Mark Scheme

Q1.

Question Number	Scheme		Marks
(a)	Area $BDE = \frac{1}{2}(5)^2(1.4)$	M1: Use of the correct formula or method for the area of the sector	M1A1
	$= 17.5\ (\text{cm}^2)$	A1: 17.5 oe	
			[2]
(b)	**Parts (b) and (c) can be marked together**		
	$6.1^2 = 5^2 + 7.5^2 - (2\times 5\times 7.5\cos DBC)$ or $\cos DBC = \frac{5^2+7.5^2-6.1^2}{2\times 5\times 7.5}$ (or equivalent)		M1
	M1: A correct statement involving the angle DBC		
	Angle $DBC = 0.943201...$	awrt 0.943	A1
	Note that work for (b) may be seen on the diagram or in part (c)		
			[2]
(c)	**Note that candidates may work in degrees in (c) (Angle $DBC = 54.04....$degrees)**		
	Area $CBD = \frac{1}{2}5(7.5)\sin(0.943)$		
	Angle $EBA = \pi - 1.4 - \text{"}0.943\text{"}$ **(Maybe seen on the diagram)**	Area $CBD = \frac{1}{2}5(7.5)\sin(\text{their } 0.943)$ or awrt 15.2. (Note area of $CBD = 15.177...$) A correct method for the area of triangle CBD which can be implied by awrt 15.2	M1
	$\pi - 1.4 -$ "their 0.943" A value for angle EBA of awrt 0.8 (from 0.7985926536... or 0.7983916536...) or value for angle EBA of $(1.74159... -$ their angle $DBC)$ would imply this mark.		M1
	$AB = 5\cos(\pi - 1.4 - \text{"}0.943\text{"})$ or $AE = 5\sin(\pi - 1.4 - \text{"}0.943\text{"})$		
		$AB = 5\cos(\pi - 1.4 - \text{their } 0.943)$ $AB = 5\cos(0.79859...) = 3.488577938...$ Allow M1 for $AB =$ awrt 3.49 Or $AE = 5\sin(\pi - 1.4 - \text{their } 0.943)$ $AE = 5\sin(0.79859...) = 3.581874365688...$ Allow M1 for $AE =$ awrt 3.58 **It must be clear that $\pi - 1.4 - \text{"}0.943\text{"}$ is being used for angle EBA.** **Note that some candidates use the sin rule here but it must be used correctly – do not allow mixing of degrees and radians.**	M1
	Area $EAB = \frac{1}{2}5\cos(\pi - 1.4 - \text{"}0.943\text{"})\times 5\sin(\pi - 1.4 - \text{"}0.943\text{"})$		
	This is dependent on the previous M1 and there must be no other errors in finding the area of triangle EAB		dM1
	Allow M1 for area $EAB =$ awrt 6.2		
	Area $ABCDE = 15.17... + 17.5 + 6.24... = 38.92...$		
		awrt 38.9	A1cso
			[5]
	Note that a sign error in (b) can give the obtuse angle (2.198....) and could lead to the correct answer in (c) – this would lose the final mark in (c)		**Total 9**

Q2.

Question	Scheme	Marks	AOs
(a)	$D = 5 + 2\sin(30\times 6.5)° =$ awrt 4.48m with units	B1	3.4
		(1)	
(b)	$3.8 = 5 + 2\sin(30t)° \Rightarrow \sin(30t)° = -0.6$	M1 A1	1.1b 1.1b
	$t = 10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

Notes:

(a)

B1: Scored for using the model ie. substituting $t = 6.5$ into $D = 5 + 2\sin(30t)°$ and stating $D = awrt$ 4.48m. The units must be seen somewhere in (a). So allow when $D = 4.482.. = 4.5$ m

Allow the mark for a correct answer without any working.

(b)

M1: For using $D = 3.8$ and proceeding to $\sin(30t)° = k$, $|k| \le 1$

A1: $\sin(30t)° = -0.6$ This may be implied by any correct answer for t such as $t = 7.2$

If the A1 implied, the calculation must be performed in degrees.

dM1: For finding **the first value** of t for their $\sin(30t)° = k$ after $t = 8.5$.

You may well see other values as well which is not an issue for this dM mark

(Note that $\sin(30t)° = -0.6 \Rightarrow 30t = 216.9°$ as well but this gives $t = 7.2$)

For the correct $\sin(30t)° = -0.6 \Rightarrow 30t = 323.1 \Rightarrow t =$ awrt 10.8

For the incorrect $\sin(30t)° = +0.6 \Rightarrow 30t = 396.9 \Rightarrow t =$ awrt 13.2

So award this mark if you see $30t = \text{invsin } their -0.6$ to give the first value of t where $30t > 255$

A1: Allow 10:46 a.m. (12 hour clock notation) or 10:46 (24 hour clock notation) oe

Allow 10:47 a.m. (12 hour clock notation) or 10:47 (24 hour clock notation) oe

DO NOT allow 646 minutes or 10 hours 46 minutes.

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Q3.

Question	Scheme	Marks	AOs
(a)	$a = 60$	B1	3.1b
	$2 = "60" - b(-20)^2 \Rightarrow b = ...$	M1	3.4
	$H = 60 - 0.145(t-20)^2$	A1	3.3
		(3)	
(b)	Height = 2 m	B1	3.4
		(1)	
(c)	$\alpha = 180$ or $\beta = 31$	M1	3.4
	$H = 29\cos(9t + 180)° + 31$	A1	3.3
		(2)	
(d)	e.g. "The model allows for more than one circuit"	B1	3.5a
		(1)	
			(7 marks)

Notes

(a)

B1: $a = 60$ (may be seen in their final equation of the model or implied by 60 substituted for a in the model)

M1: Attempts to find b by substituting in $t = 0$, $H = 2$ and their a and proceeding to a value for b.
May be seen as two simultaneous equations formed:
$2 = a - b(-20)^2$ and $60 = a - b(20-20)^2$ proceeding to a value for b

A1: $H = 60 - 0.145(t-20)^2$ or equivalent such as $H = -\frac{29}{200}t^2 + 5.8t + 2$ or $H = 60 - \frac{29}{200}(t-20)^2$ isw once a correct equation for the model is seen. Must be in terms of H and t. If they just state $a = 60$, $b = 0.145$ then A0
A correct answer with no working seen scores full marks.

