Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int\left(\frac{4}{x^3}+kx\right)dx = -\frac{2}{x^2}+\frac{1}{2}kx^2+c$	A1 A1	1.1b 1.1b
		(3)	
(b)	$\left[-\frac{2}{x^2}+\frac{1}{2}kx^2\right]_{0.5}^{2} = \left(-\frac{2}{2^2}+\frac{1}{2}k\times 4\right)-\left(-\frac{2}{(0.5)^2}+\frac{1}{2}k\times(0.5)^2\right)=8$	M1	1.1b
	$7.5+\frac{15}{8}k=8 \Rightarrow k=\ldots$	dM1	1.1b
	$k=\frac{4}{15}$ oe	A1	1.1b
		(3)	
			(6 marks)

Notes

Mark parts (a) and (b) as one

(a)

M1: For $x^n \to x^{n+1}$ for either x^{-3} or x^1. This can be implied by the sight of either x^{-2} or x^2.
Condone "unprocessed" values here. Eg. x^{-3+1} and x^{1+1}

A1: Either term correct (un simplified).
Accept $4\times\frac{x^{-2}}{-2}$ or $k\frac{x^2}{2}$ **with** the indices processed.

A1: Correct (and simplified) with $+c$.
Ignore spurious notation e.g. answer appearing with an $\int$ sign or with dx on the end.
Accept $-\frac{2}{x^2}+\frac{1}{2}kx^2+c$ or exact simplified equivalent such as $-2x^{-2}+k\frac{x^2}{2}+c$

(b)

M1: For substituting both limits into their $-\frac{2}{x^2}+\frac{1}{2}kx^2$, subtracting either way around and setting equal to 8. Allow this when using a changed function. (so the M in part (a) may not have been awarded). Condone missing brackets. Take care here as substituting 2 into the original function gives the same result as the integrated function so you will have to consider both limits.

dM1: For solving a **linear** equation in k. It is dependent upon the previous M only
Don't be too concerned by the mechanics here. Allow for a linear equation in k leading to $k=$

A1: $k=\frac{4}{15}$ or exact equivalent. Allow for $\frac{m}{n}$ where m and n are integers and $\frac{m}{n}=\frac{4}{15}$
Condone the recurring decimal $0.2\dot{6}$ but not 0.266 or 0.267
Please remember to isw after a correct answer

Q2.

Question	Scheme	Marks	AOs
	States $\left\{ \lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\, \delta x \text{ is} \right\}$ $\int_4^9 \sqrt{x}\, dx$	B1	1.2
	$= \left[\frac{2}{3} x^{\frac{3}{2}} \right]_4^9$	M1	1.1b
	$= \frac{2}{3} \times 9^{\frac{3}{2}} - \frac{2}{3} \times 4^{\frac{3}{2}} = \frac{54}{3} - \frac{16}{3}$		
	$= \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7	A1	1.1b
		(3)	
			(3 marks)

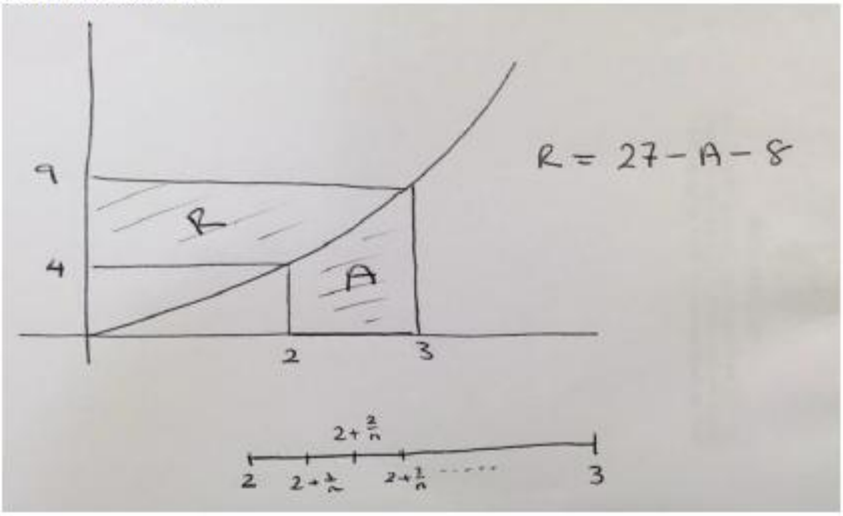
Notes for Question 5

B1:	States $\int_4^9 \sqrt{x}\, dx$ with or without the 'dx'
M1:	Integrates $\sqrt{x}$ to give $\lambda x^{\frac{3}{2}}$; $\lambda \neq 0$
A1:	See scheme
Note:	You can imply B1 for $\left[\lambda x^{\frac{3}{2}} \right]_4^9$ or for $\lambda \times 9^{\frac{3}{2}} - \lambda \times 4^{\frac{3}{2}}$
Note:	Give B0 for $\int_1^9 \sqrt{x}\, dx - \int_1^3 \sqrt{x}\, dx$ or for $\int_3^9 \sqrt{x}\, dx$ without reference to a correct $\int_4^9 \sqrt{x}\, dx$
Note:	Give B1 M1 A1 for no working leading to a correct $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\int_4^9 \sqrt{x}\, dx = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\left[\frac{2}{3} x^{\frac{3}{2}} + c \right]_4^9 = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for no working followed by an answer $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give M0 A0 for use of a trapezium rule method to give an answer of awrt 12.7, but allow B1 if $\int_4^9 \sqrt{x}\, dx$ is seen in a trapezium rule method
Note:	Otherwise, give B0 M0 A0 for using the trapezium rule to give an answer of awrt 12.7

Notes for Question Continued

Alt

The following method is correct:



$$\text{Area }(A) = \lim_{n\to\infty}\sum_{i=1}^{n}(x_i - x_{i-1})\mathrm{f}(x_i) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(2+\frac{i}{n}\right)^2$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{4i}{n}\right) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{4}{n^2}\sum_{i=1}^{n}i + \frac{1}{n^3}\sum_{i=1}^{n}i^2\right]$$

$$= \lim_{n\to\infty}\left[\frac{4n}{n} + \frac{4}{n^2}\left(\frac{1}{2}n(n+1)\right) + \frac{1}{n^3}\left(\frac{1}{6}n(n+1)(2n+1)\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{4}{n} + \frac{4n^2+4n}{2n^2} + \frac{2n^3+3n^2+n}{6n^3}\right]$$

$$= \lim_{n\to\infty}\left[4 + 2 + \frac{2}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right]$$

$$= 4 + 2 + \frac{1}{3} = \frac{19}{3}$$

So, $\lim_{\delta x\to 0}\sum_{x=4}^{9}\sqrt{x}\,\delta x = \text{Area}(R) = (3\times 9) - (2\times 4) - \frac{19}{3}$

$$= \frac{38}{3} \quad \text{or} \quad 12\frac{2}{3} \quad \text{or awrt } 12.7$$

Q3.

Question Number	Scheme	Marks
(a)	$\frac{5}{(x-1)(3x+2)} = \frac{A}{x-1} + \frac{B}{3x+2}$	
	$5 = A(3x+2) + B(x-1)$	
	$x \to 1$ $\quad 5 = 5A \Rightarrow A = 1$	M1 A1
	$x \to -\frac{2}{3}$ $\quad 5 = -\frac{5}{3}B \Rightarrow B = -3$	A1 (3)
(b)	$\int \frac{5}{(x-1)(3x+2)} dx = \int \left(\frac{1}{x-1} - \frac{3}{3x+2} \right) dx$	
	$= \ln(x-1) - \ln(3x+2) \quad (+C)$ ft constants	M1 A1ft A1ft
		(3)
(c)	$\int \frac{5}{(x-1)(3x+2)} dx = \int \left(\frac{1}{y} \right) dy$	M1
	$\ln(x-1) - \ln(3x+2) = \ln y \quad (+C)$	M1 A1
	$y = \frac{K(x-1)}{3x+2}$ depends on first two Ms in (c)	M1 dep
	Using $(2, 8)$ $\quad 8 = \frac{K}{8}$ depends on first two Ms in (c)	M1 dep
	$y = \frac{64(x-1)}{3x+2}$	A1 (6)
		[12]

Q4.

Question	Scheme	Marks	AOs
(a)	$\lim_{\delta x\to 0}\sum_{x=2.1}^{6.3}\frac{2}{x}\delta x=\int_{2.1}^{6.3}\frac{2}{x}\,dx$	B1	1.2
		(1)	
(b)	$=[2\ln x]_{2.1}^{6.3}=2\ln 6.3-2\ln 2.1$	M1	1.1b
	$=\ln 9$ CSO	A1	1.1b
		(2)	
			(3 marks)
Notes:			

Mark (a) and (b) as one

(a)

B1: States that $\int_{2.1}^{6.3}\frac{2}{x}\,dx$ or equivalent such as $2\int_{2.1}^{6.3}x^{-1}\,dx$ but must include the limits and the dx.

Condone $dx\leftrightarrow\delta x$ as it is very difficult to tell one from another sometimes

(b)

M1: Know that $\int\frac{1}{x}dx=\ln x$ and attempts to apply the limits (either way around)

Condone $\int\frac{2}{x}dx=p\ln x$ (including $p=1$) or $\int\frac{2}{x}dx=p\ln qx$ as long as the limits are applied.

Also be aware that $\int\frac{2}{x}dx=\ln x^2$, $\int\frac{2}{x}dx=2\ln|x|+c$ and $\int\frac{2}{x}dx=2\ln cx$ o.e. are also correct

$[p\ln x]_{2.1}^{6.3}=p\ln 6.3-p\ln 2.1$ is sufficient evidence to award this mark

A1: CSO $\ln 9$. Also answer $=\ln 3^2$ so $k=9$ is fine. Condone $\ln|9|$

The method mark must have been awarded. Do not accept answers such as $\ln\frac{39.69}{4.41}$

Note that solutions appearing from "rounded" decimal work when taking lns should not score the final mark. It is a "show that" question

E.g. $[2\ln x]_{2.1}^{6.3}=2\ln 6.3-2\ln 2.1=2.197=\ln k\Rightarrow k=e^{2.197}=8.998=9$

Q5.

