + \frac{4}{5}P = \frac{33}{50}g$

$1.1P = \frac{11}{50}g$

$P = 1.96\text{N}$ //

$\nwarrow \quad R = P\sin\alpha + 1.1g\cos\alpha$

$R = P \times \frac{3}{5} + 1.1g \times \frac{4}{5}$

$R = \frac{3}{5}P + \frac{22}{25}g$

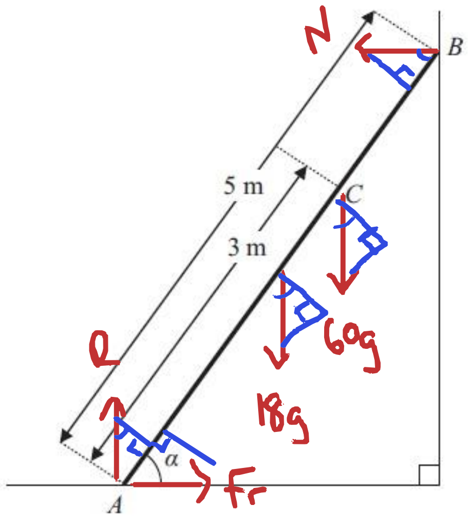
$R = \frac{3}{5}(1.96) + \frac{22}{25}(9.8)$

$R = 9.8\text{N}$ //

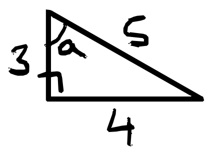
(11)

(Total 13 marks)

Q8.



$\tan a = \frac{4}{3}$



$\cos a = \frac{3}{5}$

$\sin a = \frac{4}{5}$

Figure 1

A ladder, of length 5 m and mass 18 kg, has one end A resting on rough horizontal ground and its other end B resting against a smooth vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle a with the horizontal ground, where $\tan a = \frac{4}{3}$, as shown in Figure 1. The coefficient of friction between the ladder and the ground is μ. A woman of mass 60 kg stands on the ladder at the point C, where AC = 3 m. The ladder is modelled as a uniform rod and the woman as a particle.

Find the value of μ.

$\updownarrow \quad R = 18g + 60g$

$R = 78g$

$\longleftrightarrow \quad Fr = N$

$Fr = \mu R$

$Fr = \mu \times 78g$

Moments at B

$2 \times 60g \cos a + 2.5 \times 18g \cos a + 5 \times Fr \sin a = 5 \times R \cos a$

$120g \times \frac{3}{5} + 45g \times \frac{3}{5} + 5Fr \times \frac{4}{5} = 5R \times \frac{3}{5}$

$72g + 27g + 4\mu \times 78g = 3 \times 78g$

$99 + 312\mu = 234$

$312\mu = 135$

$\mu = 0.433$

(9)

(Total 9 marks)

Q9.

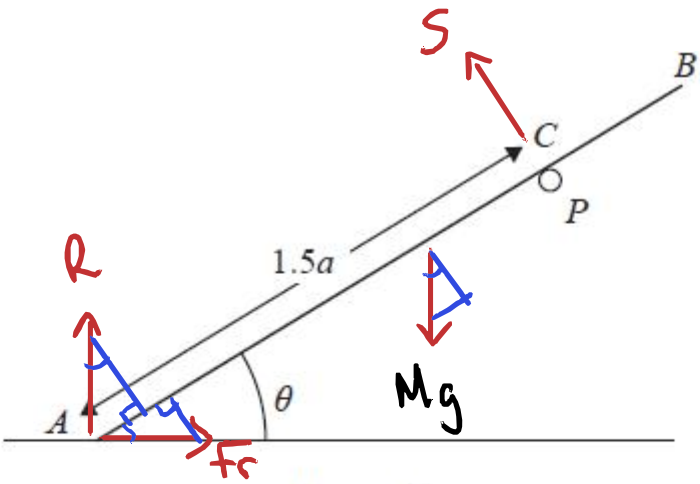
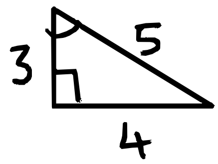


Figure 5

$\tan\theta = \frac{4}{3}$



$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

Figure 5 shows a uniform rod *AB* of mass *M* and length 2*a*.

