Exam Questions – Trig Chapter 3 and 4

Q1.

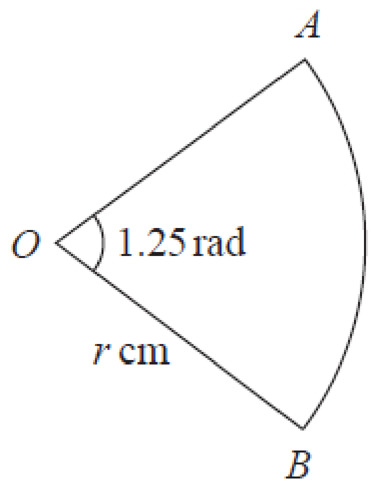

Figure 1

Figure 1 shows a sector *AOB* of a circle with centre *O* and radius *r* cm.

The angle *AOB* is 1.25 radians.

Given that the area of the sector *AOB* is 15 cm^2

(a) find the exact value of *r*,

(2)

(b) find the exact length of the perimeter of the sector. Write your answer in simplest form.

(3)

(Total for question = 5 marks)

2.

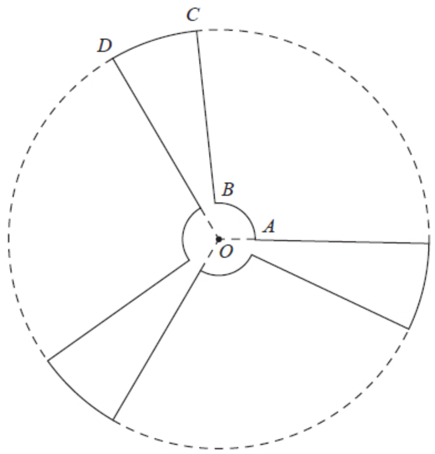

Figure 3

Figure 3 shows a sketch of the outline of the face of a ceiling fan viewed from below.

The fan consists of three identical sections congruent to *OABCDO*, shown in Figure 3, where

- *OABO* is a sector of a circle with centre *O* and radius 9 cm
- *OBCDO* is a sector of a circle with centre *O* and radius 84 cm
- angle *AOD* = $\frac{2\pi}{3}$ radians

Given that the length of the arc *AB* is 15 cm,

(a) show that the length of the arc *CD* is 35.9 cm to one decimal place.

(3)

The face of the fan is modelled to be a flat surface.

Find, according to the model,

(b) the perimeter of the face of the fan, giving your answer to the nearest cm,

(2)

(c) the surface area of the face of the fan.

Give your answer to 3 significant figures and make your units clear.

(5)

(Total for question = 10 marks)

Q3.

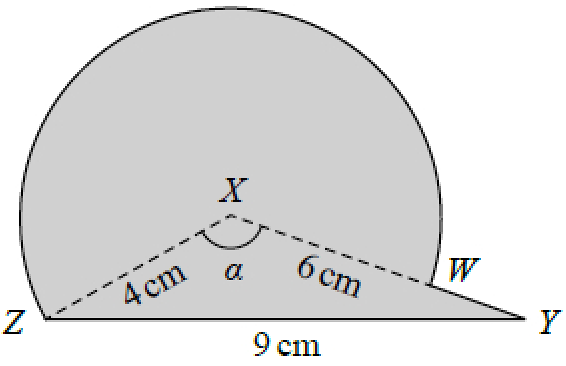

Figure 4

The triangle XYZ in Figure 4 has $XY = 6$ cm, = 9 cm, $ZX = 4$ cm and angle $ZXY = \alpha$.

The point W lies on the line XY.

The circular arc ZW, in Figure 4, is a major arc of the circle with centre X and radius 4 cm.

(a) Show that, to 3 significant figures, $\alpha = 2.22$ radians.

(2)

(b) Find the area, in cm^2, of the major sector $XZWX$.

(3)

The region, shown shaded in Figure 4, is to be used as a design for a logo.

Calculate

(c) the area of the logo

(3)

(d) the perimeter of the logo.

(4)

(Total for question = 12 marks)

Q4.

(i) (a) Find, in ascending powers of x, the 2^{nd}, 3^{rd} and 5^{th} terms of the binomial expansion of

$$(3 + 2x)^6$$

(3)

For a particular value of x, these three terms form consecutive terms in a geometric series.

(b) Find this value of x.