Question	Scheme	Marks	AOs
	$C: y = x\ln x$; l is a normal to C at $P(\mathrm{e}, \mathrm{e})$ Let x_A be the x-coordinate of where l cuts the x-axis		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ln x + x\left(\dfrac{1}{x}\right) \quad \{= 1 + \ln x\}$	M1	2.1
		A1	1.1b
	$x = \mathrm{e},\ m_T = 2 \Rightarrow m_N = -\dfrac{1}{2} \Rightarrow y - \mathrm{e} = -\dfrac{1}{2}(x - \mathrm{e})$ $y = 0 \Rightarrow -\mathrm{e} = -\dfrac{1}{2}(x - \mathrm{e}) \Rightarrow x = \ldots$	M1	3.1a
	l meets x-axis at $x = 3\mathrm{e}$ (allow $x = 2\mathrm{e} + \mathrm{e}\ln\mathrm{e}$)	A1	1.1b
	{Areas:} **either** $\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$ **or** $\dfrac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$	M1	2.1
	$\left\{\int x\ln x\,\mathrm{d}x = \right\}\ \dfrac{1}{2}x^2\ln x - \int \dfrac{1}{x}.\left(\dfrac{x^2}{2}\right)\{\mathrm{d}x\}$	M1	2.1
	$\left\{= \dfrac{1}{2}x^2\ln x - \int \dfrac{1}{2}x\,\{\mathrm{d}x\}\right\} = \dfrac{1}{2}x^2\ln x - \dfrac{1}{4}x^2$	dM1	1.1b
		A1	1.1b
	$\text{Area}(R_1) = \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$; $\text{Area}(R_2) = \dfrac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2) \quad \{= \tfrac{1}{4}\mathrm{e}^2 + \tfrac{1}{4} + \mathrm{e}^2\}$	M1	3.1a
	$\text{Area}(R) = \tfrac{5}{4}\mathrm{e}^2 + \tfrac{1}{4}$	A1	1.1b
		(10)	

Notes for Question	
M1:	Differentiates by using the product rule to give $\ln x + x(\text{their } g'(x))$, where $g(x) = \ln x$
A1:	Correct differentiation of $y = x\ln x$, which can be un-simplified or simplified
M1:	Complete strategy to find the x coordinate where their normal to C at $P(\mathrm{e}, \mathrm{e})$ meets the x-axis i.e. Sets $y = 0$ in $y - \mathrm{e} = m_N(x - \mathrm{e})$ to find $x = \ldots$
Note:	m_T is found by using calculus and $m_N \neq m_T$
A1:	l meets x-axis at $x = 3\mathrm{e}$, allowing un-simplified values for x such as $x = 2\mathrm{e} + \mathrm{e}\ln \mathrm{e}$
Note:	Allow $x =$ awrt 8.15
M1:	Scored for either • Area under curve $= \int_1^{\mathrm{e}} x\ln x \, \mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$, with limits of e and 1 and some attempt to substitute these and subtract • or Area under line $= \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$, with a valid attempt to find x_A
M1:	Integration by parts the correct way around to give $Ax^2\ln x - \int B\left(\frac{x^2}{x}\right)\{\mathrm{d}x\}$; $A \neq 0$, $B > 0$
dM1:	**dependent on the previous M mark** Integrates the second term to give $\pm\lambda x^2$; $\lambda \neq 0$
A1:	$\frac{1}{2}x^2\ln x - \frac{1}{4}x^2$
M1:	Complete strategy of finding the area of R by finding the sum of two key areas. See scheme.
A1:	$\frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$
Note:	$\text{Area}(R_2)$ can also be found by integrating the line l between limits of e and their x_A i.e. $\text{Area}(R_2) = \int_{\mathrm{e}}^{\text{their } x_A} \left(-\frac{1}{2}x + \frac{3}{2}\mathrm{e}\right) \mathrm{d}x = [\ \ldots\ ]_{\mathrm{e}}^{\text{their } x_A} = \ldots$
Note:	**<u>Calculator approach with no algebra, differentiation or integration seen:</u>** • Finding l cuts through the x-axis at awrt 8.15 is 2nd M1 2nd A1 • Finding area between curve and the x-axis between $x = 1$ and $x = \mathrm{e}$ to give awrt 2.10 is 3rd M1 • Using the above information (must be seen) to apply $\text{Area}(R) = 2.0972\ldots + 7.3890\ldots = 9.4862\ldots$ is final M1 Therefore, a maximum of 4 marks out of the 10 available.

Q6.

Question	Scheme	Marks	AOs
	$y=\dfrac{(x-2)(x-4)}{4\sqrt{x}}=\dfrac{x^2-6x+8}{4\sqrt{x}}=\dfrac{1}{4}x^{\frac{3}{2}}-\dfrac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$\int \dfrac{1}{4}x^{\frac{3}{2}}-\dfrac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}\mathrm{d}x=\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}(+c)$	dM1 A1	3.1a 1.1b
	Deduces limits of integral are 2 and 4 and applies to their $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$	M1	2.2a
	$\left(\dfrac{32}{10}-8+8\right)-\left(\dfrac{2}{5}\sqrt{2}-2\sqrt{2}+4\sqrt{2}\right)=\dfrac{16}{5}-\dfrac{12}{5}\sqrt{2}$ Area $R=\dfrac{12}{5}\sqrt{2}-\dfrac{16}{5}\ \left(\text{or } \dfrac{16}{5}-\dfrac{12}{5}\sqrt{2}\right)$	A1	2.1
		(6)	
			(6 marks)
Notes:			

M1: Correct attempt to write $\dfrac{(x-2)(x-4)}{4\sqrt{x}}$ as a sum of terms with **indices**.

Look for at least two different terms with the correct index e.g. two of $x^{\frac{3}{2}}, x^{\frac{1}{2}}, x^{-\frac{1}{2}}$ which have come from the correct places.

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\dfrac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

A1: $\dfrac{1}{4}x^{\frac{3}{2}}-\dfrac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$ which can be left unsimplified e.g. $\dfrac{1}{4}x^{2-\frac{1}{2}}-\dfrac{1}{2}x^{\frac{1}{2}}-x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$

or as e.g. $\dfrac{1}{4}\left(x^{\frac{3}{2}}-6x^{\frac{1}{2}}+8x^{-\frac{1}{2}}\right)$

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\dfrac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

dM1: Integrates $x^n \to x^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $x^{\frac{3}{2}}\to x^{\frac{5}{2}},\ x^{\frac{1}{2}}\to x^{\frac{3}{2}},\ x^{-\frac{1}{2}}\to x^{\frac{1}{2}}$

It is dependent upon the first M so at least two terms must have had a correct index.

A1: $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}(+c)$. Allow unsimplified e.g. $\dfrac{1}{4}\times\dfrac{2}{5}x^{\frac{3}{2}+1}-\dfrac{1}{2}\times\dfrac{2}{3}x^{\frac{1}{2}+1}-\dfrac{2}{3}x^{\frac{1}{2}+1}+2\times 2x^{\frac{1}{2}}$

or as e.g. $\dfrac{1}{4}\left(\dfrac{2}{5}x^{\frac{5}{2}}-4x^{\frac{3}{2}}+16x^{\frac{1}{2}}\right)(+c)$.

M1: Substitutes the limits 4 and 2 to their $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$ and subtracts either way round.

There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\dfrac{1}{10}\times 4^{\frac{5}{2}}-4^{\frac{3}{2}}+4\times 4^{\frac{1}{2}}-\dfrac{1}{10}\times 2^{\frac{5}{2}}-2^{\frac{3}{2}}+4\times 2^{\frac{1}{2}}$

This is an independent mark but the limits must be applied to an expression that is not y so they may even have differentiated.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw

Integration by parts:

$\int\frac{(x-2)(x-4)}{4\sqrt{x}}\mathrm{d}x=\int\frac{1}{4}(x-2)(x-4)x^{-\frac{1}{2}}\mathrm{d}x=\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}}-\int\frac{1}{2}(2x-6)x^{\frac{1}{2}}\mathrm{d}x$	M1 A1	1.1b 1.1b
$\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}}-\underline{\int\frac{1}{2}(2x-6)x^{\frac{1}{2}}\mathrm{d}x}=\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}}-\underline{\int x^{\frac{3}{2}}-3x^{\frac{1}{2}}\mathrm{d}x}$ $=\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}}\underline{-\frac{2}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}}$ Or e.g. $=\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}}-\frac{1}{3}x^{\frac{3}{2}}(2x-6)+\frac{4}{15}x^{\frac{5}{2}}$	dM1 A1	3.1a 1.1b
Deduces limits of integral are 2 and 4 and applies to their $\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}}-\frac{1}{3}x^{\frac{3}{2}}(2x-6)+\frac{4}{15}x^{\frac{5}{2}}$	M1	2.2a
$0-\frac{16}{3}+\frac{128}{15}-\left(0+\frac{4}{3}\sqrt{2}+\frac{16}{15}\sqrt{2}\right)$ Area $R=\frac{12}{5}\sqrt{2}-\frac{16}{5}\left(\text{or }\frac{16}{5}-\frac{12}{5}\sqrt{2}\right)$	A1	2.1
	(6)	

Notes:

M1: Applies integration by parts and reaches the form $\alpha(x-2)(x-4)x^{\frac{1}{2}}\pm\int(px+q)x^{\frac{1}{2}}\mathrm{d}x$ $\alpha, p\neq 0$

oe e.g. $\alpha\left(x^2-6x+8\right)x^{\frac{1}{2}}\pm\int(px+q)x^{\frac{1}{2}}\mathrm{d}x$ $\alpha, p\neq 0$

A1: Correct first application of parts in any form

dM1: Attempts their $\int(px+q)x^{\frac{1}{2}}\mathrm{d}x$ by expanding and integrating or may attempt parts again.

E.g. $\int(2x-6)x^{\frac{1}{2}}\mathrm{d}x=\int\left(2x^{\frac{3}{2}}-6x^{\frac{1}{2}}\right)\mathrm{d}x=...$ or e.g. $\int(2x-6)x^{\frac{1}{2}}\mathrm{d}x=\frac{2}{3}x^{\frac{3}{2}}(2x-6)-\frac{4}{3}\int x^{\frac{3}{2}}\mathrm{d}x$

If they expand then at least one term requires $x^n\rightarrow x^{n+1}$ or if parts is attempted again, the structure must be correct.

A1: Fully correct integration in any form

M1: Substitutes the limits 4 and 2 to their $=\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}}-\frac{2}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}$ and subtracts

either way round. There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $0-\frac{16}{3}+\frac{128}{15}-0+\frac{4}{3}\sqrt{2}+\frac{16}{15}\sqrt{2}$

This is an independent mark but the limits must be applied to a "changed" function.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Attempts at integration by parts "the other way round" should be sent to review.

