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Question 1

Show that $a^3 - a + 1$ is odd for all positive integer values of a. **(5)**

Question 2

Find the value of the constant k if

$$\int_1^3 6x^2 + kx \;\; dx = 8.$$

(5)

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Question 3

$$f(x) = x^2, \ x \in \mathbb{R}.$$

Use the formal definition of the derivative as a limit, to show that

$$f'(x) = 2x. \qquad \textbf{(5)}$$

Question 4

The graph of the curve with equation

$$y = 2\sin(2x + k)°,\ \ 0 \le x < 360,$$

where k is a constant so that $0 < k < 90$, passes through the points with coordinates $P(55,1)$ and $Q(\alpha, \sqrt{3})$.

a) Show, without verification, that $k = 40$. **(5)**

b) Determine the possible values of α. **(5)**

Question 5

The variables x and y are thought to obey a law of the form

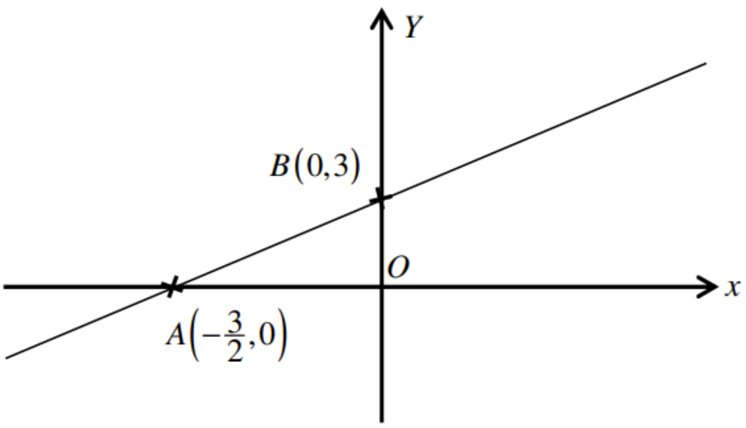
$$y = a \times k^x,$$

where a and k are positive constants.

Let $Y = \log_{10} y$.

a) Show there is a linear relationship between x and Y. **(4)**

The figure below shows the graph of Y against x.



b) Determine the value of a and the value of k. **(4)**

Question 6

The triangle ABC has $AB = 13$ cm and $BC = 15$ cm.

Given that $\measuredangle BCA = 60°$, determine the possible values of AC. **(5)**

Question 7

The points A, B and C have coordinates $(2,1)$, $(4,0)$ and $(6,4)$ respectively.

a) Determine an equation of the straight line L which passes through C and is parallel to AB. **(4)**

b) Show that the angle ABC is $90°$. **(3)**

c) Calculate the distance AC. **(2)**

A circle passes through the points A, B and C.

d) Show that the equation of this circle is given by

$$x^2 + y^2 - 8x - 5y + 16 = 0.$$ **(5)**

e) Find the coordinates of the point other than the point C where L intersects the circle. **(5)**

Question 8

A cubic curve C_1 has equation

$$y=(x-8)\left(x^2-4x+3\right).$$

A quadratic curve C_2 has equation

$$y=(2x-3)(8-x).$$

a) Sketch on separate set of axes the graphs of C_1 and C_2.

The sketches must contain the coordinates of the points where each of the curves meet the coordinate axes. **(5)**

b) Hence find the solutions of the following equation.

$$(x-8)\left(x^2-4x+3\right)=(2x-3)(8-x).$$ **(6)**

Question 9

The points A and C have coordinates $(3,2)$ and $(5,6)$, respectively.

a) Find an equation for the perpendicular bisector of AC, giving the answer in the form $ax+by=c$, where a, b and c are integers. **(5)**

The perpendicular bisector of AC crosses the y axis at the point B.

The point D is such so that $ABCD$ is a rhombus.

b) Show that the coordinates of D are $(8,2)$. **(3)**

c) Calculate the area of the rhombus $ABCD$. **(4)**

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Question 10

The point P, whose x coordinate is $\frac{1}{4}$, lies on the curve with equation

$$y = \frac{k + 4x\sqrt{x}}{7x}, \quad x \in \mathbb{R}, \quad x > 0,$$

where k is a non zero constant.