(b)

B1: 2 cao (condone lack of units) This can be scored even if their model in (a) is incorrect (they may have used symmetry to determine this value)

(c)

M1: $(\alpha =)\,180$ or $(\beta =)\,31$ Condone $(\alpha =)\,\pi$

A1: $H = 29\cos(9t + 180)° + 31$ or equivalent e.g. $H = -29\cos(9t) + 31$ isw once a correct equation for the model is seen. Must be in terms of H and t. If they just state $\alpha = 180$, $\beta = 31$ then A0.
A correct equation with no working seen scores both marks. Does not require the degree symbol.

(d)

B1: Score for a reason which makes reference to any of

- the alternative model allows repetition (allow phrases e.g. "multiple cycles", "repeated circuits", "cyclical", "periodic", "loops around", "the original model can only go up and down once")
- the alternative model after 2 minutes the carriage will be back at the start (e.g."at 2 mins, $H = 2$")
- the original/quadratic model after 40 seconds (or any time after this) will be negative (e.g. "the height will be negative which cannot happen")
- the original model after 2 minutes would not be back at the start

Do not allow vague responses on their own e.g. "the original model is a parabola"
If calculations are used then they must be correct using a correct model (allow rounded or truncated)
Look for a valid reason and ignore reference to anything else as long as it does not contradict

t	0	5	10	15	20	25	30	35	40	45	50	55	60	80	100	120
h	2	27	46	56	60	56	46	27	2	-31	-71	-118	-172	-462	-868	-1390

Q4.

Question	Scheme	Marks
(a)	$2\cot 2x+\tan x\equiv\dfrac{2}{\tan 2x}+\tan x$	B1
	$\equiv\dfrac{(1-\tan^2 x)}{\tan x}+\dfrac{\tan^2 x}{\tan x}$	M1
	$\equiv\dfrac{1}{\tan x}$	M1
	$\equiv\cot x$	A1*
		(4)
(b)	$6\cot 2x+3\tan x=\operatorname{cosec}^2 x-2\Rightarrow 3\cot x=\operatorname{cosec}^2 x-2$	
	$\Rightarrow 3\cot x=1+\cot^2 x-2$	M1
	$\Rightarrow 0=\cot^2 x-3\cot x-1$	A1
	$\Rightarrow \cot x=\dfrac{3\pm\sqrt{13}}{2}$	M1
	$\Rightarrow \tan x=\dfrac{2}{3\pm\sqrt{13}}\Rightarrow x=..$	M1
	$\Rightarrow x=0.294,\,-2.848,\,-1.277\,,1.865$	A2,1,0
		(6)
		(10 marks)
(a)alt 1	$2\cot 2x+\tan x\equiv\dfrac{2\cos 2x}{\sin 2x}+\tan x$	B1
	$\equiv 2\dfrac{\cos^2 x-\sin^2 x}{2\sin x\cos x}+\dfrac{\sin x}{\cos x}$	M1
	$\equiv\dfrac{\cos^2 x-\sin^2 x}{\sin x\cos x}+\dfrac{\sin^2 x}{\sin x\cos x}\equiv\dfrac{\cos^2 x}{\sin x\cos x}$	M1
	$\equiv\dfrac{\cos x}{\sin x}$	
	$\equiv\cot x$	A1*
(a)alt 2	$2\cot 2x+\tan x\equiv 2\dfrac{(1-\tan^2 x)}{2\tan x}+\tan x$	B1M1
	$\equiv\dfrac{2}{2\tan x}-\dfrac{2\tan^2 x}{2\tan x}+\tan x$ or $\dfrac{(1-\tan^2 x)+\tan^2 x}{\tan x}$	
	$\equiv\dfrac{2}{2\tan x}=\cot x$	M1A1*

Alt (b)	$6\cot 2x+3\tan x=\operatorname{cosec}^2 x-2\Rightarrow\dfrac{3\cos x}{\sin x}=\dfrac{1}{\sin^2 x}-2$	
	$\left(\times\sin^2 x\right)\Rightarrow 3\sin x\cos x=1-2\sin^2 x$	M1
	$\Rightarrow\dfrac{3}{2}\sin 2x=\cos 2x$	M1A1
	$\Rightarrow\tan 2x=\dfrac{2}{3}\Rightarrow x=..$	M1
	$\Rightarrow x=0.294,\,-2.848,\,-1.277\,,1.865$	A2,1,0
		(6)

(a)

B1 States or uses the identity $2\cot 2x = \dfrac{2}{\tan 2x}$ or alternatively $2\cot 2x = \dfrac{2\cos 2x}{\sin 2x}$

This may be implied by $2\cot 2x = \dfrac{1-\tan^2 x}{\tan x}$. Note $2\cot 2x = \dfrac{1}{2\tan 2x}$ is B0

M1 Uses the correct double angle identity $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

Alternatively uses $\sin 2x = 2\sin x\cos x$, $\cos 2x = \cos^2 x - \sin^2 x$ oe and $\tan x = \dfrac{\sin x}{\cos x}$

M1 Writes their two terms with a single common denominator and simplifies to a form $\dfrac{ab}{cd}$.

