Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	Considers $\frac{(x+h)^3 - x^3}{h}$	B1	2.1
	Expands $(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$	M1	1.1b
	so gradient (of chord) $= \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2$	A1	1.1b
	States as $h \to 0$, $3x^2 + 3xh + h^2 \to 3x^2$ so derivative $= 3x^2$ *	A1*	2.5
		(4 marks)	

B1: Gives the correct fraction for the gradient of the chord either $\frac{(x+h)^3 - x^3}{h}$ or $\frac{(x+\delta x)^3 - x^3}{\delta x}$

It may also be awarded for $\frac{(x+h)^3 - x^3}{x+h-x}$ oe. It may be seen in an expanded form

It does not have to be linked to the gradient of the chord

M1: Attempts to expand $(x+h)^3$ or $(x+\delta x)^3$ Look for two correct terms, most likely $x^3 + ... + h^3$ This is independent of the B1

A1: Achieves gradient (of chord) is $3x^2 + 3xh + h^2$ or exact un simplified equivalent such as $3x^2 + 2xh + xh + h^2$. Again, there is no requirement to state that this expression is the gradient of the chord

A1*: CSO. Requires correct algebra and making a link between the gradient of the chord and the gradient of the curve. See below how the link can be made. The words "gradient of the chord" do not need to be mentioned but derivative, $f'(x)$, $\frac{dy}{dx}$, y' should be. Condone invisible brackets for the expansion of $(x+h)^3$ as long as it is only seen at the side as intermediate working.

Requires either

- $f'(x) \underset{\lim h \to 0}{=} \frac{(x+h)^3 - x^3}{h} = 3x^2 + 3xh + h^2 = 3x^2$
- Gradient of chord $= 3x^2 + 3xh + h^2$ As $h \to 0$ Gradient of chord tends to the gradient of curve so derivative is $3x^2$
- $f'(x) \underset{\lim h \to 0}{=} 3x^2 + 3xh + h^2 = 3x^2$
- Gradient of **chord** $= 3x^2 + 3xh + h^2$ when $h \to 0$ gradient of **curve** $= 3x^2$
- Do not allow $h = 0$ alone without limit being considered somewhere: so don't accept $h = 0 \Rightarrow f'(x) = 3x^2 + 3x \times 0 + 0^2 = 3x^2$

Alternative: B1: Considers $\frac{(x+h)^3 - (x-h)^3}{2h}$ M1: As above A1: $\frac{6x^2h^2 + 2h^3}{2h} = 3x^2 + h^2$

Q2.

Question	Scheme	Marks	AOs
	$\frac{d}{d\theta}(\cos\theta) = -\sin\theta$; as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$		
	$\frac{\cos(\theta+h) - \cos\theta}{h}$	B1	2.1
	$= \frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$	M1	1.1b
		A1	1.1b
	$= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$		
	As $h \to 0$, $-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta \to -1\sin\theta + 0\cos\theta$	dM1	2.1
	so $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$ *	A1*	2.5
		(5)	
			(5 marks)

Notes for Question	
B1:	Gives the correct fraction such as $\frac{\cos(\theta+h) - \cos\theta}{h}$ or $\frac{\cos(\theta+\delta\theta) - \cos\theta}{\delta\theta}$ Allow $\frac{\cos(\theta+h) - \cos\theta}{(\theta+h) - \theta}$ o.e. **Note:** $\cos(\theta+h)$ or $\cos(\theta+\delta\theta)$ may be expanded
M1:	Uses the compound angle formula for $\cos(\theta+h)$ to give $\cos\theta\cos h \pm \sin\theta\sin h$
A1:	Achieves $\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$ or equivalent
dM1:	**dependent on both the B and M marks being awarded** Complete attempt to apply the given limits to the gradient of their chord
Note:	They must isolate $\frac{\sin h}{h}$ and $\left(\frac{\cos h - 1}{h}\right)$, and replace $\frac{\sin h}{h}$ with 1 and replace $\left(\frac{\cos h - 1}{h}\right)$ with 0
A1*:	cso. Uses correct mathematical language of limiting arguments to prove $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$
Note:	Acceptable responses for the final A mark include: • $\frac{d}{d\theta}(\cos\theta) = \lim_{h \to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of chord tends to the gradient of the curve, so derivative is $-\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of *curve* is $-\sin\theta$
Note:	Give final A0 for the following example which shows ***no limiting arguments***: when $h = 0$, $\frac{d}{d\theta}(\cos\theta) = -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta = -1\sin\theta + 0\cos\theta = -\sin\theta$
Note:	Do not allow the final A1 for stating $\frac{\sin h}{h} = 1$ or $\left(\frac{\cos h - 1}{h}\right) = 0$ and attempting to apply these
Note:	In this question $\delta\theta$ may be used in place of h
Note:	Condone $f'(\theta)$ where $f(\theta) = \cos\theta$ or $\frac{dy}{d\theta}$ where $y = \cos\theta$ used in place of $\frac{d}{d\theta}(\cos\theta)$

Notes for Question Continued	
Note:	Condone x used in place of θ if this is done consistently
Note:	Give final A0 for • $\frac{d}{d\theta}(\cos x) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • $\frac{d}{d\theta} = ...$ • Defining $f(x) = \cos\theta$ and applying $f'(x) = ...$ • $\frac{d}{dx}(\cos\theta)$
Note:	Give final A1 for a correct limiting argument in x, followed by $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$ e.g. $\frac{d}{d\theta}(\cos x) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin x + \left(\frac{\cos h - 1}{h}\right)\cos x\right) = -1\sin x + 0\cos x = -\sin x$ $\Rightarrow \frac{d}{d\theta}(\cos\theta) = -\sin\theta$
Note:	Applying $h \to 0$, $\sin h \to h$, $\cos h \to 1$ to give e.g. $\lim_{h\to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \left(\frac{\cos\theta(1) - \sin\theta(h) - \cos\theta}{h}\right) = \frac{-\sin\theta(h)}{h} = -\sin\theta$ is final M0 A0 for incorrect application of limits
Note:	$\lim_{h\to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right)$ $= \lim_{h\to 0}(-(1)\sin\theta + 0\cos\theta) = -\sin\theta$. So for not removing $\lim_{h\to 0}$ when the limit was taken is final A0
Note:	<u>**Alternative Method:**</u> Considers $\frac{\cos(\theta+h) - \cos(\theta-h)}{(\theta+h)-(\theta-h)}$ which simplifies to $\frac{-2\sin\theta\sin h}{2h}$

Q3.

Question Number	Scheme	Marks
(a)	$\left(\frac{dy}{dx}=\right)\frac{3}{2}x^2-\frac{27}{2}x^{\frac{1}{2}}-8x^{-2}$	M1A1A1A1 (4)
(b)	$x=4 \Rightarrow y=\frac{1}{2}\times 64-9\times 2^3+\frac{8}{4}+30$	M1
	$=32-72+2+30 \qquad = \underline{-8}$ *	A1cso (2)
(c)	$x=4 \Rightarrow y'=\frac{3}{2}\times 4^2-\frac{27}{2}\times 2-\frac{8}{16}$	M1
	$=24-27-\frac{1}{2}= \qquad -\frac{7}{2}$	A1
	Gradient of the normal $=-1\div"\frac{7}{2}"$	M1
	Equation of normal: $y--8=\frac{2}{7}(x-4)$	M1A1ft
	$\underline{7y-2x+64=0}$	A1 (6) 12

Question Number	Scheme	Marks
	<u>Notes</u>	
(a)	1[st] M1 for an attempt to differentiate $x^n \to x^{n-1}$ 1[st] A1 for one correct term in x 2[nd] A1 for 2 terms in x correct 3[rd] A1 for all correct x terms. No 30 term and no $+c$.	
(b)	M1 for substituting $x=4$ into $y=$ and attempting $4^{\frac{3}{2}}$ A1 note this is a printed answer	
(c)	1[st] M1 Substitute x = 4 into y' (allow slips) A1 Obtains −3.5 or equivalent 2[nd] M1 for correct use of the perpendicular gradient rule using their gradient. (May be slip doing the division) Their gradient must have come from y' 3[rd] M1 for an attempt at equation of tangent or normal at P 2[nd] A1ft for correct use of their changed gradient to find **normal** at P. Depends on 1[st], 2[nd] and 3[rd] Ms 3[rd] A1 for any equivalent form with integer coefficients	

Q4.

Question Number	Scheme		Marks
(a)	$(3-x^2)^2 = 9-6x^2+x^4$	An attempt to expand the numerator obtaining an expression of the form $9 \pm px^2 \pm qx^4$, $p.q \neq 0$	M1
	$9x^{-2}+x^2$	Must come from $\frac{9+x^4}{x^2}$	A1
	-6	Must come from $\frac{-6x^2}{x^2}$	A1
	Alternative 1: Writes $\frac{(3-x^2)^2}{x^2}$ as $(3x^{-1}-x)^2$ and attempts to expand = M1 then A1A1 as in the scheme.		
	Alternative 2: Sets $(3-x^2)^2 = 9+Ax^2+Bx^4$, expands $(3-x^2)^2$ and compares coefficients = M1 then A1A1 as in the scheme.		
			(3)
	$(f'(x) = 9x^{-2} - 6 + x^2)$		
(b)	$-18x^{-3}+2x$	M1: $x^n \to x^{n-1}$ on separate terms at least once. Do not award for $A \to 0$ (Integrating is M0) A1ft: $-18x^{-3} + 2"B"x$ with a numerical B and no extra terms. (A may have been incorrect or even zero)	M1 A1ft
			(2)
(c)	$f(x) = -9x^{-1} - 6x + \frac{x^3}{3}(+c)$	M1: $x^n \to x^{n+1}$ on separate terms at least once. (Differentiating is M0) A1ft: $-9x^{-1} + Ax + \frac{Bx^3}{3}(+c)$ with numerical A and B, $A, B \neq 0$	M1A1ft
	$10 = \frac{-9}{-3} - 6(-3) + \frac{(-3)^3}{3} + c$ so c $= \ldots$	Uses $x = -3$ and $y = 10$ in what they think is f(x) (They may have differentiated here) but it must be a changed function i.e. not the original $f'(x)$, to form a linear equation in c and attempts to find c. No $+c$ gets M0 and A0 unless their method implies that they are correctly finding a constant.	M1
	$c = -2$	cso	A1
	$(f(x) =) -9x^{-1} - 6x + \frac{x^3}{3} +$ their c	Follow through their c in an otherwise (possibly un-simplified) **correct expression**. Allow $-\frac{9}{x}$ for $-9x^{-1}$ or even $\frac{9x^{-1}}{-1}$.	A1ft
	Note that if they integrate in (b), no marks there but if they then go on to use their integration in (c), the marks for integration are available.		
			(5)
			[10]

Q5.