a) Determine, in terms of k, the gradient of the curve at P. **(5)**

The tangent to the curve at P is parallel to the straight line with equation

$$44x + 7y - 5 = 0.$$

b) Find an equation of the tangent to the curve at P. **(7)**

Question 11

Find the exact solutions of the equation

$$2e^{2x} - 5e^{x} + 3e^{-x} = 4 . \qquad \textbf{(8)}$$

Question 1

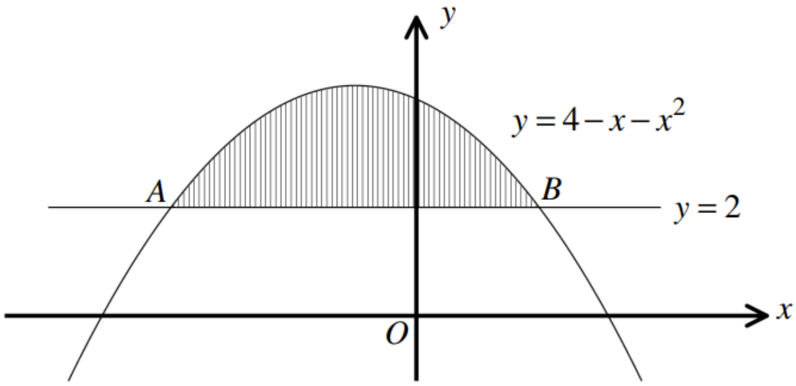
The straight line l_1 passes through the points $A(3,20)$ and $B(13,0)$.

The straight line l_2 has gradient $\frac{1}{3}$ and passes through the point $C(0,5)$.

The point D is the intersection of l_1 and l_2.

Show that the length of AD is $k\sqrt{5}$, where k is an integer. **(8)**

Question 2



The figure above shows a quadratic curve and a straight line with respective equations

$$y = 4 - x - x^2 \quad \text{and} \quad y = 2.$$

The points A and B are the points of intersection between the quadratic curve and the straight line.

a) Find the coordinates of A and B. **(3)**

b) Determine the exact area of the finite region bounded by the quadratic curve and the straight line, shown shaded in the above figure. **(5)**

Question 3

A circle C with centre at the point P and radius r, has equation

$$x^2 - 8x + y^2 - 2y = 0.$$

a) Find the value of r and the coordinates of P. **(3)**

b) Determine the coordinates of the points where C meets the coordinate axes. **(3)**

The points A, B and $Q(8,2)$ lie on C.

The straight line AB is diameter of the circle so that PQ is perpendicular to AB.

c) Calculate the coordinates of A and B. **(6)**

Question 4

A polynomial $f(x)$ is defined in terms of the constants a, b and c as

$$f(x) = 2x^3 + ax^2 + bx + c, \quad x \in \mathbb{R}.$$

It is further given that

$$f(2) = f(-1) = 0 \quad \text{and} \quad f(1) = -14.$$

a) Find the value of a, b and c. **(5)**

b) Sketch the graph of $f(x)$.

The sketch must include any points where the graph of $f(x)$ meets the coordinate axes. **(4)**

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Question 5

Solve the following trigonometric equation in the range given.

$$\frac{5\cos 2x + \sin 2x}{3\sin 2x} = 7, \quad -90° \le x < 90°. \qquad \textbf{(6)}$$

Question 6

$$x^3 - 4x + 1 = 0.$$

The above cubic equation has three real roots x_1, x_2 and x_3.

Use transformation arguments to find, in a simplified form, another cubic equation whose roots are

$$x_1 + 1, \qquad x_2 + 1, \qquad x_3 + 1.$$

(4)

Question 7

A curve C has equation

$$y = 4x^3 + 7x^2 + x + 11, \quad x \in \mathbb{R}.$$

The point P lies on C, where $x = -1$.

a) Find an equation of the tangent to C at P. **(4)**

The tangent to C at P meets C again at the point Q.

b) Determine the x coordinate of Q. **(5)**

Question 8

A quadratic curve has equation

$$f(x) \equiv 12x^2 + 4x - 161, \ x \in \mathbb{R}.$$

Express the above equation as the product of two linear factors.

A detailed method must be shown in this question. **(5)**

Question 9

Show that if x is numerically small

$$\left(2+x-x^2\right)^5 \approx A+Bx+Cx^3$$

where A, B and C are integers to be found. **(6)**

Question 10

$$f(x) = x^4 - 4x, \; x \in \mathbb{R}.$$

a) Find a simplified expression for

$$f(2+h) - f(2).$$ **(4)**

b) Use the formal definition of the derivative as a limit, to show that

$$f'(2) = 28.$$ **(3)**

Question 11

Find, **without** the use of any calculating aid, the solution of the equation

$$\frac{1}{2} \times 4^{2x} = 64^{64}. \qquad \textbf{(5)}$$

1. Sara is investigating the variation in daily maximum gust, t kn, for Camborne in June and July 1987.

She used the large data set to select a sample of size 20 from the June and July data for 1987. Sara selected the first value using a random number from 1 to 4 and then selected every third value after that.