For this to be scored the expression must be in either sin x and cos x or just tan x.

In alternative 2 it is for splitting the complex fraction into parts and simplifying to a form $\dfrac{ab}{cd}$.

You are awarding this for a correct method to proceed to terms like $\dfrac{\cos^2 x}{\sin x\cos x}$, $\dfrac{2\cos^3 x}{2\sin x\cos^2 x}$, $\dfrac{2}{2\tan x}$

A1* cso. For proceeding to the correct answer. This is a given answer and all aspects must be correct including the consistent use of variables. If the candidate approaches from both sides there must be a conclusion for this mark to be awarded. Occasionally you may see a candidate attempting to prove $\cot x - \tan x \equiv 2\cot 2x$. This is fine but again there needs to be a conclusion for the A1*

If you are unsure of how some items should be marked then please use review

(b)

M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)

WITH an attempt at using $\text{cosec}^2 x = \pm 1 \pm \cot^2 x$ to produce a quadratic equation in just $\cot x / \tan x$

A1 $\cot^2 x - 3\cot x - 1 = 0$ The = 0 may be implied by subsequent working

Alternatively accept $\tan^2 x + 3\tan x - 1 = 0$

M1 Solves a 3TQ=0 in $\cot x$ (or tan) using the formula or any suitable method for their quadratic to find at least one solution. Accept answers written down from a calculator. You may have to check these from an incorrect quadratic. FYI answers are $\cot x =$ awrt 3.30, -0.30

Be aware that $\cot x = \dfrac{3\pm\sqrt{13}}{2} \Rightarrow \tan x = \dfrac{-3\pm\sqrt{13}}{2}$

M1 For $\tan x = \dfrac{1}{\cot x}$ and using arctan producing at least one answer for x in degrees or radians.

You may have to check these with your calculator.

A1 Two of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3dp) in radians or degrees.

In degrees the answers you would accept are (awrt 2dp) $x = 16.8°, 106.8°, -73.2°, -163.2°$

A1 All four of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3 dp) with no extra solutions in the range $-\pi \leqslant x \leqslant \pi$

See main scheme for Alt to (b) using Double Angle formulae still entered M A M M A A in epen

1st M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)

then using $\cot x = \dfrac{\cos x}{\sin x}$, $\text{cosec}^2 x = \dfrac{1}{\sin^2 x}$ and $\times \sin^2 x$ to form an equation sin and cos

1st A1 For $\dfrac{3}{2}\sin 2x = \cos 2x$ or equivalent. **Attached to the next M**

2nd M1 For using both correct double angle formula

3rd M1 For moving from $\tan 2x = C$ to x=..using the correct order of operations.

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Q5.

Question Number	Scheme	Marks
(a)	$4\cos 2\theta + 2\sin 2\theta = R\cos(2\theta - \alpha)$	
	$R = \sqrt{4^2 + 2^2} = \sqrt{20} = \left(2\sqrt{5}\right)$	B1
	$\alpha = \arctan\left(\frac{1}{2}\right) = 26.565°... = \text{awrt } 26.57°$	M1A1
		(3)
(b)	$\sqrt{20}\cos(2\theta - 26.6) = 1 \Rightarrow \cos(2\theta - 26.57) = \frac{1}{\sqrt{20}}$	M1
	$\Rightarrow (2\theta - 26.57) = +77.1.. \Rightarrow \theta = ...$	dM1
	$\theta = \text{awrt } 51.8°$	A1
	$2\theta - 26.57 = \text{'}-77.1...\text{'} \Rightarrow \theta = -\text{awrt } 25.3°$	ddM1A1
		(5)
(c)	$k < -\sqrt{20}, k > \sqrt{20}$	B1ft either B1ft both
		(2)
		(10 marks)

You can marks parts (a) and (b) together as one.

(a)

B1 For $R = \sqrt{20} = 2\sqrt{5}$. Condone $R = \pm\sqrt{20}$

M1 For $\alpha = \arctan\left(\pm\frac{1}{2}\right)$ or $\alpha = \arctan(\pm 2)$ leading to a solution of α

Condone any solutions coming from $\cos\alpha = 4, \sin\alpha = 2$

Condone for this mark $2\alpha = \arctan\left(\pm\frac{1}{2}\right) \Rightarrow \alpha = ..$

If R has been used to find α award for only $\alpha = \arccos\left(\pm\frac{4}{'R'}\right)$ $\alpha = \arcsin\left(\pm\frac{2}{'R'}\right)$

A1 $\alpha = \text{awrt } 26.57°$

(b)

M1 Using part (a) and proceeding as far as $\cos(2\theta \pm \text{their } 26.57) = \frac{1}{\text{their } R}$.

This may be implied by $(2\theta \pm \text{their } 26.57) = \arccos\left(\frac{1}{\text{their} R}\right)$

Allow this mark for $\cos(\theta \pm \text{their } 26.57) = \frac{1}{\text{their } R}$

dM1 Dependent upon the first M1- it is for a correct method to find θ from their principal value
Look for the correct order of operations, that is dealing with the "26.57" before the "2".
Condone subtracting 26.57 instead of adding.

$\cos(2\theta \pm \text{their } 26.57) = ... \Rightarrow 2\theta \pm \text{their } 26.57 = \beta \Rightarrow \theta = \frac{\beta \pm \text{their } 26.57}{2}$

A1 awrt $\theta = 51.8°$

ddM1 For a correct method to find a secondary value of θ in the range
Either $2\theta \pm 26.57 = '-\beta' \Rightarrow \theta =$ OR $2\theta \pm 26.57 = 360 - '\beta' \Rightarrow \theta =$ THEN MINUS 180

A1 awrt $\theta = -25.3°$
Withhold this mark if there are extra solutions in the range.