Question Number	Scheme		Marks
	(a) $\dfrac{2}{x+2}+\dfrac{4}{x^2+5}-\dfrac{18}{(x+2)(x^2+5)}=\dfrac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$		M1A1
	$=\dfrac{2x(x+2)}{(x+2)(x^2+5)}$		M1
	$=\dfrac{2x}{(x^2+5)}$		A1*
			(4)
	(b) $\text{h}'(x)=\dfrac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$		M1A1
	$\text{h}'(x)=\dfrac{10-2x^2}{(x^2+5)^2}$	cso	A1
			(3)
	(c) Maximum occurs when $\text{h}'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$		M1
	$\Rightarrow x=\sqrt{5}$		A1
	(c) Maximum occurs when $\text{h}'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$		M1
	$\Rightarrow x=\sqrt{5}$		A1
	When $x=\sqrt{5}\Rightarrow \text{h}(x)=\dfrac{\sqrt{5}}{5}$		M1,A1
	Range of h(x) is $0\le h(x)\le\dfrac{\sqrt{5}}{5}$		A1ft
			(5)
			(12 marks)

(a) M1 Combines the three fractions to form a single fraction with a common denominator.
Allow errors on the numerator but at least one must have been adapted.
Condone 'invisible' brackets for this mark.
Accept three separate fractions with the same denominator.
Amongst possible options allowed for this method are

$\frac{2x^2+5+4x+2-18}{(x+2)(x^2+5)}$ Eg 1 An example of 'invisible' brackets

$\frac{2(x^2+5)}{(x+2)(x^2+5)}+\frac{4}{(x+2)(x^2+5)}-\frac{18}{(x+2)(x^2+5)}$ Eg 2An example of an error (on middle term), 1st term has been adapted

$\frac{2(x^2+5)^2(x+2)+4(x+2)^2(x^2+5)-18(x^2+5)(x+2)}{(x+2)^2(x^2+5)^2}$ Eg 3 An example of a correct fraction with a different denominator

A1 Award for a correct un simplified fraction with the correct (lowest) common denominator.
$\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$

Accept if there are three separate fractions with the correct (lowest) common denominator.
Eg $\frac{2(x^2+5)}{(x+2)(x^2+5)}+\frac{4(x+2)}{(x+2)(x^2+5)}-\frac{18}{(x+2)(x^2+5)}$

Note, Example 3 would score M1A0 as it does not have the correct lowest common denominator

M1 There must be a single denominator. Terms must be collected on the numerator.
A factor of $(x+2)$ must be taken out of the numerator and then cancelled with one in the denominator. The cancelling may be assumed if the term 'disappears'

A1* Cso $\frac{2x}{(x^2+5)}$ This is a given solution and this mark should be withheld if there are any errors

(b) M1 Applies the quotient rule to $\frac{2x}{(x^2+5)}$, a form of which appears in the formula book.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{(x^2+5)\times A-2x\times Bx}{(x^2+5)^2} \quad \text{where } A, B>0$$

A1 Correct unsimplified answer $h'(x)=\frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$

A1 $h'(x)=\frac{10-2x^2}{(x^2+5)^2}$ The correct simplified answer. Accept $\frac{2(5-x^2)}{(x^2+5)^2}$ $\frac{-2(x^2-5)}{(x^2+5)^2}$, $\frac{10-2x^2}{(x^4+10x^2+25)}$

DO NOT ISW FOR PART (b). INCORRECT SIMPLIFICATION IS A0

(c) M1 Sets their h'(x)=0 and proceeds with a correct method to find x. There must have been an attempt to differentiate. Allow numerical errors but do not allow solutions from 'unsolvable' equations.

A1 Finds the correct x value of the maximum point $x=\sqrt{5}$.
Ignore the solution $x=-\sqrt{5}$ but withhold this mark if other positive values found.

M1 Substitutes their answer into their h'(x)=0 in h(x) to determine the maximum value

A1 Cso-the maximum value of h(x) = $\frac{\sqrt{5}}{5}$. Accept equivalents such as $\frac{2\sqrt{5}}{10}$ **but not** 0.447

A1ft Range of h(x) is $0 \le h(x) \le \frac{\sqrt{5}}{5}$. Follow through on their maximum value if the M's have been scored. Allow $0 \le y \le \frac{\sqrt{5}}{5}$, $0 \le Range \le \frac{\sqrt{5}}{5}$, $\left[0, \frac{\sqrt{5}}{5}\right]$ but not $0 \le x \le \frac{\sqrt{5}}{5}$, $\left(0, \frac{\sqrt{5}}{5}\right)$

If a candidate attempts to work out $h^{-1}(x)$ in (b) and does all that is required for (b) in (c), then allow. Do not allow $h^{-1}(x)$ to be used for h'(x) in part (c). For this question (b) and (c) can be scored together.

Alternative to (b) using the product rule

M1 Sets $h(x)=2x(x^2+5)^{-1}$ and applies the product rule vu'+uv' with terms being $2x$ and $(x^2+5)^{-1}$
If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$(x^2+5)^{-1}\times A+2x\times \pm Bx(x^2+5)^{-2}$$

A1 Correct un simplified answer $(x^2+5)^{-1}\times 2+2x\times -2x(x^2+5)^{-2}$

A1 The question asks for h'(x) to be put in its simplest form. Hence in this method the terms need to be combined to form a single correct expression.

For a correct simplified answer accept
$h'(x)=\frac{10-2x^2}{(x^2+5)^2}=\frac{2(5-x^2)}{(x^2+5)^2}=\frac{-2(x^2-5)}{(x^2+5)^2}=(10-2x^2)(x^2+5)^{-2}$

Q6.

Question	Scheme	Marks
(a)	$x^2+x-6\overline{)x^4+x^3-3x^2+7x-6}$ with quotient x^2+3	
	$\underline{x^4+x^3-6x^2}$	
	$3x^2+7x-6$	M1 A1
	$\underline{3x^2+3x-18}$	
	$4x+12$	
	$\dfrac{x^4+x^3-3x^2+7x-6}{x^2+x-6} \equiv x^2+3+\dfrac{4(x+3)}{(x+3)(x-2)}$	M1
	$\equiv x^2+3+\dfrac{4}{(x-2)}$	A1
		(4)
(b)	$\mathrm{f}'(x)=2x-\dfrac{4}{(x-2)^2}$	M1A1ft
	Subs $x=3$ into $\mathrm{f}'(x=3)=2\times 3-\dfrac{4}{(3-2)^2}=(2)$	M1
	Uses $m=-\dfrac{1}{\mathrm{f}'(3)}=\left(-\dfrac{1}{2}\right)$ with $(3,\mathrm{f}(3))=(3,16)$ to form eqn of normal	
	$y-16=-\dfrac{1}{2}(x-3)$ or equivalent cso	M1A1
		(5)
		(9 marks)

(a)

M1 Divides $x^4+x^3-3x^2+7x-6$ by x^2+x-6 to get a quadratic quotient and a linear or constant remainder. To award this look for a minimum of the following

$$x^2+x-6\overline{)x^4+x^3-3x^2+7x-6} \quad \text{quotient } x^2(+..x)+A$$

$$\underline{x^4+x^3-6x^2}$$

$$\overline{}(Cx)+D$$

If they divide by $(x+3)$ first they must then divide their by result by $(x-2)$ before they score this method mark. Look for a cubic quotient with a constant remainder followed by a quadratic quotient and a constant remainder

Note: FYI Dividing by $(x+3)$ gives x^3-2x^2+3x-2 and $\left(x^3-2x^2+3x-2\right)\div(x-2)=x^2+3$ with a remainder of 4.

Division by $(x-2)$ first is possible but difficult.....please send to review any you feel deserves credit.

A1 Quotient $=x^2+3$ and Remainder $=4x+12$

M1 Factorises x^2+x-6 and writes their expression in the appropriate form.

$$\left(\frac{x^4+x^3-3x^2+7x-6}{x^2+x-6}\right)\equiv \text{Their Quadratic Quotient}+\frac{\text{Their Linear Remainder}}{(x+3)(x-2)}$$

It is possible to do this part by partial fractions. To score M1 under this method the terms must be correct and it must be a full method to find both "numerators"

A1 $x^2+3+\frac{4}{(x-2)}$ or $A=3, B=4$ but don't penalise after a correct statement.

(b)

M1 $x^2+A+\frac{B}{x-2}\to 2x\pm\frac{B}{(x-2)^2}$

If they fail in part (a) to get a function in the form $x^2+A+\frac{B}{x-2}$ allow candidates to pick up this method mark for differentiating a function of the form $x^2+Px+Q+\frac{Rx+S}{x\pm T}$ using the quotient rule oe.

