Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Maximise $P = 8x + 20y + 40z$	B1	3.3
	$4x + 8y + 15z \leqslant 9000$ $x + 5y + 12z \leqslant 3600$	M1 A1	3.3 1.1b
	$x + y + z \leqslant 1600$	B1	3.3
	$-x + 3y \geqslant 0$ $(x, y, z \geqslant 0)$	B1	3.3
		(5)	
(b)	$4x + 8y + 15z + s_1 = 9000$ $x + 5y + 12z - s_2 + a_1 = 3600$ $x + y + z - s_3 + a_2 = 1600$ $-x + 3y + s_4 = 0$	B1 B1	1.1b 2.5
	$I = -(a_1 + a_2)$ where $a_1 = 3600 - x - 5y - 12z + s_2$ and $a_2 = 1600 - x - y - z + s_3$	M1	2.1
	$I - 2x - 6y - 13z + s_2 + s_3 = -5200$ and $P - 8x - 20y - 40z = 0$	A1	2.2a
	e.g. b.v. | x | y | z | s_1 | s_2 | s_3 | s_4 | a_1 | a_2 | Value s_1 | 4 | 8 | 15 | 1 | 0 | 0 | 0 | 0 | 0 | 9000 a_1 | 1 | 5 | 12 | 0 | -1 | 0 | 0 | 1 | 0 | 3600 a_2 | 1 | 1 | 1 | 0 | 0 | -1 | 0 | 0 | 1 | 1600 s_4 | -1 | 3 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 P | -8 | -20 | -40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 I | -2 | -6 | -13 | 0 | 1 | 1 | 0 | 0 | 0 | -5200	M1 A1	3.4 2.2a
		(6)	

(c)	<table><tr><th>b.v.</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>s_4</th><th>Value</th><th>Row Ops</th></tr><tr><td>s_3</td><td>0</td><td>0</td><td>0</td><td>$\frac{1}{3}$</td><td>$\frac{1}{3}$</td><td>1</td><td>0</td><td>200</td><td>$\frac{1}{3}r_1$</td></tr><tr><td>z</td><td>0</td><td>$\frac{4}{11}$</td><td>1</td><td>$-\frac{1}{33}$</td><td>$-\frac{4}{33}$</td><td>0</td><td>0</td><td>$\frac{1800}{11}$</td><td>$r_2 - \frac{1}{11}R_1$</td></tr><tr><td>x</td><td>1</td><td>$\frac{7}{11}$</td><td>0</td><td>$\frac{4}{11}$</td><td>$\frac{5}{11}$</td><td>0</td><td>0</td><td>$\frac{18000}{11}$</td><td>$r_3 + \frac{12}{11}R_1$</td></tr><tr><td>s_4</td><td>0</td><td>$\frac{40}{11}$</td><td>0</td><td>$\frac{4}{11}$</td><td>$\frac{5}{11}$</td><td>0</td><td>1</td><td>$\frac{18000}{11}$</td><td>$r_4 + \frac{12}{11}R_1$</td></tr><tr><td>P</td><td>0</td><td>$-\frac{4}{11}$</td><td>0</td><td>$\frac{56}{33}$</td><td>$-\frac{40}{33}$</td><td>0</td><td>0</td><td>$\frac{216000}{11}$</td><td>$r_5 + \frac{56}{11}R_1$</td></tr></table>	B1 M1 A1 A1 B1	1.1b 2.1 1.1b 1.1b 2.4
		(5)	
(d)	(i) £20 500	B1	3.4
	(ii) 1125 + 5(375) + 12(100) = (4200 minutes so) 70 (hours)	B1	2.2a
		(2)	
(e)	e.g. there is no guarantee that all the books will be sold	B1	3.5b
		(1)	
			(19 marks)

Notes for Question

(a)

B1: Correct objective function $(8x+20y+40z)$ plus 'maximise' or 'max' but not 'maximum' (and '$P =$' is not required)

M1: Either one correct inequality (need not be simplified) **or** both $4x+8y+15z \,,\!,\, k_1$ and $x+5y+12z \ldots k_2$ where $k_1, k_2 > 0$

A1: Both correct ($4x+8y+15z \,,\!,\, 9000$, $x+5y+12z \ldots 3600$) – allow equivalent answers (provided 4 terms only and integer coefficients e.g. $2x+10y+24z - 7200 \ldots 0$)

B1: CAO ($x+y+z \ldots 1600$ oe provided 4 terms only and integer coefficients)

B1: CAO ($3y \,,\!,\, x$ oe provided 2 terms only and integer coefficients)

(b) Note in this part that the numbering of the suffices for the slack, surplus and artificial variables seen in both the candidate's equations and Simplex tableau will most likely be different from what is seen in the MS (e.g. $4x+8y+15z+s_3 = 9000$ is correct). The correct values in the rows of the tableau do NOT imply the first four marks (the question explicitly asked for the constraints as equations and the rows for the two objectives to be explicitly stated)

B1: Any two correct inequalities converted into equations correctly (condone the same letter, say s_3, being used twice) – any equivalent forms of the correct equations are acceptable (e.g. variables do not need to be on the same side)

B1: All four correct equations (using distinct slack, surplus and artificial variables) – any equivalent forms of the correct equations are acceptable

M1: Using $I = -(a_1 + a_2)$ with their expressions for a_1 and a_2 (allow slips in forming I from their two expressions for the two artificial variables) but must be a clear intention to calculate $I = -(a_1 + a_2)$) – must be **exactly two** artificial variables in their expression for I for this mark

A1: CAO for I and P (must be stated as $P - 8x - 20y - 40z = 0$ (**A0** if = 0 missing) and $I - 2x - 6y - 13z + s_2 + s_3 = -5200$ - so variables on one side and constant on the other)

