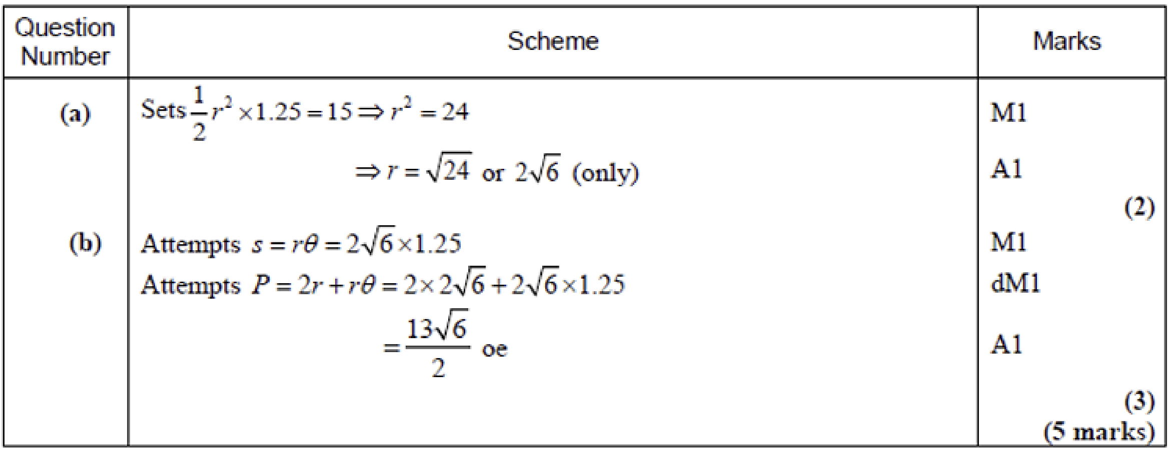
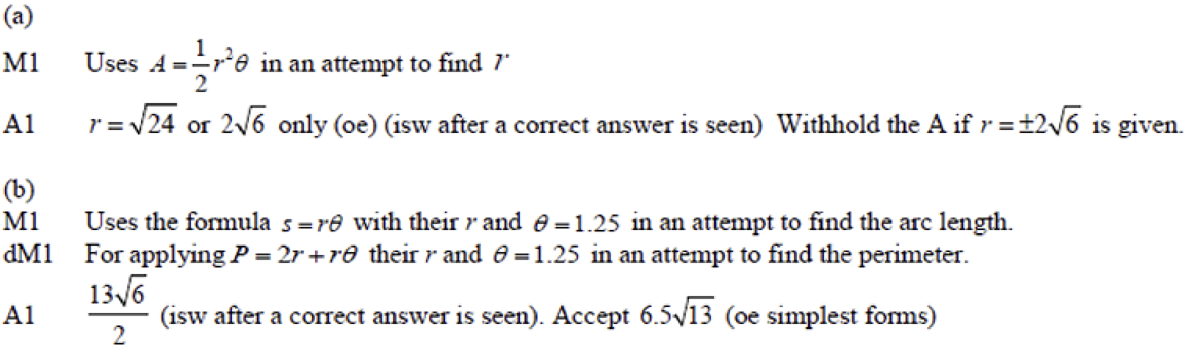
Mark Scheme

Q1.



Question Number	Scheme	Marks
(a)	Sets $\frac{1}{2}r^2\times 1.25=15\Rightarrow r^2=24$	M1
	$\Rightarrow r=\sqrt{24}$ or $2\sqrt{6}$ (only)	A1
		(2)
(b)	Attempts $s=r\theta=2\sqrt{6}\times 1.25$	M1
	Attempts $P=2r+r\theta=2\times 2\sqrt{6}+2\sqrt{6}\times 1.25$	dM1
	$=\frac{13\sqrt{6}}{2}$ oe	A1
		(3) **(5 marks)**



(a)

M1 Uses $A=\frac{1}{2}r^2\theta$ in an attempt to find r

A1 $r=\sqrt{24}$ or $2\sqrt{6}$ only (oe) (isw after a correct answer is seen) Withhold the A if $r=\pm 2\sqrt{6}$ is given.

(b)

M1 Uses the formula $s=r\theta$ with their r and $\theta=1.25$ in an attempt to find the arc length.

dM1 For applying $P=2r+r\theta$ their r and $\theta=1.25$ in an attempt to find the perimeter.

A1 $\frac{13\sqrt{6}}{2}$ (isw after a correct answer is seen). Accept $6.5\sqrt{13}$ (oe simplest forms)

Q2

Question	Scheme	Marks
(a)	With θ being the angle subtended by arc AB and ϕ being the angle subtended by arc CD	
	$15=9\times\theta\Rightarrow\theta=\frac{5}{3}=(1.67)$	M1
	Therefore $\phi=\frac{2\pi}{3}-\frac{5}{3}=(0.4277...)$	dM1
	So length of arc CD = $84\times\left(\frac{2\pi}{3}-\frac{5}{3}\right)=35.929...=35.9\,\text{cm}$ (1 d.p.)* CSO	A1*
		(3)
(b)	Perimeter $=3\times(15+35.9...)+6\times(84-9)$	M1
	$=$ awrt 603 cm (602.787…)	A1
		(2)
(c)	FOR EXAMPLE Area of a "blade" is $\frac{1}{2}\times84^2\times"\left(\frac{2\pi-5}{3}\right)"$ = awrt (1510)	M1
	Area of sector of inner circle between "blades" is $\frac{1}{2}\times9^2\times"\frac{5}{3}"=(67.5)$	dM1 A1
	Total area is $3\left(\frac{1}{2}\times84^2\times"\left(\frac{2\pi}{3}-\frac{5}{3}\right)"+\frac{1}{2}\times9^2\times"\frac{5}{3}"\right)=...(4729.577764\,\text{cm}^2)$	ddM1
	So area is awrt 0.473 m^2 or awrt 4730 cm^2	A1
		(5)
		(10 marks)
Notes:		

(a)

M1: Correct use of the arc length formula to find the angle subtended by arc *AB*.

Attempts $15 = 9 \times \theta \Rightarrow \theta = ...$ Don't be concerned by what the angle is called

dM1: Correct method to find the angle subtended by arc *CD* using their angle for arc *AB*.

Note that $\phi = \frac{1}{3}\left(2\pi - 3 \times \frac{5}{3}\right)$ is also correct. It is dependent upon the previous M

A1*: CSO Arrives at 35.9 with a correct value to at least 2 d.p. (rounded or truncated) seen first.

Alternatively sight of $84 \times \left(\frac{2\pi}{3} - \frac{5}{3}\right)$ or $84 \times$ awrt 0.4277 followed by $35.9(\text{cm})$ is fine

Note that there are equivalent methods such as $84 \times \frac{2\pi}{3} - 84 \times \frac{5}{3} = 35.9$ or $\frac{2\pi}{3} \times 84 - 140 = 35.9$

(b)

M1: Correct method to find the perimeter, it should include all six arcs and radial edges.

Look for $3 \times 15 + 3 \times 35.9 + 6 \times ...$ If no method is seen it is implied by awrt 603

A1: For awrt 603 (cm). The units need not be given.

(c) This part is now being marked M1 dM1 A1 ddM1 A1

Please look through all of the solution first. The marks can be awarded in the following way.

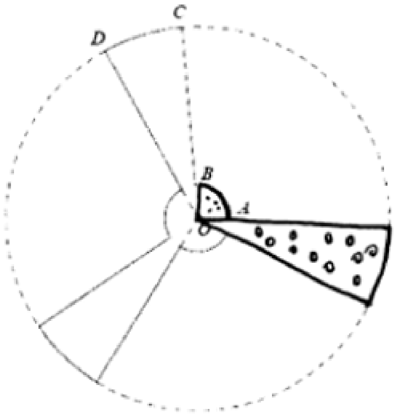
M1: A correct attempt at any relevant area

dM1: A correct attempt at a corresponding area that can be combined with the first area in some way to find the area of the fan. FT on angles found in part (a). Dependent upon previous mark

A1: Both areas correct. They do not need to be calculated but the angles must be correct to 3sf

ddM1: A correct combination of areas to find the area of the fan

A1: awrt 0.473 m^2 or awrt 4730 cm^2. Must include the units. ISW after a correct answer



Main method

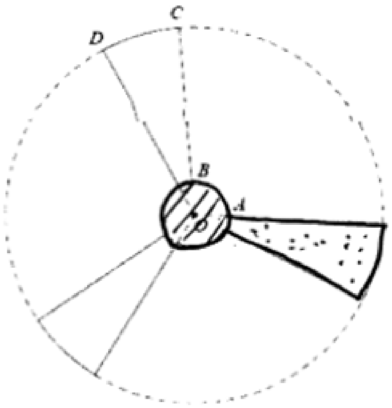
∴ M1 : One relevant area eg $\frac{1}{2} \times 9^2 \times \text{"}\frac{5}{3}\text{"}$

°° dM1 : Corresponding area $\frac{1}{2} \times 84^2 \times \left(\frac{2\pi}{3} - \text{"}\frac{5}{3}\text{"}\right)$

A1 : Both correct

ddM1 : 3×  + 3× 

A1 : Awrt 0.473 m^2 or 4730 cm^2



Alt I

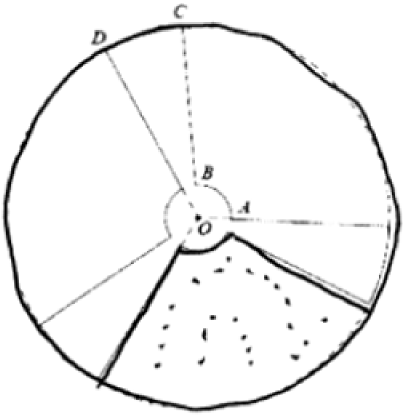
▨ M1 : One relevant area Eg $\pi \times 9^2$

⏢ dM1 : Corresponding area $\frac{1}{2} \times 84^2 \left(\frac{2\pi}{3} - \frac{5}{3}\right) - \frac{1}{2} \times 9^2 \times \left(\frac{\pi}{3} - \frac{5}{3}\right)$

A1 : Both correct

ddM1 : ▨ $+ 3\times$ ⏢

A1: Awrt 0.473 m^2 or 4730 cm^2



Alt II

○ M1 : One relevant area Eg $\pi \times 84^2$

⌔ dM1 : Corresponding area $\frac{1}{2} \times 84^2 \times \frac{5}{3} - \frac{1}{2} \times 9^2 \times \frac{5}{3}$

A1 : Both correct

ddM1 : ○ $- 3\times$ ⌔

A1 : Awrt 0.473 m^2 or 4730 cm^2

Variations are possible, e.g. 3 × area of blades (inc. circle) + area circle – area of blades within the circle, but these can be marked according to the scheme.

Q3

Question	Scheme		Marks
(a)	$9^2 = 4^2 + 6^2 - 2\times 4\times 6\cos\alpha \Rightarrow \cos\alpha = ...$	Correct use of cosine rule leading to a value for cos α	M1
	$\cos\alpha = \frac{4^2+6^2-9^2}{2\times 4\times 6}\left(=-\frac{29}{48}=-0.604..\right)$		
	$\alpha = 2.22$ * cso		A1
			(2)
	Alternative		
	$XY^2 = 4^2 + 6^2 - 2\times 4\times 6\cos 2.22 \Rightarrow XY^2 = ..$	Correct use of cosine rule leading to a value for XY^2	M1
	$XY = 9.00....$		A1
			(2)
(b)	$2\pi - 2.22(=4.06366......)$	$2\pi - 2.22$ or $2\pi - 2.2$ or awrt 4.06 (May be implied)	B1
	$\frac{1}{2}\times 4^2\times "4.06"$	Correct method for major sector area. Allow $\pi - 2.22$ for the major sector angle.	M1
	32.5	Awrt 32.5	A1
			(3)
	Alternative – Circle Minor – sector		
	$\pi\times 4^2$	Correct expression for circle area	B1
	$\pi\times 4^2 - \frac{1}{2}\times 4^2\times 2.22 = 32.5$	Correct method for circle - minor sector area	M1
	$=32.5$	Awrt 32.5	A1
			(3)
(c)	Area of triangle = $\frac{1}{2}\times 4\times 6\times\sin 2.22(=9.56)$	Correct expression for the area of triangle XYZ (allow 2.2 or awrt 2.22)	B1
	So area required = "9.56" + "32.5"	Their Triangle XYZ + part (b) or correct attempt at major sector (Not triangle ZXW)	M1
	Area of logo = 42.1 cm^2 or 42.0 cm^2	Awrt 42.1 or 42.0 (or just 42)	A1
			(3)
(d)	$\text{Arc length} = 4\times 4.06(=16.24)$ **or** $8\pi - 4\times 2.22$	M1: $4\times \textit{their}(2\pi - 2.22)$ or circumference – minor arc A1: Correct ft expression	M1 A1ft
	Perimeter = $ZY + WY$ + Arc Length	9 + 2 + Any Arc	M1
	Perimeter of logo = 27.2 or 27.3	Awrt 27.2 or awrt 27.3	A1
			(4)
			(12 marks)

Q4

Question Number	Scheme	Marks
(i)(a)	$(3+2x)^6$	
	Any correct form for any term of ${}^6C_1 3^5(2x)^1$ or ${}^6C_2 3^4(2x)^2$ or ${}^6C_4 3^2(2x)^4$	M1
	Any two correct: t(2) = $6\times 3^5\times(2x)^1$, t(3) = $15\times 3^4\times(2x)^2$ t(5) = $15\times 3^2\times(2x)^4$	A1
	All 3 terms correct 2nd term = $2916x$, 3rd term= $4860x^2$, 5th term=$2160x^4$	A1
		(3)
(b)	Method using consecutive terms of a GP e.g $\frac{2160x^4}{4860x^2}=\frac{4860x^2}{2916x}$	M1
	$\Rightarrow \ldots x=\ldots$	dM1
	$\Rightarrow x=\frac{15}{4}$	A1
		(3)
(ii) (a)	Attempts to use $S_\infty=\frac{a}{1-r}\Rightarrow\frac{8}{5}=\frac{\sin^2\theta}{1-2\cos\theta}$	M1
	$\Rightarrow\frac{8}{5}=\frac{1-\cos^2\theta}{1-2\cos\theta}$	dM1
	$\Rightarrow 8-16\cos\theta=5-5\cos^2\theta$	
	$\Rightarrow 5\cos^2\theta-16\cos\theta+3=0$	A1*
		(3)
(b)	Solves $5\cos^2\theta-16\cos\theta+3=0\Rightarrow\cos\theta=\frac{1}{5}$	B1
	Attempts $ar=\sin^2\theta\times 2\cos\theta=\left(1-\left(\frac{1}{5}\right)^2\right)\times 2\times\frac{1}{5}$	M1
	$=\frac{48}{125}$	A1
		(3) (12 marks)

(i) (a)

M1: For an attempt at using the binomial expansion to find one of the required terms.
Condone the omission of the brackets on the $2x$ terms. Allow using combination notation.
Don't be concerned over any labelling (2nd term). May be awarded from a full expansion

A1: Any two correct terms, simplified or not. The ${}^6C_2$ must now be numerical.
Don't be concerned by labelling and accept as part of a large list or expansion

A1: All three terms correct and simplified.
They may be part of a larger or complete expansion, which is fine. Look for the 2nd , 3rd and 5th terms
Accept the 3 terms with +'s between. So accept for example $2916x+4860x^2+2160x^4$

Note that the expansion can be done by $(3+2x)^6=3^6\left(1+\frac{2}{3}x\right)^6$ The unsimplified versions of the terms are:

t (2) = $3^6\times 6\left(\frac{2}{3}x\right)$ t(3) = $3^6\times\frac{6\times5}{2}\left(\frac{2}{3}x\right)^2$ and t(5) = $3^6\times\frac{6\times5\times4\times3}{4\times3\times2\times1}\left(\frac{2}{3}x\right)^4$

(i)(b)

M1: Uses the fact that the 2nd, 3rd and 5th terms of their binomial expansion in (a) are consecutive terms of a GP to set up an equation in x.