(3)

(ii) In a **different** geometric series,

- the first term is $\sin^2\theta$
- the common ratio is $2\cos\theta$
- the sum to infinity is $\frac{8}{5}$

(a) Show that

$$5\cos^2\theta - 16\cos\theta + 3 = 0$$

(3)

(b) Hence find the exact value of the 2^{nd} term in the series.

(3)

(Total for question = 12 marks)

Q5.

$$f(x) = 8 \sin x \cos x + 4 \cos^2 x - 3$$

(a) Write f(x) in the form

$$a \sin 2x + b \cos 2x + c$$

where a, b and c are integers to be found.

(3)

(b) Use the answer to part (a) to write f(x) in the form

$$R \sin (2x + \alpha) + c$$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 significant figures.

(3)

(c) Hence, or otherwise,

(i) state the maximum value of f(x)

(ii) find the **second** smallest positive value of x at which a maximum value of f(x) occurs. Give your answer to 3 significant figures.

(3)

(Total for question = 9 marks)

Q6.

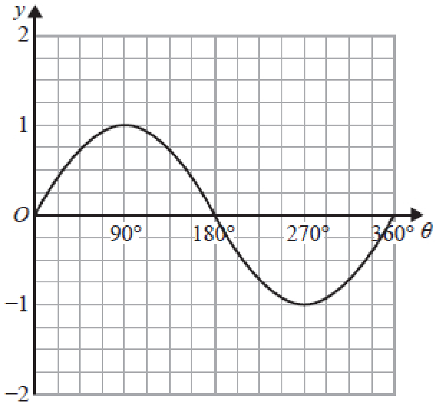

Figure 3

Figure 3 shows a plot of the curve with equation $y = \sin\theta$, $0 \leq \theta \leq 360°$

(a) State the coordinates of the minimum point on the curve with equation

$$y = 4\sin\theta, \quad 0 \leq \theta \leq 360°$$

(2)

A copy of Figure 3, called Diagram 1, is shown here.

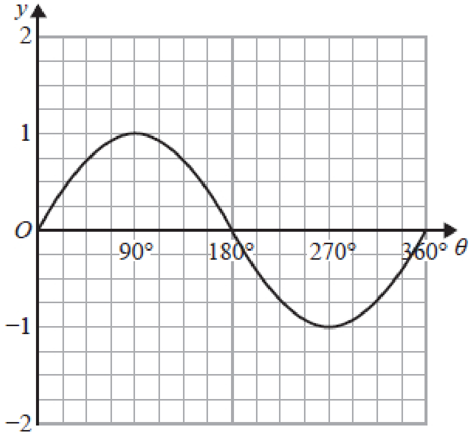

Diagram 1

(b) On Diagram 1, sketch and label the curves

(i) $y = 1 + \sin\theta$, $0 \leq \theta \leq 360°$
(ii) $y = \tan\theta$, $0 \leq \theta \leq 360°$

(2)

(c) Hence find the number of solutions of the equation

(i) $\tan\theta = 1 + \sin\theta$ that lie in the region $0 \leq \theta \leq 2160°$
(i) $\tan\theta = 1 + \sin\theta$ that lie in the region $0 \leq \theta \leq 1980°$

(3)

(Total for question = 7 marks)

Q7.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta < \pi$

$$3\text{cosec}\theta = 8\cos\theta$$

giving your answers, in radians, to 3 significant figures.

(5)

(ii) Solve, for $0 < x < 180°$

$$\frac{\tan 2x - \tan 70°}{1 + \tan 2x \tan 70°} = -\frac{3}{8}$$

giving your answers, in degrees, to one decimal place.

(4)

(Total for question = 9 marks)

Q8.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x < \pi$

$$(x - 2)\left(\sqrt{3}\sec x + 2\right) = 0$$

(3)

(ii) Solve, for $0 < \theta < 360°$

$$10 \sin\theta = 3 \cos 2\theta$$

(4)

(Total for question = 7 marks)

Q9.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Express 8 sin x − 15 cos x in the form R sin (x − α) , where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R, and give the value of α, in radians, to 4 significant figures.

(3)

$$f(x) = \frac{15}{41 + 16\sin x - 30\cos x} \qquad x > 0$$

(b) Find

(i) the minimum value of f(x)

(ii) the smallest value of x at which this minimum value occurs.