M1: setting up initial tableau – **all** six rows complete (with no blanks) and two correct rows (but ignore b.v. column for this mark)

A1: CAO (any equivalent correct form) - **note that the candidate's order in which the rows appear in the tableau (and choice of letter to represent the slack, surplus and artificial variables) may be different – check to ensure that the basic variable column is consistent with their choice of lettering for the slack and artificial variables**

(c)

B1: Pivot (top) row completely correct including change of b.v. (but not 'Row Ops' column)

M1: All **values** in one of the non-pivot rows correct (so ignore b.v. column and 'Row Ops' column) **or** one of the 'non zero and one' columns (which are y, s_1, s_2 or Value) correct (must have pivoted on the correct value)

A1: Row operations used correctly at least twice, i.e. two of the 'non zero and one' columns (s_1, s_2, y or Value) correct

A1: CAO **all** values including b.v. column – ignore 'Row Ops' column for this mark

B1: Correct row operations stated – alternatives are $\frac{1}{3}r_1, r_2 - \frac{1}{33}r_1, r_3 + \frac{12}{33}r_1, r_4 + \frac{12}{33}r_1, r_5 + \frac{56}{33}r_1$

(d)(i)

B1: CAO (20 500)

(ii)

B1: CAO (70 only)

(e)

B1: CAO – must **explicitly** mention the fact that it is possible that not all the books will be sold. Ignore reasons for why, provided they do not relate to the publisher producing less than the optimal number of books

Q2.

Question	Scheme	Marks	AOs
(a)	Constraints: $2x+3y+4z \leqslant 13$ $x-2y+2z \leqslant 8$ $3x-4z \leqslant 12$ $(x, y, z \geqslant 0)$	B1 B1	3.4 2.5
	Objective functions: Maximise $-2x+3y+z$ Minimise $2x-3y-z$	M1 A1	3.1a 2.2a
		(4)	
(b)	(Because M is big) the only negative in the objective row is the $2-4M$ so the pivot is from the x-column	B1	2.4
	The 3 in the a_2 row is the pivot as $\frac{12}{3}$ is less than both $\frac{8}{1}$ and $\frac{13}{2}$	B1	2.2a
		(2)	
			(6 marks)

Notes for Question

a1B1: One correct non-trivial inequality (allow strict inequality provided direction of inequality sign is correct) – equations with slack variables etc. scores no marks unless replaced with correct inequalities

a2B1: All three non-trivial inequalities correct

a1M1: Either expression stated correctly (allow equal to (or an inequality with) any letter e.g. $P=-2x+3y+z$ but not equal to a value e.g. = 0) – ignore any mention of maximum/minimum for this mark

a1A1: Both expressions correct including max/min correctly matched with each expression (allow equal to any letter only) – do not isw if they continue and place their expression(s) equal to a value(s)

b1B1: Correct reasoning that the pivot is a value from the x-column – as a minimum must state that the $2-4M$ is the only negative (condone most negative) in the objective row (allow profit row or P row, condone 'bottom row')

b2B1: Correct justification of why the 3 in the a_2 row or the 3 in the x column is the pivot – so **must** state the correct pivot in a clear unambiguous way (so just saying the pivot is 'the 3' is B0) **and** comparing or stating that $\frac{12}{3}$ or 4 is less than/least positive for both $\frac{8}{1}$ or 8 **and** $\frac{13}{2}$ or 6.5 – **must** see all three values so do check the table for possibly stating the θ values there. However, just stating that the 3 is the pivot because it is the smallest θ value (without seeing anywhere these θ values) is B0

Q3.

Question	Scheme	Marks	AOs
(a)	Maximise $P = 12x + 20y + 16z$	B1	3.3
	Subject to $2x+3y+z \le 80$ $4x+2y+3z \le 140$ $3x+4y+2z \le 96$ $x, y, z \ge 0$	M1 A1 A1 B1	3.3 1.1b 1.1b 3.3
		(5)	
(b)	The values must all be integers	B1	3.3
		(1)	
(c)	Variable y entered the basic variable column….	M1	2.4
	…so y was increased first	A1	2.2a
		(2)	
(d)	$(80+140+96)-(8+92) = 216$ plants	B1	3.2a
		(1)	

(e)	The next pivot must come from a column which has a negative value in the objective row so therefore the pivot must come from column z.	M1	2.4
	The pivot must be positive and the least of 92/2 = 46 and 24/0.5 = 48 so the pivot must be the 2 (from column z).	A1	2.2a
		(2)	
(f)	P + 10.5x + 3s + 3.5t = 756 so increasing x, s or t will decrease profit	B1	2.4
		(1)	
(g)	Make 1 *Drama* basket and 46 *Peaceful* baskets	B1	2.2a
		(1)	
(h)	The slack variable, r, associated with this type of plant, is currently at 31. Increasing the number of *Impact* plants by a further 20 would have no effect.	M1 A1	3.1b 3.2a
		(2)	
			(15 marks)

Notes:
(a) **B1:** Correct objective function/expression (accept in pence rather than pounds e.g. $1200x + 2000y + 1600z$) **M1:** Correct coefficients and correct right-hand side for at least one inequality – accept any inequality or equals **A1:** Two correct (non-trivial) inequalities **A1:** All three non-trivial inequalities correct **B1:** $x, y, z \geq 0$
(b) **B1:** CAO
(c) **M1:** Correct reasoning that y has become a basic variable **A1:** Correct deduction that y was therefore increased first
(d) **B1:** CAO
(e) **M1:** Correct reasoning given that the pivot value must come from column z **A1:** Correctly deduce (from correctly stated calculations) that the pivot value is the 2 in column z
(f) **B1:** States correct objective function and mention of increasing x, s or t will decrease profit
(g) **B1:** CAO – in context so not in terms of y and z
(h) **M1:** Identifies the slack variable r and its current value of 31 **A1:** Correct interpretation that increasing the number of Impact plants would have no effect

Q4.