There are a few ways in which the equation could be set up so look carefully at what is written.

The most common ways are $\frac{"2160x^4"}{"4860x^2"} = \frac{"4860x^2"}{"2916x"}$, $\frac{"4860x^2"}{"2160x^4"} = \frac{"2916x"}{"4860x^2"}$ and

$"2160x^4" = \frac{"4860x^2"}{"2916x"} \times "4860x^2"$. Condone slips e.g. 2196 for 2916

The candidates **must** be using what they believe are the 2nd, 3rd and 5th terms of the binomial expansion and not the 2nd, 3rd or 4th or any other combination.

If a list is given it must be the 2nd, 3rd and 5th terms in $a+bx+cx^2+dx^3+ex^4+....$ where $a,b,c,d \neq 0$

dM1: It is dependent upon the first M and having the correct index on the powers of x.

It is for proceeding to an equation of the form $...x = ...$

A1: $x = \frac{15}{4}$ or exact equivalent. Condone an extra $x = 0$

……

If there are unlabelled/mislabelled terms in (a) that are used in (b) and you feel that they deserve more credit than that afforded by the scheme, please use review. A common error is described below.

It seems that some candidates are finding the terms in x^2, x^3 and x^5 in (i) (a) and (b). We will treat as a SC **only when appearing in both parts**. FYI: These are $4860x^2$, $4320x^3$ and $576x^5$. These values would score M1 in (i)(a) for the x^2 term and potentially M1, dM1 (when proceeding to $...x = ...$) in (i)(b)

……

(ii)(a) Allow use of a different parameter, e.g. $\theta \leftrightarrow x$

M1: Attempts to use $S_\infty = \frac{a}{1-r} \Rightarrow \frac{8}{5} = \frac{\sin^2\theta}{1-2\cos\theta}$ Condone slips only. The formula must be correct

dM1: Uses the identity $\sin^2\theta = 1-\cos^2\theta$ to form an equation in just $\cos\theta$

A1*: Completes proof with sufficient working.

Condone one slip in notation. E.g. $\sin\theta^2 \leftrightarrow \sin^2\theta$, $\cos \leftrightarrow \cos\theta$ but the final line must be correct and include the $= 0$

(ii)(b)

B1: States or uses $\cos\theta = \frac{1}{5}$

M1: Attempts $ar = \sin^2\theta \times 2\cos\theta = \left(1 - "\left(\frac{1}{5}\right)"^2\right) \times 2 \times "\frac{1}{5}"$

Alternatively finds the value of θ from their $\cos\theta = \frac{1}{5}$ and uses this in $ar = \sin^2\theta \times 2\cos\theta$

A1: $\frac{48}{125}$ or exact equivalent. Eg. 0.384.

It cannot be scored by rounding a non exact answer or from a decimal value of θ

So, for example, answers of $\frac{48}{125}$ achieved using a value of $\theta =$ 78.46….. don't score the final A1

Q5

Question Number	Scheme	Marks
	$f(x)=8\sin x\cos x+4\cos^2 x-3$	
(a)	States or uses $\sin 2x=2\sin x\cos x$ or $\cos 2x=\pm 2\cos^2 x\pm 1$	M1
	Uses $\sin 2x=2\sin x\cos x$ and $\cos 2x=\pm 2\cos^2 x\pm 1$ in $f(x)$	dM1
	$\left(f(x)=\right)\ 8\sin x\cos x+4\cos^2 x-3=4\sin 2x+2\cos 2x-1$	A1
		(3)
(b)	$R^2=a^2+b^2\Rightarrow R=\sqrt{20}$ or $2\sqrt{5}$	B1ft
	$\tan\alpha=\frac{b}{a}\Rightarrow\alpha=\ldots\ \left(=\text{"awrt 0.464"}\right)$	M1
	$\left(f(x)=\right)\ 2\sqrt{5}\sin\left(2x+0.464\right)-1$	A1
		(3)
(c)	(i) Maximum value = "$2\sqrt{5}-1$"	B1 ft
	(ii) Solves $2x+\alpha=\frac{5\pi}{2}\Rightarrow x=\ldots$	M1
	$\left(x=\right)$ awrt 3.69 (or $\left(x=\right)$ awrt 3.70)	A1
		(3)
		(9 marks)

(a) For the method marks condone working in mixed variables provided the intention is clear

M1: **Either** states one of the correct identities $\sin 2x=2\sin x\cos x$ or $\cos 2x=2\cos^2 x-1$

or uses $\sin 2x=2\sin x\cos x$ or $\cos 2x=\pm 2\cos^2 x\pm 1$ (may be implied by use of e.g. $\cos^2 x=\pm 1\pm\sin^2 x$ and $\cos 2x=\pm 1\pm 2\sin^2 x$ or $\cos 2x=\pm\cos^2 x\pm\sin^2 x$)

Can be implied by **either** of

- $a=4$
- $b=\pm 2$ **and** $c=-5$ or -1

dM1: Uses $\sin 2x=2\sin x\cos x$ and $\cos 2x=\pm 2\cos^2 x\pm 1$ (which may be implied as above for the first method mark) in $f(x)$ to produce an expression of the required form $a\sin 2x+b\cos 2x+c$
Condone slips when substituting in.

Can be implied by $a=4$ **and** $b=\pm 2$ **and** $c=-5$ or -1

A1: $4\sin 2x+2\cos 2x-1$ Correct answer scores all three marks. Must be in terms of x.

(b) **Full marks can only be scored provided correct values for a, b and c are found in (a).**

B1ft: Finds the exact value of R using $R^2=a^2+b^2$ (or may use trigonometry using their value for α)

Can be implied by a correct exact value.

Follow through on their a or b but the correct values should give $R=\sqrt{20}$ or $2\sqrt{5}$

M1: Uses a "correct" method to find α using their a and b. Accept for example $\tan\alpha = \pm\frac{b}{a} \Rightarrow \alpha = ...$ or $\tan\alpha = \pm\frac{a}{b} \Rightarrow \alpha = ...$ (You may need to check this on your calculator if only the angle is seen)

May also be seen using e.g. $\sin\alpha = \pm\frac{b}{"R"} \Rightarrow \alpha = ...$ or $\cos\alpha = \pm\frac{a}{"R"} \Rightarrow \alpha = ...$ Allow the angle to be found in degrees for this mark. $\alpha =$"awrt 26.6°" (1dp)

A1cso: $\left(\mathrm{f}(x) =\right)\ 2\sqrt{5}\sin\left(2x + \text{awrt}\, 0.464\right) - 1$ o.e. e.g. $\left(\mathrm{f}(x) =\right)\ \sqrt{20}\sin\left(2x + 0.464\right) - 1$ (may be awarded if seen in (c)) Can only be scored provided correct values for a, b and c are found in (a).

(c) (i)

B1ft: "$2\sqrt{5} - 1$"o.e. e.g. "$\sqrt{20} - 1$" but follow through on their $R + c$ as long as **R has been correctly found** (ie awarded in (b)). Ft on their c which may be different in (a) and (b) but **cannot be 0**

Allow to be a decimal if R was given as a decimal in (b) at any point. isw following a correct answer.

(c)(ii)

M1: Solves $2x + \alpha = \frac{5\pi}{2} \Rightarrow x = ...$ using their α found in (b) (note that this is still the equation which will need to be solved if they differentiate f(x) first and set equal to 0). You may need to check this on your calculator.

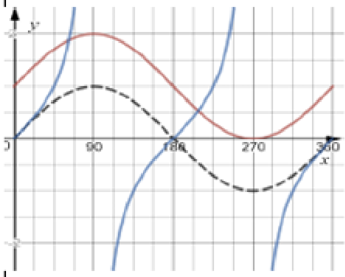
Allow this mark to be scored even if there are additional equations formed (and possibly solved).

They must be consistent in their use of degrees or radians in an equation that they are solving.

$\frac{5\pi}{2} = 7.85...$ may be used in their working.

A1: $\left(x =\right)$ awrt 3.69 but also allow $\left(x =\right)$ awrt 3.70 . If several angles are found then they must indicate which angle is their final answer by e.g. underlining, circling

Q6

Question Number	Scheme	Marks
(a)	$(270°, -4)$	B1 B1 (2)
(b)	 For $y=1+\sin\theta$ $y=\tan\theta$	B1 B1 (2)
(c)	(i) $6\times2=12$ (ii) 11	M1 A1 B1 ft (3) (7 marks)

(a)

B1 Either coordinate correct. Look for either $270^{(°)}$ or -4 in the correct position within (,) .

Alternatively look for either $x=270$ or $y=-4$ Condone $\frac{3\pi}{2}=270°$

Do not accept multiple answers unless one point is chosen or it is clearly part of their thought process. There is no need for the degrees symbol. Condone swapped coordinates, ie $(-4, 270)$ for this mark

B1 For correct coordinates.

$(270°, -4)$ with or without degrees symbol. Condone $x=270(°), y=-4$

(b) These may appear on Figure 3 rather than Diagram 1

B1 For $y=1+\sin\theta$ Score for a curve passing through $(0,1), (90°,2), (180°,1), (270°,0), (360°,1)$ with acceptable curvature. Do not accept straight lines

B1 For $y=\tan\theta$ with acceptable curvature. Must go beyond $y=1$ and -1

Score for the general shape of the curve rather than specific coordinates. See practice and qualification items for clarification.

First quadrant from $(0,0)\rightarrow(90°,\infty)$

Second and third quadrants from $(90°,-\infty)\rightarrow(270°,\infty)$ passing through $(180°,0)$

Fourth quadrant from $(270°,-\infty)\rightarrow(0,0)$

(c)(i) The question states hence so it is expected the results come from graphs.
If neither or only one graph is drawn then score for 12 in (i) for M1 A1 and 11 in (ii) B1

M1 For the calculation $\frac{2160}{360}=6$ or $\frac{2160}{180}=12$ or multiplying the number of intersections in their (b) by 6
Sight of 6 or 12 will imply this mark.

A1 12. 12 will score both marks.

(c) (ii)

B1 ft For either 11 (correct answer)
or follow through on n less than their answer to (c) (i) where n is their number of solutions in the range $180° < \theta \leqslant 360°$

Q7

Question Number	Scheme	Marks
(i)	States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	B1
	Uses both $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$	M1
	$3\operatorname{cosec}\theta = 8\cos\theta \Rightarrow \sin 2\theta = \dfrac{3}{4}$	A1
	$\Rightarrow \theta = \dfrac{1}{2}\arcsin\left(\dfrac{3}{4}\right) = \text{awrt } 0.424, \text{awrt } 1.15$	M1 A1
		(5)
(ii)	$\dfrac{\tan 2x - \tan 70^\circ}{1 + \tan 2x \tan 70^\circ} = -\dfrac{3}{8} \Rightarrow \tan(2x - 70^\circ) = -\dfrac{3}{8}$	M1 A1
	Correct order of operations $x = \dfrac{\arctan\left(-\dfrac{3}{8}\right) + 70^\circ}{2}$	dM1
	awrt 24.7°, awrt 114.7°	A1
		(4)
		(9 marks)

(i)

B1: States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

M1: Uses both $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to set up an equation in $\sin 2\theta$

A1: $\sin 2\theta = \dfrac{3}{4}$

M1: Correct method of solving an equation of the form $\sin 2\theta = k$, $|k| < 1$ to find at least one value for θ. May be solving in degrees for this mark ($24.30^\circ, 65.70^\circ$)

A1: $\theta = \text{awrt } 0.424, \text{awrt } 1.15$ and no other values in the range (isw for incorrect rounding).

(ii) Main scheme

M1: Uses the compound angle identities to reach $\tan(2x \pm 70^\circ) = \pm\dfrac{3}{8}$

A1: $\tan(2x-70°) = -\frac{3}{8}$

dM1: Correct order of operations to find a value for x. Condone if radians mode used, e.g.

$2x-70 = -0.3587... \Rightarrow x = \frac{70-0.3587...}{2} = 34.82...$ May be scored from

$2x+70 = ... \rightarrow x = \frac{-70-....}{2}$ Condone errors in finding the second solution such at $180 - PV$.

A1: Both awrt 24.7°, awrt 114.7° and no other values in the range (isw for incorrect rounding)

(ii) Alt method 1

M1: Cross multiplies and makes $\tan 2x$ the subject. Condone $\tan 70°$ being replaced by 2.75

A1: $\tan 2x = \frac{8\tan 70° - 3}{8+3\tan 70°} = \text{awrt } 1.17$

dM1: Correct order of operations to find one value for x. See note on main scheme.
Implied by a correct value for their $\tan 2x = \text{awrt } 1.17$ provided the previous M has been scored

A1: Both awrt 24.7°, awrt 114.7° and no other values in the range (isw for incorrect rounding)

(ii) Alt method 2

M1: Applies double angle formula and multiplies through to achieve a quadratic in tan x (condoning slips and not necessarily gathered)

$$\frac{\frac{2\tan x}{1-\tan^2 x}-\tan 70°}{1+\frac{2\tan x}{1-\tan^2 x}\tan 70°} = -\frac{3}{8} \Rightarrow 16\tan x - 8\tan 70°\left(1-\tan^2 x\right) = -3\left(1-\tan^2 x + 2\tan x\tan 70°\right)$$

$$\left(\Rightarrow t^2\left(8\tan 70°-3\right)+t\left(16+6\tan 70°\right)+3-8\tan 70° = 0 \quad \left(18.98t^2+32.48t-18.98=0\right)\right)$$

A1: A correct value for tan x, $\tan x = \text{awrt } 0.4604$ or awrt -2.172

dM1: Applies arctan to achieve at least one value for x.