Integration by substitution example:

$u=\sqrt{x}\left(x=u^2\right)\Rightarrow\int\frac{(x-2)(x-4)}{4\sqrt{x}}\mathrm{d}x=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ $=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}2u\mathrm{d}u$	M1 A1	1.1b 1.1b
$=\frac{1}{2}\int\left(u^4-6u^2+8\right)\mathrm{d}u=\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$	dM1 A1	3.1a 1.1b
Deduces limits of integral are $\sqrt{2}$ and 2 and applies to their $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$	M1	2.2a
$\frac{1}{2}\left(\frac{32}{5}-16+16-\left(\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)\right)$ Area $R=\frac{12}{5}\sqrt{2}-\frac{16}{5}\left(\text{or }\frac{16}{5}-\frac{12}{5}\sqrt{2}\right)$	A1	2.1
	(6)	

Notes:

M1: Applies the substitution e.g. $u=\sqrt{x}$ and attempts $k\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$

A1: Fully correct integral in terms of u in any form e.g. $\frac{1}{2}\int\left(u^2-2\right)\left(u^2-4\right)\mathrm{d}u$

dM1: Expands the bracket and integrates $u^n\to u^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $u^4\to u^5,\ u^2\to u^3,\ k\to ku$

A1: $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$. Allow unsimplified.

M1: Substitutes the limits 2 and $\sqrt{2}$ to their $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$ and subtracts either way round.

There is no requirement to evaluate but 2 and $\sqrt{2}$ must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\frac{1}{2}\left(\frac{32}{5}-16+16-\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)$

Alternatively reverses the substitution and applies the limits 4 and 2 with the same conditions.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw.

There may be other substitutions seen and the same marking principles apply.

Q7.

Question Number	Scheme	Notes	Marks
	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 1**	$\int \frac{1}{x}\,dx = \int -\frac{5}{2}\,dt$	Separates variables as shown. dx and dt should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\ln x = -\frac{5}{2}t + c$	Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ or $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$\ln x = -\frac{5}{2}t + c$, including "$+c$"	A1
	$\{t=0, x=60 \Rightarrow\}\ \ln 60 = c$ $\ln x = -\frac{5}{2}t + \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 cso
			[4]
(a) **Way 2**	$\frac{dt}{dx} = -\frac{2}{5x}$ or $t = \int -\frac{2}{5x}\,dx$	**Either** $\frac{dt}{dx} = -\frac{2}{5x}$ or $t = \int -\frac{2}{5x}\,dx$	B1
	$t = -\frac{2}{5}\ln x + c$	Integrates both sides to give **either** $t = \ldots$ **or** $\pm\alpha\ln px$; $\alpha \neq 0, p > 0$	M1
		$t = -\frac{2}{5}\ln x + c$, including "$+c$"	A1
	$\{t=0, x=60 \Rightarrow\}\ c = \frac{2}{5}\ln 60 \Rightarrow t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60$ $\Rightarrow -\frac{5}{2}t = \ln x - \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 cso
			[4]
(a) **Way 3**	$\int_{60}^{x} \frac{1}{x}\,dx = \int_{0}^{t} -\frac{5}{2}\,dt$	Ignore limits	B1
	$[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$	Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ or $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$ including the correct limits	A1
	$\ln x - \ln 60 = -\frac{5}{2}t \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 cso
			[4]

(b)	$20 = 60e^{-\frac{5}{2}t}$ or $\ln 20 = -\frac{5}{2}t + \ln 60$	Substitutes $x = 20$ into an equation in the form of either $x = \pm\lambda e^{\pm\mu t} \pm \beta$ or $x = \pm\lambda e^{\pm\mu t \pm \alpha\ln\delta x}$ or $\pm\alpha\ln\delta x = \pm\mu t \pm \beta$ or $t = \pm\lambda\ln\delta x \pm \beta$; $\alpha, \lambda, \mu, \delta \neq 0$ and β can be 0	M1
	$t = -\frac{2}{5}\ln\left(\frac{20}{60}\right)$ $\{= 0.4394449\ldots \text{ (days)}\}$ **Note: t must be greater than 0**	**dependent on the previous M mark** Uses correct algebra to achieve an equation of the form of either $t = A\ln\left(\frac{60}{20}\right)$ or $A\ln\left(\frac{20}{60}\right)$ or $A\ln 3$ or $A\ln\left(\frac{1}{3}\right)$ o.e. or $t = A(\ln 20 - \ln 60)$ or $A(\ln 60 - \ln 20)$ o.e. $(A \in \square, t > 0)$	dM1
	$\Rightarrow t = 632.8006\ldots = 633$ (to the nearest minute)	awrt 633 or 10 hours and awrt 33 minutes	A1 cso
	Note: dM1 can be implied by $t =$ awrt 0.44 from no incorrect working.		
			7

Question Number	Scheme	Notes	Marks
	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) Way 4	$\int \frac{2}{5x}\,dx = -\int dt$	Separates variables as shown. dx and dt should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\frac{2}{5}\ln(5x) = -t + c$	Integrates both sides to give **either** $\pm\alpha\ln(px)$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$; $p > 0$	M1
		$\frac{2}{5}\ln(5x) = -t + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}\ \frac{2}{5}\ln 300 = c$ $\frac{2}{5}\ln(5x) = -t + \frac{2}{5}\ln 300 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 cso
			[4]
(a) Way 5	$\left\{\frac{dt}{dx} = -\frac{2}{5x} \Rightarrow\right\}\ t = \int_{60}^{x} -\frac{2}{5x}\,dx$	Ignore limits	B1
	$t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$	Integrates both sides to give **either** $\pm k \to \pm kt$ (with respect to t) **or** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$; $k, \alpha \neq 0$	M1
		$t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$ including the correct limits	A1
	$t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60 \Rightarrow -\frac{5}{2}t = \ln x - \ln 60$ $\Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 cso
			[4]

		Question Notes
(a)	**B1**	For the correct separation of variables. E.g. $\int \frac{1}{5x}\,dx = \int -\frac{1}{2}\,dt$
	Note	B1 can be implied by seeing **either** $\ln x = -\frac{5}{2}t + c$ **or** $t = -\frac{2}{5}\ln x + c$ with or without $+c$
	Note	B1 can also be implied by seeing $[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$
	Note	Allow A1 for $x = 60\sqrt{e^{-5t}}$ or $x = \frac{60}{\sqrt{e^{5t}}}$ with no incorrect working seen
	Note	Give final A0 for $x = e^{-\frac{5}{2}t} + 60 \to x = 60e^{-\frac{5}{2}t}$
	Note	Give final A0 for writing $x = e^{-\frac{5}{2}t + \ln 60}$ as their final answer (without seeing $x = 60e^{-\frac{5}{2}t}$)
	Note	Way 1 to Way 5 do not exhaust all the different methods that candidates can give.
	Note	Give B0M0A0A0 for writing down $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with no evidence of working or integration seen.
(b)	**A1**	You can apply **cso** for the work only seen in part (b).
	Note	Give dM1(Implied) A1 for $\frac{5}{2}t = \ln 3$ followed by $t =$ awrt 633 from no incorrect working.
	Note	Substitutes $x = 40$ into their equation from part (a) is M0dM0A0

Q8.

Question Number	Scheme	Notes	Marks
	(i) $\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y,\ y>0$, (ii) $\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x,\ x=4\sin^2\theta$		
(i) **Way 1**	$\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y}+\frac{B}{(3y+2)} \Rightarrow 3y-4 = A(3y+2)+By$	See notes	M1
	$y=0 \Rightarrow -4=2A \Rightarrow A=-2$	At least one of their $A=-2$ **or** their $B=9$	A1
	$y=-\frac{2}{3} \Rightarrow -6=-\frac{2}{3}B \Rightarrow B=9$	Both their $A=-2$ **and** their $B=9$	A1
	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{-2}{y}+\frac{9}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\frac{A}{y} \to \pm\lambda \ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$, $A\neq 0,\ B\neq 0$	M1
		At least one term correctly followed through from their A or from their B	A1 ft
	$= -2\ln y + 3\ln(3y+2)\ \{+c\}$	$-2\ln y + 3\ln(3y+2)$ **or** $-2\ln y + 3\ln(y+\frac{2}{3})$ with correct bracketing, simplified or un-simplified. Can apply isw.	A1 cao
			[6]
(ii) (a) **Way 1**	$\{x=4\sin^2\theta \Rightarrow\}\ \frac{\mathrm{d}x}{\mathrm{d}\theta}=8\sin\theta\cos\theta$ or $\frac{\mathrm{d}x}{\mathrm{d}\theta}=4\sin2\theta$ or $\mathrm{d}x = 8\sin\theta\cos\theta\,\mathrm{d}\theta$		B1
	$\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}.\,8\sin\theta\cos\theta\ \{\mathrm{d}\theta\}$ or $\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}.\,4\sin2\theta\ \{\mathrm{d}\theta\}$		M1
	$= \int \underline{\underline{\tan\theta}}.\,8\sin\theta\cos\theta\ \{\mathrm{d}\theta\}$ or $\int \underline{\underline{\tan\theta}}.\,4\sin2\theta\ \{\mathrm{d}\theta\}$	$\sqrt{\left(\frac{x}{4-x}\right)} \to \pm K\tan\theta$ or $\pm K\left(\frac{\sin\theta}{\cos\theta}\right)$	$\underline{\underline{\text{M1}}}$
	$= \int 8\sin^2\theta\,\mathrm{d}\theta$	$\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1
	$3=4\sin^2\theta$ **or** $\frac{3}{4}=\sin^2\theta$ **or** $\sin\theta=\frac{\sqrt{3}}{2} \Rightarrow \theta=\frac{\pi}{3}$ $\{x=0 \to \theta=0\}$	Writes down a correct equation involving $x=3$ leading to $\theta=\frac{\pi}{3}$ and no incorrect work seen regarding limits	B1
			[5]
(ii) (b)	$=\{8\}\int\left(\frac{1-\cos2\theta}{2}\right)\mathrm{d}\theta\ \ \left\{=\int(4-4\cos2\theta)\,\mathrm{d}\theta\right\}$	Applies $\cos2\theta = 1-2\sin^2\theta$ to their integral. (See notes)	M1
	$=\{8\}\left(\frac{1}{2}\theta-\frac{1}{4}\sin2\theta\right)\ \ \{=4\theta-2\sin2\theta\}$	For $\pm\alpha\theta\pm\beta\sin2\theta,\ \alpha,\beta\neq 0$	M1
		$\sin^2\theta \to \left(\frac{1}{2}\theta-\frac{1}{4}\sin2\theta\right)$	A1
	$\left\{\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,\mathrm{d}\theta = 8\left[\frac{1}{2}\theta-\frac{1}{4}\sin2\theta\right]_0^{\frac{\pi}{3}}\right\} = 8\left(\left(\frac{\pi}{6}-\frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right)-(0+0)\right)$		
	$=\frac{4}{3}\pi-\sqrt{3}$	"two term" exact answer of e.g. $\frac{4}{3}\pi-\sqrt{3}$ or $\frac{1}{3}(4\pi-3\sqrt{3})$	A1 o.e.
			[4]
			15