Question	Scheme	Marks	AOs
(a)	Simplex can only work with ≤ constraints	B1	3.5b
		(1)	
(b)	M is an arbitrary large real number	B1	2.5
		(1)	
(c)	$x \geqslant 3 \Rightarrow x - s_3 + t_1 = 3$ where s_3 is a surplus variable and t_1 is an artificial variable	B1	2.4
		(1)	
(d)	Let $P = 3x + 2y + 2z - Mt_1$ (where M is an arbitrary large number) $\therefore P = 3x + 2y + 2z - M(3 - x + s_3)$ $= (3 + M)x + 2y + 2z - Ms_3 - 3M$ $\Rightarrow P - (3 + M)x - 2y - 2z + Ms_3 = -3M$	M1 A1	2.1 1.1b
		(2)	

(e)	b.v.	x	y	z	s_1	s_2	s_3	t_1	Value	Row Ops	M1 A1 A1	1.1b 1.1b 1.1b
	s_3	0	1	½	½	0	1	−1	19/2	$r_1 = (1/2)R_1$		
	s_2	0	3	−1/2	−1/2	1	0	0	5/2	$R_2 - r_1$		
	x	1	1	½	½	0	0	0	25/2	$R_3 + r_1$		
	P	0	1	−1/2	3/2	0	0	M	75/2	$R_4 + 3r_1$		

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value	Row Ops	M1 A1 B1	1.1b 1.1b 2.4
z	0	2	1	1	0	2	−2	19	$r_1 = 2R_1$		
s_2	0	4	0	0	1	1	−1	12	$R_2 + (1/2)r_1$		
x	1	0	0	0	0	−1	1	3	$R_3 - (1/2)r_1$		
P	0	2	0	2	0	1	$M-1$	47	$R_4 + (1/2)r_1$		

$P = 47, x = 3, y = 0, z = 19$	B1ft	1.1b
	(7)	
	(12 marks)	

Notes:

(a)

B1: Correctly states the limitation of the Simplex model – Simplex involves iterations which allow movement from one vertex in the feasible region to another vertex (in the feasible region). If all constraints are of the form $\leq$ this means that the origin is always a feasible solution and therefore can act as the initial starting point for the problem. However, the constraint $x \geqslant 3$ means that the origin is not feasible and so the algorithm is unable to begin.

(b)

B1: CAO including the correct mathematical language (must include 'arbitrary', 'large' and 'real')

(C)

(i)B1: Correctly states both the inequality $x \geq 3$ and the equation $x - s_3 + t_1 = 3$ together with an explanation of the meaning behind the variables s_3 and t_1

(ii)M1: $P = 3x + 2y + 2z - Mt_1$ and substitutes their expression for t_1

(ii)A1: Correct mathematical argument including sufficient detail to allow the line of reasoning to be followed to the correct conclusion – dependent on previous B mark in (c)

(d)

M1: Correct pivot located, attempt to divide row. If negative value used then no marks

A1: Pivot row correct (including change of b.v.) and row operations used at least once, one of columns y, z, s_1, t_1 or Value correct

A1: CAO for values (ignore b.v. column and Row Ops)

M1: Pivot row consistent (following their previous table) including change of b.v. and row operations used at least once, one of columns y, s_1, s_3, t_1 or Value correct

A1: CAO on final table (ignore Row Ops)

B1: The correct Row Operations explained either in terms of the 'old' or 'new' pivot rows

B1ft: Correctly states the final values of P, x, y and z from their correct corresponding rows of the final table

Q5.

(a) $P - 2x - 4y - 3z = 0$ (o.e.) B2,0 (2)

(b) $12x + 4y + 5z \leq 246$
$9x + 6y + 3z \leq 153$
$5x + 2y - 2z \leq 171$
B1 B1 B1 (3)

(c)

basic variable	x	y	z	r	s	t	Value
r	12	4	5	1	0	0	246
s	9	(6)	3	0	1	0	153
t	5	2	-2	0	0	1	171
P	-2	-4	-3	0	0	0	0

b.v.	x	y	z	r	s	t	Value	Row operations
r	6	0	3	1	$-\frac{2}{3}$	0	114	$R_1 - 4R_2$
y	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{6}$	0	25.5	$R_2 \div 6$
t	2	0	-3	0	$-\frac{1}{3}$	1	120	$R_3 - 2R_2$
P	4	0	-1	0	$\frac{2}{3}$	0	102	$R_4 + 4R_2$

m1 A1 m1 A1✓ B1✓

b.v.	x	y	z	r	s	t	Value	Row operations
z	2	0	1	$\frac{1}{3}$	$-\frac{2}{9}$	0	48	$R_1 \div 3$
y	$\frac{1}{2}$	1	0	$-\frac{1}{6}$	$\frac{5}{18}$	0	1.5	$R_2 - \frac{1}{2}R_1$
t	8	0	0	1	-1	1	264	$R_3 + 3R_1$
P	6	0	0	$\frac{1}{3}$	$\frac{4}{9}$	0	150	$R_4 + R_1$

m1 A1 m1 A1 (9)

(d) $P = 150$ $x = 0$ $y = 1.5$ $z = 48$
$r = 0$ $s = 0$ $t = 264$
m1 A1✓ A1✓ (3)

(e) (The third constraint) $t \neq 0$ B1✓ (1)

18

(a) B2 cao

(b) B1 cao } penalise same error once only
B1 cao } e.g. < rather than ≤, letter changes.
B1 cao } Penalise = or ≥ twice (> loses 2 also)