A1ft $x^2+A+\frac{B}{x-2}\to 2x-\frac{B}{(x-2)^2}$ oe. FT on their numerical A, B for for $x^2+A+\frac{B}{x-2}$ only

M1 Subs $x=3$ into their $f'(x)$ in an attempt to find a numerical gradient

M1 For the correct method of finding an equation of a normal. The gradient must be $-\frac{1}{\text{their } f'(3)}$ and the point must be $(3, f(3))$. Don't be overly concerned about how they found their f(3), ie accept x=3 y =.

Look for $y-f(3)=-\frac{1}{f'(3)}(x-3)$ or $(y-f(3))\times -f'(3)=(x-3)$

If the form $y=mx+c$ is used they must proceed as far as $c=$

A1 cso $y-16=-\frac{1}{2}(x-3)$ oe such as $2y+x-35=0$ but remember to isw after a correct answer.

Alt (a) attempted by equating terms.

Alt (a)	$x^4+x^3-3x^2+7x-6\equiv(x^2+A)(x^2+x-6)+B(x+3)$ Compare 2 terms (or substitute 2 values) AND solve simultaneously ie $x^2\Rightarrow A-6=-3,\quad x\Rightarrow A+B=7,\quad \text{const}\Rightarrow -6A+3B=-6$ $A=3, B=4$	M1 M1 A1,A1

1st Mark M1 Scored for multiplying by (x^2+x-6) and cancelling/dividing to achieve $x^4+x^3-3x^2+7x-6\equiv(x^2+A)(x^2+x-6)+B(x\pm3)$

3rd Mark M1 Scored for comparing two terms (or for substituting two values) **AND** solving simultaneously to get values of A and B.

2nd Mark A1 Either $A=3$ or $B=4$. One value may be correct by substitution of say $x=-3$

4th Mark A1 Both $A=3$ and $B=4$

Alt (b) is attempted by the quotient (or product rule)

ALT (b) 1st 3 marks	$f'(x)=\frac{(x^2+x-6)(4x^3+3x^2-6x+7)-(x^4+x^3-3x^2+7x-6)(2x+1)}{(x^2+x-6)^2}$ Subs $x=3$ into	M1A1 M1

M1 Attempt to use the **quotient rule** $\frac{vu'-uv'}{v^2}$ with $u=x^4+x^3-3x^2+7x-6$ and $v=x^2+x-6$ and achieves an expression of the form $f'(x)=\frac{(x^2+x-6)(..x^3........)-(x^4+x^3-3x^2+7x-6)(..x..)}{(x^2+x-6)^2}$.

Use a similar approach to the product rule with $u=x^4+x^3-3x^2+7x-6$ and $v=(x^2+x-6)^{-1}$

Note that this can score full marks from a partially solved part (a) where $f(x)\equiv x^2+3+\frac{4x+12}{x^2+x-6}$

Q7.

Question Number	Scheme	Marks
(a)	$p = 4\pi^2$ or $(2\pi)^2$	B1
		(1)
(b)	$x=(4y-\sin 2y)^2 \Rightarrow \frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	M1A1
	Sub $y=\frac{\pi}{2}$ into $\frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	
	$\Rightarrow \frac{dx}{dy}=24\pi \quad (=75.4)$ / $\frac{dy}{dx}=\frac{1}{24\pi}(=0.013)$	M1
	Equation of tangent $y-\frac{\pi}{2}=\frac{1}{24\pi}(x-4\pi^2)$	M1
	Using $y-\frac{\pi}{2}=\frac{1}{24\pi}(x-4\pi^2)$ with $x=0 \Rightarrow y=\frac{\pi}{3}$ cso	M1, A1
		(6)
		(7 marks)
Alt (b) I	$x=(4y-\sin 2y)^2 \Rightarrow x^{0.5}=4y-\sin 2y$ $\Rightarrow 0.5x^{-0.5}\frac{dx}{dy}=4-2\cos 2y$	M1A1
Alt (b) II	$x=(16y^2-8y\sin 2y+\sin^2 2y)$ $\Rightarrow 1=32y\frac{dy}{dx}-8\sin 2y\frac{dy}{dx}-16y\cos 2y\frac{dy}{dx}+4\sin 2y\cos 2y\frac{dy}{dx}$ Or $1dx=32y\,dy-8\sin 2y\,dy-16y\cos 2y\,dy+4\sin 2y\cos 2y\,dy$	M1A1

(a)

B1 $p=4\pi^2$ or exact equivalent $(2\pi)^2$

Also allow $x=4\pi^2$

(b)

M1 Uses the chain rule of differentiation to get a form $A(4y-\sin 2y)(B \pm C\cos 2y)$, $A, B, C \neq 0$ on the right hand side

Alternatively attempts to expand and then differentiate using product rule and chain rule to a form $x=\left(16y^2-8y\sin 2y+\sin^2 2y\right) \Rightarrow \frac{dx}{dy}=Py \pm Q\sin 2y \pm Ry\cos 2y \pm S\sin 2y\cos 2y \quad P, Q, R, S \neq 0$

A second method is to take the square root first. To score the method look for a differentiated expression of the form $Px^{-0.5}... = 4-Q\cos 2y$

A third method is to multiply out and use implicit differentiation. Look for the correct terms, condoning errors on just the constants.

A1 $\frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$ or $\frac{dy}{dx}=\frac{1}{2(4y-\sin 2y)(4-2\cos 2y)}$ with both sides correct. The lhs may be seen elsewhere if clearly linked to the rhs.

In the alternative $\frac{dx}{dy}=32y-8\sin 2y-16y\cos 2y+4\sin 2y\cos 2y$

M1 Sub $y=\frac{\pi}{2}$ into their $\frac{dx}{dy}$ or inverted $\frac{dx}{dy}$. Evidence could be minimal, eg $y=\frac{\pi}{2} \Rightarrow \frac{dx}{dy}=...$

It is not dependent upon the previous M1 but it must be a changed $x=(4y-\sin 2y)^2$

M1 Score for a correct method for finding the equation of the tangent at $\left('4\pi^2', \frac{\pi}{2}\right)$.

Allow for $y-\frac{\pi}{2}=\frac{1}{\text{their numerical}\left(\frac{dx}{dy}\right)}\left(x-\text{their } 4\pi^2\right)$

Allow for $\left(y-\frac{\pi}{2}\right)\times\text{their numerical}\left(\frac{dx}{dy}\right)=\left(x-\text{their } 4\pi^2\right)$

Even allow for $y-\frac{\pi}{2}=\frac{1}{\text{their numerical}\left(\frac{dx}{dy}\right)}(x-p)$

It is possible to score this by stating the equation $y=\frac{1}{24\pi}x+c$ as long as $\left('4\pi^2', \frac{\pi}{2}\right)$ is used in a subsequent line.

M1 Score for writing their equation in the form $y=mx+c$ and stating the value of 'c'

Or setting $x=0$ in their $y-\frac{\pi}{2}=\frac{1}{24\pi}\left(x-4\pi^2\right)$ and solving for y.

Alternatively using the gradient of the line segment AP = gradient of tangent.

Look for $\frac{\frac{\pi}{2}-y}{4\pi^2}=\frac{1}{24\pi} \Rightarrow y=..$ Such a method scores the previous M mark as well.

At this stage all of the constants must be numerical. It is not dependent and it is possible to score this using the "incorrect" gradient.

A1 cso $y=\frac{\pi}{3}$. You do not have to see $\left(0, \frac{\pi}{3}\right)$

Q8.

Question Number	Scheme	Marks
(i) (a)	$y=2x\left(x^2-1\right)^5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2-1\right)^5\times 2+2x\times 10x\left(x^2-1\right)^4$	M1A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2-1\right)^4\left(2x^2-2+20x^2\right)=\left(x^2-1\right)^4\left(22x^2-2\right)$	M1 A1
		(4)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}\ldots 0 \Rightarrow \left(22x^2-2\right)\ldots 0 \Rightarrow$ critical values of $\pm\frac{1}{\sqrt{11}}$	M1
	$x\ldots\frac{1}{\sqrt{11}}$ $x\,,,\, -\frac{1}{\sqrt{11}}$	A1
		(2)
(ii)	$x=\ln(\sec 2y) \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}=\frac{1}{\sec 2y}\times 2\sec 2y\tan 2y$	B1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\tan 2y}=\frac{1}{2\sqrt{\sec^2 2y-1}}=\frac{1}{2\sqrt{e^{2x}-1}}$	M1 M1 A1
		(4)
		10 marks

Alt 1 (ii)	$x=\ln(\sec 2y) \Rightarrow \sec 2y=e^x$	
	$\Rightarrow 2\sec 2y\tan 2y\frac{\mathrm{d}y}{\mathrm{d}x}=e^x$	B1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{e^x}{2\sec 2y\tan 2y}=\frac{e^x}{2e^x\sqrt{\sec^2 2y-1}}=\frac{1}{2\sqrt{e^{2x}-1}}$	M1M1A1
		(4)
Alt 2 (ii)	$y=\frac{1}{2}\arccos\left(e^{-x}\right) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}\times\frac{1}{\sqrt{1-\left(e^{-x}\right)^2}}\times -e^{-x}$	B1M1M1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{e^{2x}-1}}$	A1
		(4)

(i)(a)

M1 Attempts the product rule to differentiate $2x(x^2-1)^5$ to a form $A\left(x^2-1\right)^5+Bx^n\left(x^2-1\right)^4$ where $n=1$ or 2. and $A, B>0$ If the rule is stated it must be correct, and not with a "$-$" sign.