Radian solution: Only lose the first time it occurs.

FYI. In radians desired accuracy is awrt 2 dp (a) $\alpha = 0.46$ and (b) $\theta_1 = 0.90, \theta_2 = -0.44$

Mixing degrees and radians only scores the first M

(c)

B1ft Follow through on their R. Accept decimals here including $\sqrt{20} \approx$ awrt 4.5 .

Score for one of the ends $k > \sqrt{20}, k < -\sqrt{20}$

Condone versions such as $g(\theta) > \sqrt{20}, y > \sqrt{20}$

or both ends including the boundaries $k \geqslant \sqrt{20}, k \leqslant -\sqrt{20}$

B1 ft For both intervals in terms of k.

Accept $k > \sqrt{20}$ or $k < -\sqrt{20}$. Accept $|k| > \sqrt{20}$ Accept $k \in (\sqrt{20}, \infty)(-\infty, -\sqrt{20})$

Condone $k > \sqrt{20}, k < -\sqrt{20}$ $k > \sqrt{20}$ and $k < -\sqrt{20}$ for both marks

but $-\sqrt{20} > k > \sqrt{20}$ is B1 B0

Q6.

Question	Scheme	Marks
(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=\frac{1}{2}\Rightarrow\alpha=26.57°$	M1A1
		(3)
(b)	$\frac{2}{2\cos\theta-\sin\theta-1}=15\Rightarrow\frac{2}{\sqrt{5}\cos(\theta+26.6°)-1}=15$	
	$\Rightarrow\cos(\theta+26.6°)=\frac{17}{15\sqrt{5}}=(\text{awrt }0.507)$	M1A1
	$\theta+26.57°=59.54°$	
	$\Rightarrow\theta=\text{awrt }33.0°$ or $\text{awrt }273.9°$	A1
	$\theta+26.6°=360°-\text{their '}59.5°\text{'}$	dM1
	$\Rightarrow\theta=\text{awrt }273.9°$ and $\text{awrt }33.0°$	A1
		(5)
(c)	$\theta-\text{their }26.57°=\text{their }59.54°\Rightarrow\theta=...$	M1
	$\theta=\text{awrt }86.1°$	A1
		(2)
		(10 marks)

(a)

B1 $R=\sqrt{5}$. Condone $R=\pm\sqrt{5}$ Ignore decimals

M1 $\tan\alpha=\pm\frac{1}{2}$, $\tan\alpha=\pm\frac{2}{1}\Rightarrow\alpha=...$

If their value of R is used to find the value of α only accept $\cos\alpha=\pm\frac{2}{R}$ OR $\sin\alpha=\pm\frac{1}{R}\Rightarrow\alpha=...$

A1 $\alpha=\text{awrt }26.57°$

(b)

M1 Attempts to use part (a) $\Rightarrow\cos(\theta\pm\text{their }26.6°)=K$, $|K|\leqslant 1$

A1 $\cos(\theta\pm\text{their }26.6°)=\frac{17}{15\sqrt{5}}=(\text{awrt }0.507)$. Can be implied by $(\theta\pm\text{their }26.6°)=\text{awrt }59.5°/59.6°$

A1 One solution correct, $\theta=\text{awrt }33.0°$ or $\theta=\text{awrt }273.9°$ Do not accept 33 for 33.0.

dM1 Obtains a second solution in the range. It is dependent upon having scored the previous M.
Usually for $\theta\pm\text{their }26.6°=360°-\text{their }59.5°\Rightarrow\theta=...$

A1 Both solutions $\theta=\text{awrt }33.0°$ and $\text{awrt }273.9°$. Do not accept 33 for 33.0.
Extra solutions inside the range withhold this A1. Ignore solutions outside the range $0\leqslant\theta<360°$

(c)

M1 $\theta-\text{their }26.57°=\text{their }59.54°\Rightarrow\theta=...$
Alternatively $-\theta+\text{their }26.6°=-\text{their }59.5°\Rightarrow\theta=...$
If the candidate has an incorrect sign for α, for example they used $\cos(\theta-26.57°)$ in part (b) it would be scored for $\theta+\text{their }26.57°=\text{their }59.54°\Rightarrow\theta=...$

A1 awrt 86.1° ONLY. Allow both marks following a correct (a) and (b)
They can restart the question to achieve this result. Do not award if 86.1 was their smallest answer in (b). This occurs when they have $\cos(\theta-26.57°)$ instead of $\cos(\theta+26.57°)$ in (b)

Answers in radians: Withhold only one A mark, the first time a solution in radians appears
FYI (a) $\alpha=0.46$ (b) $\theta_1=\text{awrt }0.58$ and $\theta_2=\text{awrt }4.78$ (c) $\theta_3=\text{awrt }1.50$. Require 2 dp accuracy

Q7.