(c) M1 Correct pivot chosen – ie 6 in y column
A1 pivot row correct cao including basic variable
M1 Correct row operations used (all 3). Whole row ✓ M0, one non zero, none 1 term correct per row.
A1✓ non-pivot rows correct. ✓ one error in pivot row only
B1✓ One set of row operations correct

M1 } A1 last time (see above) must be this and following
A1 }
M1 }
A1 }

- negative pivot = M0 M0
- If already ✓ an error and a second pivot row error created M1 A0 M0
- starting with negatives in value column M0 M0

(d) M1 All 7 stated – negative = M0
A1✓ 5 correct ✓
A1✓ all 7 correct

(e) B1✓ cao must be just one non-zero slack variable

[Remove last 2 A or B marks earned]

1) chooses column x initially (pivot 9 in x column)

b.v.	x	y	z	r	s	t	value	Row ops
r	0	-4	1	1	$-\frac{4}{3}$	0	42	$R_1 - 12R_2$
x	1	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{1}{9}$	0	17	$R_2 \div 9$
t	0	$-\frac{14}{3}$	$-\frac{11}{3}$	0	$-\frac{5}{9}$	1	86	$R_3 - 5R_2$
p	0	$-\frac{8}{3}$	$-\frac{7}{3}$	0	$\frac{2}{9}$	0	34	$R_4 + 2R_2$

1st 5 marks available here

THEN

(a) chooses y next - gets main scheme 1st tableau [no mark]
chooses z next - as main scheme [last 4 mark now available]

OR

(b) chooses z next [no marks]

b.v.	x	y	z	r	s	t	value	Row ops
z	0	-4	1	1	$-\frac{4}{3}$	0	42	$R_1 \div 1$
x	1	2	0	$-\frac{1}{3}$	$\frac{5}{9}$	0	3	$R_2 - \frac{1}{3}R_1$
t	0	-16	0	$\frac{11}{3}$	$-\frac{45}{9}$	1	240	$R_3 + \frac{11}{3}R_1$
p	0	-12	0	$\frac{7}{3}$	$-\frac{56}{9}$	0	132	$R_4 + \frac{7}{3}R_1$

(i) chooses y next - gets final tableau [last 4 marks available]
(ii) chooses s next (gets one more tableau) then final tableau for which 4 marks available

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2) chooses column z initially (pivot 5 in z column)

b.v.	x	y	z	r	s	t	Value	Row ops
z	$\frac{12}{5}$	$\frac{4}{5}$	1	$\frac{1}{5}$	0	0	49.2	$R_1 \div 5$
s	$\frac{9}{5}$	$\frac{18}{5}$	-2	$-\frac{3}{5}$	1	0	5.4	$R_2 - 3R_1$
t	$\frac{49}{5}$	$\frac{18}{5}$	0	$\frac{2}{5}$	0	1	269.4	$R_3 + 2R_1$
p	$\frac{26}{5}$	$-\frac{8}{5}$	0	$\frac{3}{5}$	0	0	147.6	$R_4 + 3R_1$

1st 5 marks available here

chooses y next - gets final tableau [last 4 marks available]

Q6.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>(a)(i)</td><td>$P = x + 2y + 5z$</td><td>B1</td></tr>
<tr><td>(ii)</td><td>$15x - 2y + 3z \le 180$
$10x + y + z \le 80$
$x + 6y - 2z \le 100$</td><td>M1 A1 (3)</td></tr>
<tr><td>(b)</td><td>
<table>
<tr><td>b.v</td><td>x</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>value</td></tr>
<tr><td>r</td><td>15</td><td>-2</td><td>3</td><td>1</td><td>0</td><td>0</td><td>180</td></tr>
<tr><td>s</td><td>10</td><td>1</td><td>1</td><td>0</td><td>1</td><td>0</td><td>80</td></tr>
<tr><td>t</td><td>1</td><td>6</td><td>-2</td><td>0</td><td>0</td><td>1</td><td>100</td></tr>
<tr><td>P</td><td>-1</td><td>-2</td><td>-5</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table>
<table>
<tr><td>b.v</td><td>x</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>value</td><td>row ops</td></tr>
<tr><td>z</td><td>5</td><td>$-\frac{2}{3}$</td><td>1</td><td>$\frac{1}{3}$</td><td>0</td><td>0</td><td>60</td><td>$R_1 \div 3$</td></tr>
<tr><td>s</td><td>5</td><td>$\frac{\mathbf{5}}{\mathbf{3}}$</td><td>0</td><td>$-\frac{1}{3}$</td><td>1</td><td>0</td><td>20</td><td>$R_2 - R_1$</td></tr>
<tr><td>t</td><td>11</td><td>$\frac{14}{3}$</td><td>0</td><td>$\frac{2}{3}$</td><td>0</td><td>1</td><td>220</td><td>$R_3 + 2R_1$</td></tr>
<tr><td>P</td><td>24</td><td>$-\frac{16}{3}$</td><td>0</td><td>$\frac{5}{3}$</td><td>0</td><td>0</td><td>300</td><td>$R_4 + 5R_1$</td></tr>
</table>
<table>
<tr><td>b.v</td><td>x</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>value</td><td>row ops</td></tr>
<tr><td>z</td><td>7</td><td>0</td><td>1</td><td>$\frac{1}{5}$</td><td>$\frac{2}{5}$</td><td>0</td><td>68</td><td>$R_1 + \frac{2}{3}R_2$</td></tr>
<tr><td>y</td><td>3</td><td>1</td><td>0</td><td>$-\frac{1}{5}$</td><td>$\frac{3}{5}$</td><td>0</td><td>12</td><td>$R_2 \div \frac{5}{3}$</td></tr>
<tr><td>t</td><td>-3</td><td>0</td><td>0</td><td>$\frac{8}{5}$</td><td>$-\frac{14}{5}$</td><td>1</td><td>164</td><td>$R_3 - \frac{14}{3}R_2$</td></tr>
<tr><td>P</td><td>40</td><td>0</td><td>0</td><td>$\frac{3}{5}$</td><td>$\frac{16}{5}$</td><td>0</td><td>364</td><td>$R_4 + \frac{16}{3}R_2$</td></tr>
</table>
</td><td>M1 A1
M1 A1
M1 A1ft
M1 A1 (8)</td></tr>
<tr><td>(c)</td><td>$P = 364;\ x = 0;\ y = 12;\ z = 68;\ r = s = 0;\ t = 164$</td><td>M1 A1 (2)</td></tr>
<tr><td></td><td></td><td>13 marks</td></tr>
</table>

