

Exam Questions – Normal Distribution

Q1.

The heights of adult females are normally distributed with mean 160 cm and standard deviation 8 cm.

(a) Find the probability that a randomly selected adult female has a height greater than 170 cm.

(3)

Any adult female whose height is greater than 170 cm is defined as tall.

An adult female is chosen at random. Given that she is tall,

(b) find the probability that she has a height greater than 180 cm.

(4)

Half of tall adult females have a height greater than h cm.

(c) Find the value of h .

(5)

(Total 12 marks)

Q2.

The time taken, in minutes, by children to complete a mathematical puzzle is assumed to be normally distributed with mean μ and standard deviation σ . The puzzle can be completed in less than 24 minutes by 80% of the children. For 5% of the children it takes more than 28 minutes to complete the puzzle.

(a) Show this information on the Normal curve below.

(2)

(b) Write down the percentage of children who take between 24 minutes and 28 minutes to complete the puzzle.

(1)

(c) (i) Find two equations in μ and σ .

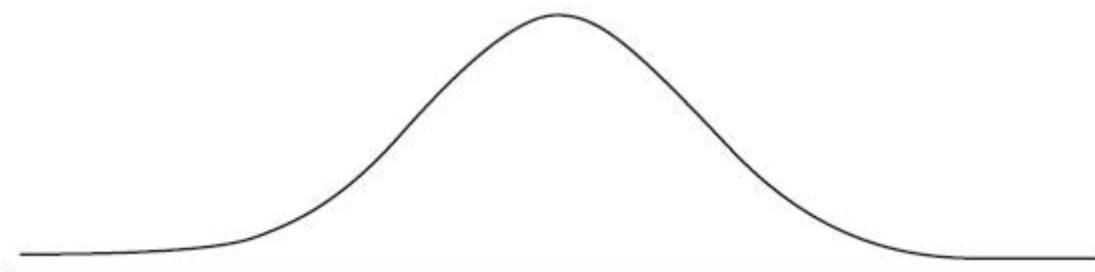
(ii) Hence find, to 3 significant figures, the value of μ and the value of σ .

(7)

A child is selected at random.

(d) Find the probability that the child takes less than 12 minutes to complete the puzzle.

(3)



(Total 13 marks)

Q3.

The random variable $Z \sim N(0, 1)$

A is the event $Z > 1.1$

B is the event $Z > -1.9$

C is the event $-1.5 < Z < 1.5$

(a) Find

(i) $P(A)$

(ii) $P(B)$

(iii) $P(C)$

(iv) $P(A \cup C)$

(6)

The random variable X has a normal distribution with mean 21 and standard deviation 5

(b) Find the value of w such that $P(X > w \mid X > 28) = 0.625$

(6)

(Total for question = 12 marks)

Q4.

The length of time, L hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

(a) Find $P(L > 127)$.

(3)

(b) Find the value of d such that $P(L < d) = 0.10$

(3)

Alice is about to go on a 6 hour journey.

Given that it is 127 hours since Alice last charged her phone,

(c) find the probability that her phone will not need charging before her journey is completed.

(4)

(Total 10 marks)

Q5.

Past records show that the times, in seconds, taken to run 100 m by children at a school can be modelled by a normal distribution with a mean of 16.12 and a standard deviation of 1.60

A child from the school is selected at random.

- (a) Find the probability that this child runs 100 m in less than 15 s.

(3)

On sports day the school awards certificates to the fastest 30% of the children in the 100 m race.

- (b) Estimate, to 2 decimal places, the slowest time taken to run 100 m for which a child will be awarded a certificate.

(4)

(Total 7 marks)

Q6.

The weight, X grams, of soup put in a tin by machine A is normally distributed with a mean of 160 g and a standard deviation of 5 g. A tin is selected at random.

- (a) Find the probability that this tin contains more than 168 g.

(3)

The weight stated on the tin is w grams.

- (b) Find w such that $P(X < w) = 0.01$

(3)

The weight, Y grams, of soup put into a carton by machine B is normally distributed with mean μ grams and standard deviation σ grams.

- (c) Given that $P(Y < 160) = 0.99$ and $P(Y > 152) = 0.90$ find the value of μ and the value of σ .

(6)

(Total 12 marks)

Q7.

Yuto works in the quality control department of a large company. The time, T minutes, it takes Yuto to analyse a sample is normally distributed with mean 18 minutes and standard deviation 5 minutes.

- (a) Find the probability that Yuto takes longer than 20 minutes to analyse the next sample.

(3)

The company has a large store of samples analysed by Yuto with the time taken for each analysis recorded. Serena is investigating the samples that took Yuto longer than 15 minutes to analyse.

She selects, at random, one of the samples that took Yuto longer than 15 minutes to analyse.

- (b) Find the probability that this sample took Yuto more than 20 minutes to analyse.

(4)

Serena can identify, in advance, the samples that Yuto can analyse in under 15 minutes and in future she will assign these to someone else.

- (c) Estimate the median time taken by Yuto to analyse samples in future.

(5)

(Total for question = 12 marks)

Q8.

The time, in minutes, taken by men to run a marathon is modelled by a normal distribution with mean 240 minutes and standard deviation 40 minutes.

- (a) Find the proportion of men that take longer than 300 minutes to run a marathon.

(3)

Nathaniel is preparing to run a marathon. He aims to finish in the first 20% of male runners.

- (b) Using the above model estimate the longest time that Nathaniel can take to run the marathon and achieve his aim.

(3)

The time, W minutes, taken by women to run a marathon is modelled by a normal distribution with mean μ minutes.

Given that $P(W < \mu + 30) = 0.82$

- (c) find $P(W < \mu - 30 \mid W < \mu)$

(3)

(Total for question = 9 marks)

Q9.

The heights of a population of women are normally distributed with mean μ cm and standard deviation σ cm. It is known that 30% of the women are taller than 172 cm and 5% are shorter than 154 cm.

- (a) Sketch a diagram to show the distribution of heights represented by this information

(3)

- (b) Show that $\mu = 154 + 1.6449\sigma$.

(3)

- (c) Obtain a second equation and hence find the value of μ and the value of σ .

(4)

A woman is chosen at random from the population.

- (d) Find the probability that she is taller than 160 cm.

(3)

(Total 13 marks)