Question	Scheme		Marks	AOs
(a)	$\frac{1}{\cos\theta}+\tan\theta=\frac{1+\sin\theta}{\cos\theta}$ or $\frac{(1+\sin\theta)\cos\theta}{\cos^2\theta}$		M1	1.1b
	$=\frac{1+\sin\theta}{\cos\theta}\times\frac{1-\sin\theta}{1-\sin\theta}=\frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}=\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$ or $\frac{(1+\sin\theta)\cos\theta}{\cos^2\theta}=\frac{(1+\sin\theta)\cos\theta}{1-\sin^2\theta}=\frac{(1+\sin\theta)\cos\theta}{(1+\sin\theta)(1-\sin\theta)}$		dM1	2.1
	$=\frac{\cos\theta}{1-\sin\theta}$ *		A1*	1.1b
			(3)	
(b)	$\frac{1}{\cos 2x}+\tan 2x=3\cos 2x$ $\Rightarrow 1+\sin 2x=3\cos^2 2x=3(1-\sin^2 2x)$	$\frac{\cos 2x}{1-\sin 2x}=3\cos 2x$ $\Rightarrow \cos 2x=3\cos 2x(1-\sin 2x)$	M1	2.1
	$\Rightarrow 3\sin^2 2x+\sin 2x-2=0$	$\Rightarrow \cos 2x(2-3\sin 2x)=0$	A1	1.1b
	$\sin 2x=\frac{2}{3},\ (-1)\Rightarrow 2x=...\Rightarrow x=...$		M1	1.1b
	$x=20.9°,\ 69.1°$		A1 A1	1.1b 1.1b
			(5)	
			(8 marks)	

Notes

(a) If starting with the LHS: Condone if another variable for θ is used except for the final mark

M1: Combines terms with a common denominator. The numerator must be correct for their common denominator.

dM1: Either:

- $\frac{1+\sin\theta}{\cos\theta}$: Multiplies numerator and denominator by $1-\sin\theta$, uses the difference of two squares and applies $\cos^2\theta=1-\sin^2\theta$
- $\frac{(1+\sin\theta)\cos\theta}{\cos^2\theta}$: Uses $\cos^2\theta=1-\sin^2\theta$ on the denominator, applies the difference of two squares

It is dependent on the previous method mark.

A1*: Fully correct proof with correct notation and no errors in the main body of their work. Withhold this mark for writing eg sin instead of $\sin\theta$ anywhere in the solution and for eg $\sin\theta^2$ instead of $\sin^2\theta$

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Alt(a) If starting with the RHS: Condone if another variable is used for θ except for the final mark

M1: Multiplies by $\frac{1+\sin\theta}{1+\sin\theta}$ leading to $\frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$ or

Multiplies by $\frac{\cos\theta}{\cos\theta}$ leading to $\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$

dM1: Applies $\cos^2\theta = 1-\sin^2\theta$ and cancels the $\cos\theta$ factor from the numerator and denominator leading to $\frac{1+\sin\theta}{\cos\theta}$ or

Applies $\cos^2\theta = 1-\sin^2\theta$ and uses the difference of two squares leading to $\frac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$

It is dependent on the previous method mark.

A1*: Fully correct proof with correct notation and no errors in the main body of their work. If they work from both the LHS and the RHS and meet in the middle with both sides the same then they need to conclude at the end by stating the original equation.

(b) ***Be aware that this can be done entirely on their calculator which is not acceptable***

M1: Either multiplies through by cos $2x$ and applies $\cos^2 2x = 1-\sin^2 2x$ to obtain an equation in sin $2x$ only or alternatively sets $\frac{\cos 2x}{1-\sin 2x} = 3\cos 2x$ and multiplies by $1-\sin 2x$

A1: Correct equation or equivalent. The = 0 may be implied by their later work (Condone notational slips in their working)

M1: Solves for sin $2x$, uses arcsin to obtain at least one value for $2x$ and divides by 2 to obtain at least one value for x. The roots of the quadratic can be found using a calculator. They cannot just write down values for x from their quadratic in sin$2x$

A1: For 1 of the required angles. Accept awrt 21 or awrt 69. Also accept awrt 0.36 rad or awrt 1.21 rad

A1: For both angles (awrt 20.9 and awrt 69.1) and no others inside the range. If they find $x = 45$ it must be rejected. (Condone notational slips in their working)

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Q8.

Question Number	Scheme		Marks
(a)	Way 1 $1-\sin^2 x = 8\sin^2 x - 6\sin x$ E.g. $9\sin^2 x - 6\sin x = 1$ or $9\sin^2 x - 6\sin x - 1 = 0$ or $9\sin^2 x - 6\sin x + 1 = 2$ So $9\sin^2 x - 6\sin x + 1 = 2$ or $(3\sin x - 1)^2 - 2 = 0$ so $(3\sin x - 1)^2 = 2$ or $2 = (3\sin x - 1)^2$ *	Way 2 $2 = (3\sin x - 1)^2$ gives $9\sin^2 x - 6\sin x + 1 = 2$ so $\sin^2 x + 8\sin^2 x - 6\sin x + 1 = 2$ so $8\sin^2 x - 6\sin x = 1 - \sin^2 x$ $8\sin^2 x - 6\sin x = \cos^2 x$ *	B1 M1 A1cso* **(3)**
(b)	Way 1: $(3\sin x - 1) = (\pm)\sqrt{2}$	Way 2: Expands $(3\sin x - 1)^2 = 2$ and uses quadratic formula on 3TQ	M1
	$\sin x = \dfrac{1 \pm \sqrt{2}}{3}$ or awrt 0.8047 **and** awrt − 0.1381		A1
	$x = 53.58$, 126.42 (or 126.41), 352.06, 187.94		dM1A1 A1 **(5)** **[8]**

	Notes
(a)	**Way 1** B1: Uses $\cos^2 x = 1 - \sin^2 x$ M1: Collects $\sin^2 x$ terms to form a three term quadratic or into a suitable completed square format. May be sign slips in the collection of terms. A1*: cso This needs an intermediate step from 3 term quadratic and no errors in answer and printed answer stated but allow $2 = (3\sin x - 1)^2$. If sin is used throughout instead of sin$x$ it is A0. **Way 2** B1: **Needs correct expansion** and **split** M1: Collects $1 - \sin^2 x$ together A1*: Conclusion and no errors seen
(b)	M1: Square roots both sides(Way 1), or expands and uses quadratic formula (Way 2) Attempts at factorization after expanding are M0. A1: **Both** correct answers for sin$x$ (need plus and minus). Need not be simplified. dM1: Uses inverse sin to give one of the given correct answers 1st A1: Need two correct angles (allow awrt) Note that the scheme allows 126.41 in place of 126.42 though 126.42 is preferred A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore) (**Premature approximation**:– in the final three marks lose first A1 then ft other angles for second A mark) Do **not** require degrees symbol for the marks **Special case: Working in radians** M1A1A0 for the *correct* 0.94, 2.21, 6.14, 3.28

Q9.