Question Number	Scheme	Marks

Notes for Question

ai1B1: CAO - allow in any equivalent form e.g. $P-x-2y-5z=0$ but not say $P=x+2y+5z=0$

aii1M1: Two inequalities (or equations with slack variables) correct

aii1A1: CAO

b1M1: Correct pivot located (3 in the z column), attempt to divide row. If choosing negative pivot then M0M0

b1A1: CAO pivot row correct including change of b.v. (so r must be changed to z)

b2M1: (ft) **All** values in one of the non-pivot rows correct **or** one of the non zero/one columns (x, y, r or value) correct following through their choice of pivot

b2A1: CAO on all values for the first iteration – ignore row ops and b.v. column for this mark

b3M1: Their correct pivot located following their first iteration, attempt to divide row. If choosing negative pivot M0M0 - however, allow recovery for the third and fourth M marks only if positive pivot chosen for the second iteration after a negative pivot chosen for the first iteration

b3A1ft: Their pivot row correct including change of b.v. following their first iteration

b4M1: (ft) **All** values in one of the non-pivot rows correct **or** one of the non zero/one columns (x, r, s or value) correct following through their choice of pivot

b4A1: CAO for all values and row operations for **both** iterations - including all eight row operations stated correctly (ignore b.v. column for this mark)

c1M1: Their correct values stated for at least P, x, y, z from their 'optimal' iteration so there must be no negatives in the profit row. Two M marks in (b) must have been awarded – the numerical value of P must be explicitly stated and not as part of an equation

c1A1: CAO for all seven values explicitly stated

If pivoting on any other positive value for the first iteration then candidates can score in (b) and (c):

(b) M0A0M1A0 M1A1M0A0 (c) M1A0 (so max. of 4/10)

Q7.

Question Number	Scheme	Marks
(a)	e.g. variable x was increased first, since it has become a basic variable	B1 (1)
(b)	<table><tr><th>b.v.</th><th>x</th><th>y</th><th>z</th><th>r</th><th>s</th><th>t</th><th>Value</th><th>row ops</th></tr><tr><td>z</td><td>0</td><td>2.5</td><td>1</td><td>0.5</td><td>-1.5</td><td>0</td><td>5</td><td>$R_1 \div 2$</td></tr><tr><td>x</td><td>1</td><td>-5.5</td><td>0</td><td>-1.5</td><td>5.5</td><td>0</td><td>3</td><td>$R_2 - 3R_1$</td></tr><tr><td>t</td><td>0</td><td>3.5</td><td>0</td><td>0.5</td><td>2.5</td><td>1</td><td>8</td><td>$R_3 + R_1$</td></tr><tr><td>P</td><td>0</td><td>13</td><td>0</td><td>2</td><td>-5</td><td>0</td><td>27</td><td>$R_4 + 4R_1$</td></tr></table>	M1 A1 M1 A1ft A1 (5)
(c)	$P+13y+2r-5s=27$	B1ft (1)
(d)	$P=27-13y-2r+5s$, so we can increase the profit by increasing s, hence not optimal	B2,1,0 (2) **9 marks**

Notes for Question

a1B1: e.g. identifies x, refers to basic variable (oe)

If pivoting on a negative value or on a value from the x or y column then no marks in (b), (c) or (d)

b1M1: Correct pivot located (2 in column z), attempt to divide row
b1A1: Pivot row correct **including change of b.v.** (so the r must be replaced with a z)
b2M1: **All** values in one of the non-pivot rows correct **or** one of the non zero-and-one columns (y, r, s or value) correct following through their choice of pivot from column z
b2A1ft: Row operations used correctly at least twice, i.e. **two** of the non zero-and-one columns (y, r, s or value) correct following through their choice of pivot from column z
b3A1: CAO – no follow through – all values and row operations correctly stated – allow if row operations given in terms of old row 1 – **ignore b.v. column for this mark**

c1B1ft: Dependent on the second M mark earned in (b) – must be an equation containing P (please note that $P=13y+2r-5s+27$ is incorrect)

d1B1: Must have gained both M marks in (b) **and** must refer to **increasing** y, r or s. Do not accept 'negatives in profit row' with no further explanation given
d2B1: CAO – dependent on the correct profit equation in (c). Specifically identifies s as the next variable that could be increased and states 'not optimal' (oe)

Pivoting on the 3 in the z column (can score a maximum of B1 M0A0M1A1A0 B1 B0B0 – so 4/9)

b.v.	x	y	z	r	s	t	Value
r	-2/3	11/3	0	1	-11/3	0	-2
z	1/3	2/3	1	0	1/3	0	6
t	1/3	5/3	0	0	13/3	1	9
P	4/3	17/3	0	0	7/3	0	31

Q8.