(4)

(c) State the y coordinate of the minimum points on the curve with equation

$$y = 2f(x) - 5 \qquad x > 0$$

(1)

(d) State the smallest value of x at which a maximum point occurs for the curve with equation

$$y = -f(2x) \qquad x > 0$$

(1)

(Total for question = 9 marks)

Q10.

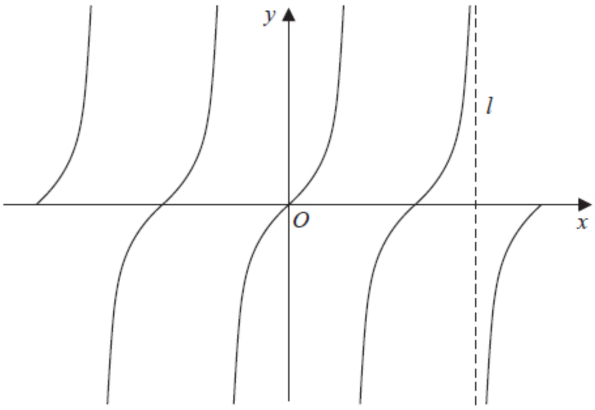

Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \tan x \qquad -2\pi \leq x \leq 2\pi$$

The line l, shown in Figure 4, is an asymptote to $y = \tan x$

(a) State an equation for l.

(1)

(b) (i) On figure 4, above, sketch the curve with equation

$$y = \frac{1}{x} + 1 \qquad -2\pi \leqslant x \leqslant 2\pi$$

stating the equation of the horizontal asymptote of this curve.

(ii) Hence, **giving a reason**, state the number of solutions of the equation

$$\tan x = \frac{1}{x} + 1$$

in the region $-2\pi \leq x \leq 2\pi$

(4)

(c) State the number of solutions of the equation $\tan x = \frac{1}{x} + 1$ in the region

(i) $0 \leq x \leq 40\pi$

(ii) $-10\pi \leq x \leq \frac{5}{2}\pi$

(2)

(Total for question = 7 marks)

Q11.

In this question you must show all stages of your working.

Solutions based entirely on calculator technology are not acceptable.

(i) Solve, for $-\frac{\pi}{2} < x < \pi$ the equation

$$5 \sin (3x + 0.1) + 2 = 0$$

giving your answers, **in radians**, to 2 decimal places.

(4)

(ii) Solve, for $0 < \theta < 360°$, the equation

$$2 \tan\theta \sin\theta = 5 + \cos\theta$$

giving your answers, **in degrees**, to one decimal place.

(5)

(Total for question = 9 marks)

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Q12.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\cot^2 x - \tan^2 x \equiv 4\cot 2x \operatorname{cosec} 2x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2 \theta$$

giving your answers to 2 decimal places.

(5)

(Total for question = 9 marks)

Q13.

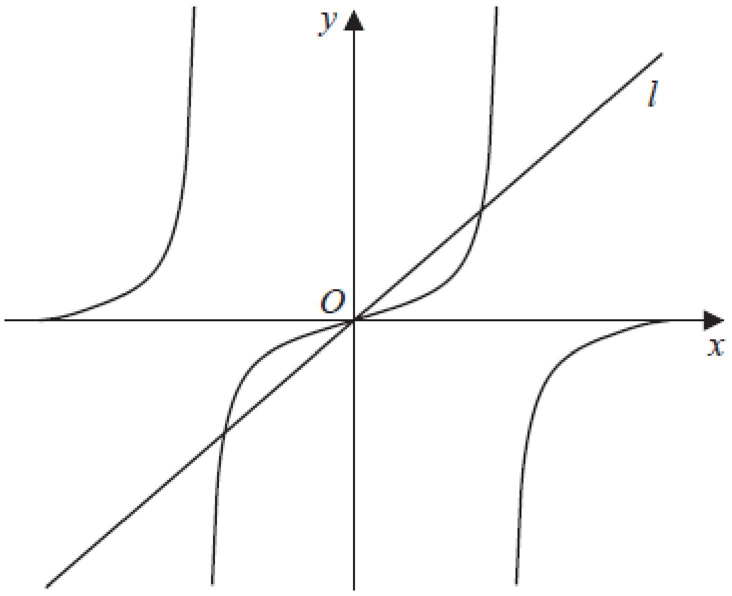

Figure 3

Figure 3 shows a sketch of

- the curve with equation $y = \tan x$
- the straight line l with equation $y = \pi x$

in the interval $-\pi < x < \pi$

(a) State the period of $\tan x$

(1)