Question Number	Scheme	Marks
(a)	$\sin 2x - \tan x = 2\sin x\cos x - \tan x$	M1
	$= \dfrac{2\sin x\cos^2 x}{\cos x} - \dfrac{\sin x}{\cos x}$	M1
	$= \dfrac{\sin x}{\cos x} \times (2\cos^2 x - 1)$	
	$= \tan x\cos 2x$	dM1 A1*
		(4)
(b)	$\tan x\cos 2x = 3\tan x\sin x \Rightarrow \tan x(\cos 2x - 3\sin x) = 0$	
	$\cos 2x - 3\sin x = 0$	M1
	$\Rightarrow 1 - 2\sin^2 x - 3\sin x = 0$	M1
	$\Rightarrow 2\sin^2 x + 3\sin x - 1 = 0 \Rightarrow \sin x = \dfrac{-3 \pm \sqrt{17}}{4} \Rightarrow x = ...$	M1
	Two of $x = 16.3°, 163.7°, 0, 180°$	A1
	All four of $x = 16.3°, 163.7°, 0, 180°$	A1
		(5)
		(9 marks)

(a)

M1 Uses a correct double angle identity involving $\sin 2x$ Accept $\sin(x+x) = \sin x\cos x + \cos x\sin x$

M1 Uses $\tan x = \dfrac{\sin x}{\cos x}$ with $\sin 2x = 2\sin x\cos x$ and attempts to combine the two terms using a common denominator. This can be awarded on two separate terms with a common denominator.
Alternatively uses $\sin x = \tan x\cos x$ and attempts to combine two terms using factorisation of $\tan x$

dM1 Both M's must have been scored. Uses a correct double angle identity involving $\cos 2x$.

A1* A fully correct solution with no errors or omissions. All notation must be correct and variables must be consistent
Withhold this mark if for instance they write $\tan x = \dfrac{\sin}{\cos}$

If the candidate $\times\cos x$ on line 1 and/or $\div\sin x$ they cannot score any more than one mark unless they are working with both sides of the equation or it is fully explained.

(b)

M1 The $\tan x$ must be cancelled or factorised out to produce $\cos 2x - 3\sin x = 0$ or $\dfrac{\cos 2x}{\sin x} = 3$ oe Condone slips

M1 Uses $\cos 2x = 1 - 2\sin^2 x$ to form a 3TQ=0 in $\sin x$ The = 0 may be implied by later work

M1 Uses the formula/completion of square or GC with invsin to produce at least one value for x
It may be implied by one correct value.
This mark **can** be scored from factorisation of their 3TQ in $\sin x$ **but only if** their quadratic factorises.

A1 Two of $x = 0, 180°$, awrt $16.3°$, awrt $163.7°$ or in radians two of awrt 0.28, 2.86, 0 and π or 3.14
This mark can be awarded as a SC for those students who just produce $0, 180°$ (or 0 and π) from $\tan x = 0$ or $\sin x = 0$.

A1 All four values in degrees $x = 0, 180°$, awrt $16.3°$, awrt $163.7°$ and no extra's inside the range $0 ,, x < 360°$.
Condone $0 = 0.0$ and $180° = 180.0°$ Ignore any roots outside range.

Online Maths

Alternatives to parts (a) and (b)

(a) Alt 1	$\tan x\cos 2x = \tan x\left(2\cos^2 x - 1\right)$	M1
	$= 2\tan x\cos^2 x - \tan x$	
	$= 2\dfrac{\sin x}{\cos x}\cos^2 x - \tan x$	M1
	$= 2\sin x\cos x - \tan x$	
	$= \sin 2x - \tan x$	dM1 A1
		(4)

a) Alt 1 Starting from the rhs

M1 Uses a correct double angle identity for $\cos 2x$. Accept any correct version including $\cos(x+x) = \cos x\cos x - \sin x\sin x$

M1 **Uses** $\tan x = \dfrac{\sin x}{\cos x}$ with $\cos 2x = 2\cos^2 x - 1$ and attempts to multiply out the bracket

dM1 Both M's must have been scored. It is for using $2\sin x\cos x = \sin 2x$

A1* A fully correct solution with no errors or omissions. All notation must be correct and variables must be consistent.

See Main scheme for how to deal with candidates who $\div \tan x$

(a) Alt 2	$\sin 2x - \tan x \equiv \tan x\cos 2x$	
	$2\sin x\cos x - \tan x \equiv \tan x(2\cos^2 x - 1)$	M1
	$2\sin x\cos x - \cancel{\tan x} \equiv 2\tan x\cos^2 x - \cancel{\tan x}$	
	$2\sin x\cos x \equiv 2\dfrac{\sin x}{\cos x}\cos^2 x$	M1
	$2\sin x\cos x \equiv 2\sin x\cos x$	dM1
	+statement that it must be true	A1*

a) Alt 2 Candidates who use both sides

M1 Uses a correct double angle identity involving $\sin 2x$ **or** $\cos 2x$. Can be scored from either side Accept $\sin(x+x) = \sin x\cos x + \cos x\sin x$ or $\cos(x+x) = \cos x\cos x - \sin x\sin x$

M1 **Uses** $\tan x = \dfrac{\sin x}{\cos x}$ with $\cos 2x = 2\cos^2 x - 1$ and cancels the $\tan x$ term from both sides

dM1 Uses a correct double angle identity involving $\sin 2x$ Both previous M's must have been scored

A1* A fully correct solution with no errors or omissions AND statement "hence true", "a tick", "QED". W[5] All notation must be correct and variables must be consistent

....