Question Number	Scheme	Marks
(a)	b.v. | x | y | z | r | s | t | Value | Row ops r | 0 | -5 | 5 | 1 | $-\frac{1}{2}$ | 0 | 5 | $R_1 - 2R_2$ x | 1 | $\frac{1}{2}$ | -2 | 0 | $\frac{1}{4}$ | 0 | 5 | $R_2 \div 4$ t | 0 | $-\frac{3}{2}$ | 6 | 0 | $-\frac{1}{4}$ | 1 | 3 | $R_3 - R_2$ P | 0 | $\frac{7}{2}$ | 1 | 0 | $\frac{3}{4}$ | 0 | 15 | $R_4 + 3R_2$	M1 A1 M1 A1ft A1 (5)
(b)	$P + \frac{7}{2}y + z + \frac{3}{4}s = 15$ $r = 5,\ s = 0,\ t = 3$	B1ft B1 (2) **7 marks**

Notes for Question

a1M1: Correct pivot located (4 in column x), attempt to divide row
a1A1: Pivot row correct **including change of b.v.**
a2M1: **All** values in one of the non-pivot rows correct **or** one of the non zero and one columns (y, z, s or value) correct following through their choice of pivot from column x
a2A1ft: Row operations used correctly at least twice, i.e. **two** of the non zero and one columns (y, z, s or value) correct following through their choice of pivot from column x
a3A1: CAO – no follow through – all values and row operations correctly stated – allow if row operations given in terms of old row 2 – **ignore b.v. column for this mark**

b1B1ft: Follow their profit equation from (a) dependent on scoring **both** M marks in (a)
b2B1: CAO (no follow through) for slack variables ($r = 5, s = 0, t = 3$)

Pivoting on the 1 in the x-column

b.v.	x	y	z	r	S	t	V
r	0	-2	-7	1	0	-2	-1
s	0	6	-24	0	1	-4	-12
x	1	-1	4	0	0	1	8
P	0	-1	19	0	0	3	24

Pivoting on the 2 in the x-column

b.v.	x	y	z	r	s	t	V
x	1	-2	0.5	0.5	0	0	7.5
s	0	10	-10	-2	1	0	-10
t	0	1	3.5	-0.5	0	1	0.5
P	0	-4	8.5	1.5	0	0	22.5

Q9.

Question Number	Scheme	Marks
(a)		

b.v	x	y	z	r	s	t	value	θ values
r	4	3	$\frac{5}{2}$	1	0	0	50	16.67
s	1	2	1	0	1	0	30	15
t	0	5	1	0	0	1	80	16
P	–25	–40	–35	0	0	0	0	

b.v.	x	y	z	r	s	t	value	Row ops
r	$\frac{5}{2}$	0	1	1	$-\frac{3}{2}$	0	5	R1 – 3R2
y	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	15	R2 ÷ 2
t	$-\frac{5}{2}$	0	$-\frac{3}{2}$	0	$-\frac{5}{2}$	1	5	R3 – 5R2
P	–5	0	–15	0	20	0	600	R4 + 40R2

M1 A1
B1
M1 A1

b.v.	x	y	z	r	s	t	value	Row ops
z	$\frac{5}{2}$	0	1	1	$-\frac{3}{2}$	0	5	R1stet
y	$-\frac{3}{4}$	1	0	$-\frac{1}{2}$	$\frac{5}{4}$	0	12.5	R2 – $\frac{1}{2}$R1
t	$\frac{5}{4}$	0	0	$\frac{3}{2}$	$-\frac{19}{4}$	1	12.5	R3 + $\frac{3}{2}$R1
P	32.5	0	0	15	–2.5	0	675	R4 + 15R1

B1
B1
M1 A1 **(9)**

(b)	$P + 32.5x + 15r – 2.5s = 675$	B1 **(1)**
(c)	$P = 675 – 32.5x – 15r + 2.5s$, so can increase profit by increasing s, hence not optimal.	B2,1,0 **(2)**
		12 marks

Notes for Question

a1M1: Correct pivot located, attempt to divide row. If choosing negative pivot no marks.
a1A1: Pivot row correct including change of b.v.
a1B1: Row operations CAO – allow if given in terms of old row 2.
a2M1: (ft) Correct row operations used at least once, column x, z, s or value correct.
a2A1: CAO on numbers (ignore row operations and b.v.).
a2B1: Correct pivot located and b.v. changed. If choosing negative pivot 2B0 3M0.
a3B1: Row operations CAO.
a3M1: (ft) Correct row operations used at least once, column x, r, s or value correct.
a3A1: CAO on numbers (ignore row operations and b.v.).
b1B1: CAO
c1B1ft: **Explanation**. Must have gained at least 2 M marks in (a) must refer to increasing x, r and s, (condone no ref to $y = z = t = 0$), must have correct signs in equation in (b). Do not accept 'negatives in profit row' o.e. alone.
c2DB1: CAO – dependent on correct equation in (b). Specifically identifies s as the next variable that could be increased.

Q10.

<table>
<tr><th>Qu</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>(a)</td><td>$3x+4y\leqslant 20\Rightarrow 3x+4y+s_1=20$

$3x+y\leqslant 8\Rightarrow 3x+y+s_4=8$</td><td>B1</td><td>2.5</td></tr>
<tr><td></td><td>$x\geqslant 1\Rightarrow x-s_2+a_1=1$

$2x+3y\geqslant 6\Rightarrow 2x+3y-s_3+a_2=6$</td><td>B1

B1</td><td>2.5

1.1b</td></tr>
<tr><td></td><td>$P=11x+ky-M(a_1+a_2)$
$a_1+a_2=7-3x-3y+s_2+s_3$
$P=11x+ky-M(7-3x-3y+s_2+s_3)$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$P-(11+3M)x-(k+3M)y+Ms_2+Ms_3=-7M$</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td>e.g.
<table>
<tr><th>b.v.</th><th>x</th><th>y</th><th>s_1</th><th>s_2</th><th>s_3</th><th>s_4</th><th>a_1</th><th>a_2</th><th>Value</th></tr>
<tr><td>s_1</td><td>3</td><td>4</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>20</td></tr>
<tr><td>a_1</td><td>1</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td></tr>
<tr><td>a_2</td><td>2</td><td>3</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>1</td><td>6</td></tr>
<tr><td>s_4</td><td>3</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>8</td></tr>
<tr><td>P</td><td>$-(11+3M)$</td><td>$-(k+3M)$</td><td>0</td><td>M</td><td>M</td><td>0</td><td>0</td><td>0</td><td>$-7M$</td></tr>
</table></td><td>M1