A1 Any unsimplified but correct form $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=2\left(x^2-1\right)^5+20x^2\left(x^2-1\right)^4$

M1 For taking a common factor of $\left(x^2-1\right)^4$ out of a suitable expression

Look for $A\left(x^2-1\right)^5 \pm Bx^n\left(x^2-1\right)^4=\left(x^2-1\right)^4\left\{A\left(x^2-1\right)\pm Bx^n\right\}$ but you may condone missing brackets

It can be scored from a vu'-uv' or similar.

A1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\left(x^2-1\right)^4\left(22x^2-2\right)$ Expect $\mathrm{g}(x)$ to be simplified but accept $\frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2-1\right)^4 2\left(11x^2-1\right)$

There is no need to state g(x) and remember to isw after a correct answer. This must be in part (a).

(i)(b)

M1 Sets their $\frac{\mathrm{d}y}{\mathrm{d}x} \geqslant 0, >0$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and proceeds to find one of the critical values for **their** $\mathrm{g}(x)$ or their $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ rearranged and $\div\left(x^2-1\right)^4$ if g(x) not found. $\mathrm{g}(x)$ should be at least a 2TQ with real roots. If g(x) is factorised, the usual rules apply. The M cannot be awarded from work **just** on $\left(x^2-1\right)^4 \geqslant 0$ ie $x=\pm 1$

You may see and accept decimals for the M.

A1 cao $x \geqslant \frac{1}{\sqrt{11}}$ $x \leqslant -\frac{1}{\sqrt{11}}$ or exact equivalent only. Condone $x > \frac{1}{\sqrt{11}}$ $x < -\frac{1}{\sqrt{11}}$, with $x \geqslant 1, x \leqslant -1$

Accept exact equivalents such as $x \geqslant \frac{\sqrt{11}}{11}$ $x \leqslant -\frac{\sqrt{11}}{11}$; $|x| \geqslant \frac{1}{\sqrt{11}}$; $\left\{\left(-\infty,-\frac{\sqrt{11}}{11}\right]\cup\left[\frac{\sqrt{11}}{11},\infty\right)\right\}$

Condone the word "and" appearing between the two sets of values.

Withhold the final mark if $x \geqslant \frac{1}{\sqrt{11}}$ $x \leqslant -\frac{1}{\sqrt{11}}$, appears with values not in this region eg $x \leqslant 1, x \geqslant -1$

(ii)

B1 Differentiates and achieves a correct line involving $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$

Accept $\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{1}{\sec 2y}\times 2\sec 2y\tan 2y$, $\frac{\mathrm{d}x}{\mathrm{d}y}=-\frac{1}{\cos 2y}\times -2\sin 2y$ $\quad 2\sec 2y\tan 2y\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^x$

M1 For inverting their expression for $\frac{\mathrm{d}x}{\mathrm{d}y}$ to achieve an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The variables (on the rhs) must be consistent, you may condone slips on the coefficients but not the terms.
In the alternative method it is for correctly changing the subject

M1 Scored for using $\tan^2 2y=\pm 1\pm\sec^2 2y$ **and** $\sec 2y=\mathrm{e}^x$ to achieve $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x

Alternatively they could use $\sin^2 2y+\cos^2 2y=1$ with $\cos 2y=\mathrm{e}^{-x}$ to achieve $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x

For the M mark you may condone $\sec^2 2y=\left(\mathrm{e}^x\right)^2$ appearing as e^{x^2}

A1 cso $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$ Final answer, do not allow if students then simplify this to eg. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\mathrm{e}^x-1}$

Condone $\frac{\mathrm{d}y}{\mathrm{d}x}=\pm\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$ but do not allow $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$

Allow a misread on $x=\ln(\sec y)$ for the two method marks only

Q9.

Question Number	Scheme	Marks
(a)	$x^2-9=(x+3)(x-3)$	B1
	$\frac{4x-5}{(2x+1)(x-3)}-\frac{2x}{(x+3)(x-3)}$	
	$=\frac{(4x-5)(x+3)}{(2x+1)(x-3)(x+3)}-\frac{2x(2x+1)}{(2x+1)(x+3)(x-3)}$	M1
	$=\frac{5x-15}{(2x+1)(x-3)(x+3)}$	M1A1
	$=\frac{5(x-3)}{(2x+1)(x-3)(x+3)}=\frac{5}{(2x+1)(x+3)}$	A1*
		(5)
(b)	$f(x)=\frac{5}{2x^2+7x+3}$	
	$f'(x)=\frac{-5(\mathbf{4x+7})}{(2x^2+7x+3)^2}$	M1**M1**A1
	$f'(-1)=-\frac{15}{4}$	M1A1
	Uses $m_1m_2=-1$ to give gradient of normal$=\frac{4}{15}$	M1
	$\frac{y-(-\frac{5}{2})}{(x--1)}=\textit{their }\frac{4}{15}$	M1
	$y+\frac{5}{2}=\frac{4}{15}(x+1)$ or any equivalent form	A1
		(8)
		13 Marks

Q10

Question	Scheme	Marks	AOs
(a)	$f(x)=\dfrac{2x-3}{x^2+4} \Rightarrow f'(x)=\dfrac{2(x^2+4)-2x(2x-3)}{(x^2+4)^2}$ or $f(x)=(2x-3)(x^2+4)^{-1} \Rightarrow f'(x)=2(x^2+4)^{-1}-2x(2x-3)(x^2+4)^{-2}$	M1 A1	1.1b 1.1b
	$f'(x)=\dfrac{-2x^2+6x+8}{(x^2+4)^2}$	A1	1.1b
		(3)	
(b)	$-2x^2+6x+8=0 \Rightarrow -2(x+1)(x-4)=0 \Rightarrow x=-1,\ 4$	B1ft (M1 on EPEN)	1.1b
	Chooses correct region for their numerator and their critical values $x<-1$ or $x>4$	M1 A1	1.1b 2.2a
		(3)	
			(6 marks)

Notes

(a)

M1: Attempts the quotient rule to obtain an expression of the form $\dfrac{P(x^2+4)-Qx(2x-3)}{(x^2+4)^2}$ $P,Q>0$ condoning bracketing errors/omissions or minor slips (e.g. $2x+3$ or $x+4$).

Condone, e.g. $\{f'(x)=\}\dfrac{2(x^2+4)-2x(2x-3)}{(x^2+4)}$ provided an incorrect formula is not quoted.

May also see the product rule applied to $(2x-3)(x^2+4)^{-1}$ to obtain an expression of the form $\{f'(x)=\}P(x^2+4)^{-1}-Qx(2x-3)(x^2+4)^{-2}$ $P,Q>0$ condoning bracketing errors/omissions or minor slips (e.g. $2x+3$ or $x+4$)

A1: Fully correct differentiation in any form with correct bracketing which may be implied by subsequent working.

A1: $f'(x)=\dfrac{-2x^2+6x+8}{(x^2+4)^2}$ or simplified equivalent, e.g. numerator terms in a different order.

Allow recovery from "invisible" brackets earlier and apply isw once a correct answer is seen. Note that the complete form of the answer is not given so allow candidates to go from e.g.

$f'(x)=\dfrac{2(x^2+4)-2x(2x-3)}{(x^2+4)^2} \rightarrow \dfrac{-2x^2+6x+8}{(x^2+4)^2}$ for the final mark. The denominator $(x^2+4)^2$ may go "missing" on an intermediate line provided it is present in their initial derivative and recovered in the final answer. Allow recovery from incorrect expansion of the denominator.

The $f'(x)=$ must appear at some point but allow e.g. "$\dfrac{dy}{dx}=$"

Note that just e.g. $f'(x)=\dfrac{-2(x^2-3x-4)}{(x^2+4)^2}$ without sight of a correct derivative in the correct form scores A0.

(b) **Note:** it is possible to score B0M1A1 in this question due to the demand to "use algebra".
Note: if their numerator from (a) is not a 3 term quadratic then no marks can be scored in (b).

B1ft: Uses algebra to solve their $ax^2+bx+c\ldots0$ with $a,b,c\neq0$ where … is any equality or inequality, finding the correct, real critical values for their 3TQ.
The … 0 may be implied by their method.
They must show their working for this mark, so expect to see factorisation, substitution into the correct quadratic formula or completing the square.
Correct values for their quadratic do **not** imply this mark.
Approaches via factorisation must have completely correct factorisation, e.g.

$-2x^2+6x+8\{=0\}\Rightarrow -2(x+1)(x-4)\{=0\}\Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\}\Rightarrow (2x+2)(4-x)\{=0\}\Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\}\Rightarrow x^2-3x-4\{=0\}\Rightarrow (x+1)(x-4)\{=0\}\Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\}\Rightarrow (2x+2)(x-4)\{=0\}\Rightarrow x=-1,\ 4$ scores B0ft

$-2x^2+6x+8\{=0\}\Rightarrow (x+1)(x-4)\{=0\}\Rightarrow x=-1,\ 4$ scores B0ft

M1: Selects the "correct" region for their critical values and their a from part (a). Must be x not f(x)
CVs may have been found using a calculator and may be implied if they are correct for their 3TQ. CVs may be incorrect due to errors in their calculations (but not errors in their method).