(a) State the sampling technique Sara used.

(1)

(b) From your knowledge of the large data set explain why this process may not generate a sample of size 20.

(1)

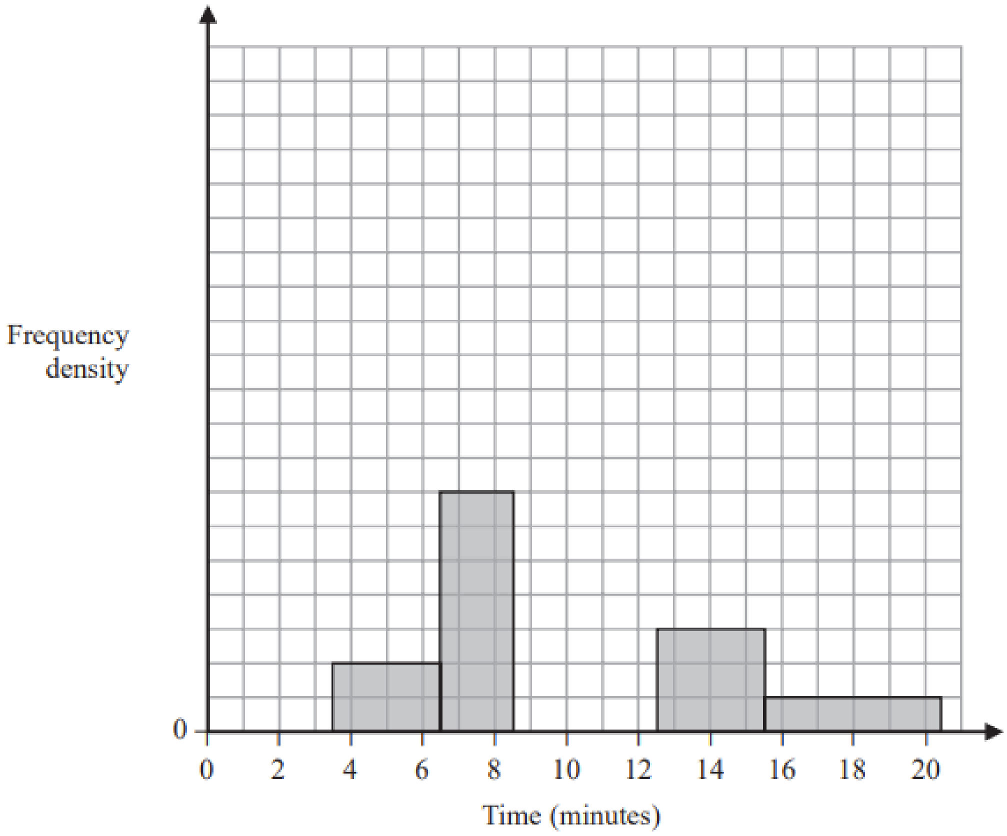
The data Sara collected are summarised as follows

$$n = 20 \qquad \sum t = 374 \qquad \sum t^2 = 7600$$

(c) Calculate the standard deviation.

(2)

2. The partially completed histogram and the partially completed table show the time, to the nearest minute, that a random sample of motorists was delayed by roadworks on a stretch of motorway.



Delay (minutes)	Number of motorists
4 – 6	6
7 – 8	
9	17
10 – 12	45
13 – 15	9
16 – 20	

Estimate the percentage of these motorists who were delayed by the roadworks for between 8.5 and 13.5 minutes.

(5)

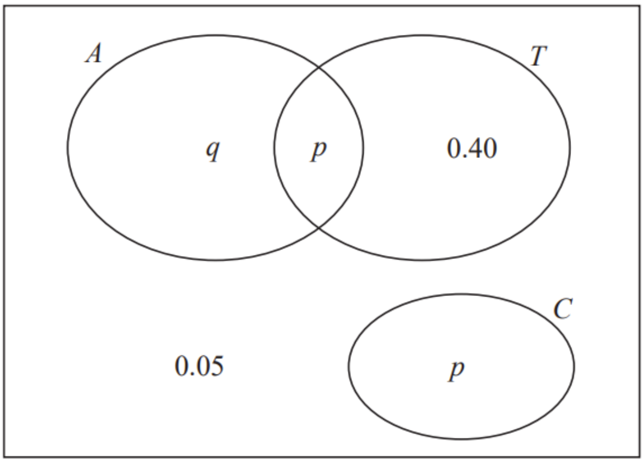
3. The Venn diagram shows the probabilities for students at a college taking part in various sports.