A1</td><td>3.3

2.2a</td></tr>
<tr><td></td><td></td><td>(7)</td><td></td></tr>
</table>

(b)	Using $x=\frac{18}{7}, y=\frac{2}{7}$ or stating $P=\frac{198}{7}+\frac{2}{7}k$	**B1**	3.4
	If optimal after the third iteration, then $\frac{11}{7}-\frac{3}{7}k \geqslant 0$ and $\frac{33}{7}-\frac{2}{7}k \geqslant 0$	**M1**	3.1a
	$(0<)\ k \leqslant \frac{11}{3}$	**A1**	2.2a
	Maximum P when $k=\frac{11}{3}$ and $P=11\left(\frac{18}{7}\right)+\frac{11}{3}\left(\frac{2}{7}\right)$	**dM1**	3.4
	Optimal value of P is $\frac{88}{3}$ accept answers from a stated value of k from $(0<)\ k \leqslant \frac{11}{3}$ so $\frac{198}{7} < P \leqslant \frac{88}{3}$	**A1**	2.2a
		(5)	
			(12 marks)

Notes:

If correct they must use two slack, two surplus and two artificial variables. Accept alternative letters for these as long as the artificial variables are clearly identifiable

Please check suffices on *s* and *a* terms carefully – they may be in a different order – check that these are consistent

a1B1: Correctly re-writing the two $\leqslant$ inequalities as equations with slack variables (can be implied by the corresponding two correct rows in Simplex tableau – our row 1 and 4)

a2B1: Correctly re-writing one of the $\geqslant$ inequalities as an equation with surplus and artificial variables (can be implied by a correct corresponding row in Simplex tableau – our row 2 or 3)

a3B1: Correctly re-writing both $\geqslant$ inequalities

a1M1: Forming an objective of the form $P = 11x + ky - M(a_1 + a_2)$ and substituting for a_1 and a_2 (we must see the substitution but this does not need to be a correct expression for this mark)

a1A1: CAO for new objective (accept equivalent equation with terms in x and y collected) (M1 A1 may be implied by a correct objective row in the tableau)

a2M1: Any two rows correct on the ft from the candidate's stated equations (ignore b.v. for this mark)

a2A1: CAO (including consistent b.v. column) – note that the candidate's order in which the rows appear in the tableau (and choice of slack variable) may be different

A fully correct tableau implies all marks in (a) provided that there are no errors seen in the formation of the objective function

b1B1: Either using the correct values of x and y in the objective function or stating $P = \frac{198}{7} + \frac{2}{7}k$

b1M1: Considering at least one of the expressions (s_3 or s_4 columns) in the P row that involve k and compare with 0 (accept any correct inequality or equals)

b1A1: Correct range of values for k (condone missing $0 <$) but must have considered both possibilities and chosen 11/3 (may be implied by subsequent working) (allow $k = 11/3$ stated as the maximum value)

b2dM1: Dependent on previous M mark – using their k in given P which must come from a correct inequality or equation (they may choose a value of k from the correct range e.g. $k = 3$)

b2A1: CAO - Correct value of P **for their choice of** k

(accept answers in the range $\frac{198}{7} < P \leqslant \frac{88}{3}$ if $k = 3$ $P = 204/7$)

Q11.

<table>
<tr><th>Qn</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>(a)</td><td>$x-y\geqslant 100\Rightarrow x-y-s_1+a_1=100$</td><td rowspan="3">B3,2,1</td><td>3.3</td></tr>
<tr><td></td><td>$x-5y\leqslant 0\Rightarrow x-5y+s_2=0$</td><td>3.3</td></tr>
<tr><td></td><td>$2x+3y\geqslant 350\Rightarrow 2x+3y-s_3+a_2=350$</td><td>3.3</td></tr>
<tr><td></td><td>$P=-x-y-M(a_1+a_2)$ and substitute expressions for a_1 and a_2
$\left(P+(1-3M)x+(1-2M)y+Ms_1+Ms_3=-450M\right)$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>e.g.
<table><tr><th>b.v</th><th>x</th><th>y</th><th>s_1</th><th>s_2</th><th>s_3</th><th>a_1</th><th>a_2</th><th>Value</th></tr><tr><td>a_1</td><td>1</td><td>-1</td><td>-1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>100</td></tr><tr><td>s_2</td><td>1</td><td>-5</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>a_2</td><td>2</td><td>3</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>1</td><td>350</td></tr><tr><td>P</td><td>$1-3M$</td><td>$1-2M$</td><td>M</td><td>0</td><td>M</td><td>0</td><td>0</td><td>$-450M$</td></tr></table></td><td>M1
A1</td><td>3.3
2.2a</td></tr>
<tr><td></td><td></td><td>(6)</td><td></td></tr>
<tr><td>(b)
(i)</td><td>$x=130$</td><td>B1</td><td>3.4</td></tr>
<tr><td>(ii)</td><td>When $x=130\Rightarrow y\leqslant 30, y\geqslant 26, y<130$ and $y\geqslant 30$</td><td>M1</td><td>3.1a</td></tr>
<tr><td></td><td>$y=30$</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4">(9 marks)</td></tr>
</table>

(a) NOTE: if correct they must use one slack, two surplus and two artificial variables. Accept alternative letters for these as long as the artificial variables are clearly identifiable