A1: As main scheme.

Alt (i)	States or uses $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ (or equivalent identity)	B1
	$3\operatorname{cosec}\theta = 8\cos\theta \Rightarrow 9\operatorname{cosec}^2\theta = 64\cos^2\theta$ and uses both $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ and $\sin^2\theta+\cos^2\theta = 1$ to get equation in one trig term	M1
	$\Rightarrow 9 = 64\sin^2\theta\left(1-\sin^2\theta\right) \Rightarrow \sin^2\theta = \frac{4\pm\sqrt{7}}{8}$	A1
	$\Rightarrow \theta = \arcsin\left(\sqrt{\frac{4\pm\sqrt{7}}{8}}\right) = \text{awrt } 0.424, \text{awrt } 1.15$	M1 A1
		(5)

B1: States or uses $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ or in variations a similar correct reciprocal identity (e.g. $\cot\theta = \frac{\cos\theta}{\sin\theta}$) during the proof.

M1: Squares both sides and uses both reciprocal identity and Pythagorean identity to set up an equation in one trig term only.

A1: $\sin^2\theta = \frac{4\pm\sqrt{7}}{8}$ note it is the same roots for $\cos^2\theta$

M1: Correct method of solving an equation of the form $\sin^2\theta = k$ or $\cos^2\theta = k$, $0 < k < 1$ to find at least one value for θ. May be solving in degrees for this mark $(24.30°, 65.70°)$

A1: $\theta = \text{awrt } 0.424, \text{awrt } 1.15$ and no other values in the range. Under this method the extra solutions formed via squaring will need to be rejected to score this mark.

Q8

Question Number	Scheme	Marks
(i)	States $x = 2$	B1
	$\sqrt{3}\sec x + 2 = 0 \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = ...$	M1
	$x = \frac{5\pi}{6}$	A1
		(3)
(ii)	Attempts to use $\cos 2\theta = 1 - 2\sin^2\theta$	M1
	$6\sin^2\theta + 10\sin\theta - 3 = 0$	A1
	$\sin\theta = \frac{-5 \pm \sqrt{43}}{6}(= -1.926..., 0.2595...) \Rightarrow \theta = \arcsin(...)$	M1
	$\theta = 15.0°, 165°$	A1
		(4) (7 marks)

Notes

(i)

B1 States $x = 2$. May be seen anywhere in (i) and don't be concerned where it comes from.

M1 For a correct process to solve $\sqrt{3}\sec x + 2 = 0$ E.g. $\sec x = \frac{1}{\cos x} \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = ...$ Allow slips in rearranging but must attempt to solve $\cos x = k, |k| < 1$ or $\sec x = k, |k| > 1$ Degree value ($150°$) following a correct equation implies the M mark. Note some may use $\sec^2 x = 1 + \tan^2 x$ and form a quadratic in $\tan x$. These will need a correct identity, correct method to solve a quadratic (which may be by calculator) and attempt to solve $\tan x = k,\ k \neq 0$

A1 $x = \frac{5\pi}{6}$ and no other extra solutions in the range. Accept awrt 2.62 (and isw).

Note that $\sqrt{3}\sec x + 2 = 0 \rightarrow x = \frac{5\pi}{6}$ can score M1A1 as no incorrect work is seen, method implied.

Question required working to be shown $x = \frac{5\pi}{6}$ without seeing at least $\sqrt{3}\sec x + 2 = 0$ extracted first is M0A0.

(ii)

M1 Attempts to use $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ to form a quadratic equation in $\sin\theta$. If using alternative forms for the identity, must also use $\cos^2\theta = 1 - \sin^2\theta$ before gaining this mark.

A1 Correct 3 term quadratic equation $6\sin^2\theta + 10\sin\theta - 3 = 0$ or a multiple of this. Alternatively may be scored for $6\sin^2\theta + 10\sin\theta = 3$ if followed by completing the square on LHS to solve.

M1 Full attempt to find one value for θ from a quadratic in $\sin\theta$. Must involved

- correct method to solve the quadratic in $\sin\theta$ (usual rules, may use calculator) to produce a value for $\sin\theta$
- use of arcsin(..) to reach the value for θ (you may need to check the values if arcsin(…) is not shown). Radian answers can imply the mark (awrt 0.263, 2.88 if correct).

May be scored from an incorrect identity as long as a quadratic is achieved. Accept arcsin expression for the M

A1 θ=awrt $15.0°, 165°$ and no other solutions in the range. Accept just $15°$ for $15.0°$ (but not awrt $15°$ if it does not round to $15.0°$)

Condone a different variable used than θ throughout.

Q9

Question	Scheme	Marks
(a)	$R=17$	B1
	$\tan\alpha=\frac{15}{8}$	M1
	$\alpha=1.081$	A1
		(3)
(b)	(i) Min f(x) $=\frac{15}{41+2\times"17"}$	M1
	$=\frac{1}{5}$	A1
	(ii) Occurs when $\sin(x-1.081)=1\Rightarrow x-1.081=\frac{\pi}{2}\Rightarrow x=\ldots$	M1
	$x=$ awrt 2.65	A1
		(4)
(c)	$-\frac{23}{5}$ (or -4.6)	B1ft
		(1)
(d)	Awrt 1.33	B1ft
		(1)
		(9 marks)
Notes:		

(a)

B1: For 17 only.

M1: Attempts an equation in *α*. Accept $\tan\alpha=\pm\frac{15}{8}$ or $\tan\alpha=\pm\frac{8}{15}$. If using *R* accept $\cos\alpha=\pm\frac{8}{"17"}$ or $\sin\alpha=\pm\frac{15}{"17"}$. Implied by a correct value for *α*

A1: Awrt 1.081. Must be in radians.

(b)(i)

M1: Attempts to apply the result from (a) to find the minimum. Allow for $\frac{15}{41\pm2\times"\text{their } R"}$

A1: cao.

(ii)

M1: Attempts to solve $x\pm"\alpha"=\frac{\pi}{2}$

A1: cao

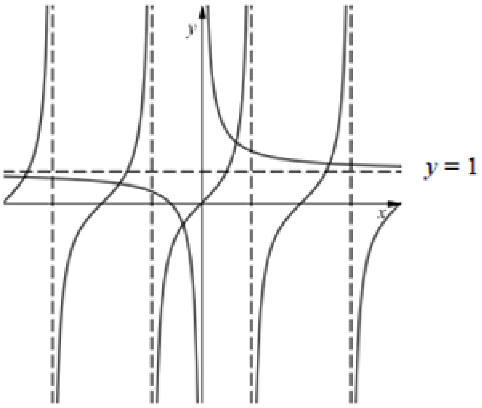
(c)

B1ft: Correct answer or follow through $2\times\left(\text{their}\frac{1}{5}\right)-5$ and no other solutions.

(d)

B1ft: For awrt 1.33 or follow through $0.5\times$ their 2.65 and no other solutions.

Q10

Question Number	Scheme	Marks
(a)	$x=\frac{3\pi}{2}$ oe	B1
		(1)
(b)(i)		B1B1
(ii)	5 (solutions)	B1
	Number of solutions are the number of points of intersections between the graphs	B1
		(4)
(c) (i)	(Number of solutions) $= 40$	B1ft
(ii)	(Number of solutions) $= 14$	B1
		(2)
		(7 marks)

(a)

B1 $x=\frac{3\pi}{2}$ oe and no others. Do not accept in degrees. It may be labelled on the graph, but it must be an equation. If multiple answers are given then $x=\frac{3\pi}{2}$ oe must be identified (eg may be circled)

(b)(i)

B1 For the shape of a $\frac{1}{x}$ type curve in Quadrant 1. It must not cross either axis and have acceptable curvature – do not penalise candidates unless it is clear that a minimum point was intended.

B1 Correct shape and position for both branches with an asymptote in the correct position **and** labelled as $y = 1$ or stated in their work. Again, do not penalise the sketch unless it is clear that turning points are intended. The asymptote line/dashed line does not need to be drawn on the sketch.

(ii)

B1 5 only

B1 Number of solutions are the number of points of intersections between the graphs. (Do not allow if they mention where they cross the x-axis).

(c)

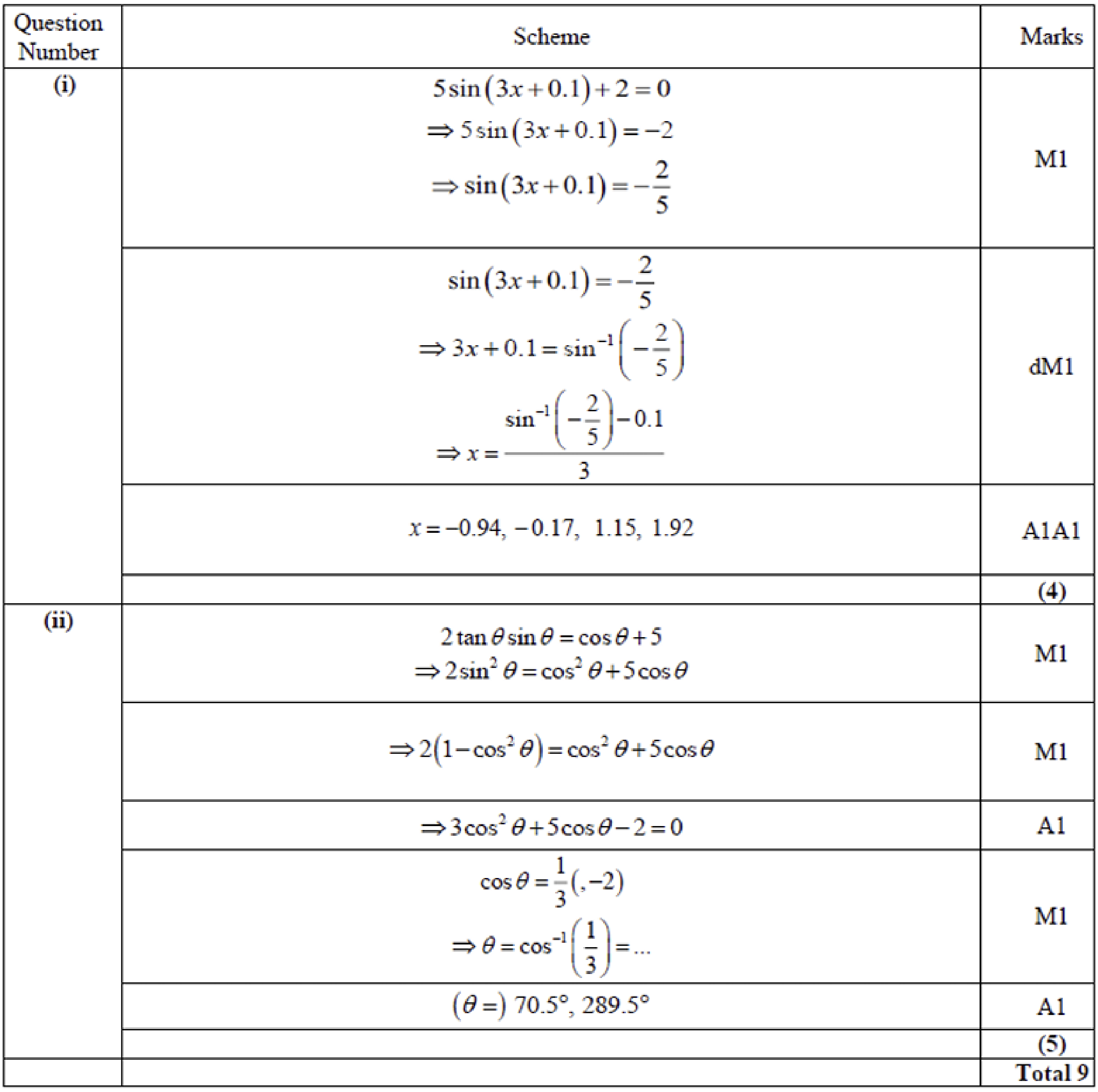
(i)

B1ft 40 Follow through from their sketch
(eg number of intersections in first quadrant $\times 20$)

(ii)

B1 14

Q11



Question Number	Scheme	Marks
(i)	$5\sin(3x+0.1)+2=0$ $\Rightarrow 5\sin(3x+0.1)=-2$ $\Rightarrow \sin(3x+0.1)=-\frac{2}{5}$	M1
	$\sin(3x+0.1)=-\frac{2}{5}$ $\Rightarrow 3x+0.1=\sin^{-1}\left(-\frac{2}{5}\right)$ $\Rightarrow x=\frac{\sin^{-1}\left(-\frac{2}{5}\right)-0.1}{3}$	dM1
	$x=-0.94,\ -0.17,\ 1.15,\ 1.92$	A1A1
		(4)
(ii)	$2\tan\theta\sin\theta=\cos\theta+5$ $\Rightarrow 2\sin^2\theta=\cos^2\theta+5\cos\theta$	M1
	$\Rightarrow 2\left(1-\cos^2\theta\right)=\cos^2\theta+5\cos\theta$	M1
	$\Rightarrow 3\cos^2\theta+5\cos\theta-2=0$	A1
	$\cos\theta=\frac{1}{3}(,-2)$ $\Rightarrow \theta=\cos^{-1}\left(\frac{1}{3}\right)=\ldots$	M1
	$(\theta=)\ 70.5°,\ 289.5°$	A1
		(5)
		Total 9

Answers with no working score 0 marks

(i)

M1 For ±2 and then dividing by 5 to reach $\sin(3x+0.1)=\pm\frac{2}{5}$ oe

May be implied by $3x+0.1=\pm 0.411...$ or allow a different variable to be used such that $y=3x+0.1 \Rightarrow \sin(y)=\pm\frac{2}{5}$.

Condone $3x+0.1=\pm 23.57...$ if they have incorrectly worked in degrees for this mark only.

dM1 Correct strategy for finding x. Allow $x=\dfrac{\pm 2\pi n+\sin^{-1}\left(\pm\frac{2}{5}\right)\pm 0.1}{3}$ or

e.g. $x=\dfrac{\pi-\sin^{-1}\left(\pm\frac{2}{5}\right)\pm 0.1}{3}$ May be implied by a correct angle, but they must have proceeded as far as $\sin(3x+0.1)=-\frac{2}{5}$ before achieving an angle.

Must be working in radians OR entirely in degrees (if the 0.1 radians is converted first)
It is dependent on the first method mark.

A1 Two of awrt $-0.94,\ -0.17,\ 1.15,\ 1.92$. Must be in radians.

