Questions

Q1.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Write $\dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$ in the form $a\sqrt{3}+b\sqrt{5}$ where *a* and *b* are integers to be found.

(3)

(b) Hence, or otherwise, solve

$$\left(x+5\sqrt{3}\right)\sqrt{5}=40-2x\sqrt{3}$$

giving your answer in simplest form.

(3)

(Total for question = 6 marks)

Q2.

Given that

$$(3pq^2)^4 \times 2p\sqrt{q^8} \equiv ap^bq^c$$

find the values of the constants *a*, *b* and *c*.

(Total for question = 3 marks)

Q3.

Given $y = 3^x$, express each of the following in terms of *y*. Write each expression in its simplest form.

(a) 3^{3x}

(1)

(b) $\dfrac{1}{3^{x-2}}$

(2)

(c) $\dfrac{81}{9^{2-3x}}$

(2)

(Total for question = 5 marks)

Q4.

A curve with equation $y = f(x)$ passes through the point (9, 10).

Given that

$$f'(x) = 27x^2 - \frac{21x^3 - 5x}{2\sqrt{x}} \qquad x > 0$$

find $f(x)$, fully simplifying each term.

(Total for question = 6 marks)

Q5.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

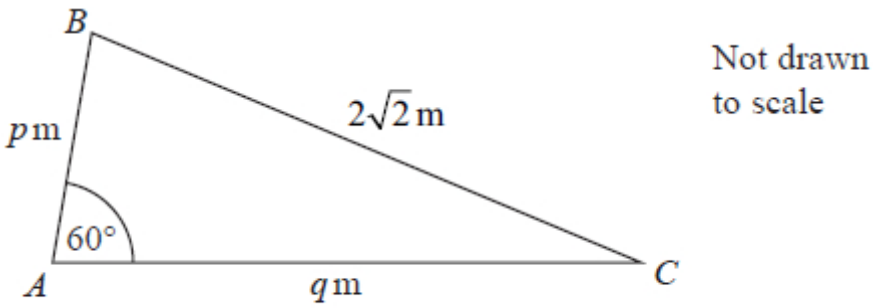

Figure 1

Figure 1 shows the plan view of a flower bed.
The flowerbed is in the shape of a triangle *ABC* with

- *AB* = *p* metres
- *AC* = *q* metres
- *BC* = $2\sqrt{2}$ metres
- angle *BAC* = 60°

(a) Show that

$$p^2 + q^2 - pq = 8$$

(2)

Given that side *AC* is 2 metres longer than side *AB*, use algebra to find

(b) (i) the exact value of *p*,

(ii) the exact value of q.

(5)

Using the answers to part (b),

(c) calculate the exact area of the flower bed.

(2)

(Total for question = 9 marks)

Q6.

Solve, using algebra, the equation

$$x - 6x^{\frac{1}{2}} + 4 = 0$$

Fully simplify your answers, writing them in the form $a + b\sqrt{c}$, where a, b and c are integers to be found.

(5)

(Total for question = 5 marks)

Q7.

Given

$$\frac{3^x}{3^{4y}} = 27\sqrt{3}$$

find y as a simplified function of x.

(Total for question = 3 marks)

Q8.

$$f(x) = 11 - 4x - 2x^2$$

(a) Express $f(x)$ in the form

$$a + b(x + c)^2$$

where a, b and c are integers to be found.

(3)

(b) Sketch the graph of the curve C with equation $y = f(x)$, showing clearly the coordinates of the point where the curve crosses the y-axis.

(2)

(c) Write down the equation of the line of symmetry of C.

(1)

(Total for question = 6 marks)

Q9.

The curve C_1 has equation

$$y = x^2 + kx - 9$$

and the curve C_2 has equation

$$y = -3x^2 - 5x + k$$

where k is a constant.

Given that C_1 and C_2 meet at a single point P

(a) show that

$$k^2 + 26k + 169 = 0$$

(3)

(b) Hence find the coordinates of P

(3)

(Total for question = 6 marks)

Q10.

(a) Express $3x^2 + 12x + 13$ in the form

$$a(x + b)^2 + c$$

where a, b and c are integers to be found.