B1: one correct equation or two correct inequalities (do not accept strict inequalities)

B1: two correct equations or three correct inequalities

B1: all three equations correct (please check suffices on s and a terms carefully – they may be in a different order)

M1: setting up the new objective which must be $P = -x - y - M(a_1 + a_2)$ and an attempt to substitute for their a_1 and a_2 (accept any equivalent form, which may not be fully simplified) (accept the use of Q instead of P throughout)

M1: setting up initial tableau – all four rows complete with two correct rows (but ignore b.v. column for this mark) **(Note the order of rows may be different from above) Check that the slack, surplus and artificial variables correspond to their equations**

A1: CAO (any equivalent correct form, but the terms in the objective row must be simplified)

(b)

(i) B1: CAO ($x = 130$)

(ii) M1: Substitute $x = 130$ into candidate's inequalities from (a) (at least 3 inequalities seen or both $y \geqslant 30$ and $y \leqslant 30$) (condone $y \leqslant 130$)

A1: CAO ($y = 30$) we must see both $y \geqslant 30$ and $y \leqslant 30$ explicitly stated for this mark (must not follow from any incorrect working)

Q12.

<table>
<tr><th>Qu</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>(a)</td><td>Simplex can only be applied when the non-negativity constraints are $\leqslant$</td><td>B1</td><td>3.5b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>$3x+y+2z\leq 30\Rightarrow 3x+y+2z+s_1=30$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>$x-y+z\geq 8\Rightarrow x-y+z-s_2+a_1=8$</td><td>B1</td><td>2.5</td></tr>
<tr><td></td><td>$4y+2z\geq 15\Rightarrow 4y+2z-s_3+a_2=15$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>$P=2x+2y-z\Rightarrow P=2x+2y-z-M(a_1+a_2)$ together with $a_1+a_2=23-x-3y-3z+s_2+s_3$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$P-(2+M)x-(2+3M)y-(-1+3M)z+Ms_2+Ms_3=-23M$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>
<table>
<tr><td>b.v</td><td>x</td><td>y</td><td>z</td><td>s_1</td><td>s_2</td><td>s_3</td><td>a_1</td><td>a_2</td><td>Value</td></tr>
<tr><td>s_1</td><td>3</td><td>1</td><td>2</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>30</td></tr>
<tr><td>a_1</td><td>1</td><td>-1</td><td>1</td><td>0</td><td>-1</td><td>0</td><td>1</td><td>0</td><td>8</td></tr>
<tr><td>a_2</td><td>0</td><td>4</td><td>2</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>1</td><td>15</td></tr>
<tr><td>P</td><td>$-(2+M)$</td><td>$-(2+3M)$</td><td>$-(3M-1)$</td><td>0</td><td>M</td><td>M</td><td>0</td><td>0</td><td>$-23M$</td></tr>
</table>
</td><td>M1
A1</td><td>3.3
2.2a</td></tr>
<tr><td></td><td></td><td>(7)</td><td></td></tr>
<tr><td>(c)</td><td>$s_1=26.25, a_1=11.75, y=3.75, x=z=s_2=s_3=a_2=0$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(d)</td><td>The solution after the 1st iteration is not feasible because $a_1=11.75$ is an artificial variable which must be zero in a feasible solution</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(e)</td><td>The most negative value in the objective row is $2-1.5M$ so the pivot is a value from the z-column</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>The 0.5 in the y row is the pivot because $\frac{3.75}{0.5}$ is less than both $\frac{26.25}{1.5}$ and $\frac{11.75}{1.5}$</td><td>dB1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td colspan="4">(12 marks)</td></tr>
</table>

Notes for Question

(a)

B1: CAO – e.g. not all of the constraints are $\leq$, the origin is not a (basic feasible) solution of the LP

(b)

B1: CAO $3x+y+2z+s_1=30$ (may be seen in the simplex tableau – allow any s_i (or s) for s_1)

B1: CAO $x-y+z-s_2+a_1=8$ (may be seen in the simplex tableau – allow any consistent s_i for s_2 (or t say) but not the same s_i as in the previous mark and allow any a_i for a_1)

B1: CAO $4y+2z-s_3+a_2=15$ (may be seen in the simplex tableau – same conditions as above)

M1: setting up the new objective which must be $P=2x+2y-z-M(a_1+a_2)$ and substituting for their a_1 and a_2 (if no working then the **correct** objective line in the tableau implies this mark)

A1: CAO $P-(2+M)x-(2+3M)y-(-1+3M)z+Ms_2+Ms_3=-23M$ (any equivalent form – need not be factorised and does not need to be re-arranged into this form - if no working then the **correct** objective line in the tableau implies this mark)

M1: setting up initial tableau – all four rows complete with two correct rows (but ignore b.v. column for this mark)

A1: CAO (any equivalent correct form)

(c)

B1: CAO $s_1=26.25, a_1=11.75, y=3.75, x=z=s_2=s_3=a_2=0$ (ignore expression for P if given)

(d)

B1: correct reasoning of why the solution is not feasible e.g. a_1 is not zero but B0 for just stating that the artificial variable is non-zero (so must see either a_1 or 11.75 being stated as non-zero)

(e)

B1: correct reasoning of why the pivot comes from a value from the z-column so must say that the most negative value (in the objective row) is $2-1.5M$ (or this expression clearly implied)

dB1: correct justification of why the 0.5 in the third row is the next pivot (dependent on previous B mark) – so must compare or state that $\frac{3.75}{0.5}$ or 7.5 is less than both $\frac{26.25}{1.5}$ or 17.5 and $\frac{11.75}{1.5}$ or 7.8(3333….) – just stating that the 0.5 in the third row is the next pivot without reasoning is no marks in this part