A1 All of awrt $-0.94,\ -0.17,\ 1.15,\ 1.92$ and no extras in range

Beware of $5\sin(3x+0.1)=-2 \Rightarrow 15\sin x+0.5=-2 \Rightarrow x=-0.17$ which scores 0 marks

Note: There are other credit worthy methods such as squaring $5\sin(3x+0.1)=-2$ and using $\pm\sin^2\theta=\pm 1\pm\cos^2\theta$, then solving a quadratic in cos θ. For the first M1 they would have to proceed as far $3x+0.1=...$ but allow the mark if slips in rearranging are made.

(ii)

M1 For using $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ and attempting to multiply by cos θ (may even be $\cos^2\theta$ or higher which is acceptable). Look for the denominator being removed from $\dfrac{\sin^2\theta}{\cos\theta}$ and multiplying at least one other term by cos θ (or $\cos^2\theta$ etc).

M1 Attempts to use $\pm\sin^2\theta=\pm 1\pm\cos^2\theta$ and proceeds to an equation in $\cos\theta$ only or $\sin\theta$ only

A1 $3\cos^2\theta+5\cos\theta-2\ (=0)$. Not all need to be on the same side of the equation and condone the omission of = 0 if all on one side. Condone poor notation for $\cos^2\theta$ eg $\cos\theta^2$

M1 Solves their 3TQ in cos θ and takes inverse cos of one of their roots to obtain at least one value for θ. As a minimum, expect to see the root(s) to their 3TQ before proceeding to an angle which may need to be checked. Condone their angle to be in radians eg typically awrt 1.23 or awrt 5.05

A1 awrt 70.5, awrt 289.5 and no others in the range. Must be in degrees not radians.

Q12

Question Number	Scheme	Marks
(a) **Way One**	$\cot^2 x - \tan^2 x \equiv \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\cos^2 x} \equiv \frac{\cos^4 x - \sin^4 x}{\sin^2 x \cos^2 x}$	M1
	$\equiv \frac{\left(\cos^2 x - \sin^2 x\right)\left(\cos^2 x + \sin^2 x\right)}{\sin^2 x \cos^2 x} \equiv \frac{\cos 2x}{\ldots}$ or $\frac{\ldots}{\left(\frac{1}{2}\sin 2x\right)^2}$	dM1
	$\equiv \frac{\cos 2x}{\left(\frac{1}{2}\sin 2x\right)^2}$	A1
	$\equiv 4\frac{\cos 2x}{\sin 2x \sin 2x} \equiv 4\cot 2x \operatorname{cosec} 2x\,*$	A1*
		(4)
(b)	$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2\theta \Rightarrow \cot^2\theta - \tan^2\theta = 2\tan^2\theta \Rightarrow \cot^2\theta - 3\tan^2\theta = 0$	M1
	$\cot^2\theta - 3\tan^2\theta = 0 \Rightarrow \frac{1}{\tan^2\theta} - 3\tan^2\theta = 0 \Rightarrow \tan^4\theta = \frac{1}{3}$	A1
	$\tan^4\theta = "\frac{1}{3}" \Rightarrow \tan\theta = \pm\sqrt[4]{"\frac{1}{3}"} = "\pm 0.7598.." \Rightarrow \theta = \ldots$	M1
	$\theta =$ awrt $0.65,\ -0.65$	A1A1
		(5)
		Total 9

(a) Way One LHS to RHS

M1: Changes the LHS to sin x and cos x and attempts to make a single fraction using a correct common denominator. Condone errors/slips on the numerator

dM1: Attempts/ applies

- Either $\cos^4 x - \sin^4 x = \left(\cos^2 x - \sin^2 x\right)\left(\cos^2 x + \sin^2 x\right) = \cos^2 x - \sin^2 x = \cos 2x$ on the numerator
- Or $\sin 2x = 2\sin x \cos x$ to the denominator condoning bracketing slip

A1: Applies both of the above correctly to achieve a correct expression in terms of cos2x and sin2x

A1*: Reaches the right hand side with sufficient working shown. Expect to see $\sin^2 2x$ split into $\sin 2x \sin 2x$ Penalise consistent (not once or twice) use of poor notation on this mark only.

Examples $\cos^2 x \leftrightarrow \cos x^2$, $\sin^2 \leftrightarrow \sin^2 x$, constantly switching between $x \leftrightarrow \theta$

(b) Allow use of $x \leftrightarrow \theta$

M1: Uses part (a) and attempt to collects terms. (See Appendix III for ways not using part (a))
See below for equations in $\sin\theta$ or $\cos\theta$ where this mark is awarded for an equation in just $\sin\theta$ or $\cos\theta$

A1: Reaches a correct equation in a single term, usually $\tan\theta$. Look for $\tan^4\theta = \frac{1}{3}$ o.e. such as $3\tan^4\theta = 1$

Other correct intermediate forms are $2\sin^4\theta + 2\sin^2\theta - 1 = 0$ and $2\cos^4\theta - 6\cos^2\theta + 3 = 0$

M1: Takes the 4th root of their $\frac{1}{3}$ (o.e) and uses $\tan^{-1}$ (you may need to check) to obtain at least one value for θ

For the other intermediate forms look for working such as $\sin^2\theta = \frac{\sqrt{3}-1}{2} \Rightarrow \sin\theta = \sqrt{\frac{\sqrt{3}-1}{2}} \Rightarrow \theta = \ldots$

A1: Either awrt 0.65 or awrt −0.65. Allow either answer in degrees, so awrt $\pm 37.2°$

A1: Both answers in radians, awrt ± 0.65, and no extras in range

There are many different ways to do part (a). Generally, this is how they will be marked.
Most cases can be aligned to one of the three cases.

RHS to LHS

(a) Way 2	$4\cot 2x\,\text{cosec}2x \equiv 4\frac{\cos 2x}{\sin 2x}\times\frac{1}{\sin 2x}\equiv\frac{4\left(\cos^2 x-\sin^2 x\right)}{\ldots}$ or $\frac{\ldots}{4\sin^2 x\cos^2 x}$	M1
	$\frac{\cos^2 x-\sin^2 x}{\sin^2 x\cos^2 x}\equiv\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}\equiv\text{cosec}^2 x-\sec^2 x$	dM1A1
	$\equiv 1+\cot^2 x-1-\tan^2 x\equiv\cot^2 x-\tan^2 x\,*$	A1*

M1: Changeto sin $2x$ and cos $2x$ or tan$2x$ and sin$2x$ and attempts single angles in sin$x$ and cos$x$
dM1: Changs to single angles throughout and splits into 2 separate fractions (which don't need to be simplified)
A1: Correct xpression in terms of the single angles cosec x and sec x
A1*: Reache the left hand side with sufficient working shown

Working on oth sides: One possible way

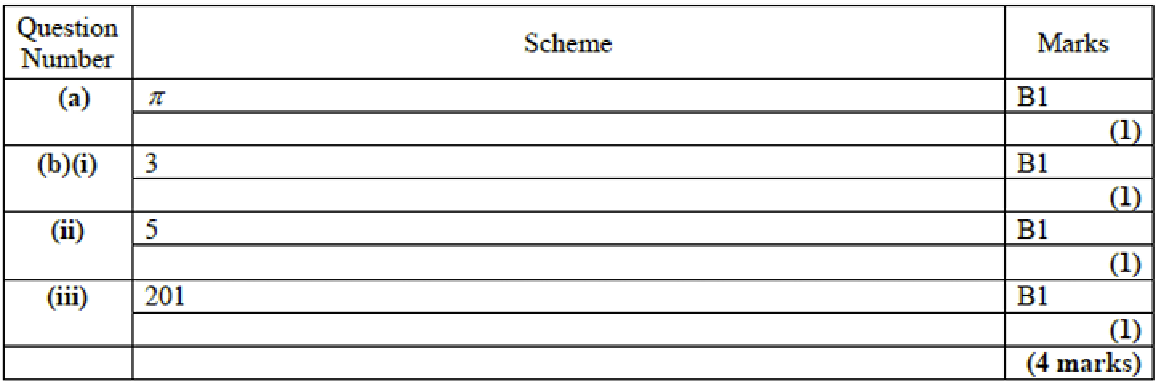
(a) Way 3	$\cot^2 x-\tan^2 x\equiv 4\cot 2x\,\text{cosec}2x$ $\frac{\cos^2 x}{\sin^2 x}-\frac{\sin^2 x}{\cos^2 x}\equiv 4\times\frac{\cos 2x}{\sin 2x}\times\frac{1}{\sin 2x}$ $\cos^4 x-\sin^4 x\equiv 4\sin^2 x\cos^2 x\frac{\cos 2x}{\sin 2x}\times\frac{1}{\sin 2x}$	M1
	$\cos^4 x-\sin^4 x\equiv 4\times\sin^2 x\cos^2 x\frac{\left(\cos^2 x-\sin^2 x\right)}{\left(2\sin x\cos x\right)^2}$	dM1A1
	$\left(\cos^4 x-\sin^4 x\right)\equiv\left(\cos^2 x-\sin^2 x\right)$ $\left(\cos^2 x-\sin^2 x\right)\underset{1}{\cancel{\left(\cos^2 x+\sin^2 x\right)}}\equiv\left(\cos^2 x-\sin^2 x\right)$ Hence true	A1*

M1: Changes to sin x, cos x, sin $2x$ and cos$2\,x$ and attempts to cross multiply
dM1: Applies

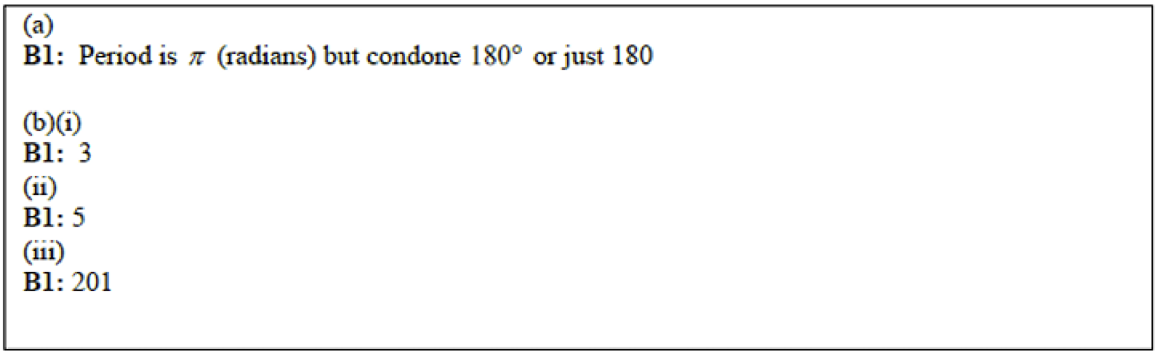
- either cos^2x - sin^2x = cos$2x$ to the numerator
- or sin$2x$ = 2sin$x$ cos$x$ to the denominator

A1: Correct identity in terms of cos x and sin x
A1*: Reaches a point where both sides are equal and makes a minimal comment
Example of how you mark a "different" approach.

Q13



Question Number	Scheme	Marks
(a)	π	B1
		(1)
(b)(i)	3	B1
		(1)
(ii)	5	B1
		(1)
(iii)	201	B1
		(1)
		(4 marks)



(a)
B1: Period is π (radians) but condone $180°$ or just 180

(b)(i)
B1: 3
(ii)
B1: 5
(iii)
B1: 201

Q14

Question Number	Scheme	Marks
(i)	$\tan^2\left(2x+\frac{\pi}{4}\right)=3\Rightarrow 2x+\frac{\pi}{4}=\pm\frac{\pi}{3},\pm\frac{2\pi}{3}$	M1
	$\Rightarrow x=\left(...-\frac{\pi}{4}\right)\div 2$	dM1
	$\Rightarrow x=\frac{\pi}{24},-\frac{11\pi}{24};\frac{5\pi}{24},-\frac{7\pi}{24}$	A1; A1, A1
		(5)
(ii)	$(2\sin\theta-\cos\theta)^2=1\Rightarrow 4\sin^2\theta-4\sin\theta\cos\theta+\cos^2\theta=1$	
	Attempts to use $\sin^2\theta+\cos^2\theta=1\ \Rightarrow 4\sin^2\theta-4\sin\theta\cos\theta=\sin^2\theta$	M1
	$\Rightarrow\sin\theta(3\sin\theta-4\cos\theta)=0$	dM1
	$\tan\theta=\frac{4}{3}$	A1
	$\theta=53.1°,233.1°,\underline{180°}$	A1, <u>B1</u>
		(5)
		(10 marks)

(i)

M1: Correct order of operations. Takes square root followed by arctan.

Implied by $2x+\frac{\pi}{4}=\frac{\pi}{3}$, $2x+\frac{\pi}{4}=1.047$. Condone for this mark only $2x+\frac{\pi}{4}=60$.

Allow if they use θ for $2x+\frac{\pi}{4}$

A longer method is to do $\frac{\sin^2\left(2x+\frac{\pi}{4}\right)}{\cos^2\left(2x+\frac{\pi}{4}\right)}=3\Rightarrow\sin^2\left(2x+\frac{\pi}{4}\right)=3\cos^2\left(2x+\frac{\pi}{4}\right)$ and use $\sin^2\theta+\cos^2\theta=1$ to produce an equation in either sin or cos, before taking the inverse. They must have a correct method up to slips in rearranging, and reach the stage of taking arcsin or arccos in order to score the M.

dM1: Complete attempt to find one value for x.

This would involve an attempt to move the $\frac{\pi}{4}$ before dividing by 2. Condone

$x=\left(1.047\pm\frac{\pi}{4}\right)\div 2$

A1: One value of $\frac{\pi}{24},-\frac{11\pi}{24}$. Condone decimals here awrt 0.13, −1.44

A1: One value of $\frac{5\pi}{24},-\frac{7\pi}{24}$. Condone decimals here awrt 0.65, −0.92

A1: $\frac{\pi}{24},\frac{5\pi}{24},-\frac{7\pi}{24},-\frac{11\pi}{24}$ and no other values in the range

(ii)

M1: Attempts to multiply out to at least three terms, and use $\sin^2\theta+\cos^2\theta=1$ somewhere in the equation.

dM1: Cancels or factorises out the $\sin\theta$ term to produce a factor $a\sin\theta\pm b\cos\theta$ or an equation of the form $a\sin\theta\pm b\cos\theta=0$ oe (the "=0" may be implied)

A1: $\tan\theta=\frac{4}{3}$

A1: $\theta=53.1°, 233.1°$ and no others in the range

(Note: 0 and 360° are outside the range so ignore if given as solutions.)

B1: $\theta=180°$ Award when seen and allow however it arises.