- the rod has its end *A* on rough horizontal ground
- the rod rests in equilibrium against a small smooth fixed horizontal peg *P*
- the point *C* on the rod, where *AC* = 1.5*a*, is the point of contact between the rod and the peg
- the rod is at an angle θ to the ground, where $\tan\theta = \frac{4}{3}$

The rod lies in a vertical plane perpendicular to the peg.

The magnitude of the normal reaction of the peg on the rod at *C* is *S*.

(a) Show that $S = \frac{2}{5}Mg$

Moments at A

$a \times Mg\cos\theta = 1.5a \times S$

$Mg \times \frac{3}{5} = 1.5 \times S$

$\frac{2}{5}Mg = S$

$S = \frac{2}{5}Mg$ //

(3)

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The coefficient of friction between the rod and the ground is μ.

Given that the rod is in limiting equilibrium,

(b) find the value of μ.

$\nwarrow\!\!\searrow$ ① $S + R\cos\theta = Fr\sin\theta + Mg\cos\theta$

$\swarrow\!\!\nearrow$ ② $R\sin\theta + Fr\cos\theta = Mg\sin\theta$

$Fr = \mu R \qquad S = \frac{2}{5}Mg$

① $\frac{2}{5}Mg + R \times \frac{3}{5} = \mu R \times \frac{4}{5} + Mg \times \frac{3}{5}$

$R\left(\frac{3}{5} - \mu \times \frac{4}{5}\right) = \frac{3}{5}Mg - \frac{2}{5}Mg$

$R\left(\frac{3}{5} - \frac{4}{5}\mu\right) = \frac{1}{5}Mg$

$R(3 - 4\mu) = Mg$

$R = \frac{Mg}{3 - 4\mu}$

② $R \times \frac{4}{5} + \mu R \times \frac{3}{5} = Mg \times \frac{4}{5}$

$R(4 + 3\mu) = 4Mg$

(6)

$\frac{Mg}{3-4\mu} = \frac{4Mg}{4+3\mu}$

(Total for question = 9 marks)

$\frac{1}{3-4\mu} = \frac{4}{4+3\mu}$

$4 + 3\mu = 4(3 - 4\mu)$

$4 + 3\mu = 12 - 16\mu$

$19\mu = 8$

$\mu = \frac{8}{19}$ //

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Q10.

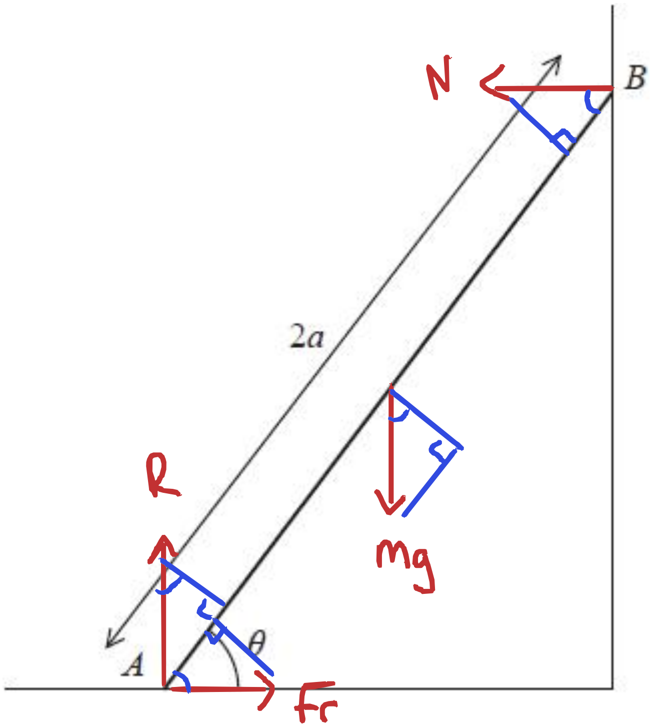


Figure 2

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geq \frac{1}{2} \cot \theta$