(3)

(b) Hence sketch the curve with equation $y = 3x^2 + 12x + 13$

On your sketch show clearly

- the coordinates of the y intercept
- the coordinates of the turning point of the curve

(3)

(Total for question = 6 marks)

Q11.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = ax^3 + (6a + 8)x^2 - a^2x$$

where a is a positive constant.

Given $f(-1) = 32$

(a) (i) show that the only possible value for a is 3

(ii) Using $a = 3$ solve the equation

$$f(x) = 0$$

(5)

(b) Hence find all real solutions of

(i) $3y + 26y^{\frac{2}{3}} - 9y^{\frac{1}{3}} = 0$

(ii) $3\left(9^{3z}\right) + 26\left(9^{2z}\right) - 9\left(9^{z}\right) = 0$

(5)

(Total for question = 10 marks)

Q12.

Solve the simultaneous equations

$$y + 4x + 1 = 0$$

$$y^2 + 5x^2 + 2x = 0$$

(Total for question = 6 marks)

Q13.

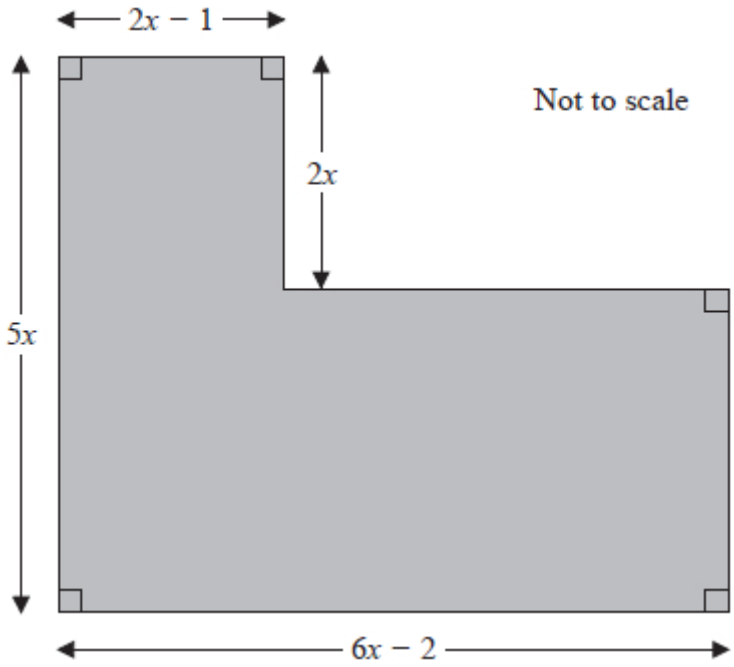

Figure 1

Figure 1 shows the plan of a garden. The marked angles are right angles.

The six edges are straight lines.

The lengths shown in the diagram are given in metres.

Given that the perimeter of the garden is greater than 29 m,

(a) show that $x > 1.5$ m

(3)

Given also that the area of the garden is less than 72 m^2,

(b) form and solve a quadratic inequality in x.

(5)

(c) Hence state the range of possible values of x.

(1)

(Total for question = 9 marks)

Q14.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) By substituting $p = 2^x$, show that the equation

$$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4$$

can be written in the form

$$4p^2 - 33p + 8 = 0$$

(3)

(b) Hence solve

$$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4$$

(3)

(Total for question = 6 marks)

Q15.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(i)

$$f(x) = (x + \sqrt{2})^2 + (3x - 5\sqrt{8})^2$$

Express f(x) in the form $ax^2 + bx\sqrt{2} + c$ where a, b and c are integers to be found.

(3)

(ii) Solve the equation

$$\sqrt{3}(4y - 3\sqrt{3}) = 5y + \sqrt{3}$$

giving your answer in the form $p + q\sqrt{3}$ where p and q are simplified fractions to be found.

(4)

(Total for question = 7 marks)

Q16.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Expand and simplify

$$\left(r - \frac{1}{r}\right)^2$$

(2)

(b) Express $\frac{1}{3 + 2\sqrt{2}}$ in the form $p + q\sqrt{2}$ where p and q are integers.

(2)

(c) Use the results of parts (a) and (b), or otherwise, to show that

$$\sqrt{3 + 2\sqrt{2}} - \frac{1}{\sqrt{3 + 2\sqrt{2}}} = 2$$

(3)

(Total for question = 7 marks)