- For $a<0$ and roots $\alpha<\beta$ they need e.g. $x<\alpha$, $x>\beta$ (or e.g. $x\leqslant\alpha$ or $x\geqslant\beta$)
- For $a>0$ and roots $\alpha<\beta$ they need e.g. $\alpha<x<\beta$ (or e.g. $x\geqslant\alpha,\ x\leqslant\beta$)

Do not be overly concerned about their use of = , > , < in reference to their $-2x^2+6x+8\ldots0$ for this mark or for the A1.
Indicating the region on a sketch is not sufficient. Allow , / or / and / ∪ / ∩ for the M1.
If they have complex roots (or they use the discriminant to find there are no real roots) then they can score this mark for concluding:

- if $a<0$, "all values for x (have f decreasing)" or "f is always decreasing" or $x\in\mathbb{R}$
- if $a>0$, "no values for x (have f decreasing)" or "f is never decreasing"

A1: Correct solution $x<-1$ or $x>4$ (allow $x\leqslant-1$ or $x\geqslant4$) coming from the correct numerator.
Do not isw if they go on to select e.g. $x>4$ or combine incorrectly to $4<x<-1$
Allow full marks to be scored in (b) from an incorrect denominator (but it must be positive for all x), e.g. from $f'(x)=\dfrac{-2x^2+6x+8}{(x+4)^2}$ or $f'(x)=\dfrac{-2x^2+6x+8}{4x^2}$ or $f'(x)=\dfrac{-2x^2+6x+8}{x^2+16}$

Examples: Just "$x<-1$ or $x>4$" stated scores B0M1A1

$-2x^2+6x+8\{=0\}\Rightarrow -2(x+1)(x-4)=0\Rightarrow x<-1, x>4$ scores B1ftM1A1

$-2x^2+6x+8\{=0\}\Rightarrow x^2-3x-4\{=0\}\Rightarrow (x+1)(x-4)=0\Rightarrow x\leqslant-1,\ x\geqslant4$ scores B1ftM1A1

$-2x^2+6x+8\{=0\}\Rightarrow (2x+2)(x-4)\Rightarrow x<-1,\ x>4$ scores B0ftM1A1

$-2x^2+6x+8<0\Rightarrow x^2-3x-4<0\Rightarrow (x+1)(x-4)<0\Rightarrow x<-1, x>4$ scores B1ftM1A1 (as this has correct factorisation shown, the region follows from $a<0$ (M1) and we condone reference to $x^2-3x-4<0$ as part of their working to find critical values (A1).)

Acceptable notation: allow a ",", "or", "and" or "∪" to link the two regions, which may also be in set notation. e.g. $x<-1$ or $x>4$; $x\leqslant-1$, $x\geqslant4$; $x<-1$ and $x>4$
$x\leqslant-1\cup x\geqslant4$; $\{x: x<-1\cup x>4\}$; $\{x\in\mathbb{R}: x\leqslant-1\}\cup\{x\in\mathbb{R}: x\geqslant4\}$;
$x\in(-\infty,-1)\cup(4,\infty)$; $(-\infty,-1]\cup[4,\infty)$ etc.
Do not accept $4<x<-1$ or use of the ∩ symbol e.g. $(-\infty,-1]\cap[4,\infty)$ for the final mark, but they may be condoned for the M1. Note also that $[-\infty,-1]\cup[4,\infty]$ scores A0.

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Q11.

Question Number	Scheme	Notes	Marks
	$x = 3t - 4,\ y = 5 - \frac{6}{t},\quad t > 0$		
(a)	$\frac{dx}{dt} = 3,\quad \frac{dy}{dt} = 6t^{-2}$		
	$\frac{dy}{dx} = \frac{6t^{-2}}{3}\ \left\{= \frac{6}{3t^2} = 2t^{-2} = \frac{2}{t^2}\right\}$	their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ to give $\frac{dy}{dx}$ in terms of t or their $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$ to give $\frac{dy}{dx}$ in terms of t	M1
		$\frac{6t^{-2}}{3}$, simplified or un-simplified, in terms of t. **See note.**	A1 isw
	Award **Special Case 1st M1 if both** $\frac{dx}{dt}$ **and** $\frac{dy}{dt}$ are stated **correctly** and **explicitly**.		[2]
	Note: You can recover the work for part (a) in part (b).		
(a) **Way 2**	$y = 5 - \frac{18}{x+4} \Rightarrow \frac{dy}{dx} = \frac{18}{(x+4)^2} = \frac{18}{(3t)^2}$	Writes $\frac{dy}{dx}$ in the form $\frac{\pm\lambda}{(x+4)^2}$, ***and*** writes $\frac{dy}{dx}$ as a function of t.	M1
		Correct un-simplified or simplified answer, in terms of t. **See note.**	A1 isw
			[2]
(b)	$\left\{t = \frac{1}{2} \Rightarrow\right\} P\left(-\frac{5}{2}, -7\right)$	$x = -\frac{5}{2}, y = -7$ **or** $P\left(-\frac{5}{2}, -7\right)$ seen or implied.	B1
	$\frac{dy}{dx} = \frac{2}{\left(\frac{1}{2}\right)^2}$ **and** either • $y - "-7" = "8"\left(x - "-\frac{5}{2}"\right)$ • $"-7" = ("8")("-\frac{5}{2}") + c$ So, $y = (\text{their } m_T)x + "c"$	**Some** attempt to substitute $t = 0.5$ into their $\frac{dy}{dx}$ which contains t in order to find m_T **and either** applies $y - (\text{their } y_P) = (\text{their } m_T)(x - \text{their } x_P)$ **or** finds c from $(\text{their } y_P) = (\text{their } m_T)(\text{their } x_P) + c$ and uses their numerical c in $y = (\text{their } m_T)x + c$	M1
	T: $y = 8x + 13$	$y = 8x + 13$ or $y = 13 + 8x$	A1 cso
	Note: their x_P, their y_P and their m_T must be numerical values in order to award M1		[3]

(c) **Way 1**	$\left\{t = \frac{x+4}{3} \Rightarrow\right\} y = 5 - \frac{6}{\left(\frac{x+4}{3}\right)}$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	$\Rightarrow y = 5 - \frac{18}{x+4} \Rightarrow y = \frac{5(x+4) - 18}{x+4}$		
	So, $y = \frac{5x+2}{x+4},\ \{x > -4\}$	$y = \frac{5x+2}{x+4}$ (or implied equation)	A1 cso
			[3]
(c) **Way 2**	$\left\{t = \frac{6}{5-y} \Rightarrow\right\} x = \frac{18}{5-y} - 4$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	$\Rightarrow (x+4)(5-y) = 18 \Rightarrow 5x - xy + 20 - 4y = 18$		
	$\{\Rightarrow 5x + 2 = y(x+4)\}$ So, $y = \frac{5x+2}{x+4},\ \{x > -4\}$	$y = \frac{5x+2}{x+4}$ (or implied equation)	A1 cso
			[3]
	Note: Some or all of the work for part (c) can be recovered in part (a) or part (b)		8

Question Number	Scheme	Notes	Marks
(c) **Way 3**	$y = \frac{3at-4a+b}{3t-4+4} = \frac{3at}{3t} - \frac{4a-b}{3t} = a - \frac{4a-b}{3t} \Rightarrow a = 5$	A full method leading to the value of a being found	M1
		$y = a - \frac{4a-b}{3t}$ and $a = 5$	A1
	$\frac{4a-b}{3} = 6 \Rightarrow b = 4(5) - 6(3) = 2$	**Both** $a = 5$ **and** $b = 2$	A1
			[3]

		Question Notes
(a)	**Note**	Condone $\frac{dy}{dx} = \frac{\left(\frac{6}{t^2}\right)}{3}$ for A1
	Note	You can ignore subsequent working following on from a correct expression for $\frac{dy}{dx}$ in terms of t.
(b)	**Note**	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$ or $-\left(\text{their } \frac{dy}{dx}\right)$) is M0.
	Note	**Final A1:** A correct solution is required from a correct $\frac{dy}{dx}$.
	Note	**Final A1:** You can ignore subsequent working following on from a correct solution.
(c)	**Note**	**1st M1:** A full attempt to eliminate t is defined as either • rearranging one of the parametric equations to make t the subject and substituting for t in the other parametric equation (only the RHS of the equation required for M mark) • rearranging both parametric equations to make t the subject and putting the results equal to each other.
	Note	Award M1A1 for $\frac{6}{5-y} = \frac{x+4}{3}$ or equivalent.

Q12.

Question Number	Scheme	Notes	Marks
	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \quad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 1**	$\frac{dx}{dt} = 4\sec^2 t, \ \frac{dy}{dt} = 10\sqrt{3}\cos 2t$ $\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t} \quad \left\{= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t\right\}$	**Either both** x and y are differentiated correctly with respect to t **or** their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **or** applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
		Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\left\{\text{At } P\left(4\sqrt{3}, \frac{15}{2}\right), \ t = \frac{\pi}{3}\right\}$		
	$\frac{dy}{dx} = \frac{10\sqrt{3}\cos\left(\frac{2\pi}{3}\right)}{4\sec^2\left(\frac{\pi}{3}\right)}$	**dependent on the previous M mark** *Some evidence* of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]
(b)	$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$		
	So $x = 4\tan\left(\frac{\pi}{4}\right), \ y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4\tan\left(\frac{\pi}{4}\right)$ **or** $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ **or** $x = 4$ **or** $y = 5\sqrt{3}$ **or** $y = \text{awrt } 8.7$	M1
	Coordinates are $\left(4, 5\sqrt{3}\right)$	$\left(4, 5\sqrt{3}\right)$ **or** $x = 4, \ y = 5\sqrt{3}$	A1
			[2]
			6

		Question Notes
(a)	1st A1	Correct $\frac{dy}{dx}$. E.g. $\frac{10\sqrt{3}\cos 2t}{4\sec^2 t}$ or $\frac{5}{2}\sqrt{3}\cos 2t\cos^2 t$ or $\frac{5}{2}\sqrt{3}\cos^2 t(\cos^2 t - \sin^2 t)$ or any equivalent form.
	Note	Give the final A0 for a final answer of $-\frac{10}{32}\sqrt{3}$ **without reference to** $-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$
	Note	Give the final A0 for more than one value stated for $\frac{dy}{dx}$
(b)	**Note**	Also allow M1 for either $x = 4\tan(45)$ or $y = 5\sqrt{3}\sin\left(2(45)\right)$
	Note	M1 can be gained by ignoring previous working in part (a) and/or part (b)
	Note	Give A0 for stating more than one set of coordinates for Q.
	Note	Writing $x = 4, \ y = 5\sqrt{3}$ followed by $\left(5\sqrt{3}, 4\right)$ is A0.