$\updownarrow \quad R = mg \qquad \longleftrightarrow \quad F_r = N$

Moments at B

$a \times mg\cos\theta + 2a \times F_r \sin\theta = 2a \times R\cos\theta$

$mg\cos\theta + 2F_r\sin\theta = 2 \times mg\cos\theta$

$F_r \geq \mu R \qquad 2F_r\sin\theta = mg\cos\theta$

$F_r = \frac{1}{2} mg\cot\theta$

(5)

$\frac{1}{2} mg\cot\theta \geq \mu \times mg$

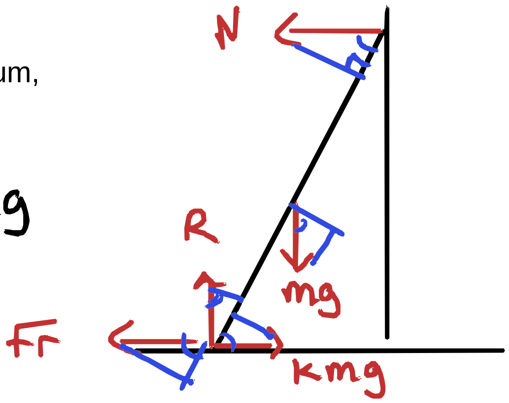
$\frac{1}{2}\cot\theta \geq \mu \qquad \mu \leq \frac{1}{2}\cot\theta$ //

A horizontal force of magnitude *kmg*, where *k* is a constant, is now applied to the beam at *A*.

This force acts in a direction that is perpendicular to the wall and towards the wall.

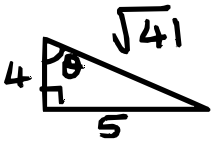
Given that $\tan\theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of *k*.



$\updownarrow$ $R = mg$ $\quad\longleftrightarrow$ $N + F_r = kmg$

$$N + \tfrac{1}{2}mg = kmg$$
$$N = kmg - \tfrac{1}{2}mg$$
$$N = mg(k - \tfrac{1}{2})$$



$\tan\theta = \frac{5}{4}$

$\sin\theta = \frac{5}{\sqrt{41}}$

$\cos\theta = \frac{4}{\sqrt{41}}$

Moments at A

$$a \times mg\cos\theta = 2a \times N\sin\theta$$
$$mg \times \frac{4}{\sqrt{41}} = 2 \times N \times \frac{5}{\sqrt{41}}$$
$$4mg = 10\left(mg(k - \tfrac{1}{2})\right)$$
$$\frac{4}{10} = k - \frac{1}{2}$$
$$\frac{9}{10} = k$$
$$k = \frac{9}{10}$$

(5)

(Total for question = 10 marks)

Q11.

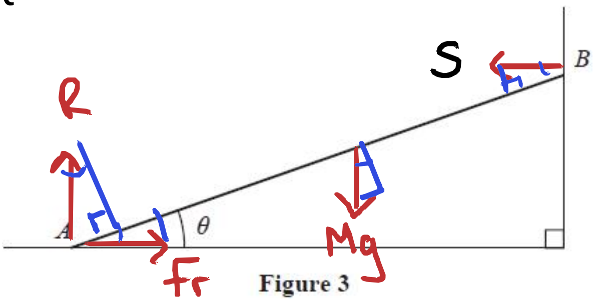


Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium.

(a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer**.

Friction must go right, to balance out the reaction of the wall on B //

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (b), (c) and (d).

(b) By taking moments about A, show that

$$S = \frac{1}{2}Mg\cot\theta$$

Moments at A

$$a \times Mg \times \cos\theta = 2a \times S \times \sin\theta$$

$$Mg\cos\theta = 2S\sin\theta$$

$$Mg\cot\theta = 2S$$

$$\frac{1}{2}Mg\cot\theta = S$$

$$S = \frac{1}{2}Mg\cot\theta \,//$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan\theta = \frac{3}{4}$