Question Number	Scheme	Marks
(ii) **Alt**	$(2\sin\theta-\cos\theta)^2=1\Rightarrow 2\sin\theta-\cos\theta=\pm1$	
	$\Rightarrow 2\sin\theta=\cos\theta\pm1$	
	$\Rightarrow 4\sin^2\theta=\cos^2\theta\pm2\cos\theta+1$	
	then attempts to use $\sin^2\theta+\cos^2\theta=1$ $\Rightarrow 4-4\cos^2\theta=\cos^2\pm2\cos\theta+1$	M1
	$\Rightarrow 5\cos^2\theta\pm2\cos\theta-3=0\Rightarrow\cos\theta=\ldots$	dM1
	$\cos\theta=\frac{3}{5},-1$ or $\cos\theta=-\frac{3}{5},1$	A1
	$\theta=53.1°, 233.1°, \underline{180°}$	A1, $\underline{\text{B1}}$
		(5) **(10 marks)**

M1: Attempts at least one of $2\sin\theta-\cos\theta=\pm1$, rearranges and squares then attempts to use $\sin^2\theta+\cos^2\theta=1$ to produce an equation in just $\cos\theta$ or just $\sin\theta$. FYI in $\sin\theta$ it is $4\sin^2\theta\pm4\sin\theta+1=1-\sin^2\theta$

dM1: Solves their quadratic in $\cos\theta$ or $\sin\theta$ to obtain at least one value for $\cos\theta$ or $\sin\theta$

A1: One correct pair of solutions. FYI in $\sin\theta$ it is $\sin\theta=0,\frac{4}{5}$ or $\sin\theta=0,-\frac{4}{5}$

A1: $\theta=53.1°, 233.1°$ and no others in the range. (Note via this method some extra solutions in the range are likely)

B1: $\theta=180°$ Award when seen and allow however it arises.

It is possible someone could have studied WMA13.

M1: Attempts to write $2\sin\theta-\cos\theta$ in the form $R\sin(\theta-\alpha)$ o.e. FYI $2\sin\theta-\cos\theta=\sqrt{5}\sin(\theta-26.6°)$

dM1: Proceeds from $R^2\sin^2(\theta-\alpha)=1$ to $\sin(\theta-\alpha)=\pm\frac{1}{R}$

A1: $\sin(\theta-26.6°)=\pm\frac{1}{\sqrt{5}}$

A1: $\theta=53.1°, 233.1°$ and no others in the range

(Note: 0 and 360° are outside the range so ignore if given as solutions.)

B1: $\theta=180°$ Award when seen and allow however it arises.

Q15

Question Number	Scheme	Marks
(a)	$2\sin(\theta-30°)=5\cos\theta \Rightarrow 2\sin\theta\cos30°-2\cos\theta\sin30°=5\cos\theta$	M1
	$\div\cos\theta \qquad \Rightarrow 2\tan\theta\cos30°-2\sin30°=5$	dM1
	$\Rightarrow 2\tan\theta\times\frac{\sqrt{3}}{2}-2\times\frac{1}{2}=5$	A1
	$\Rightarrow \sqrt{3}\tan\theta=6 \Rightarrow \tan\theta=2\sqrt{3}$ *	A1*
		(4)
(b)	Attempts $\arctan 2\sqrt{3}$ ..and then subtracts $20°$	M1
	$\Rightarrow x=$ awrt $53.9°, 233.9°$	A1, A1
		(3)
		(7 marks)

(a)

M1 Attempts to use $\sin(\theta-30°)=\sin\theta\cos(\pm30°)\pm\cos\theta\sin(\pm30°)$ within the given equation

Condone the omission of a 2 on the second term and a slip on the 5 of $5\cos\theta$

dM1 Divides by $\cos\theta$ to set up an equation in just $\tan\theta$.

They may collect terms in $\sin\theta$ and $\cos\theta$ before dividing by $\cos\theta$ to set up an equation in just $\tan\theta$

An equation with $\cos30°$ and $\sin30°$ still not processed is acceptable.

A1 Fully correct equation in $\tan\theta$ with the $\cos30°$ and $\sin30°$ processed

($\sqrt{3}\sin\theta=6\cos\theta\Rightarrow\tan\theta=2\sqrt{3}$ is acceptable for both A marks)

(Note If they proceed directly to the final answer from

$\tan\theta=\frac{5+2\sin30°}{2\cos30°}\Rightarrow\tan\theta=2\sqrt{3}$ then maximum M1dM1A0A0 unless

$\tan\theta=\frac{5+1}{\sqrt{3}}$ or equivalent is seen before the final given answer.

A1* Correctly proceeds to given answer.

(b) Answers with no working scores 0 marks

M1 Attempts to find a value for x.

Allow $\arctan 2\sqrt{3}$ ...followed by adding or subtracting $20°$. Which may be implied by

$\tan(x+20)=2\sqrt{3}\Rightarrow x=\arctan(2\sqrt{3})\pm20=\ldots$

Alternatively, attempts to use $\sin(x-10°)=\sin x\cos10°\pm\cos x\sin10°$ within the given equation, divides by $\cos x$ to set up an equation in just $\tan x$ and proceeds to find an angle for x

$\tan x=\frac{5\cos20+2\sin10}{2\cos10+5\sin20}\Rightarrow x=\ldots$

A1 One value provided M1 has been scored. Allow either awrt $54°$ or $234°$ (or in radians awrt 0.94 or 4.08)

A1 $x=$ awrt $53.9°, 233.9°$ and no others inside the range provided M1 has been scored. Ignore any angles outside the range. Must be in degrees

Q16

Question	Scheme	Marks
	$3\tan(\theta+43°)=2\cos(\theta+43°)\Rightarrow 3\dfrac{\sin(\theta+43°)}{\cos(\theta+43°)}=2\cos(\theta+43°)$ $\Rightarrow 3\sin(\theta+43°)=2\cos^2(\theta+43°)$	M1
	$\Rightarrow 3\sin(\theta+43°)=2(1-\sin^2(\theta+43°))$	M1
	$\Rightarrow 2\sin^2(\theta+43°)+3\sin(\theta+43°)-2=0$ $\Rightarrow(2\sin(\theta+43°)-1)(\sin(\theta+43°)+2)=0\Rightarrow\sin(\theta+43°)=...$	M1
	$\sin(\theta+43°)=\dfrac{1}{2}$	A1
	$\theta=\arcsin\text{"}\dfrac{1}{2}\text{"}-43°$	M1
	$\theta=-13°,\ 107°$	A1
		(6)
		(6 marks)

Notes:

M1: Uses $\tan..=\dfrac{\sin..}{\cos..}$ and multiplies through to form an equation of the form $A\sin...=B\cos^2...$

Condone poor notation e.g.:

$3\tan(\theta+43°)=2\cos(\theta+43°)\Rightarrow 3\dfrac{\sin}{\cos}(\theta+43°)=2\cos(\theta+43°)$

$\Rightarrow 3\sin(\theta+43°)=2\cos^2(\theta+43°)$ (with or without brackets)

M1: Applies Pythagorean identity to obtain a 3 term quadratic equation in sin.

Allow use of $\cos^2...=\pm1\pm\sin^2...$

M1: Solves a 3 term quadratic in $\sin(\theta+43°)$ by any valid means.

This may be implied by at least one correct root for their quadratic.

Allow if they have $\sin(\theta+43°)=x$ or another variable or e.g. $\sin\alpha$ where $\alpha=\theta+43°$

A1: Correct value of $\sin(\theta+43°)$. If $\sin(\theta+43°)=x$ is used, it must be clear they mean $\sin(\theta+43°)$ but this may be implied if they have e.g. sin α = ½ where $\alpha=\theta+43°$

If $x=\dfrac{1}{2}$ is left as a final answer it is A0.

M1: Correct method for solving $\sin(\theta+43°)=k, |k|<1$, look for use of inverse sine followed by **subtraction** of 43 from $\sin^{-1}$(their k). Implied by one correct solution for their k

Do **not** allow mixing of degrees and radians for this mark.

A1: Correct solutions and no others in the range.

Q17

Question Number	Scheme	Marks
(i)	States or uses $\tan x = \frac{\sin x}{\cos x} \Rightarrow 5\sin x \times \frac{\sin x}{\cos x} + 13 = \cos x$	B1
	$5\sin^2 x + 13\cos x = \cos^2 x \Rightarrow 5\left(1-\cos^2 x\right) + 13\cos x = \cos^2 x$	M1
	$\Rightarrow 6\cos^2 x - 13\cos x - 5 = 0$	A1
	$\Rightarrow (3\cos x + 1)(2\cos x - 5) = 0 \Rightarrow \cos x = -\frac{1}{3}$	M1
	$\Rightarrow x = 1.91$	A1
		(5)
(ii) (a)	$20 = 10 + 12\sin(6k+18)° \Rightarrow \sin(6k+18)° = \frac{5}{6}$	M1
	$\Rightarrow (6k+18) = 56.4, 123.6$	dM1
	$\Rightarrow k = 6.41, 17.59$	A1, A1
		(4)
(b)	22°C	B1
		(1)
(c)	$"6.41"t + 18 = 90 \Rightarrow t = 11.23$ Time of day = 11:14	M1, A1
		(2)
		(12 marks)

(i)

B1: States or uses $\tan x = \frac{\sin x}{\cos x}$ E.g. $5\sin x \times \frac{\sin x}{\cos x} + 13 = \cos x$

M1: Attempts to use $\tan x = \frac{\sin x}{\cos x}$, $\sin^2 x + \cos^2 x = 1$ and multiply by $\cos x$ to form a quadratic equation in $\cos x$ (allow if there are slips in coefficients but the trig terms must be correct)

A1: Correct simplified quadratic $6\cos^2 x - 13\cos x - 5 (= 0)$ The "=0" may be implied.

M1: Solves a 3TQ in $\cos x$ leading to at least one value for $\cos x$

A1: awrt $x = 1.91$ (following $\cos x = -\frac{1}{3}$) and no other values in the range.

Note: Answers only with no working score no marks. From $6\cos^2 x - 13\cos x - 5 (= 0)$ to 1.91 directly, score final M0A0.

(ii) (a)

M1: Attempts to use the given information and proceeds to $\sin(6k+18)° = c$, or may be implied by

$A\sin(6k+18)° = B \rightarrow 6k + 18 = \arcsin\frac{B}{A}$ (evaluated) (which scores M1dM1)

dM1: Takes arcsin leading to a value for $6k + 18$. Accept radian values seen for this mark. (0.985, 2.16). Allow awrt 2 s.f. answers for evidence.

A1: One value for k: 6.41 or 17.59 Must be in degrees.

SC Allow for 6.4 and 17.6 both given if no more accurate answers are stated.

A1: awrt $k = 6.41, 17.59$ and no other values

Note answers only scores no marks. – the requirements of the M marks must be seen to be able to score them.

(ii)(b)

B1: cao 22°C Condone just 22.

(ii)(c)

M1: Sets their $"6.41"t + 18 = 90 \Rightarrow t = \ldots$ May be implied by 11.23.

Note $"6.41"t + 18 = \frac{\pi}{2}$ is M0.

A1: cao Time of day = 11:14 o.e. (e.g. 11h 14m is acceptable).

Q18

Question Number	Scheme	Marks
(a)	Starting with the LHS: $2\text{cosec}^2 2\theta(1-\cos 2\theta)=\dfrac{2-2\cos 2\theta}{\sin^2 2\theta}$	M1
	$=\dfrac{2-2(1-2\sin^2\theta)}{4\sin^2\theta\cos^2\theta}$	M1dM1
	$=\sec^2\theta=1+\tan^2\theta\equiv\text{RHS}$ *	A1*
		(4)
(b)	$\sec^2 x-3\sec x-4=0\Rightarrow\sec x=\ldots$	M1
	$\cos x=\dfrac{1}{4}$ (ignore -1)	A1
	$\cos x=\dfrac{1}{4}\Rightarrow x=\ldots$	dM1
	$x=75.5°,\ 284.5°$	A1
		(4)
		(8 marks)

Notes

(a) **The most common method and where to award marks is as follows:**

M1: Uses $\text{cosec}\,2\theta=\dfrac{1}{\sin 2\theta}$ (oe) at some stage in the proof to convert the cosec into sine.

M1 Attempts to use one of $\cos 2\theta=\pm1\pm2\sin^2\theta$ or $\sin 2\theta=2\sin\theta\cos\theta$ but condone $\sin^2 2\theta=2\sin^2\theta\cos^2\theta$ as an attempt at squaring.

dM1: Depends on previous M. Attempts to use both $\cos 2\theta=1-2\sin^2\theta$ oe and $\sin 2\theta=2\sin\theta\cos\theta$

A1*: Achieves the RHS with no mathematical errors seen and all stages of working shown. Must see the $\sec^2\theta$ before the final answer. Condone minor notational errors (e.g. a missing θ) but not consistent poor notation throughout. There will be other solutions but if they work from both sides then they need a conclusion at the end.

For other methods or variations apply scheme as follows:

M1: Uses a correct identity for either cosec 2θ or $\text{cosec}^2 2\theta$ or $\cot^2 2\theta$ to get the equation in terms of sin and cosine only or to introduce cosec2θ if working in reverse. Condone the 2 becoming ½.

M1: Attempts to apply a double angle identity correct up to sign errors, e.g. $\cos 2\theta=\pm1\pm2\sin^2\theta$

dM1: Depends on previous M. Use of at least two **correct** double angle identities and/or Pythagorean identities to *progress sufficiently towards the goal* (e.g. reduces all terms to single argument form in cosine or sine only, or identify an appropriate factor to cancel to such, or to get all double arguments if working in reverse with the sin 2θ identified). There may be algebraic slips, but all trig identities used up to this point must be correct (e.g. see stag reach in main scheme).

A1*: Fully correct proof with sufficient stages shown including use of $\sec^2\theta=1+\tan^2\theta$ or equivalent identity (e.g. other Pythagorean relation), and if necessary, a conclusion given.

For example $1+\tan^2\theta=\sec^2\theta=\dfrac{1}{\cos^2\theta}=\dfrac{2}{1+\cos 2\theta}$ 2nd M1, use of a suitable double angle identity

$=\dfrac{2(1-\cos 2\theta)}{(1+\cos 2\theta)(1-\cos 2\theta)}=\dfrac{2(1-\cos 2\theta)}{1-\cos^2 2\theta}=\dfrac{2(1-\cos 2\theta)}{\sin^2 2\theta}$ 3rd dM1, correct initial double angles identity and Pythagorean identity used to identify the sin in the denominator, all identities correct

$=2\text{cosec}^2 2\theta(1-\cos 2\theta)$ 1st M1, use of correct identity for $\text{cosec}^2 2\theta$. (A1 if all correct)

(b) Note: allow with x or θ and allow recovery of mixed variables in this part.