(b) Write down the number of roots of the equation

(i) $\tan x = (\pi + 2)x$ in the interval $-\pi < x < \pi$

(1)

(ii) $\tan x = \pi x$ in the interval $-2\pi < x < 2\pi$

(1)

(iii) $\tan x = \pi x$ in the interval $-100\pi < x < 100\pi$

(1)

(Total for question = 4 marks)

Q14.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$$\tan^2\left(2x + \frac{\pi}{4}\right) = 3$$

(5)

(ii) Solve, for $0 < \theta < 360°$

$$(2\sin\theta - \cos\theta)^2 = 1$$

giving your answers, as appropriate, to one decimal place.

(5)

(Total for question = 10 marks)

Q15.

In this question you should show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\sin(\theta - 30°) = 5\cos\theta$$

can be written in the form

$$\tan\theta = 2\sqrt{3}$$

(4)

(b) Hence, or otherwise, solve for $0 \le x \le 360°$

$$2\sin(x - 10°) = 5\cos(x + 20°)$$

giving your answers to one decimal place.

(3)

(Total for question = 7 marks)

Q16.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Solve, for $-180° < \theta \leq 180°$, the equation

$$3\tan(\theta + 43°) = 2\cos(\theta + 43°)$$

(Total for question = 6 marks)

Q17.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x < \pi$, the equation

$$5 \sin x \tan x + 13 = \cos x$$

giving your answer in radians to 3 significant figures.

(5)

(ii) The temperature inside a greenhouse is monitored on one particular day.

The temperature, H°C, inside the greenhouse, t hours after midnight, is modelled by the equation

$$H = 10 + 12 \sin(kt + 18)° \qquad 0 \leq t < 24$$

where k is a constant.

Use the equation of the model to answer parts (a) to (c).

Given that

- the temperature inside the greenhouse was 20 °C at 6 am
- $0 < k < 20$

(a) find all possible values for k, giving each answer to 2 decimal places.

(4)

Given further that $0 < k < 10$

(b) find the maximum temperature inside the greenhouse,

(1)

(c) find the time of day at which this maximum temperature occurs.

Give your answer to the nearest minute.

(2)

(Total for question = 12 marks)

Q18.

(a) Prove that

$$2 \operatorname{cosec}^2 2\theta (1 - \cos 2\theta) \equiv 1 + \tan^2\theta$$

(4)

(b) Hence solve for $0 < x < 360°$, where $x \neq (90n)°$, $n \in \mathbb{N}$, the equation

$$2 \operatorname{cosec}^2 2x (1 - \cos 2x) = 4 + 3 \sec x$$

giving your answers to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

(Total for question = 8 marks)

Q19.

(i) Show that

$$\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta} \qquad \theta \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(3)

(ii) Solve, for $0 \leq x < 90°$, the equation

$$3\cos^2(2x + 10°) = 1$$

giving your answers in degrees to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)

(Total for question = 7 marks)

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Q20.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} \equiv \operatorname{cosec} x \qquad x \neq \frac{n\pi}{2} \; n \in \mathbb{Z}$$

(3)

(b) Hence solve, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$7 + \frac{\sin 4\theta}{\cos 2\theta} + \frac{\cos 4\theta}{\sin 2\theta} = 3\cot^2 2\theta$$

giving your answers in radians to 3 significant figures where appropriate.

(6)

(Total for question = 9 marks)

Q21.

Solutions based entirely on graphical or numerical methods are not acceptable in this question.

(i) Solve, for $0 \leq \theta < 180°$, the equation

$$3 \sin (2\theta - 10°) = 1$$

giving your answers to one decimal place.

(4)

(ii) The first three terms of an arithmetic sequence are

$$\sin\alpha, \quad \frac{1}{\tan\alpha} \quad \text{and} \quad 2\sin\alpha$$

where α is a constant.

(a) Show that $2 \cos \alpha = 3 \sin^2\alpha$

(3)

Given that $\pi < \alpha < 2\pi$,

(b) find, showing all working, the value of α to 3 decimal places.

(5)

(Total for question = 12 marks)

Q22.

