

Binomial Hypothesis testing Exam Questions

Q1.

In a manufacturing process 25% of articles are thought to be defective. Articles are produced in batches of 20

(a) A batch is selected at random. Using a 5% significance level, find the critical region for a two tailed test that the probability of an article chosen at random being defective is 0.25

You should state the probability in each tail which should be as close as possible to 0.025

(5)

The manufacturer changes the production process to try to reduce the number of defective articles. She then chooses a batch at random and discovers there are 3 defective articles.

(b) Test at the 5% level of significance whether or not there is evidence that the changes to the process have reduced the percentage of defective articles. State your hypotheses clearly.

(5)

(Total 10 marks)

Q2.

A potter believes that 20% of pots break whilst being fired in a kiln. Pots are fired in batches of 25.

(a) Let X denote the number of broken pots in a batch. A batch is selected at random. Using a 10% significance level, find the critical region for a two tailed test of the potter's belief. You should state the probability in each tail of your critical region.

(4)

The potter aims to reduce the proportion of pots which break in the kiln by increasing the size of the batch fired. He now fires pots in batches of 50. He then chooses a batch at random and discovers there are 6 pots which broke whilst being fired in the kiln.

(b) Test, at the 5% level of significance, whether or not there is evidence that increasing the number of pots in a batch has reduced the percentage of pots that break whilst being fired in the kiln. State your hypotheses clearly.

(5)

(Total for question = 9 marks)

Q3.

The proportion of houses in Radville which are unable to receive digital radio is 25%. In a survey of a random sample of 30 houses taken from Radville, the number, X , of houses which are unable to receive digital radio is recorded.

- (a) Find $P(5 \leq X < 11)$

(3)

A radio company claims that a new transmitter set up in Radville will reduce the proportion of houses which are unable to receive digital radio. After the new transmitter has been set up, a random sample of 15 houses is taken, of which 1 house is unable to receive digital radio.

- (b) Test, at the 10% level of significance, the radio company's claim. State your hypotheses clearly.

(5)

(Total for question = 8 marks)

Q4.

Sue throws a fair coin 15 times and records the number of times it shows a head.

- (a) State the distribution to model the number of times the coin shows a head.

(2)

Find the probability that Sue records

- (b) exactly 8 heads,
(c) at least 4 heads.

(2)

Sue has a different coin which she believes is biased in favour of heads. She throws the coin 15 times and obtains 13 heads.

- (d) Test Sue's belief at the 1% level of significance. State your hypotheses clearly.

(6)

(Total for question = 10 marks)

Q5.

Linda regularly takes a taxi to work five times a week. Over a long period of time she finds the taxi is late once a week. The taxi firm changes her driver and Linda thinks the taxi is late more often. In the first week, with the new driver, the taxi is late 3 times.

You may assume that the number of times a taxi is late in a week has a Binomial distribution.

Test, at the 5% level of significance, whether or not there is evidence of an increase in the proportion of times the taxi is late. State your hypotheses clearly.

(7)

(Total 7 marks)

Q6.

Before Roger will use a tennis ball he checks it using a "bounce" test. The probability that a ball from Roger's usual supplier fails the bounce test is 0.2. A new supplier claims that the probability of one of their balls failing the bounce test is less than 0.2. Roger checks a random sample of 40 balls from the new supplier and finds that 3 balls fail the bounce test.

Stating your hypotheses clearly, use a 5% level of significance to test the new supplier's claim.

(5)

(Total 5 marks)

Q7.

A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

(a) Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample.

(2)

(b) Using a 5% significance level, find the critical region for a two-tailed test of the hypothesis that the

probability of a bolt being faulty is $\frac{1}{4}$. The probability of rejection in either tail should be as close as possible to 0.025

(3)

(c) Find the actual significance level of this test.

(2)

In the sample of 50 the actual number of faulty bolts was 8.

(d) Comment on the company's claim in the light of this value. Justify your answer.

(2)

The machine making the bolts was reset and another sample of 50 bolts was taken. Only 5 were found to be faulty.

(e) Test at the 1% level of significance whether or not the probability of a faulty bolt has decreased. State your hypotheses clearly.

(6)

(Total 15 marks)

Q8.

Dhriti grows tomatoes. Over a period of time, she has found that there is a probability 0.3 of a ripe tomato having a diameter greater than 4 cm. She decides to try a new fertiliser. In a random sample of 40 ripe tomatoes, 18 have a diameter greater than 4 cm. Dhriti claims that the new fertiliser has increased the probability of a ripe tomato being greater than 4 cm in diameter.

Test Dhriti's claim at the 5% level of significance. State your hypotheses clearly.

(7)

(Total 7 marks)

Q9.

(a) Define the critical region of a test statistic.

(2)

A discrete random variable X has a Binomial distribution $B(30, p)$. A single observation is used to test

$H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(b) Using a 1% level of significance find the critical region of this test. You should state the probability of rejection in each

tail which should be as close as possible to 0.005

(5)

(c) Write down the actual significance level of the test.

(1)

The value of the observation was found to be 15.

(d) Comment on this finding in light of your critical region.

(2)

(Total 10 marks)

Q10.

The proportion of left-handed adults in a country is 10%

Freya believes that the portion of left-handed adults under the age of 25 in this country is different from 10%

She takes a random sample of 40 adults under the age of 25 from this country to investigate her belief.

(a) Find the critical region for a suitable test to assess Freya's belief.

You should

- state your hypotheses clearly
- use a 5% level of significance
- state the probability of rejection in each tail

(4)

(b) Write down the actual significance level of your test in part (a)

(1)

In Freya's sample 7 adults were left-handed.

(c) With reference to your answer in part (a) comment on Freya's belief.

(1)

(Total for question = 6 marks)