M1: Uses part (a) to forms a 3 term quadratic in $\sec x$ or $\cos x$, solves their quadratic and finds $\cos x = \dots$ or $\sec x = \dots$ for at least one solution. For reference the quadratic in cos is $4\cos^2 x + 3\cos x - 1 = 0$

A1: $\cos x = \frac{1}{4}$ (ignore any reference to $\cos x = -1$) Must have a correct cosine or implied by a correct answer after reaching a secant if no incorrect working seen. May be implied if not seen explicitly.

dM1: Correct method to find a value for x from $\cos x = k$ where $|k| < 1$ as a solution for their equation. May be implied by one correct angle from $\sec x = k$ or $\cos x = k$. Note radian answer awrt 1.3 implies dM1.

A1: awrt 75.5 ° and awrt 284.5 ° Allow with or without 180° included, but there must be no other angles in given interval. All previous marks must have been scored. Ignore answers outside the domain.

(b)Alt	$1+\tan^2 x = 4+3\sec x \Rightarrow 9\sec^2 x = \tan^4 x - 6\tan^2 x + 9 \Rightarrow \tan^4 x - 15\tan^2 x = 0$ $\Rightarrow \tan^2 x = \dots (\neq 0)$	M1
	$\tan x = (\pm)\sqrt{15}$	A1
	$\tan x = k \neq 0 \Rightarrow x = \dots$	dM1
	$x = 75.5°, 284.5°$	A1
		(4)

Notes

(b) Alt

M1: Makes sec x the subject, squares and solves the resulting quadratic in $\tan^2 x$ leading to a value for $\tan^2 x$. Note there may be variations on this approach, such as $1+\tan^2 x = 4+3\sqrt{1+\tan^2 x}$ being used before squaring, or solving for "$1+\tan^2 x$". Score for a correct approach leading to a value for $\tan^2 x$.

A1: Correct value for tan x. Need not give both values.

dM1: Solves from their $\tan x = k$, $k \neq 0$ to find a value for x

A1: Both correct values obtained and no invalid solutions in the range (ignore 180° as main scheme). Must reject extra solutions.

Note: other methods or variations may be seen, which can be marked according to the same principles, first M for correct approach to find a value for a trig ratio, A1 correct non-trivial value, dM1 solves for x.

Q19

Question Number	Scheme	Notes	Marks
(i)	$\tan\theta+\frac{1}{\tan\theta}=\frac{\sin\theta}{\cos\theta}+\frac{1}{\frac{\sin\theta}{\cos\theta}}$ or $\tan\theta+\frac{1}{\tan\theta}=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$	Uses $\tan\theta\equiv\frac{\sin\theta}{\cos\theta}$ on both terms	M1
	$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$ or $\frac{\sin^2\theta}{\sin\theta\cos\theta}+\frac{\cos^2\theta}{\sin\theta\cos\theta}$ Uses $\tan\theta\equiv\frac{\sin\theta}{\cos\theta}$ and $\frac{1}{\tan\theta}\equiv\frac{\cos\theta}{\sin\theta}$ and attempts common denominator of $\sin\theta\cos\theta$ with a 2 term numerator one of which is correct. Or attempts 2 separate fractions with a denominator of $\sin\theta\cos\theta$ one of which is correct. **Depends on the first mark.**		dM1
	$=\frac{1}{\sin\theta\cos\theta}$ * or $\frac{1}{\cos\theta\sin\theta}$	Correct proof with no notation errors or missing variables but allow “≡” instead of “=”. If there are any spurious “= 0”’s alongside the proof score A0.	A1*
			(3)

	Alternative 1 for (i)		
	$\tan\theta+\frac{1}{\tan\theta}=\frac{\tan^2\theta+1}{\tan\theta}\left(\text{or } \frac{\tan^2\theta}{\tan\theta}+\frac{1}{\tan\theta}\right)$	Attempts common denominator of $\tan\theta$	M1
	$=\frac{\sec^2\theta}{\tan\theta}=\frac{1}{\cos^2\theta}\times\frac{\cos\theta}{\sin\theta}$ Or $=\frac{\frac{\sin^2\theta}{\cos^2\theta}+1}{\frac{\sin\theta}{\cos\theta}}=\frac{1}{\cos^2\theta}\times\frac{\cos\theta}{\sin\theta}$	Applies appropriate and correct identities to obtain in terms of $\sin\theta$ and $\cos\theta$ only and eliminates “double decker” fractions if necessary	dM1
	$=\frac{1}{\sin\theta\cos\theta}$ * or $\frac{1}{\cos\theta\sin\theta}$	Correct proof with no notation errors or missing variables but allow “≡” instead of “=”. If there are any spurious “= 0”’s alongside the proof score A0.	A1*
	Alternative 2 for (i)		
	$\tan\theta+\frac{1}{\tan\theta}=\frac{1}{\sin\theta\cos\theta}\Rightarrow\frac{\sin^2\theta}{\cos\theta}+\cos\theta=\frac{1}{\cos\theta}$ Uses $\tan\theta\equiv\frac{\sin\theta}{\cos\theta}$ and multiplies through by $\sin\theta$ or $\cos\theta$		M1
	$\Rightarrow\sin^2\theta+\cos^2\theta=1$ Uses $\tan\theta\equiv\frac{\sin\theta}{\cos\theta}$ and multiplies through by $\sin\theta$ and $\cos\theta$		dM1
	$\sin^2\theta+\cos^2\theta=1$ is true hence proved	Fully correct work reaching a correct identity with a conclusion. If there are any spurious “= 0”’s alongside the proof score A0.	A1*

(ii)	$3\cos^2(2x+10^\circ)=1 \Rightarrow \cos^2(2x+10^\circ)=\frac{1}{3} \Rightarrow \cos(2x+10^\circ)=(\pm)\sqrt{\frac{1}{3}}$ Divides by 3 and takes square root **of both sides**. The "±" is not required.		M1
	$2x+10^\circ=\cos^{-1}\left(\text{"}(\pm)\sqrt{\frac{1}{3}}\text{"}\right)$ $\Rightarrow x=\frac{\cos^{-1}\left(\text{"}(\pm)\sqrt{\frac{1}{3}}\text{"}\right)-10^\circ}{2}$	Applies $x=\frac{\cos^{-1}\left(\text{"}(\pm)\sqrt{\frac{1}{3}}\text{"}\right)\pm10^\circ}{2}$ You may need to check their values if no working is shown.	M1
	For reference $2x+10^\circ=54.735...,\ 125.264...$		
	$x=22.4^\circ$ or $x=57.6^\circ$	Awrt one of these	A1
	$x=22.4^\circ$ and $x=57.6^\circ$	Awrt both with no extras in range	A1
	If mixing degrees and radians allow the method marks.		
			(4)

	Alternative 1 for part (b)		
	$3\cos^2(2x+10^\circ)=1 \Rightarrow 3\left(1-\sin^2(2x+10^\circ)\right)=1 \Rightarrow$ $\Rightarrow \sin^2(2x+10^\circ)=\frac{2}{3} \Rightarrow \sin(2x+10^\circ)=(\pm)\sqrt{\frac{2}{3}}$ Uses a correct identity, rearranges and takes square root **of both sides**. The "±" is not required.		M1
	$2x+10^\circ=\sin^{-1}\left(\text{"}(\pm)\sqrt{\frac{2}{3}}\text{"}\right)$ $\Rightarrow x=\frac{\sin^{-1}\left(\text{"}(\pm)\sqrt{\frac{2}{3}}\text{"}\right)-10^\circ}{2}$	Applies $x=\frac{\sin^{-1}\left(\text{"}(\pm)\sqrt{\frac{2}{3}}\text{"}\right)\pm10^\circ}{2}$ You may need to check their values if no working is shown.	M1
	$x=22.4^\circ$ or $x=57.6^\circ$	Awrt one of these	A1
	$x=22.4^\circ$ and $x=57.6^\circ$	Awrt both with no extras in range	A1
	Alternative 2 for part (b)		
	$3\cos^2(2x+10^\circ)=3\left(\frac{1+\cos(4x+20)}{2}\right) \Rightarrow \cos(4x+20)=-\frac{1}{3}$ Uses a correct identity, rearranges to make $\cos(4x+20)$ the subject		M1
	$2x+10^\circ=\cos^{-1}\left(\text{"}-\frac{1}{3}\text{"}\right)$ $\Rightarrow x=\frac{\cos^{-1}\left(\text{"}-\frac{1}{3}\text{"}\right)-20^\circ}{4}$	Applies $\Rightarrow x=\frac{\cos^{-1}\left(\text{"}-\frac{1}{3}\text{"}\right)-20^\circ}{4}$ You may need to check their values if no working is shown.	M1
	For reference $4x+20^\circ=109.47...,\ 250.52...$		
	$x=22.4^\circ$ or $x=57.6^\circ$	Awrt one of these	A1
	$x=22.4^\circ$ and $x=57.6^\circ$	Awrt both with no extras in range	A1

		Total 7

Q20

Question Number	Scheme	Marks
(a)	Uses $\sin 2x = 2\sin x\cos x$ **AND** $\cos 2x = 1-2\sin^2 x$ o.e. in $\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}$	M1
	$\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}=\frac{2\sin x\cos x}{\cos x}+\frac{1-2\sin^2 x}{\sin x}$ $=\frac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}+\frac{1}{\sin x}-\frac{2\sin^{\cancel{2}} x}{\cancel{\sin x}}=\frac{1}{\sin x}=\operatorname{cosec} x$ *	dM1 A1*
		(3)
(b)	Uses part (a) $\Rightarrow 7+\operatorname{cosec} 2\theta = 3\cot^2 2\theta$	B1
	Either Uses $\pm1\pm\cot^2 2\theta=\pm\operatorname{cosec}^2 2\theta \rightarrow$ 3TQ in $\operatorname{cosec} 2\theta$ **Or alternatively** replaces $\operatorname{cosec}2\theta$ with $1/\sin 2\theta$, $\cot^2 2\theta$ with $\cos^2 2\theta/\sin^2 2\theta$, multiplies by $\sin^2 2\theta$ and uses $\pm\cos^2 2\theta=\pm1\pm\sin^2 2\theta \rightarrow$ 3TQ in $\sin 2\theta$	M1
	$3\operatorname{cosec}^2 2\theta-\operatorname{cosec} 2\theta-10=0$ **or** $10\sin^2 2\theta+\sin 2\theta-3=0$	A1
	$(3\operatorname{cosec} 2\theta+5)(\operatorname{cosec} 2\theta-2)=0$ **or** $(5\sin 2\theta+3)(2\sin 2\theta-1)=0$ $\Rightarrow \operatorname{cosec} 2\theta=-\frac{5}{3}, 2$ **or** $\Rightarrow \sin 2\theta=-\frac{3}{5}, \frac{1}{2}$ $\Rightarrow \sin 2\theta=-\frac{3}{5}, \frac{1}{2}\Rightarrow \theta=\ldots$	dM1
	$\theta=\frac{\pi}{12}(0.262),\ \frac{5\pi}{12}(1.31),\ -0.322,\ -1.25$ (awrt these values)	A1, A1
		(6)
		Total 9

(a)

M1: Uses

- $\sin 2x = 2\sin x\cos x$
- AND $\cos 2x = 1-2\sin^2 x$ or equivalent. Condone sign slips on the versions of $\cos 2x$

in an attempt to write $\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}$ as an expression in $\sin x$ and $\cos x$

dM1: Adopts a valid approach that can be followed and completes the proof.

All necessary steps may not be shown and condone errors such as writing cos for $\cos x$ or $\sin x^2$ for $\sin^2 x$

A1*: Correct proof showing all necessary intermediate steps with no errors (seen within the body of the solution) or omissions of any of the steps shown. The LHS starting point does not need to be seen

See main mark scheme and below for examples showing all steps and scoring full marks

$$\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}=\frac{\sin x\sin 2x+\cos x\cos 2x}{\sin x\cos x}$$
$$=\frac{2\sin^2 x\cos x+\cos x\left(1-2\sin^2 x\right)}{\sin x\cos x}$$
$$=\frac{\cancel{\cos x}}{\sin x\cancel{\cos x}}$$
$$=\text{cosec}x$$

$$\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}=\frac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}+\frac{\cos 2x}{\sin x}$$
$$=\frac{2\sin^2 x+\left(\cos^2 x-\sin^2 x\right)}{\sin x}$$
$$=\frac{\sin^2 x+\cos^2 x}{\sin x}=\frac{1}{\sin x}$$
$$=\text{cosec}x$$

Alt part (a)

M1: For using compound angle formula $\sin x\sin 2x+\cos x\cos 2x=\cos(2x-x)$

dM1: As in the main scheme, it is for adopting a valid approach that can be followed and completing the proof

A1: Correct proof showing all necessary steps (See below) with no errors or omissions

$$\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}=\frac{\sin x\sin 2x+\cos x\cos 2x}{\sin x\cos x}$$
$$=\frac{\cos(2x-x)}{\sin x\cos x}$$
$$=\frac{\cancel{\cos x}}{\sin x\cancel{\cos x}}$$
$$=\text{cosec}x$$

……

(b)

B1: States $7+\text{cosec}\ 2\theta=3\cot^2 2\theta$ or exact equivalent which may be implied by subsequent work

OR $7+\text{cosec}\ x=3\cot^2 x$ with $x=2\theta$

Watch for and do not allow $7+\text{cosec}\ \theta 2\theta 3\cot^2$

M1: Attempts to use part (a) and uses $\pm1\pm\cot^2 2\theta=\pm\text{cosec}^2 2\theta$ to form a 3TQ in $\text{cosec}\ 2\theta$

Condone $3\cot^2 2\theta$ being replaced by $3\times\pm\text{cosec}^2 2\theta\pm1$ with or without the bracket.

Condone when the "7" is missing but these attempts will score a maximum of 2 marks. This mark and dM1

The terms don't need to be collected for this mark.

Alternatively replaces $\text{cosec}2\theta$ with $1/\sin 2\theta$, $\cot^2 2\theta$ with $\cos^2 2\theta/\sin^2 2\theta$ within an equation of the form $a+b\text{cosec}\ 2\theta=c\cot^2 2\theta$ multiplies by $\sin^2 2\theta$ and uses $\pm\cos^2 2\theta=\pm1\pm\sin^2 2\theta$ → 3TQ in $\sin\ 2\theta$

A1: Correct equation $3\text{cosec}^2 2\theta-\text{cosec}\ 2\theta-10=0$ or $10\sin^2 2\theta+\sin\ 2\theta-3=0$

The = 0 may be implied by further work, e.g solution of the equation

Allow this mark even for the correct equation in a different forms. E.g. $3\text{cosec}^2 2\theta-\text{cosec}\ 2\theta=10$

dM1: For a correct attempt to solve their 3TQ $\sin\ 2\theta$ or $\text{cosec}\ 2\theta$ leading to a value for θ

If they state that $\sin\ \theta=-\frac{3}{5},\ \frac{1}{2}$ and do not proceed to take arcsin and ÷2 it is M0

A1: For two of awrt $\theta=\frac{\pi}{12}(0.262),\ \frac{5\pi}{12}(1.31),\ -0.322,\ -1.25$

A1: For awrt $\theta=\frac{\pi}{12}(0.262),\ \frac{5\pi}{12}(1.31),\ -0.322,\ -1.25$ with no additional values within the range.