(c) find the value of μ

$\updownarrow \quad R = Mg \qquad \leftrightarrow \quad S = F_r$

$F_r = \mu R$

$S = \mu \times Mg$

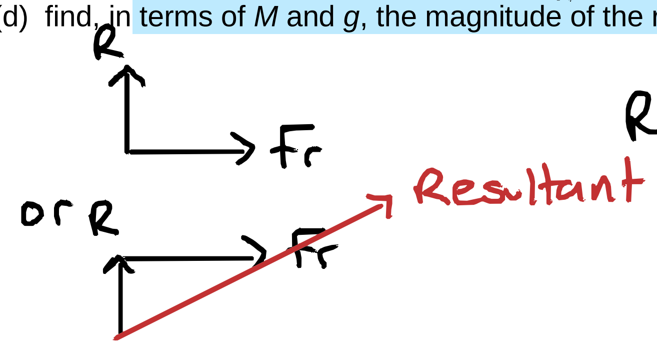
$\frac{1}{2} Mg \cot\theta = \mu \times Mg$

$\frac{1}{2} \times \frac{4}{3} = \mu$

$\mu = \frac{2}{3}$ //

(5)

(d) find, in terms of M and g, the magnitude of the resultant force acting on the rod at A.



$$\text{Resultant} = \sqrt{(Mg)^2 + \left(\tfrac{2}{3}Mg\right)^2}$$

$$= \sqrt{(Mg)^2\left(1 + \tfrac{4}{9}\right)}$$

$$= Mg\sqrt{\tfrac{13}{9}} \qquad = \tfrac{1}{3}Mg\sqrt{13} \text{ //}$$

$$= Mg\,\tfrac{\sqrt{13}}{3}$$

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A.

A new value for S is calculated using this new model, with $\tan\theta = \frac{3}{4}$

(e) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer**.

S would be larger as when taking moments from A we would multiply Mg cos θ by a larger distance from pivot. //

(Total for question = 12 marks)

Q12.

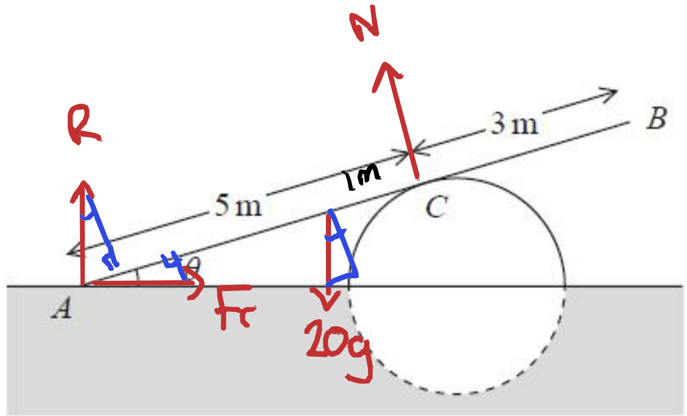


Figure 2

A ramp, *AB*, of length 8 m and mass 20 kg, rests in equilibrium with the end *A* on rough horizontal ground.

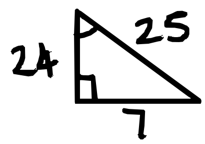
The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as *A*.

The point of contact between the ramp and the drum is *C*, where *AC* = 5 m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum,

at an angle θ to the horizontal, where $\tan\theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.



$\tan\theta = \frac{7}{24}$

$\cos\theta = \frac{24}{25}$

$\sin\theta = \frac{7}{25}$

(a) Explain why the reaction from the drum on the ramp at point *C* acts in a direction which is perpendicular to the ramp.

Drum is smooth + no friction, so force is perpendicular

(1)

(b) Find the magnitude of the resultant force acting on the ramp at *A*.

Moments at C

$1 \times 20g\cos\theta + 5 \times Fr\sin\theta = 5 \times R\cos\theta$

$R\sin\theta + Fr\cos\theta = 20g\sin\theta$

$\frac{7R}{25} + Fr \times \frac{24}{25} = 20g \times \frac{7}{25}$

$24F = 140g - 7R$

$F_r = \frac{140g - 7R}{24}$

$F_r = 42.148\text{N}$

$20g \times \frac{24}{25} + 5F_r \times \frac{7}{25} = 5R \times \frac{24}{25}$

$480g + 35F_r = 120R$

$480g + 35\left(\frac{140g - 7R}{24}\right) = 120R$

$480g + \frac{1225}{6}g - \frac{245}{24}R = 120R$

$\frac{4105}{6}g = \frac{3125}{24}R$

$R = 51.493\text{N}$

$\sqrt{42.148^2 + 51.493^2} = 66.5\text{N}$

(9)

The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to A than to B,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C.