		Question Notes
(i)	1st M1	Writing $\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)}$ and a complete method for finding the value of at least one of their A or their B.
	Note	M1A1 can be implied *for writing down* either $\frac{3y-4}{y(3y+2)} \equiv \frac{-2}{y} + \frac{\text{their } B}{(3y+2)}$ or $\frac{3y-4}{y(3y+2)} \equiv \frac{\text{their } A}{y} + \frac{9}{(3y+2)}$ with no working.
	Note	Correct bracketing is not necessary for the penultimate A1ft, but is required for the final A1 in (i)
	Note	**Give** 2nd M0 for $\frac{3y-4}{y(3y+2)}$ going directly to $\pm\alpha\ln(3y^2+2y)$
	Note	**…but allow** 2nd M1 for either $\frac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ or $\frac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$
(ii)(a)	1st M1	Substitutes $x = 4\sin^2\theta$ and their $\mathrm{d}x$ $\left(\text{from their correctly rearranged } \frac{\mathrm{d}x}{\mathrm{d}\theta}\right)$ into $\sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x$
	Note	$\mathrm{d}x \neq \lambda\,\mathrm{d}\theta$. For example $\mathrm{d}x \neq \mathrm{d}\theta$
	Note	Allow substituting $\mathrm{d}x = 4\sin 2\theta$ for the 1st M1 after a correct $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$ or $\mathrm{d}x = 4\sin 2\theta\,\mathrm{d}\theta$
	2nd M1	Applying $x = 4\sin^2\theta$ to $\sqrt{\left(\frac{x}{4-x}\right)}$ to give $\pm K\tan\theta$ or $\pm K\left(\frac{\sin\theta}{\cos\theta}\right)$
	Note	Integral sign is not needed for this mark.
	1st A1	Simplifies to give $\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$
	2nd B1	Writes down a correct equation involving $x = 3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits
	Note	Allow 2nd B1 for $x = 4\sin^2\left(\frac{\pi}{3}\right) = 3$ **and** $x = 4\sin^2 0 = 0$
	Note	Allow 2nd B1 for $\theta = \sin^{-1}\left(\sqrt{\frac{x}{4}}\right)$ followed by $x = 3, \theta = \frac{\pi}{3}; x = 0, \theta = 0$

(ii)(b)	M1	Writes down a correct equation involving $\cos 2\theta$ and $\sin^2\theta$ **E.g.:** $\cos 2\theta = 1 - 2\sin^2\theta$ or $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ or $K\sin^2\theta = K\left(\frac{1-\cos 2\theta}{2}\right)$ and *applies* it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0, \beta \neq 0$ (can be simplified or un-simplified).
	1st A1	Integrating $\sin^2\theta$ to give $\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta$, un-simplified or simplified. Correct solution only. Can be implied by $k\sin^2\theta$ giving $\frac{k}{2}\theta - \frac{k}{4}\sin 2\theta$ or $\frac{k}{4}(2\theta - \sin 2\theta)$ un-simplified or simplified.
	2nd A1	A *correct solution in part (ii)* leading to a "two term" exact answer of e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{8}{6}\pi - \sqrt{3}$ or $\frac{4}{3}\pi - \frac{2\sqrt{3}}{2}$ or $\frac{1}{3}(4\pi - 3\sqrt{3})$
	Note	A decimal answer of 2.456739397… (without a correct **exact** answer) is A0.
	Note	Candidates can work in terms of λ (note that λ is not given in (ii)) and gain the 1st three marks (i.e. M1M1A1) in part (b).
	Note	If they incorrectly obtain $\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,\mathrm{d}\theta$ in part (i)(a) (or correctly guess that $\lambda = 8$) then the final A1 is available for a correct solution in part (ii)(b).

	Scheme		Notes	Marks
(i) **Way 2**	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{6y+2}{3y^2+2y}\,dy - \int \frac{3y+6}{y(3y+2)}\,dy$			
	$\frac{3y+6}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y+6 = A(3y+2) + By$		See notes	M1
	$y = 0 \Rightarrow 6 = 2A \Rightarrow A = 3$		At least one of their $A = 3$ **or** their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$		Both their $A = 3$ **and** their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy$ $= \int \frac{6y+2}{3y^2+2y}\,dy - \int \frac{3}{y}\,dy + \int \frac{6}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$ $M \neq 0, A \neq 0, B \neq 0$		M1
		At least one term correctly followed through		A1 ft
	$= \ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)\,\{+c\}$	$\ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified		A1 cao
				[6]
(i) **Way 3**	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{3y+1}{3y^2+2y}\,dy - \int \frac{5}{y(3y+2)}\,dy$			
	$\frac{5}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 5 = A(3y+2) + By$		See notes	M1
	$y = 0 \Rightarrow 5 = 2A \Rightarrow A = \frac{5}{2}$		At least one of their $A = \frac{5}{2}$ **or** their $B = -\frac{15}{2}$	A1
	$y = -\frac{2}{3} \Rightarrow 5 = -\frac{2}{3}B \Rightarrow B = -\frac{15}{2}$		Both their $A = \frac{5}{2}$ **and** their $B = -\frac{15}{2}$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy$ $= \int \frac{3y+1}{3y^2+2y}\,dy - \int \frac{\frac{5}{2}}{y}\,dy + \int \frac{\frac{15}{2}}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$ $M \neq 0, A \neq 0, B \neq 0$		M1
		At least one term correctly followed through		A1 ft
	$= \frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)\,\{+c\}$	$\frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)$ with correct bracketing, simplified or un-simplified		A1 cao
				[6]

	Scheme	Notes	
(i) Way 4	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{3y}{y(3y+2)}\,\mathrm{d}y - \int \frac{4}{y(3y+2)}\,\mathrm{d}y$		
	$= \int \frac{3}{(3y+2)}\,\mathrm{d}y - \int \frac{4}{y(3y+2)}\,\mathrm{d}y$		
	$\frac{4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 4 = A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow 4 = 2A \Rightarrow A = 2$	At least one of their $A = 2$ or their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$	Both their $A = 2$ and their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y$ $= \int \frac{3}{3y+2}\,\mathrm{d}y - \int \frac{2}{y}\,\mathrm{d}y + \int \frac{6}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\frac{C}{(3y+2)} \to \pm\alpha\ln(3y+2)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$, $A \neq 0, B \neq 0, C \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \ln(3y+2) - 2\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y+2) - 2\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 cao
			[6]

	Alternative methods for B1M1M1A1 in (ii)(a)		
(ii)(a) Way 2	$\left\{x = 4\sin^2\theta \Rightarrow\right\} \frac{\mathrm{d}x}{\mathrm{d}\theta} = 8\sin\theta\cos\theta$	As in Way 1	B1
	$\int \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,8\sin\theta\cos\theta\ \{\mathrm{d}\theta\}$	As before	M1
	$= \int \sqrt{\frac{\sin^2\theta}{(1-\sin^2\theta)}}\,.\,8\cos\theta\sin\theta\ \{\mathrm{d}\theta\}$		
	$= \int \frac{\sin\theta}{\sqrt{(1-\sin^2\theta)}}\,.\,8\sqrt{(1-\sin^2\theta)}\sin\theta\ \{\mathrm{d}\theta\}$		
	$= \int \sin\theta\,.\,8\sin\theta\ \{\mathrm{d}\theta\}$	Correct method leading to $\sqrt{(1-\sin^2\theta)}$ being cancelled out	M1
	$= \int 8\sin^2\theta\,\mathrm{d}\theta$	$\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1 cso
(ii)(a) Way 3	$\left\{x = 4\sin^2\theta \Rightarrow\right\} \frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$	As in Way 1	B1
	$x = 4\sin^2\theta = 2 - 2\cos 2\theta,\ \ 4 - x = 2 + 2\cos 2\theta$		
	$\int \sqrt{\frac{2-2\cos 2\theta}{2+2\cos 2\theta}}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\}$		M1
	$= \int \frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2+2\cos 2\theta}}\,.\,\frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2-2\cos 2\theta}} 4\sin 2\theta\ \{\mathrm{d}\theta\} = \int \frac{2-2\cos 2\theta}{\sqrt{4-4\cos^2 2\theta}}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\}$		
	$= \int \frac{2-2\cos 2\theta}{2\sin 2\theta}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\} = \int 2(2-2\cos 2\theta).\ \{\mathrm{d}\theta\}$	Correct method leading to $\sin 2\theta$ being cancelled out	M1
	$= \int 8\sin^2\theta\,\mathrm{d}\theta$	$\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1 cso

Q9.

Question Number	Scheme	Marks
	May mark (a) and (b) together	
(a)	Expands to give $10x^{\frac{3}{2}} - 20x$	B1
	Integrates to give $\frac{10}{"\frac{5}{2}"}x^{"\frac{5}{2}"} + \frac{-"20"x^2}{2}$ $(+c)$	M1 A1ft
	Simplifies to $4x^{\frac{5}{2}} - 10x^2 (+c)$	A1cao (4)
(b)	Use limits 0 and 4 either way round on their integrated function (may only see 4 substituted)	M1
	Use limits 4 and 9 either way round on their integrated function	dM1
	Obtains either ± -32 or $\pm$ 194 needs at least one of the previous M marks for this to be awarded	A1
	(So area = $\left\lvert\int_0^4 ydx\right\rvert + \int_4^9 ydx$) i.e. 32 + 194, = 226	ddM1,A1 (5) [9]

Notes

(a) **B1**: Expands the bracket correctly

M1: Correct integration process **on at least one term** after attempt at multiplication. (Follow correct expansion or one slip resulting in $10x^k - 20x$ where k may be $\frac{1}{2}$ or $\frac{5}{2}$ or resulting in $10x^{\frac{3}{2}} - Bx$, where B may be 2 or 5)

So $x^{\frac{3}{2}} \to \frac{x^{\frac{5}{2}}}{5/2}$ or $x^{\frac{1}{2}} \to \frac{x^{\frac{3}{2}}}{3/2}$ or $x^{\frac{5}{2}} \to \frac{x^{\frac{7}{2}}}{7/2}$ and/or $x \to \frac{x^2}{2}$.