……

If you see other worthwhile solutions and the scheme cannot be applied, e.g *t* formula, please send to review

……

How to mark when other variables are used, e.g. $x=2\theta$

B1: $7+\text{cosec}\ x=3\cot^2 x$

M1: Uses $\pm1\pm\cot^2 x=\pm\text{cosec}^2 x$ to form 3TQ in $\text{cosec}\ x$ ....or the equivalent in $\sin x$

A1: Correct equation $3\text{cosec}^2 x-\text{cosec}\ x-10=0$ or $10\sin^2 x+\sin x-3=0$

ddM1: For this to be scored there must be an attempt to halve the values, otherwise M0.

Allow full marks to be scored for a candidate who uses a different variable correctly and reaches 4 correct answers

……

Q21

Question Number	Scheme	Marks
(i)	$3\sin(2\theta-10°)=1 \Rightarrow (2\theta-10°)=\arcsin\left(\frac{1}{3}\right)$	M1
	$\theta=\frac{19.47+10}{2},\frac{160.53+10}{2}$	dM1
	$\theta=$ awrt $14.7°, 85.3°$	A1, A1
		(4)
(ii) (a)	Writes $\frac{1}{\tan\alpha}-\sin\alpha=2\sin\alpha-\frac{1}{\tan\alpha}$ oe	M1
	$\frac{2}{\tan\alpha}=3\sin\alpha \Rightarrow \frac{2\cos\alpha}{\sin\alpha}=3\sin\alpha \Rightarrow 2\cos\alpha=3\sin^2\alpha$ *	dM1 A1*
		(3)
(b)	$2\cos\alpha=3\sin^2\alpha \Rightarrow 2\cos\alpha=3\left(1-\cos^2\alpha\right)$	M1
	$3\cos^2\alpha+2\cos\alpha-3=0$	A1
	Attempts to solve $3\cos^2\alpha+2\cos\alpha-3=0 \Rightarrow \cos\alpha=\frac{-2\pm\sqrt{40}}{6}$ oe	dM1 A1
	$\alpha=5.517$ radians	A1
		(5)
		(12 marks)

(i)

M1 For proceeding to $x = \arcsin\left(\frac{1}{3}\right)$ which may be implied by the sight of awrt $19.5°$ or awrt $160.5°$ (Allow awrt 0.340 (radians) for this mark)

dM1 For correct order of operations leading to one answer for θ. May be implied by $14.7°/14.8°$ but cannot be scored by adding 10 to an angle in radians but may be implied by awrt $0.257/0.258$ rad

A1 One of $\theta =$ awrt $14.7°, 85.3°$ ignore any others.

A1 Both of $\theta =$ awrt $14.7°, 85.3°$ and no others in the range.

Note that solutions based entirely on graphical or numerical methods are not acceptable so answers only will score 0 marks.

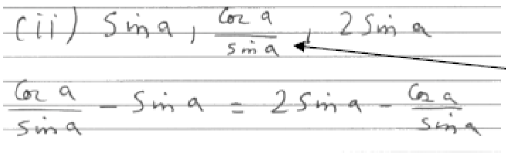
(ii)(a)

M1 Uses the terms of an AP to set up a correct equation. Eg $\frac{1}{\tan\alpha} - \sin\alpha = 2\sin\alpha - \frac{1}{\tan\alpha}$, $\frac{\cos\alpha}{\sin\alpha} - \sin\alpha = 2\sin\alpha - \frac{\cos\alpha}{\sin\alpha}$. They may also do $2\left(\frac{1}{\tan\alpha} - \sin\alpha\right) = 2\sin\alpha - \sin\alpha$. Condone mixed variables and poor notation for the method marks.

dM1 Uses $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ within an equation involving $\frac{1}{\tan\alpha}, \sin\alpha \;\&\; 2\sin\alpha$. This mark can be implied but do not award if they just proceed straight to the final answer. It is dependent on the previous method mark. A candidate starting with $\frac{\cos\alpha}{\sin\alpha} - \sin\alpha = 2\sin\alpha - \frac{\cos\alpha}{\sin\alpha}$ scores M1dM1 straight away.

A1* Proceeds to given answer with no errors or omissions. The equation must start with an equation involving $\tan\alpha$ but see the note below for further guidance. Withold this mark for incorrect notation eg $\sin\alpha^2$ or if they had mixed variables on the same line. Eg α and θ.

Note: In the example below they can score full marks as $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ is implied by their second term listed on the first line. Had they not written these first three terms or stated somewhere in their solution $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ in their working then we would withhold the final mark.



(ii) Sin a, $\frac{\text{Cos a}}{\text{Sin a}}$, 2 Sin a

$\frac{\text{Cos a}}{\text{Sin a}}$ − Sin a = 2 Sin a − $\frac{\text{Cos a}}{\text{Sin a}}$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ implied here

(ii)(b)

M1 Attempts to use $\sin^2\alpha + \cos^2\alpha = 1$ Eg. $2\cos\alpha = 3\sin^2\alpha \Rightarrow 2\cos\alpha = 3(\pm 1 \pm \cos^2\alpha)$. Beware that a candidate may use this identity in part (ii)(a) which would not gain the mark in this part.

A1 $3\cos^2\alpha + 2\cos\alpha - 3 = 0$ The "=0" may be implied by later work but the terms must be collected on one side. Evidence of this may be awarded for correct coefficients substituted into the quadratic formula.

dM1 Attempts to solve their $3\cos^2\alpha + 2\cos\alpha - 3 = 0$ by the formula/completing the square, usual rules for solving a 3TQ apply but do not award for attempted factorisation unless their quadratic factorises. Award for $(\cos\alpha =)\ \frac{-1 \pm \sqrt{10}}{3}$ or $(\cos\alpha =)$ awrt 0.72 or awrt −1.4 or equivalent. You may need to check decimal values on your calculator.

A1 $(\cos\alpha =)\ \frac{-2 + \sqrt{40}}{6}$ oe (typically $(\cos\alpha =)\ \frac{-1 + \sqrt{10}}{3}$). Implied by $(\cos\alpha =)$ awrt 0.721 or $(\alpha =)$ awrt 5.52 radians.

A1 $(\alpha =)$ awrt 5.517 radians and no others in the given range

Q22

Question Number	Scheme	Marks
(a)(i)	$P(-180,-4)$	B1, B1
(ii)	$Q(450,0)$	B1
		(3)
(b)	$R(360,7)$	B1, B1
		(2)
		(5 marks)

For all parts condone missing brackets and check the graph/next to the question for answers.

Condone use of the degree symbol for their x values eg $(-180°,...)$ instead of $(-180,...)$

(a)(i)

B1 $(-180,...)$ or $(..., -4)$ or $x=-180$ or $y=-4$ condone x in radians

B1 $(-180,-4)$ or $x=-180,\ y=-4$ Must be in degrees

SC1 $(-4,-180)$ (on EPEN this would be scored B1B0)

(a)(ii)

B1 $(450,0)$ or $x=450,\ y=0$ condone $\left(\frac{5}{2}\pi,0\right)$

(b)

B1 $(360,...)$ or $(...,7)$ $x=360$ or $y=7$ condone x in radians

B1 $(360,7)$ or $x=360,\ y=7$ Must be in degrees. Ignore any reference to $(0,7)$

SC1 $(7, 360)$ (on EPEN this would be scored B1B0)

Note if radians used throughout then max score:

(a)(i) $(-\pi,-4)$ B1B0 **(a)(ii)** $\left(\frac{5}{2}\pi,0\right)$ B1 **(b)** $(2\pi,7)$ B1B0

Q23

Question Number	Scheme	Marks
(a)	$\sqrt{2}\sin(x+45°)=\cos(x-60°)$	
	$\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°)=\cos x\cos 60°+\sin x\sin 60°$	M1 A1
	$\sin x+\cos x=\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x$	
	$\cos x=(\sqrt{3}-2)\sin x$	M1
	$\tan x\left(=\frac{1}{\sqrt{3}-2}=\frac{\sqrt{3}+2}{-1}\right)=-2-\sqrt{3}$ *	A1*
		(4)
(b)	States or uses $x+45°=2\theta$ o.e.	B1
	Proceeds from e.g. $\tan(2\theta-45°)=-2-\sqrt{3}\Rightarrow 2\theta-45°=105°,285°$	M1
	Correct order of operations to find one angle	dM1
	$\theta=75°,165°$	A1
		(4)
		(8 marks)

(a) Condone working in mixed variables

M1: Attempts to use the compound angle identities to produce an equation in $\sin x$ and $\cos x$

Look for $\sqrt{2}\times(\sin x\cos 45°\pm\cos x\sin 45°)=\cos x\cos 60°\pm\sin x\sin 60°$ o.e.

May be implied by a first line of e.g. $\sin x\pm\cos x=\frac{1}{2}\cos x\pm\frac{\sqrt{3}}{2}\sin x$ o.e.

Condone missing brackets.

A1: Correct equation in $\sin x$ and $\cos x$.

E.g. $\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°)=\cos x\cos 60°+\sin x\sin 60°$ o.e. (may be implied by further work). May also see use of equivalent angles e.g. $\cos(-60)°=\cos 60°$

M1: For an attempt to solve the problem. Look for an attempt (condoning slips) at two of the three elements required to complete the proof, namely

- use of correct exact trigonometric values for $\sin 45°, \cos 45°, \cos 60°, \sin 60°$
- collection of like terms in $\sin x$ and $\cos x$ or $\tan x$
- use of $\tan x = \frac{\sin x}{\cos x}$

A1*: Proceeds to the given answer with all previous marks scored.
There should be no errors in the manipulation and no bracket omissions, other than a missing trailing bracket.
e.g. $\sqrt{2}(\sin x \cos 45° + \cos x \sin 45° = \cos x \cos 60° + \sin x \sin 60°$
When proceeding from $A\sin x = B\cos x$ (where A or B may be 1) to the given answer it is acceptable to proceed from $\frac{1}{2}\cos x = \frac{-2+\sqrt{3}}{2}\sin x$ to the given answer as evidence of the use of $\tan x = \frac{\sin x}{\cos x}$.
Condone poor notation to be recovered provided the final answer line is written correctly. Condone working in mixed variables provided the final given answer is all in terms of x.
Allow surd work to be done via a calculator.

(b)

B1: States or uses $x + 45° = 2\theta$ o.e. e.g. $\theta = \frac{x+45°}{2}$

This is implied for sight of the equation $\tan(2\theta - 45°) = -2 - \sqrt{3}$

M1: Proceeds from $\tan(2\theta \pm \alpha°) = -2 - \sqrt{3} \Rightarrow 2\theta \pm \alpha° = 105°$ or $285°$ where $\alpha \neq 0$

The attempt must either achieve an angle of 105° or 285° or equivalent expression (Allow in radians (3sf) which are 1.83, 4.97), or allow a general solution of e.g. $2\theta \pm \alpha° = -75° + (180n)°$

Maybe implied by further work which is not 75° or 165°

dM1: Correct order of operations to solve their $2\theta \pm \alpha° = ...$
This is dependent on the previous method mark.
Note that $\tan(2\theta - 45°) = -2 - \sqrt{3} \Rightarrow 75°$ does not imply this mark. We must see either e.g. $2\theta - 45° = 105° \Rightarrow \theta = ...$ or some intermediate stage before seeing 75°

A1: Both angles $\theta = 75°, 165°$ with no others given within the range

Note that $\tan(2\theta - 45°) = -2 - \sqrt{3} \Rightarrow 2\theta - 45° = 105° \Rightarrow \theta = 75°, 165°$ is acceptable for full marks

Alt (b) via use of $\cos(2\theta - 105°) = \cos 2\theta \cos 105° + \sin 2\theta \sin 105°$

$$\sqrt{2}\sin 2\theta = \cos(2\theta - 105°) \Rightarrow \tan 2\theta = \frac{\cos 105°}{\sqrt{2} - \sin 105°} \Rightarrow \theta = 75°, 165°$$

Note the order of the marks needs to match up to the main scheme so 0110 is possible.

B1: For achieving $\tan 2\theta = -\frac{\sqrt{3}}{3}$ o.e. so allow $\tan 2\theta = \frac{\cos 105°}{\sqrt{2} - \sin 105°} =$ awrt -0.58

Or via double angle identities $\sqrt{3}\tan^2\theta - 6\tan\theta - \sqrt{3} = 0$ o.e.

M1: Attempts to use the compound angle identities (allowing sign slips when using them) to reach a form $\tan 2\theta = k$ where k is a constant not $-2-\sqrt{3}$ (or expression in trigonometric terms such as $\cos 105°$ as seen above). Allow $2\theta = -30°$ o.e. (allow in radians $-\frac{\pi}{6}$) to imply this mark. Do not be concerned by the mechanics of their rearrangement.
Alternatively, via double angle identities reaches a 3TQ in $\tan\theta$

dM1: Correct order of operations from $\tan 2\theta = k$ proceeding to $\theta = ...$ e.g. $2\theta = -30° \Rightarrow \theta = -15°$ **(which must be in degrees)** can score this mark. You may need to check this if $\theta = \frac{\tan^{-1} k}{2} = ...$ is not written

Alternatively, correctly solves their $\sqrt{3}\tan^2\theta - 6\tan\theta - \sqrt{3} = 0$ proceeding to $\theta = ...$

A1: Both angles $\theta = 75°, 165°$ with no others given within the range

Q24

Question Number	Scheme	Marks
(a)	$12\tan 2x+5\cot x\sec^{2}x=0$	
	$12\times\frac{2t}{1-t^{2}}+5\times\frac{1}{t}\left(1+t^{2}\right)=0$	B1 M1 A1
	$12\times 2t^{2}+5\left(1+t^{2}\right)\left(1-t^{2}\right)=0\rightarrow 5t^{4}-24t^{2}-5=0$ *	A1*
		(4)
(b)	$5t^{4}-24t^{2}-5=0$	
	$\left(5t^{2}+1\right)\left(t^{2}-5\right)=0$	M1
	Correct order of operations $t=(\pm)\sqrt{5}\Rightarrow x=..$	dM1
	Two of awrt $x=66°,114°,246°,294°$	A1
	All four of awrt $x=65.9°,114.1°,245.9°,294.1°$	A1
		(4)
		8 marks

(a)

B1 Any **correct** identity used within the given equation either in terms of $\tan x$ or in terms of t

Eg: Attempts to replace either $\tan 2x=\frac{2\tan x}{1-\tan^{2}x}$, $\cot x=\frac{1}{\tan x}$ or $\sec^{2}x=1+t^{2}$

M1 Uses $\tan 2x=\frac{2\tan x}{1\pm\tan^{2}x}$, $\cot x=\frac{1}{\tan x}$, and $\sec^{2}x=\pm1\pm\tan^{2}x$ or with $t=\tan x$ to produce an equation in terms of t or $\tan x$

A1 Correct intermediate equation in t or $\tan x$ The $=0$ may be implied by later work

A1* For proceeding to the correct answer with a correct intermediate line. Must have $=0$.
There cannot be any notational or bracketing errors within the body of the solution if this mark is to be awarded. A notational error is $\tan x^{2}\leftrightarrow\tan^{2}x$
The intermediate line should be of a form in which the given answer could immediately follow. See main scheme for such an example; the fractional terms have been dealt with in this case.
Condone partially completed lines if the candidate is only working on one side of the equation

(b)

M1 Correct attempt to solve.
Allow an attempt to factorise $5t^{4}-24t^{2}-5=0\Rightarrow\left(at^{2}+b\right)\left(ct^{2}+d\right)=0$ with $ac=\pm5, bd=\pm5$

Alt lets $u=t^{2}$ and attempts to factorise $5u^{2}-24u-5=0\Rightarrow$ with usual rules.
Allow use of calculator giving $t^{2}=5$ or $\tan^{2}x=5$ (You may ignore the negative root).
It is also implied by $t=\sqrt{5}$ or $t=-\sqrt{5}$ Watch out for $\tan x=5$ which is M0

dM1 For using the correct order of operations and finding one value of x for their $t^{2}=k$ where k is a positive constant. Allow accuracy to either the nearest degree or correct to 1dp in radians.
It is dependent upon them having scored the previous M1.