Question Number	Scheme	Notes	Marks
	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \quad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 2**	$\tan t = \frac{x}{4} \Rightarrow \sin t = \frac{x}{\sqrt{(x^2+16)}},\ \cos t = \frac{4}{\sqrt{(x^2+16)}} \Rightarrow y = \frac{40\sqrt{3}x}{x^2+16}$		
	$\left\{\begin{matrix} u = 40\sqrt{3}x & v = x^2+16 \\ \frac{du}{dx} = 40\sqrt{3} & \frac{dv}{dx} = 2x \end{matrix}\right\}$		
	$\frac{dy}{dx} = \frac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2}\ \left\{= \frac{40\sqrt{3}(16-x^2)}{(x^2+16)^2}\right\}$	$\frac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified	A1
	$\frac{dy}{dx} = \frac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	**dependent on the previous M mark** *Some evidence* of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]
(a) **Way 3**	$y = 5\sqrt{3}\sin\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)$		
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{2}{1+\left(\frac{x}{4}\right)^2}\right)\left(\frac{1}{4}\right)$	$\frac{dy}{dx} = \pm A\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{1}{1+x^2}\right)$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified.	A1
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}(\sqrt{3})\right)\left(\frac{2}{1+3}\right)\left(\frac{1}{4}\right)\ \left\{= 5\sqrt{3}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\right\}$	**dependent on the previous M mark** *Some evidence* of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]

Q13.

Question	Scheme	Marks	AOs
(a)	Attempts $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{3}\sin 2t}{\sin t} \quad (= 2\sqrt{3}\cos t)$	A1	1.1b
		(2)	
(b)	Substitutes $t = \frac{2\pi}{3}$ in $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{3}\sin 2t}{\sin t} = (-\sqrt{3})$	M1	2.1
	Uses gradient of normal $= -\frac{1}{\mathrm{d}y/\mathrm{d}x} = \left(\frac{1}{\sqrt{3}}\right)$	M1	2.1
	Coordinates of $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$	B1	1.1b
	Correct form of normal $y + \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x+1)$	M1	2.1
	Completes proof $\Rightarrow 2x - 2\sqrt{3}y - 1 = 0$ *	A1*	1.1b
		(5)	
(c)	Substitutes $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$	M1	3.1a
	Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic in $\cos t$	M1	3.1a
	$\Rightarrow 12\cos^2 t - 4\cos t - 5 = 0$	A1	1.1b
	Finds $\cos t = \frac{5}{6}$, ~~$\frac{1}{2}$~~	M1	2.4
	Substitutes their $\cos t = \frac{5}{6}$ into $x = 2\cos t,\ y = \sqrt{3}\cos 2t$,	M1	1.1b
	$Q = \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$	A1	1.1b
		(6)	
			(13 marks)

Notes:

(a)

M1: Attempts $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ and achieves a form $k\frac{\sin 2t}{\sin t}$ Alternatively candidates may apply the double angle identity for $\cos 2t$ and achieve a form $k\frac{\sin t \cos t}{\sin t}$

A1: Scored for a correct answer, either $\frac{\sqrt{3}\sin 2t}{\sin t}$ or $2\sqrt{3}\cos t$

(b)

M1: For substituting $t = \frac{2\pi}{3}$ in their $\frac{dy}{dx}$ which must be in terms of t

M1: Uses the gradient of the normal is the negative reciprocal of the value of $\frac{dy}{dx}$. This may be seen in the equation of l.

B1: States or uses (in their tangent or normal) that $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$

M1: Uses their numerical value of $-1/\frac{dy}{dx}$ with their $\left(-1, -\frac{\sqrt{3}}{2}\right)$ to form an equation of the normal at P

A1*: This is a proof and all aspects need to be correct. Correct answer only $2x - 2\sqrt{3}y - 1 = 0$

(c)

M1: For substituting $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$ to produce an equation in t. Alternatively candidates could use $\cos 2t = 2\cos^2 t - 1$ to set up an equation of the form $y = Ax^2 + B$.

M1: Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic equation in $\cos t$
In the alternative method it is for combining their $y = Ax^2 + B$ with $2x - 2\sqrt{3}y - 1 = 0$ to get an equation in just one variable

A1: For the correct quadratic equation $12\cos^2 t - 4\cos t - 5 = 0$
Alternatively the equations in x and y are $3x^2 - 2x - 5 = 0$ $12\sqrt{3}y^2 + 4y - 7\sqrt{3} = 0$

M1: Solves the quadratic equation in $\cos t$ (or x or y) and rejects the value corresponding to P.

M1: Substitutes their $\cos t = \frac{5}{6}$ or their $t = \arccos\left(\frac{5}{6}\right)$ in $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$
If a value of x or y has been found it is for finding the other coordinate.

A1: $Q = \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$. Allow $x = \frac{5}{3}, y = \frac{7}{18}\sqrt{3}$ but do not allow decimal equivalents.

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Q14.

Question Number	Scheme	Marks
(a)	Either: (Cost of polishing top and bottom (two circles) is) $3\times 2\pi r^2$ **or** (Cost of polishing curved surface area is) $2\times 2\pi rh$ or both - just need to see at least one of these products	B1
	Uses volume to give $(h=)\dfrac{75\pi}{\pi r^2}$ or $(h=)\dfrac{75}{r^2}$ (simplified) (if V is misread – see below)	B1ft
	$(C)=6\pi r^2+4\pi r\left(\dfrac{75}{r^2}\right)$ — Substitutes expression for h into area or cost expression of form Ar^2+Brh	M1
	$C=6\pi r^2+\dfrac{300\pi}{r}$ *	A1* (4)
(b)	$\left\{\dfrac{dC}{dr}=\right\}\ 12\pi r-\dfrac{300\pi}{r^2}$ or $12\pi r-300\pi r^{-2}$ (then isw)	M1 A1 ft
	$12\pi r-\dfrac{300\pi}{r^2}=0$ so $r^k=$ value where $k=\pm2,\pm3,\pm4$	dM1
	Use **cube** root to obtain $r=\left(their\ \frac{300}{12}\right)^{\frac{1}{3}}$ $(=2.92)$ - allow $r=3$, and thus $C=$	ddM1
	Then $C=$ awrt 483 or 484	A1cao (5)
(c)	$\left\{\dfrac{d^2C}{dr^2}=\right\}\ 12\pi+\dfrac{600\pi}{r^3}>0$ **so minimum**	B1ft (1)
		[10]

Notes

(a) **B1:** States $3\times 2\pi r^2$ or states $2\times 2\pi rh$

B1ft: Obtains a **correct** expression for h in terms of r (ft only follows misread of V)

M1: Substitutes their expression for h into **area or cost** expression of form Ar^2+Brh

A1*: Had correct expression for C and achieves **given** answer in part (a) including "$C=$" or "Cost=" and **no errors seen** such as $C=$ area expression without multiples of (£)3 and (£)2 at any point. Cost and area must be perfectly distinguished at all stages for this A mark.

N.B. Candidates using Curved Surface Area $=\dfrac{2V}{r}$ - please send to review

(b) **M1:** Attempts to differentiate as evidenced by at least one term differentiated correctly

A1ft: Correct derivative – allow $12\pi r-300\pi r^{-2}$ then isw if the power is misinterpreted (ft only for misread)

dM1: Sets their $\dfrac{dC}{dr}$ to 0, and obtains $r^k=$ value where $k=2$, 3 or 4 (needs correct collection of powers of r from their original derivative expression – allow errors dividing by 12π)

ddM1: Uses **cube** root to find r **or** see $r=$ awrt 3 as evidence of cube root and substitutes into correct expression for C to obtain value for C

A1: Accept awrt 483 or 484

(c) **B1ft: Finds** correct expression for $\dfrac{d^2C}{dr^2}$ and deduces value of $\dfrac{d^2C}{dr^2}>0$ so minimum (r may have been wrong)

OR checks gradient to left and right of 2.92 and shows gradient goes from negative to zero to positive so minimum

OR checks value of C to left and right of 2.92 and shows that $C>483$ so deduces minimum (i.e. uses shape of graph) Only ft on misread of V for each ft mark (see below)

N..B. Some candidates have **misread** the volume as 75 instead of 75π. PTO for marking instruction.

Following this misread candidates cannot legitimately obtain the printed answer in part (a). Either they obtain $C = 6\pi r^2 + \frac{300}{r}$ or they "fudge" their working to appear to give the printed answer.

The policy for a misread is **to subtract 2 marks from A or B marks**. In this case the A mark is to be subtracted from part (a) and the final A mark is to be subtracted from part (b)
The maximum mark for part (a) following this misread is 3 marks. The award is B1 B1 M1 A0 as a maximum.

(a) B1: as before

B1: Uses volume to give $(h =)\frac{75}{\pi r^2}$

M1: $(C) = 6\pi r^2 + 4\pi r\left(\frac{75}{\pi r^2}\right)$

A0: Printed answer is not obtained without error

Most Candidates may then adopt the printed answer and gain up to full marks for the rest of the question so 9 of the 10 marks maximum in all.

Any candidate who proceeds with **their** answer $C = 6\pi r^2 + \frac{300}{r}$ may be awarded up to 4 marks in part (b). These are M1A1dM1ddM1A0 and then the candidate may also be awarded the B1 mark in part (c). So 8 of the 10 marks maximum in all.