It is possible to solve part (b) without using the given identity. There are various ways of doing this, one of which is shown below.

$\sin 2x - \tan x = 3\tan x\sin x \Rightarrow \quad 2\sin x\cos x - \dfrac{\sin x}{\cos x} = 3\dfrac{\sin x}{\cos x}\sin x$

$2\sin x\cos^2 x - \sin x = 3\sin^2 x$	M1 Equation in $\sin x$ and $\cos x$
$2\sin x\left(1-\sin^2 x\right) - \sin x = 3\sin^2 x$	M1 Equation in $\sin x$ only
$\left(2\sin^2 x + 3\sin x - 1\right)\sin x = 0$	
$x = ..$	M1 Solving equation to find at least one x
Two of $x = 16.3°, 163.7°, 0, 180°$	A1
All four of $x = 16.3°, 163.7°, 0, 180°$ and no extras A1	

....

Q10.

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td></td><td colspan="2">(i) $9\sin(\theta+60^\circ)=4;\ 0\le\theta<360^\circ$
(ii) $2\tan x-3\sin x=0;\ -\pi\le x<\pi$</td><td></td></tr>
<tr><td>(i)</td><td>$\sin(\theta+60^\circ)=\frac{4}{9}$, so $(\theta+60^\circ)=26.3877...$
$(\alpha=26.3877...)$</td><td>Sight of $\sin^{-1}\left(\frac{4}{9}\right)$ or awrt $26.4°$ or 0.461^c
Can also be implied for $\theta=$ awrt -33.6 (i.e. $26.4-60$)</td><td>M1</td></tr>
<tr><td></td><td>So, $\theta+60^\circ=\{153.6122...,\ 386.3877...\}$</td><td>$\theta+60^\circ$ = either "$180-$ their α" or "$360^\circ+$ their α" and not for $\theta=$ either "$180-$ their α" or "$360^\circ+$ their α". This can be implied by later working. The candidate's α could also be in radians but do not allow mixing of degrees and radians.</td><td>M1</td></tr>
<tr><td></td><td>and $\theta=\{93.6122...,\ 326.3877...\}$</td><td>A1: At least one of awrt 93.6° or awrt 326.4°
A1: Both awrt 93.6° and awrt 326.4°</td><td>A1 A1</td></tr>
<tr><td></td><td colspan="2">Both answers are cso and must come from correct work</td><td></td></tr>
<tr><td></td><td colspan="2">Ignore extra solutions outside the range.
In an otherwise fully correct solution deduct the final A1for any extra solutions in range</td><td></td></tr>
<tr><td></td><td></td><td></td><td>[4]</td></tr>
<tr><td>(ii)</td><td>$2\left(\frac{\sin x}{\cos x}\right)-3\sin x=0$</td><td>Applies $\tan x=\frac{\sin x}{\cos x}$</td><td>M1</td></tr>
<tr><td></td><td colspan="2">Note: Applies $\tan x=\frac{\sin x}{\cos x}$ can be implied by $2\tan x-3\sin x=0\Rightarrow\tan x(2-3\cos x)$</td><td></td></tr>
<tr><td></td><td colspan="2">$2\sin x-3\sin x\cos x=0$</td><td></td></tr>
<tr><td></td><td colspan="2">$\sin x(2-3\cos x)=0$</td><td></td></tr>
<tr><td></td><td>$\cos x=\frac{2}{3}$</td><td>$\cos x=\frac{2}{3}$</td><td>A1</td></tr>
<tr><td></td><td>$x=$ awrt $\{0.84,\ -0.84\}$</td><td>A1: One of either awrt 0.84 or awrt -0.84
A1ft: You can apply ft for $x=\pm\alpha$, where $\alpha=\cos^{-1}k$ and $-1\le k\le1$</td><td>A1A1ft</td></tr>
<tr><td></td><td colspan="2">In this part of the solution, if there are any extra answers in range in an otherwise correct solution withhold the A1ft.</td><td></td></tr>
<tr><td></td><td>$\{\sin x=0\Rightarrow\}\ x=0$ and $-\pi$</td><td>Both $x=0$ and $-\pi$ or awrt -3.14 from $\sin x=0$
In this part of the solution, ignore extra solutions in range.</td><td>B1</td></tr>
<tr><td></td><td colspan="2">Note solutions are: $x=\{-3.1415...,\ -0.8410...,\ 0,\ 0.8410...\}$
Ignore extra solutions outside the range</td><td></td></tr>
<tr><td></td><td colspan="2">For all answers in degrees in (ii) M1A1A0A1ftB0 is possible</td><td></td></tr>
<tr><td></td><td colspan="2">Allow the use of θ in place of x in (ii)</td><td></td></tr>
<tr><td></td><td></td><td></td><td>[5]</td></tr>
<tr><td></td><td colspan="2"></td><td>Total 9</td></tr>
<tr><td></td><td colspan="2"></td><td></td></tr>
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Q11.