A1 Any two of awrt $x=66°,114°,246°,294°$ May be implied by awrt two of 1.15, 1.99, 4.29, 5.13

A1 All four of awrt $x=65.9°,114.1°,245.9°,294.1°$ AND no extras within the range.

Q25

Question Number	Scheme	Marks
(a)	$R=\sqrt{41}$	B1
	$\tan\alpha=\frac{4}{5}\Rightarrow\alpha=\text{awrt }0.675$	M1A1
		(3)
(b)	(i) Describes stretch: stretch in the y direction by "$\sqrt{41}$"	B1 ft
	(ii) Describes translation: E.g. translate by $\begin{pmatrix}-\arctan\frac{4}{5}\\0\end{pmatrix}$	B1 ft
		(2)
(c)	Attempts either $g(\theta)=\frac{90}{4+\left(\sqrt{41}\right)^2}$ OR $g(\theta)=\frac{90}{4}$	M1
	Range $2\leqslant g(\theta)\leqslant 22.5$	A1
		(2)
		7 marks

(a)

B1 $R=\sqrt{41}$

Condone $R=\pm\sqrt{41}$ (Do not allow decimals for this mark Eg 6.40 but remember to isw after $\sqrt{41}$)

M1 $\tan\alpha=\pm\frac{4}{5}$, $\tan\alpha=\pm\frac{5}{4}\Rightarrow\alpha=\ldots$ Condone $\sin\alpha=4,\ \cos\alpha=5\Rightarrow\tan\alpha=\frac{4}{5}$

If R is used to find α accept $\sin\alpha=\pm\frac{4}{R}$ or $\cos\alpha=\pm\frac{5}{R}\Rightarrow\alpha=\ldots$

A1 $\alpha=\text{awrt }0.675$ Note that the degree equivalent $\alpha=\text{awrt }38.7°$ is A0

(b)(i)

B1ft Fully describes the stretch. Follow through on their R. **Requires the size and the direction**

Allow responses such as

- stretch in the y direction by "$\sqrt{41}$"
- multiplies all the y coordinates/values by "$\sqrt{41}$"
- stretch in ↑ direction by "$\sqrt{41}$"
- vertical stretch by "$\sqrt{41}$"
- Scale Factor "$\sqrt{41}$" in just the y direction

Do not award for y is translated/transformed by "$\sqrt{41}$"

(b)(ii)

B1 ft Fully describes the translation. **Requires the size and the direction**

Follow through on their 0.675 or $\alpha =$ awrt 38.7° or $\arctan\frac{4}{5}$

Allow responses such as

- translates left by 0.675
- horizontal by -0.675
- condone "transforms" left by 0.675. (question asks for the translation)
- moves ← by 38.7°
- x values move back by 0.675
- shifts in the negative x direction by $\arctan\frac{4}{5}$
- $\begin{pmatrix} -0.675 \\ 0 \end{pmatrix}$

Do **not** award for translates left by -0.675 (double negative...wrong direction)
horizontal shift of 0.675 (no direction)

If there are no labels score in the order given but do allow these to be written in any order as long as the candidate clearly states which one they are answering. For example it is fine to write ....
translation is...............
stretch is ....
If the candidate does not label correctly, or states which one they are doing, but otherwise gets both completely correct then award SC B1 B0

(c)

M1 Score for either end achieved by a correct method

Look for $\frac{90}{4}$ (implied by 22.5) , $\frac{90}{4+\text{their}\left(\sqrt{41}\right)^2}$, $g\ldots 22.5$ or $g\ldots 2$ etc

A1 See scheme but allow 22.5 to be written as $\frac{90}{4}$

Accept equivalent ways of writing the interval such as $[2, 22.5]$

Condone $2 \leqslant g(x) \leqslant 22.5$ or $2 \leqslant y \leqslant 22.5$

Q26

Question Number	Scheme	Marks
(a)	$(A=)-3$	B1
		(1)
(b)	$y=3$	B1
	eg $x=30+5\times180$ or $x=210+720$ or $x=180+2\times360+30$	M1
	$x=930$	A1
		(3)
		Total 4

Check for answers next to the questions and on the graph. If there are contradictions then the answers given in the main body of the work takes precedence

(a)

B1: $(A=)-3$

(b)

B1: Correct y coordinate only (others must be discarded)

M1: Correct strategy for the x coordinate. See scheme for examples.
Values embedded is sufficient for the mark.

A1: Correct x coordinate only (others must be discarded). Isw. Note $(930,\ 3)$ with no incorrect working and no other coordinates scores full marks.

Special case: If they give $(3,\ 930)$ or $\left(\frac{31}{6}\pi,\ 3\right)$ rather than $(930,\ 3)$ score B1M1A0

Q27

Question Number	Scheme	Marks
(a)	Attempts $f\left(-\frac{3}{2}\right)=6\left(-\frac{3}{2}\right)^3+17\left(-\frac{3}{2}\right)^2+4\left(-\frac{3}{2}\right)-12$	M1
	$=0\Rightarrow(2x+3)$ is a factor *	A1*
		(2)
(b)	$6x^3+17x^2+4x-12=(2x+3)(3x^2+4x-4)$	M1 A1
	$=(2x+3)(3x-2)(x+2)$	dM1 A1
		(4)
(c)	Solves $\tan\theta=-\frac{3}{2}$ or "-2" or "$\frac{2}{3}$"	M1
	$\theta=$ awrt 2.03, 2.16	A1
		(2) (8 marks)

(a)

M1 Allow for an attempt at finding a value of $f\left(-\frac{3}{2}\right)=6\left(-\frac{3}{2}\right)^3+17\left(-\frac{3}{2}\right)^2+4\left(-\frac{3}{2}\right)-12$

Sight of embedded values is sufficient. Attempted division is M0

A1* Correctly shows $f\left(-\frac{3}{2}\right)=0$ and states "hence factor" (or substantial equivalent conclusion).

They may state "factor if $f\left(-\frac{3}{2}\right)=0$" in a preamble, in which case accept a minimal conclusion such as //

The M must be scored so there needs to be evidence of either embedded $\left(-\frac{3}{2}\right)$'s or calculations.

(b) Allow for work done in part (a) if referred to in part (b).

M1 Attempt to divide or factorise out $(2x+3)$

By factorisation look for $6x^3+17x^2+4x-12=(2x+3)(3x^2+kx\pm4)$

By division look for

$$\begin{array}{r} 3x^2\ \pm4x\ldots\ldots\ldots \\ 2x+3\overline{\smash{)}6x^3+17x^2+4x-12} \\ \underline{6x^3+9x^2}\qquad\qquad\quad \end{array}$$

By comparing coefficients look for $6x^3+17x^2+4x-12=(2x+3)(ax^2+bx+c)\Rightarrow a=3, c=\pm4, b=\ldots$

A1 Correct quadratic factor $(3x^2+4x-4)$ found. May be within the long division.

dM1 Attempts to factorise their $(3x^2+4x-4)$ usual rules.

A1 $(2x+3)(3x-2)(x+2)$ written out as a product of factors (not as a list) and isw.

Allow $3(2x+3)\left(x-\frac{2}{3}\right)(x+2)$ oe as long as there are three linear factors.

Factors given but no working shown scores M0A0dM0A0

(c)

M1 Solves $\tan\theta=k$ where k is any of their roots to their cubic. Allow if $\tan^{-1}$ seen followed by an answer, or it can be implied by awrt -0.98 or -1.1 or 0.59 or 0.58 truncated (or awrt -56.3 or -63.4 or 33.7 (degrees))

A1 $\theta=$ awrt 2.03, 2.16 and no other solutions inside the range.

Accept 0.6476π and 0.687π as these give correct answers to 3s.f. but not 0.648π as this is not correct to 3s.f.

Q28

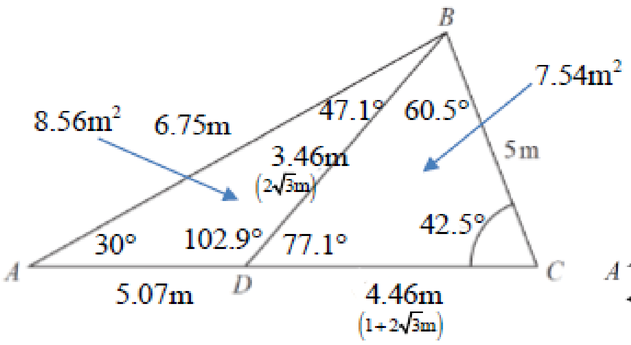
Question Number	Scheme	Marks
a	e.g. $(\cos\theta=)\ \frac{5^2+(1+x)^2-x^2}{2\times5(1+x)}\ \left(=\frac{25+1+2x+x^2-x^2}{2\times5(1+x)}\right)$ or e.g. $x^2=5^2+(1+x)^2-2\times5(1+x)\cos\theta$	M1
	$\cos\theta=\frac{13+x}{5+5x}$ *	A1*
		(2)
b	$\theta=\text{awrt }42°\quad(42.470747\ldots.)$	B1
	-- Attempts to find AB, AD or AC: e.g. $\frac{AB}{\sin"42"}=\frac{5}{\sin30}\Rightarrow AB=\ldots$ or e.g. $\angle ABC=180-30-"42.5"="107.6"$ $\frac{\sin DBC}{1+2\sqrt{3}}=\frac{\sin"42.5"}{2\sqrt{3}}\Rightarrow\angle DBC="60.5",\ \angle ABD="107.6"-"60.5"="47.1"$ $\frac{AD}{\sin("47.1")}=\frac{2\sqrt{3}}{\sin30}\Rightarrow AD=\ldots$ or e.g. $\frac{AC}{\sin"108"}=\frac{5}{\sin30}\Rightarrow AC=\ldots$ --	M1
	$AB=6.75\ldots$ or $AD=5.07\ldots$ or $AC=9.53\ldots$	A1
	Area $=\frac{1}{2}\times5\times"6.75"\times\sin(180-30-"42.5")$ or $=\frac{1}{2}\times5\times"9.54"\times\sin("42.5")$	dM1
	$=\text{awrt }16.1\ (\text{m}^2)$	A1
		(5)
		(7 marks)

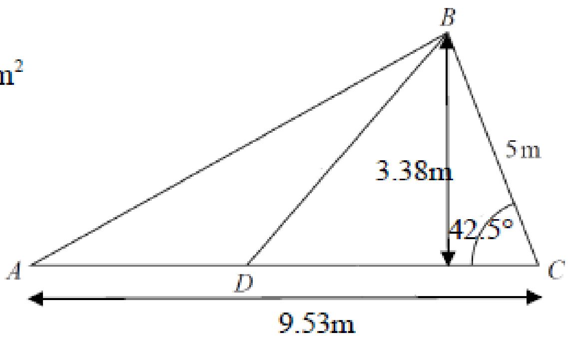
(a)

M1: Uses the cosine rule to form an equation for $\cos\theta$. Condone just an expression for $\cos\theta$. (You may not see $\cos\theta =$)
Condone invisible brackets for this mark.
Be aware of longer versions which may involve splitting triangle *BCD* into two right angled triangles. They would still need to proceed to an equation involving $\cos\theta$ to score this mark.

A1*: Achieves the given answer with no errors seen including invisible brackets. There must be at least one stage of intermediate working between their starting equation and achieving the given answer. Condone $\cos\theta$ not appearing in the answer line, provided it is seen correctly as the subject on an earlier line.

(b) Note there are a variety of different methods to finding the area of triangle *ABC*





B1: $\theta =$ awrt $42°$ seen or implied. Accept 42.5. May work in radians (awrt 0.73/0.74 radians)

M1: Attempts to find *AB*, *AD* or *AC* using a correct method. Angles and lengths must be in the correct positions in the relevant formula or formulae. Condone slips in any rearrangement, calculations and substituting in $x = 2\sqrt{3}$ provided the method is correct. Condone working in radians provided the angles are consistently in degrees or radians within the expression or formula.

A1: awrt 6.75/6.76 or awrt 9.53/9.54 or awrt 5.07/5.08 (may be implied by further work).

dM1: Correct full method to find the total area (the expression is sufficient. It is dependent on the previous method mark. Condone use of incorrectly rounded angles/slips and may work in radians.

e.g. $\text{Area} = \frac{1}{2}\times 5\times "6.75"\times \sin\left(\pi - \frac{\pi}{6} - "0.74"\right)$

May find the areas of the two separate triangles *ABD* and *BDC* and add them together.

e.g. $\text{Area} = \frac{1}{2}\times "5.07"\times "6.75"\times \sin 30 + \frac{1}{2}\times "4.46"\times 5\times \sin "42.5"$

Use the diagrams above to help with the various methods. Invisible brackets may be implied by further work or their answer.
Note that if, as part of their method to find the total area, they find angle *ADB* but incorrectly deduce this as an acute angle then this is dM0

A1: awrt 16.1 m^2 (condone lack of units)