(b) M1 A1: $\left\{\frac{dC}{dr} =\right\} 12\pi r - \frac{300}{r^2}$ or $12\pi r - 300r^{-2}$ (then isw)

dM1: $12\pi r - \frac{300}{r^2} = 0$ so r^k = value where $k = 2, 3$ or 4 or $12\pi r - \frac{300}{r^2} = 0$ so r^k = value

ddM1: Use **cube** root to obtain $r = \left(their\ \frac{300}{12\pi}\right)^{\frac{1}{3}}$ $(=1.996)$ - allow $r = 2$, and thus $C = \ldots$ must use $C = 6\pi r^2 + \frac{300}{r}$

A0: Cannot obtain $C = 483$ or 484

(c) B1: $\left\{\frac{d^2C}{dr^2} =\right\} 12\pi + \frac{600}{r^3} > 0$ so **minimum** OR checks gradient to left and right of 1.966 and shows gradient goes from negative to zero to positive so minimum
OR checks value of C to left and right of 1.966 and shows that $C > 225.4$ so deduces minimum (i.e. uses shape of graph)
There is an example in Practice of this misread.

Q15.

Question Number	Scheme		Marks
(a)	$\frac{1}{2}(9x+6x)4x$ or $2x \times 15x$ or $\left(\frac{1}{2}4x \times (9x-6x) + 6x \times 4x\right)$ or $6x^2 + 24x^2$ or $\left(9x \times 4x - \frac{1}{2}4x \times (9x-6x)\right)$ or $36x^2 - 6x^2$	M1: Correct attempt at the area of a trapezium. Note that $30x^2$ on its own or $30x^2$ from incorrect work e.g. $5x \times 6x$ is M0. If there is a clear intention to find the area of the trapezium correctly allow the M1 but the A1 can be withheld if there are any slips.	M1A1cso
	$\Rightarrow 30x^2 y = 9600 \Rightarrow y = \frac{9600}{30x^2} \Rightarrow y = \frac{320}{x^2}$ *	A1: Correct proof with at least one intermediate step and no errors seen. "$y =$" **is required.**	
			[2]
(b)	$(S =)\frac{1}{2}(9x+6x)4x + \frac{1}{2}(9x+6x)4x + 6xy + 9xy + 5xy + 4xy$		M1A1
	M1: An attempt to find the area of six faces of the prism. The 2 trapezia may be combined as $(9x+6x)4x$ or $60x^2$ and the 4 other faces may be combined as $24xy$ but all six faces must be included. There must be attempt at the areas of two trapezia that are dimensionally correct. A1: Correct expression in any form. Allow just $(S =) 60x^2 + 24xy$ for M1A1		
	$y = \frac{320}{x^2} \Rightarrow (S =) 30x^2 + 30x^2 + 24x\left(\frac{320}{x^2}\right)$		M1
	Substitutes $y = \frac{320}{x^2}$ into their expression for S (may be done earlier). S should have at least one x^2 term and one xy term but there may be other terms which may be dimensionally incorrect.		
	So, $(S =) 60x^2 + \frac{7680}{x}$ *	Correct solution only. "$S =$" is **not** required here.	A1* cso
			[4]

(c)	$\frac{dS}{dx} = 120x - 7680x^{-2}$ $\left\{= 120x - \frac{7680}{x^2}\right\}$	M1: Either $60x^2 \to 120x$ or $\frac{7680}{x} \to \frac{\pm\lambda}{x^2}$	M1
		A1: Correct differentiation (need not be simplified).	A1 aef
	$120x - \frac{7680}{x^2} = 0$ $\Rightarrow x^3 = \frac{7680}{120}; = 64 \Rightarrow x = 4$	M1: $S' = 0$ and "their $x^3 = \pm$value" or "their $x^{-3} = \pm$value" Setting their $\frac{dS}{dx} = 0$ and "candidate's ft *correct* power of x = a value". **The power of x must be consistent with their differentiation.** If inequalities are used this mark cannot be gained until candidate states value of x or S from their x without inequalities. $S' = 0$ can be implied by $120x = \frac{7680}{x^2}$. Some may spot that $x = 4$ gives $S' = 0$ and provided they clearly show $S'(4) = 0$ allow this mark as long as S' is correct. (If S' is incorrect this method is allowed if their derivative is clearly zero for their value of x)	M1A1cso
		A1: $x = 4$ **only** ($x^3 = 64 \Rightarrow x = \pm 4$ scores A0) Note that the value of x is not explicitly required so the use of $x = \sqrt[3]{64}$ to give $S = 2880$ would imply this mark.	
	Note some candidates stop here and do not go on to find S – maximum mark is 4/6		
	$\{x = 4,\}$ $S = 60(4)^2 + \frac{7680}{4} = 2880$ (cm^2)	Substitute candidate's value of $x\ (\neq 0)$ into a formula for S. **Dependent on both previous M marks.**	ddM1
		2880 cso **(Must come from correct work)**	A1 cao and cso
			[6]
(d)	$\frac{d^2S}{dx^2} = 120 + \frac{15360}{x^3} > 0$ $\Rightarrow$ Minimum	M1: Attempt S'' $(x^n \to x^{n-1})$ and considers sign. This mark requires an attempt at the second derivative **and some consideration of its sign.** There does not necessarily need to be any substitution. An attempt to solve $S'' = 0$ is M0	M1A1ft
		A1: $120 + \frac{15360}{x^3}$ and > 0 and conclusion. Requires a <u>correct</u> second derivative of $120 + \frac{15360}{x^3}$ (need not be simplified) <u>and</u> a valid reason (e.g. > 0), <u>and</u> conclusion. Only follow through a correct second derivative i.e. x may be incorrect **but must be positive** and/or S'' may have been <u>evaluated</u> incorrectly.	
	A correct S'' followed by $S''("4") = "360"$ therefore minimum would score no marks in (d) **A correct S'' followed by $S''("4") = "360"$ which is positive therefore minimum would score both marks**		
			[2]
	Note parts (c) and (d) can be marked together.		
			Total 14

Q16.

Question number	Scheme	Marks	
(a)	$(h=)\dfrac{60}{\pi x^2}$ or **equivalent exact** (not decimal) expression e.g. $(h=)60\div\pi x^2$	B1	(1)
(b)	$(A=)2\pi x^2+2\pi xh$ or $(A=)2\pi r^2+2\pi rh$ or $(A=)2\pi r^2+\pi dh$ may not be simplified and may appear on separate lines	B1	
	Either $(A)=2\pi x^2+2\pi x\left(\dfrac{60}{\pi x^2}\right)$ or As $\pi xh=\dfrac{60}{x}$ then $(A=)2\pi x^2+2\left(\dfrac{60}{x}\right)$	M1	
	$A=2\pi x^2+\left(\dfrac{120}{x}\right)$ *	A1 cso	(3)
(c)	$(\dfrac{dA}{dx})=4\pi x-\dfrac{120}{x^2}$ or $=4\pi x-120x^{-2}$	M1 A1	
	$4\pi x-\dfrac{120}{x^2}=0$ implies $x^3=$ (Use of >0 or <0 is M0 then M0A0)	M1	
	$x=\sqrt[3]{\dfrac{120}{4\pi}}$ or answers which round to 2.12 (-2.12 is A0)	dM1 A1	(5)
(d)	$A=2\pi(2.12)^2+\dfrac{120}{2.12}$, $=85$ (only ft $x=2$ or 2.1 – both give 85)	M1, A1	(2)
(e)	**Either** $\dfrac{d^2A}{dx^2}=4\pi+\dfrac{240}{x^3}$ and sign considered (May appear in (c)) — **Or** *(method 2)* considers gradient to left and right of their 2.12 (e.g at 2 and 2.5) **Or** (*method 3*) considers value of A either side	M1	
	which is >0 and therefore minimum (must substitute 2.12 but it is not essential to see a substitution) (may appear in (c)) — Finds numerical values for gradients and observes gradients go from negative to zero to positive so concludes minimum **OR** finds numerical values of A, observing greater than minimum value and draws conclusion	A1	(2)
		13 marks	
Notes	(a) **B1**: This expression must be correct and in part (a) $\dfrac{60}{\pi r^2}$ is B0 (b) **B1**: Accept any equivalent correct form – may be on two or more lines. **M1** : substitute their expression for h in terms of x into Area formula **of the form** kx^2+cxh **A1**: There should have been no errors in part (b) in obtaining this printed answer (c) **M1**: At least one power of x decreased by 1 **A1** accept any equivalent correct answer **M1**: Setting $\dfrac{dA}{dx}=0$ and finding a value for x^3 ($x^3=$ may be implied by answer). Allow $\dfrac{dy}{dx}=0$ **dM1**: Using **cube root** to find x **A1** : For any equivalent correct answer (need 3sf or more) Correct answer implies previous M mark (d) **M1** : Substitute the (+ve) x value found in (c) into equation for A and evaluate . **A1** is for **85 only** (e) **M1**: Complete method, usually one of the three listed in the scheme. For first method $A''(x)$ must be attempted and sign considered **A1**: Clear statements and conclusion. (numerical substitution of x is not necessary in first method shown, and x or calculation could be wrong but $A''(x)$ **must be correct** . Must **not** see 85 substituted)		

Q17.