Magnitude of the normal reaction at C will decrease.

(1)

(Total for question = 11 marks)

Q13.

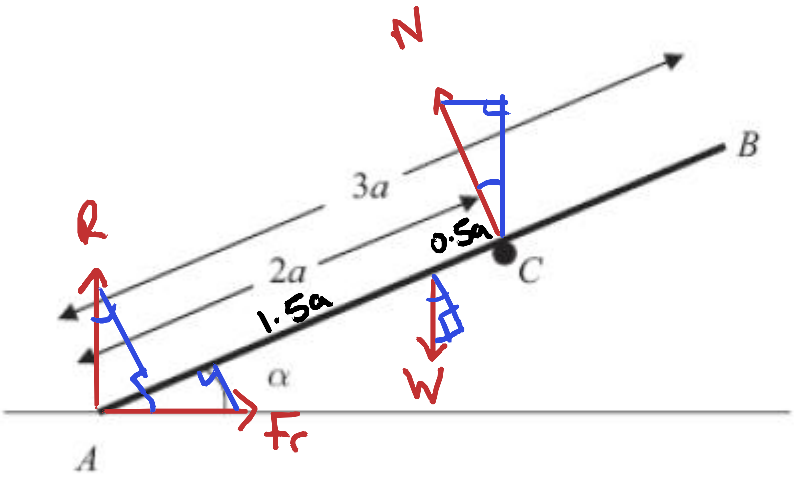


Figure 2

A plank rests in equilibrium against a fixed horizontal pole. The plank is modelled as a uniform rod *AB* and the pole as a smooth horizontal peg perpendicular to the vertical plane containing *AB*. The rod has length 3*a* and weight *W* and rests on the peg at *C*, where *AC* = 2*a*. The end *A* of the rod rests on rough horizontal ground and *AB* makes an angle *α* with the ground, as shown in Figure 2.

(a) Show that the normal reaction on the rod at *A* is $\frac{1}{4}(4 - 3\cos^2\alpha)W$.

Moments at A

$1.5a\, W\cos\alpha = 2a\, N$

$\frac{3}{2} W\cos\alpha = 2\left(\frac{W-R}{\cos\alpha}\right)$

$\frac{3}{2} W\cos^2\alpha = 2(W-R)$

$\frac{3}{4} W\cos^2\alpha = W - R$

$R = W - \frac{3}{4} W\cos^2\theta$

$\updownarrow \quad R + N\cos\alpha = W$

$N = \frac{W-R}{\cos\alpha}$

$\leftrightarrow \quad Fr = N\sin\alpha$

$R = \frac{1}{4}(4 - 3\cos^2\theta)W$ //

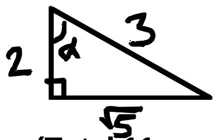
(6)

Given that the rod is in limiting equilibrium and that $\cos\alpha = \frac{2}{3}$,

(b) find the coefficient of friction between the rod and the ground.

$\cos\alpha = \frac{2}{3}$

$\sin\alpha = \frac{\sqrt{5}}{3}$



$Fr = \mu \times R$

$N\sin\alpha = \mu \times \frac{1}{4}(4 - 3\cos^2\theta)W$

$\left(\frac{W-R}{2/3}\right) \times \frac{\sqrt{5}}{3} = \mu \times \frac{1}{4}\left(4 - 3 \times \frac{4}{9}\right)W$

$\frac{\sqrt{5}}{2}W - \frac{\sqrt{5}}{2}R = \mu \times \frac{2}{3}W$

$\frac{\sqrt{5}}{2}W - \frac{\sqrt{5}}{2} \times \frac{2}{3}W = \mu \times \frac{2}{3}W$

$\frac{\sqrt{5}}{2} - \frac{\sqrt{5}}{3} = \frac{2}{3}\mu$

$\mu = \frac{\sqrt{5}}{4}$

$\mu = 0.559$ //

(5)

(Total 11 marks)