A1: Correct unsimplified follow through for both terms of their integration. Does not need (+ c)

A1: Must be simplified and correct– allow answer in scheme or $4x^{2\frac{1}{2}} - 10x^2$. Does not need (+ c)

(b) **M1**: (does not depend on first method mark) Attempt to substitute 4 into their integral (however obtained but must not be differentiated) or seeing their evaluated number (usually 32) is enough – do not need to see minus zero.

dM1: (depends on first method mark in (a)) Attempt to subtract either way round using the limits 4 and 9 $A \times 9^{\frac{5}{2}} - B \times 9^2$ with $A \times 4^{\frac{5}{2}} - B \times 4^2$ is enough – or seeing 162 –(-32) {but not 162 – 32 }

A1: At least one of the values (32 and 194) correct (needs just one of the two previous M marks in (b)) or may see 162 + 32 + 32 or 162 + 64 or may be implied by correct final answer if not evaluated until last line of working

ddM1: Adds 32 and 194 (may see 162 + 32 + 32 or may be implied by correct final answer if not evaluated until last line of working). This depends on everything being correct to this point.

A1cao: Final answer of 226 not (- 226)

Common errors: $4\times4^{\frac{5}{2}} - 10\times4^2 + 4\times9^{\frac{5}{2}} - 10\times9^2 - 4\times4^{\frac{5}{2}} - 10\times4^2 = \pm$ 162 obtains M1 M1 A0 (neither 32 nor 194 seen and final answer incorrect) then M0 A0 so 2/5

Uses correct limits to obtain -32 +162 +32 = +/-162 is M1 M1 A1 (32 seen) M0 A0 so 3/5

Special case: In part (b) Uses limits 9 and 0 = 972 – 810 - 0 = 162 M0 M1 A0 M0A0 scores 1/5

This also applies if 4 never seen.

Q10.

Question Number	Scheme		Marks
(a)	$y = 4x - xe^{\frac{1}{2}x},\ x \geqslant 0$ $\left\{y = 0 \Rightarrow 4x - xe^{\frac{1}{2}x} = 0 \Rightarrow x(4 - e^{\frac{1}{2}x}) = 0 \Rightarrow\right\}$		
	$e^{\frac{1}{2}x} = 4 \Rightarrow x_A = 4\ln 2$	Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda \ln \mu$ where $\mu > 0$	M1
		$4\ln 2$ **cao** (Ignore $x = 0$)	A1
			[2]
(b)	$\left\{\int xe^{\frac{1}{2}x}\,dx\right\} = 2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\{dx\}$	$\alpha xe^{\frac{1}{2}x} - \beta\int e^{\frac{1}{2}x}\{dx\},\ \alpha > 0,\ \beta > 0$	M1
		$2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\{dx\}$, with or without dx	A1 (M1 on ePEN)
	$= 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\ \{+c\}$	$2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$ o.e. with or without $+c$	A1
			[3]
(c)	$\left\{\int 4x\,dx\right\} = 2x^2$	$4x \to 2x^2$ or $\frac{4x^2}{2}$ o.e.	B1
	$\left\{\int_0^{4\ln 2}(4x - xe^{\frac{1}{2}x})\,dx\right\} = \left[2x^2 - \left(2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right)\right]_0^{4\ln 2 \text{ or } \ln 16 \text{ or their limits}}$		
	$= \left(2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 4e^{\frac{1}{2}(4\ln 2)}\right) - \left(2(0)^2 - 2(0)e^{\frac{1}{2}(0)} + 4e^{\frac{1}{2}(0)}\right)$	**See notes**	M1
	$= (32(\ln 2)^2 - 32(\ln 2) + 16) - (4)$ $= 32(\ln 2)^2 - 32(\ln 2) + 12$	$32(\ln 2)^2 - 32(\ln 2) + 12$, **see notes**	A1
			[3] **8**

		Question Notes
(a)	**M1**	Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda \ln \mu$ where $\mu > 0$
	A1	$4\ln 2$ **cao** stated in part (a) only (Ignore $x = 0$)
(b)	**NOTE**	**Part (b) appears as M1M1A1 on ePEN, but is now marked as M1A1A1.**
	M1	Integration by parts is applied in the form $\alpha xe^{\frac{1}{2}x} - \beta\int e^{\frac{1}{2}x}\{dx\}$, where $\alpha > 0,\ \beta > 0$. (must be in this form) with or without dx
	A1	$2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\{dx\}$ or equivalent, with or without dx. **Can be un-simplified.**
	A1	$2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$ or equivalent with or without $+ c$. **Can be un-simplified.**
	Note	You can also allow $2e^{\frac{1}{2}x}(x-2)$ or $e^{\frac{1}{2}x}(2x-4)$ for the final A1.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	SPECIAL CASE: A candidate who uses $u = x$, $\frac{dv}{dx} = e^{\frac{1}{2}x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1. (Applying their v counts for one consistent error.)

(c)	B1	$4x \rightarrow 2x^2$ or $\frac{4x^2}{2}$ oe
	M1	**Complete** method of applying limits of their x_A and 0 to all terms of an expression of the form $\pm Ax^2 \pm Bxe^{\frac{1}{2}x} \pm Ce^{\frac{1}{2}x}$ (where $A \neq 0, B \neq 0$ and $C \neq 0$) and subtracting the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	Note	$\ln 16$ or $2\ln 4$ or equivalent is fine as an upper limit.
	A1	A correct three term exact quadratic expression in $\ln 2$. For example allow for A1 • $32(\ln 2)^2 - 32(\ln 2) + 12$ • $8(2\ln 2)^2 - 8(4\ln 2) + 12$ • $2(4\ln 2)^2 - 32(\ln 2) + 12$ • $2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 12$
	Note	Note that the constant term of 12 needs to be combined from $4e^{\frac{1}{2}(4\ln 2)} - 4e^{\frac{1}{2}(0)}$ o.e.
	Note	Also allow $32\ln 2(\ln 2 - 1) + 12$ or $32\ln 2\left(\ln 2 - 1 + \frac{12}{32\ln 2}\right)$ for A1.
	Note	***Do not apply "ignore subsequent working" for incorrect simplification.*** Eg: $32(\ln 2)^2 - 32(\ln 2) + 12 \rightarrow 64(\ln 2) - 32(\ln 2) + 12$ or $32(\ln 4) - 32(\ln 2) + 12$
	Note	**Bracketing error:** $32\ln 2^2 - 32(\ln 2) + 12$, unless recovered is final A0.
	Note	**Notation:** Allow $32(\ln^2 2) - 32(\ln 2) + 12$ for the final A1.
	Note	5.19378... without seeing $32(\ln 2)^2 - 32(\ln 2) + 12$ is A0.
	Note	5.19378... following from a correct $2x^2 - \left(2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right)$ is M1A0.
	Note	5.19378... from no working is M0A0.

Q11.

Question Number	Scheme		Marks
(a)	$A = \int_0^3 \sqrt{(3-x)(x+1)}\,\mathrm{d}x\ ,\ x = 1 + 2\sin\theta$		
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$ or $2\cos\theta$ used correctly in their working. Can be implied.	B1
	$\left\{\int \sqrt{(3-x)(x+1)}\,\mathrm{d}x \text{ or } \int \sqrt{(3+2x-x^2)}\,\mathrm{d}x\right\}$		
	$= \int \sqrt{(3-(1+2\sin\theta))((1+2\sin\theta)+1)}\ 2\cos\theta\,\{\mathrm{d}\theta\}$	Substitutes for both x and $\mathrm{d}x$, where $\mathrm{d}x \neq \lambda \mathrm{d}\theta$. Ignore $\mathrm{d}\theta$	M1
	$= \int \sqrt{(2-2\sin\theta)(2+2\sin\theta)}\ 2\cos\theta\,\{\mathrm{d}\theta\}$ $= \int \sqrt{(4-4\sin^2\theta)}\ 2\cos\theta\,\{\mathrm{d}\theta\}$		
	$= \int \sqrt{(4-4(1-\cos^2\theta)}\ 2\cos\theta\,\{\mathrm{d}\theta\}$ or $\int \sqrt{4\cos^2\theta}\ 2\cos\theta\,\{\mathrm{d}\theta\}$	Applies $\cos^2\theta = 1-\sin^2\theta$ see notes	M1
	$= 4\int \cos^2\theta\,\mathrm{d}\theta,\ \{k = 4\}$	$4\int \cos^2\theta\mathrm{d}\theta$ or $\int 4\cos^2\theta\mathrm{d}\theta$ Note: $\mathrm{d}\theta$ is required here.	A1
	$0 = 1+2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2} \Rightarrow \theta = -\frac{\pi}{6}$ and $3 = 1+2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$	See notes	B1
			[5]
(b)	$\left\{k\int \cos^2\theta\,\{\mathrm{d}\theta\}\right\} = \{k\}\int \left(\frac{1+\cos 2\theta}{2}\right)\{\mathrm{d}\theta\}$	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral	M1
	$= \{k\}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$	Integrates to give $\pm\alpha\theta \pm \beta\sin 2\theta,\ \alpha \neq 0,\ \beta \neq 0$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$	M1 (A1 on ePEN)
	$\left\{\text{So } 4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\mathrm{d}\theta = \left[2\theta + \sin 2\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}}\right\}$ $= \left(2\left(\frac{\pi}{2}\right) + \sin\left(\frac{2\pi}{2}\right)\right) - \left(2\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{2\pi}{6}\right)\right)$		
	$\left\{= (\pi) - \left(-\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right\} = \frac{4\pi}{3} + \frac{\sqrt{3}}{2}$	$\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ or $\frac{1}{6}\left(8\pi + 3\sqrt{3}\right)$	A1 cao cso
			[3] 8