A represents the event that a student takes part in Athletics.

T represents the event that a student takes part in Tennis.

C represents the event that a student takes part in Cricket.

p and q are probabilities.



The probability that a student selected at random takes part in Athletics or Tennis is 0.75

(a) Find the value of p.

(1)

(b) State, giving a reason, whether or not the events A and T are statistically independent. Show your working clearly.

(3)

(c) Find the probability that a student selected at random does not take part in Athletics or Cricket.

(1)

4. Sara was studying the relationship between rainfall, r mm, and humidity, h%, in the UK. She takes a random sample of 11 days from May 1987 for Leuchars from the large data set.

She obtained the following results.

h	93	86	95	97	86	94	97	97	87	97	86
r	1.1	0.3	3.7	20.6	0	0	2.4	1.1	0.1	0.9	0.1

Sara examined the rainfall figures and found

$$Q_1 = 0.1 \qquad Q_2 = 0.9 \qquad Q_3 = 2.4$$

A value that is more than 1.5 times the interquartile range (IQR) above Q_3 is called an outlier.

(a) Show that $r = 20.6$ is an outlier.

(1)

(b) Give a reason why Sara might:

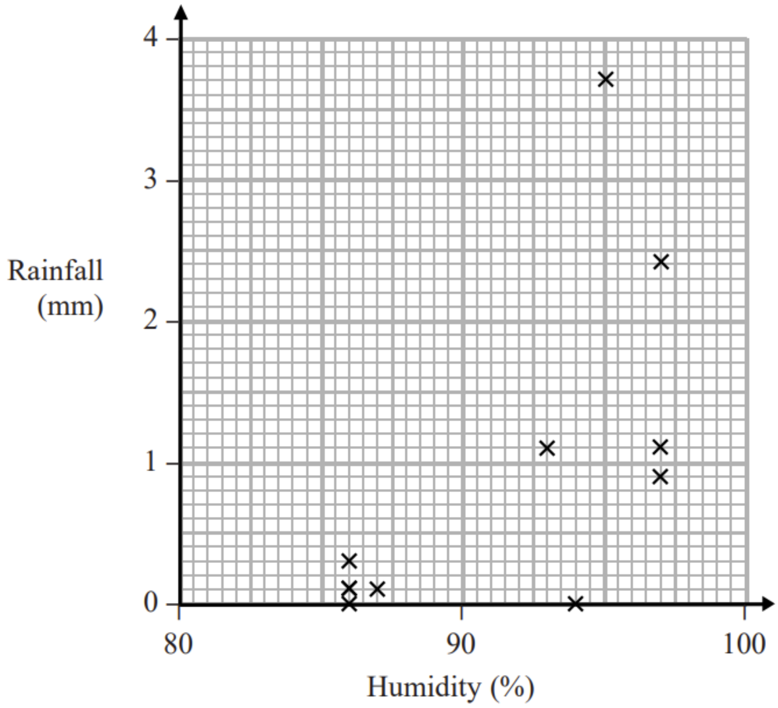
(i) include

(ii) exclude

this day's reading.

(2)

Sara decided to exclude this day's reading and drew the following scatter diagram for the remaining 10 days' values of r and h.



(c) Give an interpretation of the correlation between rainfall and humidity.

(1)

Question 4 continued

The equation of the regression line of r on h for these 10 days is $r = -12.8 + 0.15h$

(d) Give an interpretation of the gradient of this regression line.

(1)

(e) (i) Comment on the suitability of Sara's sampling method for this study.

(ii) Suggest how Sara could make better use of the large data set for her study.

(2)

5. (a) The discrete random variable $X \sim B(40, 0.27)$

Find $P(X \geqslant 16)$

(2)

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(b) Write down the hypotheses that should be used to test the manager's suspicion.

(1)

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(3)

(d) Find the actual significance level of a test based on your critical region from part (c).

(1)

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

(e) Comment on the manager's suspicion in the light of this observation.

(1)

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

(f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(1)

6.

