

Question 1

Show that $a^3 - a + 1$ is odd for all positive integer values of a . (5)

Question 2

Find the value of the constant k if

$$\int_1^3 (6x^2 + kx) \, dx = 8. \quad (5)$$

Question 3

$$f(x) = x^2, \quad x \in \mathbb{R}.$$

Use the formal definition of the derivative as a limit, to show that

$$f'(x) = 2x. \quad (5)$$

Question 4

The graph of the curve with equation

$$y = 2 \sin(2x + k)^\circ, \quad 0 \leq x < 360,$$

where k is a constant so that $0 < k < 90$, passes through the points with coordinates $P(55, 1)$ and $Q(\alpha, \sqrt{3})$.

a) Show, without verification, that $k = 40$. **(5)**

b) Determine the possible values of α . **(5)**

Question 5

The variables x and y are thought to obey a law of the form

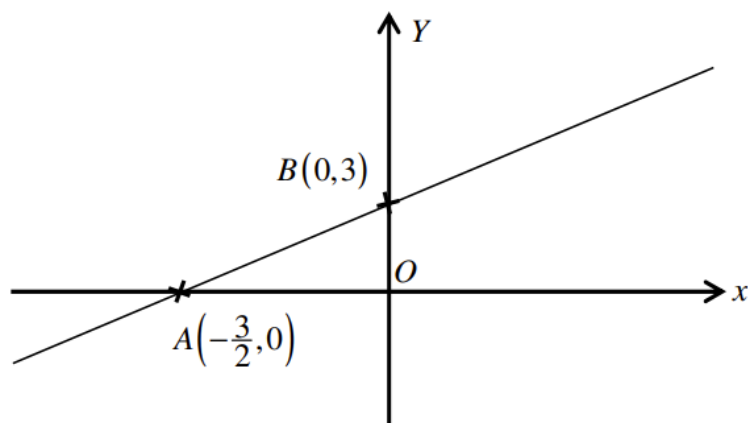
$$y = a \times k^x,$$

where a and k are positive constants.

Let $Y = \log_{10} y$.

- a) Show there is a linear relationship between x and Y . (4)

The figure below shows the graph of Y against x .



- b) Determine the value of a and the value of k . (4)

Question 6

The triangle ABC has $AB = 13$ cm and $BC = 15$ cm.

Given that $\angle BCA = 60^\circ$, determine the possible values of AC . (5)

Question 7

The points A , B and C have coordinates $(2,1)$, $(4,0)$ and $(6,4)$ respectively.

- a) Determine an equation of the straight line L which passes through C and is parallel to AB . **(4)**
- b) Show that the angle ABC is 90° . **(3)**
- c) Calculate the distance AC . **(2)**

A circle passes through the points A , B and C .

- d) Show that the equation of this circle is given by

$$x^2 + y^2 - 8x - 5y + 16 = 0. \quad (5)$$

- e) Find the coordinates of the point other than the point C where L intersects the circle. (5)

Question 8

A cubic curve C_1 has equation

$$y = (x-8)(x^2 - 4x + 3).$$

A quadratic curve C_2 has equation

$$y = (2x-3)(8-x).$$

- a)** Sketch on separate set of axes the graphs of C_1 and C_2 .

The sketches must contain the coordinates of the points where each of the curves meet the coordinate axes. **(5)**

- b)** Hence find the solutions of the following equation.

$$(x-8)(x^2 - 4x + 3) = (2x-3)(8-x). \quad \mathbf{(6)}$$

Question 9

The points A and C have coordinates $(3,2)$ and $(5,6)$, respectively.

- a) Find an equation for the perpendicular bisector of AC , giving the answer in the form $ax + by = c$, where a , b and c are integers. **(5)**

The perpendicular bisector of AC crosses the y axis at the point B .

The point D is such so that $ABCD$ is a rhombus.

- b) Show that the coordinates of D are $(8,2)$. **(3)**

- c) Calculate the area of the rhombus $ABCD$. **(4)**

Question 10

The point P , whose x coordinate is $\frac{1}{4}$, lies on the curve with equation

$$y = \frac{k + 4x\sqrt{x}}{7x}, \quad x \in \mathbb{R}, \quad x > 0,$$

where k is a non zero constant.

- a) Determine, in terms of k , the gradient of the curve at P . (5)

The tangent to the curve at P is parallel to the straight line with equation

$$44x + 7y - 5 = 0.$$

- b) Find an equation of the tangent to the curve at P . (7)

Question 11

Find the exact solutions of the equation

$$2e^{2x} - 5e^x + 3e^{-x} = 4. \quad (8)$$