	Question Notes	
(a)	B1	$\frac{dx}{d\theta} = 2\cos\theta$. Also allow $dx = 2\cos\theta d\theta$. This mark can be implied by later working.
	Note	You can give B1 for $2\cos\theta$ used correctly in their working.
	M1	Substitutes $x = 1 + 2\sin\theta$ and their dx $\left(\text{from their rearranged}\frac{dx}{d\theta}\right)$ into $\sqrt{(3-x)(x+1)}\,dx$.
	Note	Condone bracketing errors here.
	Note	$dx \neq \lambda d\theta$. For example $dx \neq d\theta$.
	Note	Condone substituting $dx = \cos\theta$ for the 1st M1 after a correct $\frac{dx}{d\theta} = 2\cos\theta$ or $dx = 2\cos\theta d\theta$
	M1	Applies either • $1 - \sin^2\theta = \cos^2\theta$ • $\lambda - \lambda\sin^2\theta$ or $\lambda(1-\sin^2\theta) = \lambda\cos^2\theta$ • $4 - 4\sin^2\theta = 4 + 2\cos 2\theta - 2 = 2 + 2\cos 2\theta = 4\cos^2\theta$ to their expression where λ is a numerical value.
	A1	Correctly proves that $\int\sqrt{(3-x)(x+1)}\,dx$ is equal to $4\int\cos^2\theta d\theta$ or $\int 4\cos^2\theta d\theta$
	Note	All three previous marks must have been awarded before A1 can be awarded.
	Note	Their final answer must include $d\theta$.
	Note	You can ignore limits for the final A1 mark.
	B1	Evidence of a correct equation in $\sin\theta$ or $\sin^{-1}\theta$ for both x-values leading to both θ values. Eg: • $0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2}$ which then leads to $\theta = -\frac{\pi}{6}$, **and** • $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1$ which then leads to $\theta = \frac{\pi}{2}$
	Note	Allow B1 for $x = 1 + 2\sin\left(-\frac{\pi}{6}\right) = 0$ **and** $x = 1 + 2\sin\left(\frac{\pi}{2}\right) = 3$
	Note	Allow B1 for $\sin\theta = \left(\frac{x-1}{2}\right)$ or $\theta = \sin^{-1}\left(\frac{x-1}{2}\right)$ followed by $x = 0,\ \theta = -\frac{\pi}{6};\ x = 3,\ \theta = \frac{\pi}{2}$
(b)	NOTE	**Part (b) appears as M1A1A1 on ePEN, but is now marked as M1M1A1.**
	M1	Writes down a correct equation involving $\cos 2\theta$ and $\cos^2\theta$ **Eg:** $\cos 2\theta = 2\cos^2\theta - 1$ or $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ or $\lambda\cos^2\theta = \lambda\left(\frac{1+\cos 2\theta}{2}\right)$ and *applies* it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0, \beta \neq 0$ (can be simplified or un-simplified).
	A1	A *correct solution in part (b)* leading to a "two term" exact answer. Eg: $\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ or $\frac{8\pi}{6} + \frac{\sqrt{3}}{2}$ or $\frac{1}{6}\left(8\pi + 3\sqrt{3}\right)$
	Note	5.054815… from no working is M0M0A0.
	Note	Candidates can work in terms of k (note that k is not given in (a)) for the M1M1 marks in part (b).
	Note	If they incorrectly obtain $4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}}\cos^2\theta d\theta$ in part (a) (or guess $k = 4$) then the final A1 is available for a correct solution in part (b) only.

Q12.

Question Number	Scheme	Marks
	$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x}$	
	$x\sqrt{x} = x^{\frac{3}{2}}$	B1
	$x^n \to x^{n+1}$	M1
	$y = \frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}(+c)$	A1, A1
	Use $x=4, y=37$ to give equation in c, $\quad 37 = 12\sqrt{4} + \frac{2}{5}(\sqrt{4})^5 + c$	M1
	$\Rightarrow c = \frac{1}{5}$ or equivalent eg. 0.2	A1
	$(y) = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$	A1
		(7 marks)

B1 $\quad x\sqrt{x} = x^{\frac{3}{2}}$. This may be implied by $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ oe in the subsequent work.

M1 $\quad x^n \to x^{n+1}$ in at least one case so see either $x^{\frac{1}{2}}$ or $x^{\frac{5}{2}}$ or both

A1 $\quad$ One term integrated correctly. It does not have to be simplified Eg. $\frac{6}{\frac{1}{2}}x^{\frac{1}{2}}$ or $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$.

No need for $+c$

A1 $\quad$ Other term integrated correctly. See above. No need to simplify nor for $+c$. Need to see

$\frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ or a simplified correct version

M1 $\quad$ Substitute $x = 4, y = 37$ to produce an equation in c.

A1 $\quad$ Correctly calculates $c = \frac{1}{5}$ or equivalent e.g. 0.2

A1 $\quad$ cso $\quad y = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$. Allow $5y = 60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1$ and accept fully simplified equivalents.

e.g. $y = \frac{1}{5}(60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1)$, $y = 12\sqrt{x} + \frac{2}{5}\sqrt{x^5} + \frac{1}{5}$

Q13.

Question Number	Scheme		Marks
	$\int_0^4 \frac{1}{2+\sqrt{(2x+1)}}\,dx$, $u = 2+\sqrt{(2x+1)}$		
	$\frac{du}{dx} = (2x+1)^{-\frac{1}{2}}$ or $\frac{dx}{du} = u-2$	Either $\frac{du}{dx} = \pm K(2x+1)^{-\frac{1}{2}}$ or $\frac{dx}{du} = \pm\lambda(u-2)$	M1
		Either $\frac{du}{dx} = (2x+1)^{-\frac{1}{2}}$ or $\frac{dx}{du} = (u-2)$	A1
	$\left\{\int \frac{1}{2+\sqrt{(2x+1)}}\,dx\right\} = \int \frac{1}{u}(u-2)\,du$	Correct substitution (Ignore integral sign and du).	A1
	$= \int\left(1-\frac{2}{u}\right)du$	An attempt to divide each term by u.	dM1
	$= u - 2\ln u$	$\pm Au \pm B\ln u$	ddM1
		$u - 2\ln u$	A1 ft
	$\left\{\text{So } [u-2\ln u]_3^5\right\} = (5-2\ln 5)-(3-2\ln 3)$	Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round.	M1
	$= 2+2\ln\left(\frac{3}{5}\right)$	$2+2\ln\left(\frac{3}{5}\right)$	A1 cao cso
			[8] 8

Notes for Question

M1: Also allow $du = \pm\lambda\frac{1}{(u-2)}dx$ or $(u-2)du = \pm\lambda\,dx$

Note: The expressions must contain du and dx. They can be simplified or un-simplified.

A1: Also allow $du = \frac{1}{(u-2)}dx$ or $(u-2)du = \pm\lambda\,dx$

Note: The expressions must contain du and dx. They can be simplified or un-simplified.

A1: $\int\frac{1}{u}(u-2)\,du$. (Ignore integral sign and du).

dM1: An attempt to divide each term by u.
Note that this mark is dependent on the previous M1 mark being awarded.
Note that this mark can be implied by later working.

ddM1: $\pm Au \pm B\ln u$, $A \neq 0$, $B \neq 0$
Note that this mark is dependent on the two previous M1 marks being awarded.

A1ft: $u - 2\ln u$ or $\pm Au \pm B\ln u$ being correctly followed through, $A \neq 0$, $B \neq 0$

M1: Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round.

A1: cso and cao. $2+2\ln\left(\frac{3}{5}\right)$ or $2+2\ln(0.6)$, $\left(= A+2\ln B, \text{ so } A=2, B=\frac{3}{5}\right)$

Note: $2-2\ln\left(\frac{3}{5}\right)$ is A0.

Notes for Question Continued

ctd	**Note:** $\int\frac{1}{u}(u-2)\,du = u-2\ln u$ with no working is 2nd M1, 3rd M1, 3rd A1.
	but **Note:** $\int\frac{1}{u}(u-2)\,du = (u-2)\ln u$ with no working is 2nd M0, 3rd M0, 3rd A0.

Q14.

Question Number	Scheme		Marks
(a)	1.0981		B1 cao
			[1]
(b)	Area $\approx \frac{1}{2}\times 1 \,;\times \left[0.5 + 2(0.8284 + \text{their } 1.0981) + 1.3333\right]$		B1; M1
	$= \frac{1}{2}\times 5.6863 = 2.84315 = 2.843$ (3 dp)	2.843 or awrt 2.843	A1
			[3]
(c)	$\{u = 1+\sqrt{x}\} \Rightarrow \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{dx}{du} = 2(u-1)$		B1
	$\left\{\int \frac{x}{1+\sqrt{x}}\,dx = \right\} \int \frac{(u-1)^2}{u}.\,2(u-1)\,du$	$\int \frac{(u-1)^2}{u}\ldots\ldots$	M1
		$\int \frac{(u-1)^2}{u}.\,2(u-1)$	A1
	$= 2\int \frac{(u-1)^3}{u}\,du = \{2\}\int \frac{(u^3-3u^2+3u-1)}{u}\,du$	Expands to give a "four term" cubic in u. Eg: $\pm Au^3 \pm Bu^2 \pm Cu \pm D$	M1
	$= \{2\}\int \left(u^2 - 3u + 3 - \frac{1}{u}\right)du$	An attempt to divide at least three terms in ***their cubic*** by u. See notes.	M1
	$= \{2\}\left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u\right)$	$\int \frac{(u-1)^3}{u} \to \left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u\right)$	A1
	$\text{Area}(R) = \left[\frac{2u^3}{3} - 3u^2 + 6u - 2\ln u\right]_2^3$		
	$= \left(\frac{2(3)^3}{3} - 3(3)^2 + 6(3) - 2\ln 3\right) - \left(\frac{2(2)^3}{3} - 3(2)^2 + 6(2) - 2\ln 2\right)$	Applies limits of 3 and 2 in u or 4 and 1 in x and subtracts either way round.	M1
	$= \frac{11}{3} + 2\ln 2 - 2\ln 3$ or $\frac{11}{3} + 2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3} - \ln\left(\frac{9}{4}\right)$, etc	Correct exact answer or equivalent.	A1
			[8]
			12

(a) **B1:** 1.0981 correct answer only. Look for this on the table or in the candidate's working.

(b) **B1:** Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$

M1: For structure of trapezium rule [............]

A1: anything that rounds to 2.843

Note: Working must be seen to demonstrate the use of the trapezium rule. **Note**: actual area is 2.85573645…

Note: Award B1M1 A1 for $\frac{1}{2}(0.5 + 1.3333) + (0.8284 + \text{their } 1.0981) = 2.84315$

Bracketing mistake: **Unless the final answer implies that the calculation has been done correctly**

Award B1M0A0 for $\frac{1}{2}\times 1 + 0.5 + 2(0.8284 + \text{their } 1.0981) + 1.3333$ (nb: answer of 6.1863).

Award B1M0A0 for $\frac{1}{2}\times 1\ (0.5+1.3333)+2(0.8284+\text{their } 1.0981)$ (nb: answer of 4.76965).