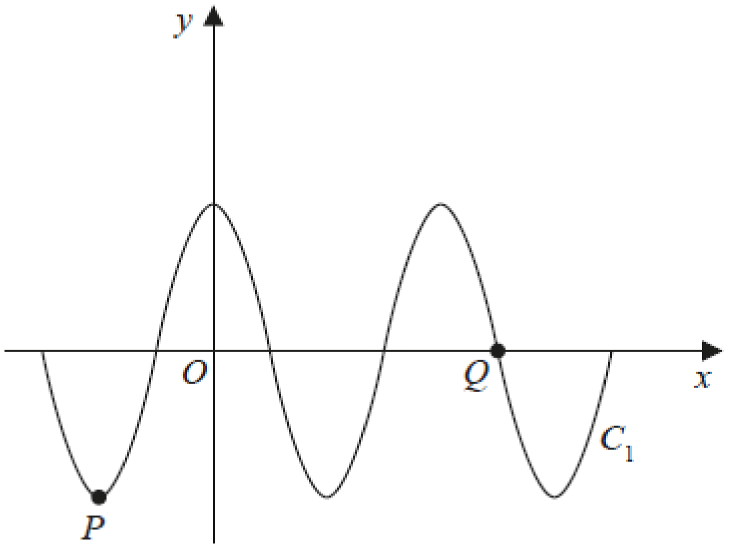

Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation $y = 4 \cos x°$

The point P and the point Q lie on C_1 and are shown in Figure 1.

(a) State

(i) the coordinates of P,
(ii) the coordinates of Q.

(3)

The curve C_2 has equation $y = 4 \cos x° + k$, where k is a constant.

Curve C_2 has a minimum y value of -1

The point R is the maximum point on C_2 with the smallest positive x coordinate.

(b) State the coordinates of R.

(2)

(Total for question = 5 marks)

Q23.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$\sqrt{2}\sin(x + 45°) = \cos(x - 60°)$$

show that

$$\tan x = -2 - \sqrt{3}$$

(4)

(b) Hence or otherwise, solve, for $0 \leq \theta < 180°$

$$\sqrt{2}\sin(2\theta) = \cos(2\theta - 105°)$$

(4)

(Total for question = 8 marks)

Q24.

(a) Use the substitution $t = \tan x$ to show that the equation

$$12 \tan 2x + 5 \cot x \sec^2 x = 0$$

can be written in the form

$$5t^4 - 24t^2 - 5 = 0$$

(4)

(b) Hence solve, for $0 \leq x < 360°$, the equation

$$12 \tan 2x + 5 \cot x \sec^2 x = 0$$

Show each stage of your working and give your answers to one decimal place.

(4)

(Total for question = 8 marks)

Q25.

$$f(\theta) = 5\cos\theta - 4\sin\theta \qquad \theta \in \mathbb{R}$$

(a) Express f(θ) in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

The curve with equation $y = \cos\theta$ is transformed onto the curve with equation $y = f(\theta)$ by a sequence of two transformations.

Given that the first transformation is a stretch and the second a translation,

(b) (i) describe fully the transformation that is a stretch,

(ii) describe fully the transformation that is a translation.

(2)

Given

$$g(\theta) = \frac{90}{4 + (f(\theta))^2} \qquad \theta \in \mathbb{R}$$

(c) find the range of g.

(2)

(Total for question = 7 marks)

Q26.