Question	Scheme	Marks	AOs
(a)	States or uses $\tan x = \frac{\sin x}{\cos x}$	B1	1.2
	$4\sin x = 5\cos^2 x \Rightarrow 4\sin x = 5\left(1-\sin^2 x\right)$	M1	1.1b
	$5\sin^2 x + 4\sin x - 5 = 0$ *	A1*	2.1
		(3)	
(b)	Attempts to solve $5\sin^2 x + 4\sin x - 5 = 0 \Rightarrow \sin x = ...$	M1	1.1b
	$\sin x = \frac{-2 \pm \sqrt{29}}{5}$ $(\sin x = \text{awrt } 0.677)$	A1	1.1b
	Takes $\sin^{-1}$ leading to at least one answer in the range	dM1	1.1b
	$x = \text{awrt } 42.6\{°\}$ and $x = \text{awrt } 137.4\{°\}$ only	A1	1.1b
		(4)	
(c)	$15 \times "2" = 30$ following through on their "2"	B1ft	2.2a
	Explains either "mathematically" by stating $3 \times 5 \times$ their number in range 0 to 360° or 'in words" e.g., stating 3 × "2" values every 360° and 5 lots of 360°	B1ft	2.4
		(2)	
		(9 marks)	

Notes:	
(a)	**Allow use of e.g. θ but the final mark requires the equation to be in terms of x**
B1:	States or uses $\tan x = \dfrac{\sin x}{\cos x}$ e.g., $4\tan x = 5\cos x \Rightarrow 4\dfrac{\sin x}{\cos x} = 5\cos x$ Allow e.g. $\tan x = \dfrac{\sin\theta}{\cos\theta}$
M1:	Multiplies by $\cos x$ and uses $\cos^2 x = 1 - \sin^2 x$ to set up a quadratic equation in just $\sin x$ Condone mixed arguments here.
A1*:	Proceeds to $5\sin^2 x + 4\sin x - 5 = 0$ with correct notation and algebra, showing all key steps. The $= 0$ must be present in the final answer line. Condone a single slip in notation, e.g., $\sin x^2$ or $\sin\theta$ seen once.
(b)	
M1:	Attempts to solve $5\sin^2 x + 4\sin x - 5 = 0 \Rightarrow$ **$\sin x = ...$** using the usual rules. $\sin x =$ may be implied later. Allow solution(s) from a calculator but one must be correct (0.6 or 0.7 or -1.4 or -1.5)
A1:	Achieves $\sin x = \dfrac{-4 \pm \sqrt{116}}{10}$ ($\sin x =$ awrt 0.677) $\sin x =$ may be implied later.
dM1:	Finds one value of x in the range 0 to 360° from their $\sin x =$ May be scored for working in radians. If using $\sin x =$ 0.677 they should have awrt 0.744 or awrt 2.40 If they have made a slip in solving the quadratic, e.g., by the formula, then their values will need checking both in degrees and radians to see if this mark can be implied.

A1:	$x =$ awrt $42.6\{^\circ\}$ and $x =$ awrt $137.4\{^\circ\}$ only. Ignore any values outside of 0 to 360° isw if they round their values to e.g., 3sf after stating acceptable answers. There must be some evidence that the quadratic has been solved.
(c)	
B1ft:	Follow through on 15 multiplied by the number of solutions in (b) in the range 0 to 360° If working in radians in (b), they must state 30 (solutions).
B1ft:	Explains either mathematically **or** in words. See scheme. Note that you might see arguments expanding the range from 1800 to 5400 to account for the stretch parallel to the x axis. $\dfrac{5400}{360} = 15$ and $15 \times 2 = 30$ which is also acceptable.
Note:	If candidates list 30 values and conclude that there are 30 solutions, score B1ftB1ft There is no need to check their 30 values are correct, but there must be 30.

Q12.

Question Number	Scheme		Marks
(a)	$9^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos\alpha \Rightarrow \cos\alpha = ...$	Correct use of cosine rule leading to a value for cos α	M1
	$\cos\alpha = \frac{4^2+6^2-9^2}{2\times4\times6}\left(= -\frac{29}{48} = -0.604..\right)$		
	$\alpha = 2.22$ *	Cso (2.22 must be seen here)	A1
	(NB $\alpha = 2.219516005$)		**(2)**
(a) Way 2	$XY^2 = 4^2 + 6^2 - 2\times4\times6\cos 2.22 \Rightarrow XY^2 = ..$	Correct use of cosine rule leading to a value for XY^2	M1
	$XY^2 = 81.01....$		
	$XY = 9.00....$		A1
			(2)
(b)	$2\pi - 2.22 (= 4.06366......)$	$2\pi - 2.22$ or awrt 4.06	B1
	$\frac{1}{2}\times 4^2 \times "4.06"$	Correct method for major sector area.	M1
	32.5	Awrt 32.5	A1
			(3)
(b) Way2	**Circle – Minor sector**		
	$\pi \times 4^2$	Correct expression for circle area	B1
	$\pi \times 4^2 - \frac{1}{2}\times 4^2 \times 2.22 = 32.5$	Correct method for circle - minor sector area	M1
	$= 32.5$	Awrt 32.5	A1
			(3)
(c)	Area of triangle = $\frac{1}{2}\times4\times6\times\sin 2.22 (= 9.56)$	Correct expression for the area of triangle XYZ	B1
	So area required = "9.56" + "32.5"	Their Triangle XYZ + (part (b) answer or correct attempt at major sector)	M1
	$= 42.1$ cm^2 or 42.0 cm^2	Awrt 42.1 or 42.0 (Or just 42)	A1
			(3)
(d)	Arc length $= 4\times 4.06 (= 16.24)$ Or $8\pi - 4\times 2.22$	M1: $4 \times their(2\pi - 2.22)$ Or circumference – minor arc A1: Correct ft expression	M1A1ft
	Perimeter = $ZY + WY$ + Arc Length	9 + 2 + Any Arc	M1
	Perimeter = 27.2 or 27.3	Awrt 27.2 or awrt 27.3	A1
			(4)
			[12]