(b) ctd

Alternative method for part (b): Adding individual trapezia

$$\text{Area} \approx 1\times\left[\frac{0.5+0.8284}{2}+\frac{0.8284+1.0981}{2}+\frac{1.0981+1.3333}{2}\right] = 2.84315$$

B1: 1 and a divisor of 2 on all terms inside brackets.
M1: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
A1: anything that rounds to 2.843

(c)

B1: $\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$ or $du=\frac{1}{2\sqrt{x}}dx$ or $2\sqrt{x}\,du = dx$ or $dx = 2(u-1)du$ or $\frac{dx}{du}=2(u-1)$ oe.

1st M1: $\frac{x}{1+\sqrt{x}}$ becoming $\frac{(u-1)^2}{u}$ (Ignore integral sign).

1st A1 (B1 on epen): $\frac{x}{1+\sqrt{x}}dx$ becoming $\frac{(u-1)^2}{u}.2(u-1)\{du\}$ or $\frac{(u-1)^2}{u}.\frac{2}{(u-1)^{-1}}\{du\}$.

You can ignore the integral sign and the du.

2nd M1: Expands to give a "four term" cubic in u, $\pm Au^3 \pm Bu^2 \pm Cu \pm D$
where $A\neq 0$, $B\neq 0$, $C\neq 0$ and $D\neq 0$ The cubic does not need to be simplified for this mark.

3rd M1: An attempt to divide at least three terms in ***their cubic*** by u.

Ie. $\frac{(u^3-3u^2+3u-1)}{u}\rightarrow u^2-3u+3-\frac{1}{u}$

2nd A1: $\int\frac{(u-1)^3}{u}du\rightarrow\left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right)$

4th M1: Some evidence of limits of 3 and 2 in u and subtracting either way round.

3rd A1: Exact answer of $\frac{11}{3}+2\ln 2-2\ln 3$ or $\frac{11}{3}+2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3}-\ln\left(\frac{9}{4}\right)$ or $2\left(\frac{11}{6}+\ln 2-\ln 3\right)$

or $\frac{22}{6}+2\ln\left(\frac{2}{3}\right)$, etc. **Note:** that fractions must be combined to give either $\frac{11}{3}$ or $\frac{22}{6}$ or $3\frac{2}{3}$

Alternative method for 2nd M1 and 3rd M1 mark

$$\{2\}\int\frac{(u-1)^2}{u}.(u-1)\,du = \{2\}\int\frac{(u^2-2u+1)}{u}.(u-1)\,du$$

$$=\{2\}\int\left(u-2+\frac{1}{u}\right).(u-1)\,du = \{2\}\int\left(u^2-...\right)du$$

$$=\{2\}\int\left(u^2-3u+3-\frac{1}{u}\right)du$$

An attempt to expand $(u-1)^2$, then divide the result by u and then go on to multiply by $(u-1)$. — **2nd M1**

(c) ctd

Final two marks in part (c): $u=1+\sqrt{x}$

$$\text{Area}(R)=\left[\frac{2(1+\sqrt{x})^3}{3}-3(1+\sqrt{x})^2+6(1+\sqrt{x})-2\ln(1+\sqrt{x})\right]_1^4$$

$$=\left(\frac{2(1+\sqrt{4})^3}{3}-3(1+\sqrt{4})^2+6(1+\sqrt{4})-2\ln(1+\sqrt{4})\right)$$

$$-\left(\frac{2(1+\sqrt{1})^3}{3}-3(1+\sqrt{1})^2+6(1+\sqrt{1})-2\ln(1+\sqrt{1})\right)$$

M1: Applies limits of 4 and 1 in x and subtracts either way round.

$$=(18-27+18-2\ln 3)-\left(\frac{16}{3}-12+12-2\ln 2\right)$$

$$=\frac{11}{3}+2\ln 2-2\ln 3 \text{ or } \frac{11}{3}+2\ln\left(\frac{2}{3}\right) \text{ or } \frac{11}{3}-\ln\left(\frac{9}{4}\right), \text{ etc}$$

A1: Correct exact answer or equivalent.

Alternative method for the final 5 marks in part (b)

$$\int \frac{(u-1)^3}{u}\,\mathrm{d}u\,, \qquad \left\{\begin{array}{lll} "u" = u^{-1} & \Rightarrow & \frac{\mathrm{d}"u"}{\mathrm{d}x} = -u^{-2} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = (u-1)^3 & \Rightarrow & v = \frac{(u-1)^4}{4} \end{array}\right\}$$

$$= \frac{(u-1)^4}{4u} - -\frac{1}{4}\int \frac{(u-1)^4}{u^2}\,\mathrm{d}u$$

$$= \frac{(u-1)^4}{4u} + \frac{1}{4}\int \frac{u^4 - 4u^3 + 6u^2 - 4u + 1}{u^2}\,\mathrm{d}u$$

M1: Applies integration by parts and expands to give a five term quartic.

$$= \frac{(u-1)^4}{4u} + \frac{1}{4}\int u^2 - 4u + 6 - \frac{4}{u} + \frac{1}{u^2}\,\mathrm{d}u$$

M1: Dividing at least 4 terms.

$$= \frac{(u-1)^4}{4u} + \frac{1}{4}\left(\frac{u^3}{3} - 2u^2 + 6u - 4\ln u - \frac{1}{u}\right)$$

A1: Correct Integration.

$$\int_2^3 \frac{(u-1)^3}{u}\,\mathrm{d}u = \left[\frac{(u-1)^4}{4u} + \frac{u^3}{12} - \frac{u^2}{2} + \frac{3u}{2} - \ln u - \frac{1}{4u}\right]_2^3$$

$$= \left(\frac{16}{12} + \frac{27}{12} - \frac{9}{2} + \frac{9}{2} - \ln 3 - \frac{1}{12}\right) - \left(\frac{1}{8} + \frac{8}{12} - \frac{4}{2} + \frac{6}{2} - \ln 2 - \frac{1}{8}\right)$$ **M1**

$$= (7 - \ln 3) - \left(\frac{5}{3} - \ln 2\right)$$

$$= \frac{11}{6} + \ln\frac{2}{3}$$

$$\text{Area}(R) = 2\int_2^3 \frac{(u-1)^3}{u}\,\mathrm{d}u = 2\left(\frac{11}{6} + \ln\frac{2}{3}\right)$$ **A1**

Q15.

Question Number	Scheme		Marks
(a)	0.73508		B1 cao [1]
(b)	Area $\approx \frac{1}{2}\times\frac{\pi}{8}$; $\times\left[0+2\left(\text{their } 0.73508+1.17157+1.02280\right)+0\right]$		B1 M1
	$=\frac{\pi}{16}\times 5.8589... = 1.150392325... = 1.1504$ (4 dp)	awrt 1.1504	A1 [3]
(c)	$\{u=1+\cos x\} \Rightarrow \frac{du}{dx}=-\sin x$		B1
	$\left\{\int \frac{2\sin 2x}{(1+\cos x)}\,dx=\right\} \int \frac{2(2\sin x\cos x)}{(1+\cos x)}\,dx$	$\sin 2x = 2\sin x\cos x$	B1
	$=\int \frac{4(u-1)}{u}.(-1)\,du \quad \left\{= 4\int \frac{(1-u)}{u}\,du\right\}$		M1
	$=4\int\left(\frac{1}{u}-1\right)du = 4\left(\ln u - u\right)+c$		dM1
	$=4\ln(1+\cos x)-4(1+\cos x)+c \quad = 4\ln(1+\cos x)-4\cos x+k$	AG	A1 cso [5]
(d)	$=\left[4\ln\left(1+\cos\frac{\pi}{2}\right)-4\cos\frac{\pi}{2}\right]-\left[4\ln(1+\cos 0)-4\cos 0\right]$	Applying limits $x=\frac{\pi}{2}$ and $x=0$ either way round.	M1
	$=[4\ln 1-0]-[4\ln 2-4]$		
	$=4-4\ln 2 \ \{=1.227411278...\}$	$\pm 4(1-\ln 2)$ or $\pm(4-4\ln 2)$ or awrt ± 1.2, *however* found.	A1
	Error $=\lvert(4-4\ln 2)-1.1504...\rvert$ $=0.0770112776... = 0.077$ (2sf)	awrt ± 0.077 or awrt $\pm 6.3(\%)$	A1 cso [3]
			12

(a)	**B1:** 0.73508 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1:** Outside brackets $\frac{1}{2}\times\frac{\pi}{8}$ or $\frac{\pi}{16}$ or awrt 0.196
	M1: For structure of trapezium rule [.............]; (0 can be implied).
	A1: anything that rounds to 1.1504
	Bracketing mistake: **Unless the final answer implies that the calculation has been done correctly**
	Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8} + 2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 6.0552).
	Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8}\ (0+0)+2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 5.8589).
	Alternative method for part (b): Adding individual trapezia
	Area $\approx \frac{\pi}{8}\times\left[\frac{0+0.73508}{2}+\frac{0.73508+1.17157}{2}+\frac{1.17157+1.02280}{2}+\frac{1.02280+0}{2}\right] = 1.150392325...$
	B1: $\frac{\pi}{8}$ and a divisor of 2 on all terms inside brackets.
	M1: One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.
	A1: anything that rounds to 1.1504
(c)	**B1:** $\frac{du}{dx}=-\sin x$ or $du=-\sin x\,dx$ or $\frac{dx}{du}=\frac{1}{-\sin x}$ oe.
	B1: For seeing, applying or implying $\sin 2x = 2\sin x\cos x$.

M1: After applying substitution candidate achieves $\pm k\int \frac{(u-1)}{u}(\mathrm{d}u)$ or $\pm k\int \frac{(1-u)}{u}(\mathrm{d}u)$.

Allow M1 for "invisible" brackets here, eg: $\pm \int \frac{(\lambda u-1)}{u}(\mathrm{d}u)$ or $\pm \int \frac{(-\lambda+u)}{u}(\mathrm{d}u)$, where λ is a positive constant.

dM1: An attempt to divide through each term by u and $\pm k\int\left(\frac{1}{u}-1\right)\mathrm{d}u \rightarrow \pm k(\ln u - u)$ with/without $+c$. Note that this mark is dependent on the previous M1 mark being awarded.

Alternative method: Candidate can also gain this mark for applying integration by parts followed by a correct method for integrating $\ln u$. (See below).