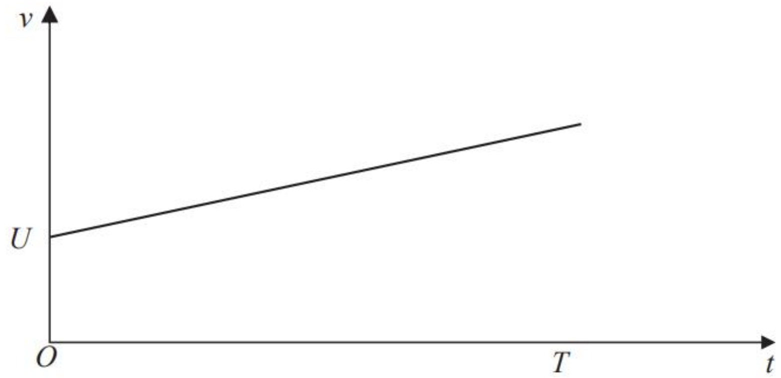


Figure 1

A car moves along a straight horizontal road. At time $t = 0$, the velocity of the car is U m s^{-1}. The car then accelerates with constant acceleration a m s^{-2} for T seconds. The car travels a distance D metres during these T seconds.

Figure 1 shows the velocity-time graph for the motion of the car for $0 \leqslant t \leqslant T$.

Using the graph, show that $D = UT + \frac{1}{2} aT^2$.

(No credit will be given for answers which use any of the kinematics (*suvat*) formulae listed under Mechanics in the AS Mathematics section of the formulae booklet.)

(4)

7. A car is moving along a straight horizontal road with constant acceleration. There are three points A, B and C, in that order, on the road, where $AB = 22$ m and $BC = 104$ m. The car takes 2 s to travel from A to B and 4 s to travel from B to C.

Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes A.

(7)

8. A bird leaves its nest at time $t = 0$ for a short flight along a straight line.

The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, s metres, of the bird from its nest at time t seconds is given by

$$s = \frac{1}{10}(t^4 - 20t^3 + 100t^2), \quad \text{where } 0 \leqslant t \leqslant 10$$

(a) Explain the restriction, $0 \leqslant t \leqslant 10$

(3)

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

(6)

9.

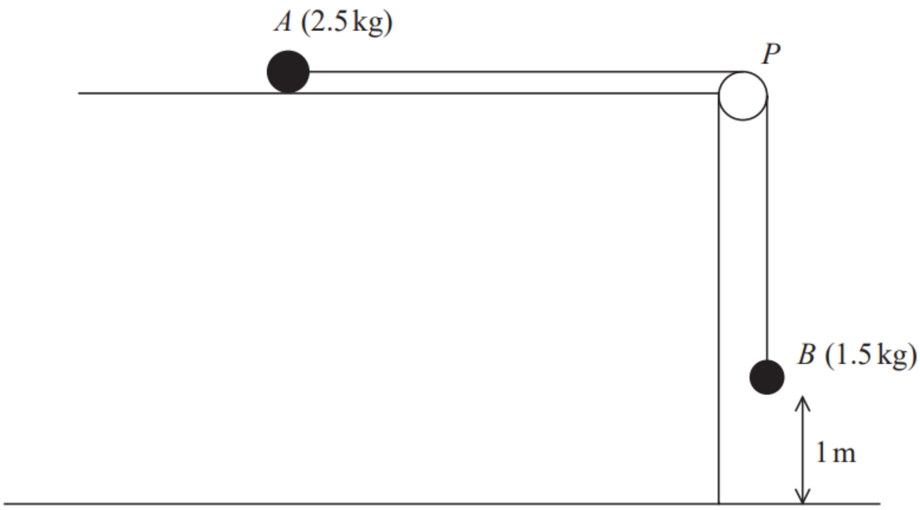


Figure 2

A small ball A of mass 2.5 kg is held at rest on a rough horizontal table.

The ball is attached to one end of a string.

The string passes over a pulley P which is fixed at the edge of the table. The other end of the string is attached to a small ball B of mass 1.5 kg hanging freely, vertically below P and with B at a height of 1 m above the horizontal floor.

The system is release from rest, with the string taut, as shown in Figure 2.

The resistance to the motion of A from the rough table is modelled as having constant magnitude 12.7 N. Ball B reaches the floor before ball A reaches the pulley.

The balls are modelled as particles, the string is modelled as being light and inextensible, the pulley is modelled as being small and smooth and the acceleration due to gravity, g, is modelled as being $9.8\,\mathrm{m\,s^{-2}}$.

(a) (i) Write down an equation of motion for A.

(ii) Write down an equation of motion for B.

(4)

(b) Hence find the acceleration of B.

(2)

(c) Using the model, find the time it takes, from release, for B to reach the floor.

(2)

(d) Suggest two improvements that could be made in the model.

(2)