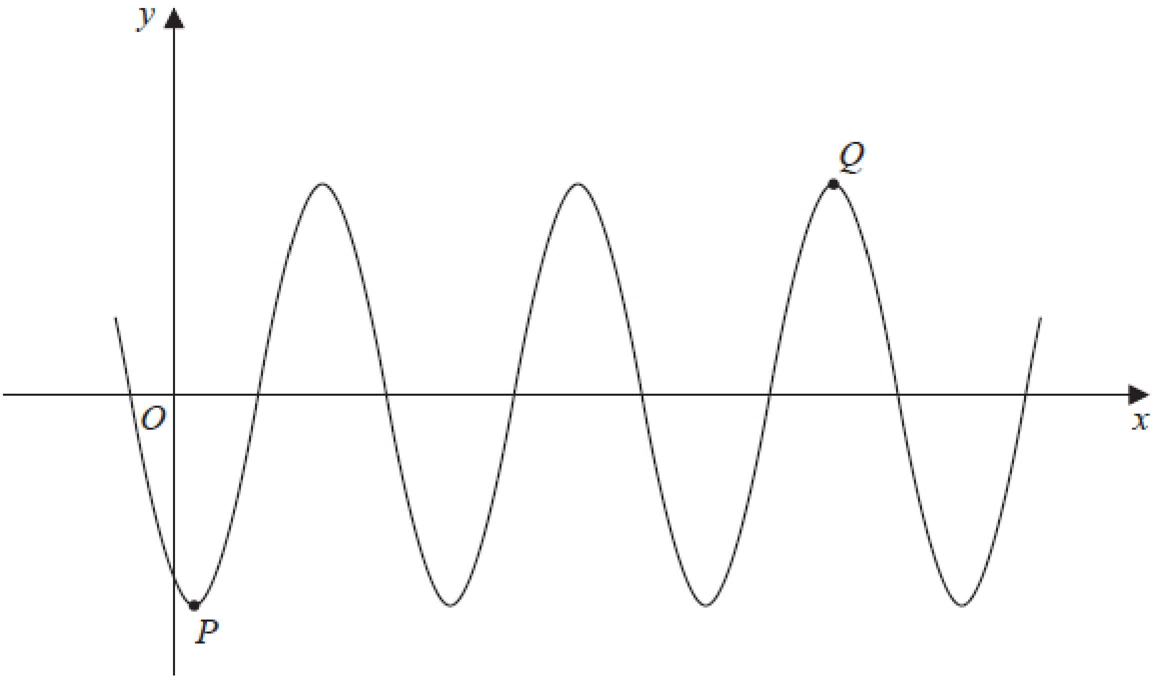

Figure 4

Figure 4 shows part of the curve with equation

$$y = A\cos(x - 30)°$$

where A is a constant.

The point P is a minimum point on the curve and has coordinates (30, –3) as shown in Figure 4.

(a) Write down the value of A.

(1)

The point Q is shown in Figure 4 and is a maximum point.

(b) Find the coordinates of Q.

(3)

(Total for question = 4 marks)

Q27.

$$f(x) = 6x^3 + 17x^2 + 4x - 12$$

(a) Use the factor theorem to show that $(2x + 3)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, write $f(x)$ as a product of three linear factors.

(4)

(c) Solve, for $\frac{\pi}{2} < \theta < \pi$, the equation

$$6\tan^3\theta + 17\tan^2\theta + 4\tan\theta - 12 = 0$$

giving your answers to 3 significant figures.

(2)

(Total for question = 8 marks)

Q28.

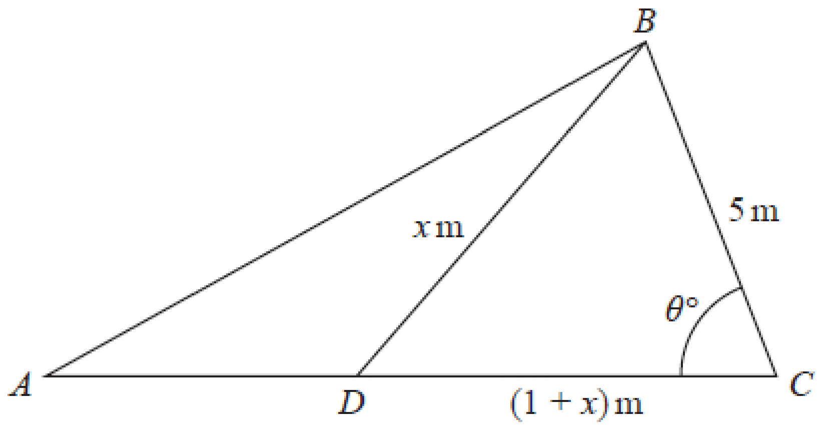

Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows the plan view of a frame for a flat roof.

The shape of the frame consists of triangle *ABD* joined to triangle *BCD*.

Given that

- $BD = x$ m
- $CD = (1 + x)$ m
- $BC = 5$ m
- angle $BCD = \theta°$

(a) show that $\cos\theta° = \dfrac{13 + x}{5 + 5x}$

(2)

Given also that

- $x = 2\sqrt{3}$
- angle $BAC = 30°$
- *ADC* is a straight line

(b) find the area of triangle *ABC*, giving your answer, in m^2, to one decimal place.

(5)

(Total for question = 7 marks)