A1: Correctly combines their $+c$ and "-4" together to give $\underline{4\ln(1+\cos x)-4\cos x+k}$

As a minimum candidate must write either $4\ln(1+\cos x)-4(1+\cos x)+c \rightarrow 4\ln(1+\cos x)-4\cos x+k$ or $4\ln(1+\cos x)-4(1+\cos x)+k \rightarrow 4\ln(1+\cos x)-4\cos x+k$

Note: that this mark is also for a correct solution only.

Note: those candidates who attempt to find the value of k will usually achieve A0.

(d)

M1: Substitutes limits of $x=\frac{\pi}{2}$ and $x=0$ into $\{4\ln(1+\cos x)-4\cos x\}$ or their answer from part (c) and subtracts the either way round. **Note that:** $\left[4\ln\left(1+\cos\frac{\pi}{2}\right)-4\cos\frac{\pi}{2}\right]-[0]$ is M0.

A1: $4(1-\ln 2)$ or $4-4\ln 2$ or awrt 1.2, however found.

This mark can be implied by the final answer of either awrt ± 0.077 or awrt ± 6.3

A1: For either awrt ± 0.077 or awrt ± 6.3 (for percentage error). Note this mark is for a **correct solution only**. Therefore if there if a candidate substitutes limits the incorrect way round and final achieves (usually fudges) the final correct answer then this mark can be withheld. Note that awrt 6.7 (for percentage error) is A0.

Alternative method for dM1 in part (c)

Alternative method for dM1 in part (c)

$$\int \frac{(1-u)}{u}\mathrm{d}u = \left((1-u)\ln u - \int -\ln u\,du\right) = \left((1-u)\ln u + u\ln u - \int \frac{u}{u}\mathrm{d}u\right) = \left((1-u)\ln u + u\ln u - u\right)$$

or $$\int \frac{(u-1)}{u}\mathrm{d}u = \left((u-1)\ln u - \int \ln u\,\mathrm{d}u\right) = \left((u-1)\ln u - \left(u\ln u - \int \frac{u}{u}\mathrm{d}u\right)\right) = \left((u-1)\ln u - u\ln u + u\right)$$

So dM1 is for $\int \frac{(1-u)}{u}\mathrm{d}u$ going to $((1-u)\ln u + u\ln u - u)$ or $((u-1)\ln u - u\ln u + u)$ oe.

Alternative method for part (d)

M1A1 for $\left\{4\int_2^1\left(\frac{1}{u}-1\right)\mathrm{d}u = \right\} 4\,[\ln u - u]_2^1 = 4\left[(\ln 1 - 1)-(\ln 2 - 2)\right] = 4(1-\ln 2)$

Alternative method for part (d): Using an extra constant λ from their integration.

$$\left[4\ln\left(1+\cos\frac{\pi}{2}\right)-4\cos\frac{\pi}{2}+\lambda\right]-\left[4\ln(1+\cos 0)-4\cos 0+\lambda\right]$$

λ is usually -4, but can be a value of k that the candidate has found in part (d).

Note: The extra constant λ should cancel out and so the candidate can gain all three marks using this method, even the final A1 cso.

Q16.

Question Number	Scheme	Marks
(a)	$x^2+2x+2=10 \Rightarrow x^2+2x-8=0$ (so $(x+4)(x-2)=0$) $\Rightarrow x=$........	M1
	$x=-4,\ 2$	A1 (2)
(b) Way 1	$\int(x^2+2x+2)\,\mathrm{d}x=\frac{x^3}{3}+\frac{2x^2}{2}+2x(+C)$	M1A1A1
	$\left[\frac{x^3}{3}+\frac{2x^2}{2}+2x\right]_{"-4"}^{"2"}=\left(\frac{8}{3}+\frac{8}{2}+4\right)-\left(-\frac{64}{3}+\frac{32}{2}-8\right)\quad(=24)$	M1
	Rectangle: $10\times(2--4)=60$	B1 cao
	$R="60"-"24"$	M1
	$=36$	A1 (7)
		Total 9
(b) Way 2	$\int(8-x^2-2x)\,\mathrm{d}x=8x-\frac{x^3}{3}-\frac{2x^2}{2}(+C)$	M1 A1ft A1
	$\left[8x-\frac{x^3}{3}-\frac{2x^2}{2}\right]_{"-4"}^{"2"}=\left(16-\frac{8}{3}-4\right)-\left(-32+\frac{64}{3}-16\right)$ =(9.3 – (- 26.7))	M1
	Implied by final answer of 36 after correct work	B1
	$10-(x^2+2x+2)=8-x^2-2x,\ =36$	M1, A1
	Notes for Question	
(a)	M1 Set the curve equation equal to 10 and collect terms. Solves quadratic to $x=$ ..... A1 cao : Both values correct – allow $A=-4$, $B=2$	
(b)	M1: One correct integration A1: Two correct integrations(ft slips subtracting in Way 2) A1: All 3 terms **correct (penalise subtraction errors here in Way 2)** M1: Substitute their limits from (a) into the **integrated** function and subtract (either way round) B1: Way 1:Find area under the line by integration or area of rectangle – should be 60 here (no follow through) Way 2: (implied by final correct answer in second method) M1: Subtract one area from the other (implied by subtraction of functions in second method)- award even after differentiation A1: Must be 36 not –36.	
	Special case 1: Combines both methods. Uses Way 2 integration, but continues after reaching "36" to subtract "36" from rectangle giving answer as "24" This loses final M1 A1 *Special case* 2: Integrates (x^2+2x-8) between limits –4 and 2 to get -36 and then changes sign and obtains 36. Do not award final A mark – so M1A1A1M1B1M1A0 If the answer is left as -36, then M1A1A1M1B0M1A0 N.B. Allow full marks for modulus used earlier in working e.g. $\left\vert\int_{-4}^{2}x^2+2x-2\,\mathrm{d}x-\int_{-4}^{2}10\,\mathrm{d}x\right\vert$	

Q17.

Question	Scheme		Marks
Method 1 5 (a)	Puts $10 - x = 10x - x^2 - 8$ and rearranges to give three term quadratic	Or puts $y = 10(10 - y) - (10 - y)^2 - 8$ and rearranges to give three term quadratic	M1
	Solves their "$x^2 - 11x + 18 = 0$" using acceptable method as in general principles to give $x =$	Solves their "$y^2 - 9y + 8 = 0$" using acceptable method as in general principles to give $y =$	M1
	Obtains $x = 2$, $x = 9$ (may be on diagram or in part (b) in limits)	Obtains $y = 8$, $y = 1$ (may be on diagram)	A1
	Substitutes their x into a given equation to give $y =$ (may be on diagram)	Substitutes their y into a given equation to give $x=$ (may be on diagram or in part (b))	M1
	$y = 8$, $y = 1$	$x = 2$, $x = 9$	A1 (5)
(b)	$\int (10x - x^2 - 8)\,dx = \frac{10x^2}{2} - \frac{x^3}{3} - 8x \{+ c\}$		M1 A1 A1
	$\left[\frac{10x^2}{2} - \frac{x^3}{3} - 8x\right]_2^9 = (\ldots\ldots) - (\ldots\ldots)$		dM1
	$= 90 - \frac{4}{3} = 88\frac{2}{3}$ or $\frac{266}{3}$		
	Area of trapezium $= \frac{1}{2}(8+1)(9-2) = 31.5$		B1
	So area of R is $88\frac{2}{3} - 31.5 = 57\frac{1}{6}$ or $\frac{343}{6}$		M1A1 cao (7)
			12 marks

Notes (a)

First **M1:** See scheme Second **M1**: See notes relating to solving quadratics

Third **M1**: This may be awarded if one substitution is made

Two correct Answers following tables of values, or from Graphical calculator are 5/5

Just one pair of correct coordinates – no working or from table is M0M0A0M1A0

(b)

M1: $x^n \to x^{n+1}$ for any one term.

1st **A1**: at least two out of three terms correct 2nd **A1**: All three correct

dM1: Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round

(NB: If candidate **changes all signs** to get $\int (-10x + x^2 + 8)\,dx = -\frac{10x^2}{2} + \frac{x^3}{3} + 8x \{+ c\}$ This is M1 A1 A1

Then uses limits dM1 and trapezium is B1

Needs to *change sign of value obtained* from integration for final M1A1 so $-88\frac{2}{3} - 31.5$ is M0A0)

B1: Obtains 31.5 for area under line using any correct method (could be integration) or triangle minus triangle $\frac{1}{2}\times 8\times 8 - \frac{1}{2}$ or rectangle plus triangle [may be implied by correct 57 1/6]

M1: Their Area under curve – Their Area under line (if integrate both need same limits)

A1: Accept 57.16recurring but not 57.16

PTO for Alternative method

Method 2 for (b)	Area of R $= \int_2^9 (10x - x^2 - 8) - (10 - x)\, dx$ $\int_2^9 -x^2 + 11x - 18 dx$ $= -\frac{x^3}{3} + \frac{11x^2}{2} - 18x \{+ c\}$ $\left[-\frac{x^3}{3} + \frac{11x^2}{2} - 18x\right]_2^9 = (......) - (......)$	3^{rd} M1 (in (b)): Uses difference between two functions in integral. M: $x^n \to x^{n+1}$ for any one term. A1 at least two out of these three simplified terms Correct integration. (Ignore + c). Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	M1 A1 A1 dM1
	This mark is implied by final answer which rounds to 57.2 ***See above working(allow bracketing errors) to decide to award 3rd M1 mark for (b) here:*** $40.5 - (-16\frac{2}{3}) = 57\frac{1}{6}$ cao		B1 M1 A1 (7)
Special case of above method	$\int_2^9 x^2 - 11x + 18 dx = \frac{x^3}{3} - \frac{11x^2}{2} + 18x \{+ c\}$ $\left[\frac{x^3}{3} - \frac{11x^2}{2} + 18x\right]_2^9 = (......) - (......)$ This mark is implied by final answer which rounds to 57.2 (not -57.2) Difference of functions implied (see above expression) $40.5 - (-16\frac{2}{3}) = 57\frac{1}{6}$ cao		M1A1A1 DM1 B1 M1 A1 (7)
Special Case 2	Integrates expression in y e.g. "$y^2 - 9y + 8 = 0$": This can have first M1 in part (b) and no other marks. (It is not a method for finding this area)		
Notes	Take away trapezium again having used Method 2 loses last two marks Common Error: Integrates $-x^2 + 9x - 18$ is likely to be M1A1A0dM1B0M1A0 Integrates $2 - 11x - x^2$ is likely to e M1A0A0dM1B0M1A0 Writing $\int_2^9 (10x - x^2 - 8) - (10 - x)\, dx$ only earns final M mark		