Question Number	Scheme	Notes	Marks
	$4x^2 - y^3 - 4xy + 2^y = 0$		
(a) Way 1	$\left\{\frac{dy}{dx} \times\right\} 8x - 3y^2\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} + 2^y \ln 2\frac{dy}{dx} = 0$		M1 A1 M1 B1
	$8(-2) - 3(4)^2\frac{dy}{dx} - 4(4) - 4(-2)\frac{dy}{dx} + 2^4 \ln 2\frac{dy}{dx} = 0$	**dependent on the first M mark**	dM1
	$-16 - 48\frac{dy}{dx} - 16 + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} = 0$		
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$	or exact equivalent	A1 cso
	NOTE: You can recover work for part (a) in part (b)		**[6]**
(b)	e.g. $m_N = \frac{-40+16\ln 2}{-32}$ or $\frac{40-16\ln 2}{32}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N **Can be implied by later working**	M1
	• $y - 4 = \left(\frac{40-16\ln 2}{32}\right)(x - -2)$ Cuts y-axis $\Rightarrow x = 0 \Rightarrow y - 4 = \left(\frac{40-16\ln 2}{32}\right)(2)$ • $4 = \left(\frac{40-16\ln 2}{32}\right)(-2) + c$	Using a numerical m_N ($\neq m_T$), either $y - 4 = m_N(x - -2)$ **and** sets $x = 0$ in their normal equation **or** $4 = (\text{their } m_N)(-2) + c$	M1
	$\left\{\Rightarrow c = 4 + \frac{40-16\ln 2}{16}, \text{ so } y = \frac{104-16\ln 2}{16} \Rightarrow\right\}$		
	y (or c) $= \frac{13}{2} - \ln 2$	$\frac{104}{16} - \ln 2$ **or** $\frac{13}{2} - \ln 2$ **or** $-\ln 2 + \frac{13}{2}$	A1 cso isw
	Note: Allow exact equivalents in the form $p - \ln 2$ for the final A mark		**[3]**
			9

(a) Way 2	$\left\{\frac{dx}{dy} \times\right\} 8x\frac{dx}{dy} - 3y^2 - 4y\frac{dx}{dy} - 4x + 2^y \ln 2 = 0$		M1 A1 M1 B1
	$8(-2)\frac{dx}{dy} - 3(4)^2 - 4(4)\frac{dx}{dy} - 4(-2) + 2^4 \ln 2 = 0$	**dependent on the first M mark**	dM1
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$	or exact equivalent	A1 cso
	Note: You must be clear that Way 2 is being applied before you use this scheme		**[6]**

		Question Notes
(a)	Note	**For the first four marks**
		Writing down ***from no working*** • $\frac{dy}{dx} = \frac{4y - 8x}{-3y^2 - 4x + 2^y \ln 2}$ or $\frac{8x - 4y}{3y^2 + 4x - 2^y \ln 2}$ scores M1A1M1B1 • $\frac{dy}{dx} = \frac{8x - 4y}{-3y^2 - 4x + 2^y \ln 2}$ or $\frac{4y - 8x}{3y^2 + 4x - 2^y \ln 2}$ scores M1A0M1B1
		Writing $8x\,dx - 3y^2\,dy - 4y\,dx - 4x\,dy + 2^y \ln 2\,dy = 0$ scores M1A1M1B1

	Question Notes Continued	
(a)	1st M1	Differentiates implicitly to include *either* $\pm 4x\frac{dy}{dx}$ *or* $-y^3 \to \pm \lambda y^2 \frac{dy}{dx}$ *or* $2^y \to \pm \mu 2^y \frac{dy}{dx}$ (Ignore $\left(\frac{dy}{dx} = \right)$). λ, μ are constants which can be 1
	1st A1	**Both** $4x^2 - y^3 \to 8x - 3y^2\frac{dy}{dx}$ **and** $= 0 \to = 0$
	Note	e.g. $8x - 3y^2\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} + 2^y \ln 2\frac{dy}{dx} \to -3y^2\frac{dy}{dx} - 4x\frac{dy}{dx} + 2^y \ln 2\frac{dy}{dx} = 4y - 8x$ or e.g. $-16 - 48\frac{dy}{dx} - 16 + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} \to -48\frac{dy}{dx} + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} = 32$ will get 1st A1 (implied) as the "$= 0$" can be implied by the rearrangement of their equation.
	2nd M1	$-4xy \to -4y - 4x\frac{dy}{dx}$ or $4y - 4x\frac{dy}{dx}$ or $-4y + 4x\frac{dy}{dx}$ or $4y + 4x\frac{dy}{dx}$
	B1	$2^y \to 2^y \ln 2\frac{dy}{dx}$ or $2^y \to e^{y\ln 2} \ln 2\frac{dy}{dx}$
	Note	If an extra term appears then award 1st A0
	3rd dM1	**dependent on the first M mark** For substituting $x = -2$ and $y = 4$ into an equation involving $\frac{dy}{dx}$
	Note	M1 can be gained by seeing at least one example of substituting $x = -2$ and at least one example of substituting $y = 4$ **unless** it is clear that they are instead applying $x = 4$ and $y = -2$ Otherwise, you will NEED to check (with your calculator) that $x = -2,\ y = 4$ that has been substituted into their equation involving $\frac{dy}{dx}$
	Note	**A1 cso:** If the candidate's solution is not completely correct, then do not give this mark.
	Note	**isw:** You can, however, ignore subsequent working following on from correct solution.
(b)	Note	The 2nd M1 mark can be implied by later working. **Eg. Award 1st M1 and 2nd M1** for $\frac{y-4}{2} = \frac{-1}{\text{their } m_T \text{ evaluated at } x = -2 \text{ and } y = 4}$
	Note	**A1:** Allow the alternative answer $\{y =\} \ln\left(\frac{1}{2}\right) + \frac{13}{2\ln 2}(\ln 2)$ which is in the form $p + q\ln 2$

(a) **Way 2**	1st M1	Differentiates implicitly to include *either* $\pm 4y\frac{dx}{dy}$ *or* $4x^2 \to \pm \lambda x\frac{dx}{dy}$ (Ignore $\left(\frac{dx}{dy} = \right)$). λ is a constant which can be 1
	1st A1	**Both** $4x^2 - y^3 \to 8x\frac{dx}{dy} - 3y^2$ **and** $= 0 \to = 0$
	2nd M1	$-4xy \to -4y\frac{dx}{dy} - 4x$ or $4y\frac{dx}{dy} - 4x$ or $-4y\frac{dx}{dy} + 4x$ or $4y\frac{dx}{dy} + 4x$
	B1	$2^y \to 2^y \ln 2$
	3rd dM1	**dependent on the first M mark** For substituting $x = -2$ and $y = 4$ into an equation involving $\frac{dx}{dy}$

Q18.

Question Number	Scheme	Marks
	$x^2 - 3xy - 4y^2 + 64 = 0$	
(a)	$\left\{\frac{\cancel{dy}}{\cancel{dx}} = \right\}$ $\underline{2x} - \left(\underline{\underline{3y + 3x\frac{dy}{dx}}}\right) \underline{- 8y\frac{dy}{dx}} = \underline{0}$	M1A1 M1
	$2x - 3y + (-3x - 8y)\frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ o.e.	A1 cso
		[5]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}$ $2x - 3y = 0$	M1
	$y = \frac{2}{3}x$ | $x = \frac{3}{2}y$	A1ft
	$x^2 - 3x\left(\frac{2}{3}x\right) - 4\left(\frac{2}{3}x\right)^2 + 64 = 0$ | $\left(\frac{3}{2}y\right)^2 - 3\left(\frac{3}{2}y\right)y - 4y^2 + 64 = 0$	dM1
	$x^2 - 2x^2 - \frac{16}{9}x^2 + 64 = 0 \Rightarrow -\frac{25}{9}x^2 + 64 = 0$ | $\frac{9}{4}y^2 - \frac{9}{2}y^2 - 4y^2 + 64 = 0 \Rightarrow -\frac{25}{4}y^2 + 64 = 0$	
	$\left\{\Rightarrow x^2 = \frac{576}{25} \Rightarrow\right\}$ $x = \frac{24}{5}$ or $-\frac{24}{5}$ | $\left\{\Rightarrow y^2 = \frac{256}{25} \Rightarrow\right\}$ $y = \frac{16}{5}$ or $-\frac{16}{5}$	A1 cso
	When $x = \pm\frac{24}{5}$, $y = \frac{2}{3}\left(\frac{24}{5}\right)$ and $-\frac{2}{3}\left(\frac{24}{5}\right)$ | When $y = \pm\frac{16}{5}$, $x = \frac{3}{2}\left(\frac{16}{5}\right)$ and $-\frac{3}{2}\left(\frac{16}{5}\right)$	
	$\left(\frac{24}{5}, \frac{16}{5}\right)$ and $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ or $x = \frac{24}{5}$, $y = \frac{16}{5}$ and $x = -\frac{24}{5}$, $y = -\frac{16}{5}$ cso	ddM1 A1
		[6] 11
	Alternative method for part (a)	
(a)	$\left\{\frac{\cancel{dx}}{\cancel{dy}} = \right\}$ $\underline{2x\frac{dx}{dy}} - \left(\underline{\underline{3y\frac{dx}{dy} + 3x}}\right) \underline{- 8y} = \underline{0}$	M1A1 M1
	$(2x - 3y)\frac{dx}{dy} - 3x - 8y = 0$	dM1
	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ o.e.	A1 cso
		[5]

		Question Notes
(a) General	Note	Writing down $\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ from no working is full marks
	Note	Writing down $\frac{dy}{dx} = \frac{2x - 3y}{-3x - 8y}$ or $\frac{3y - 2x}{3x + 8y}$ from no working is M1A0B1M1A0
	Note	Few candidates will write $2x\,dx - 3y\,dx - 3x\,dy - 8y\,dy = 0$ leading to $\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$, o.e. This should get full marks.

(a)	M1	Differentiates implicitly to include either $\pm 3x\frac{dy}{dx}$ or $-4y^2 \to \pm ky\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).
	A1	**Both** $x^2 \to \underline{2x}$ **and** $... - 4y^2 + 64 = 0 \to \underline{-8y\frac{dy}{dx} = 0}$
	Note	If an extra term appears then award A0.
	M1	$-3xy \to -3x\frac{dy}{dx} - 3y$ or $-3x\frac{dy}{dx} + 3y$ or $3x\frac{dy}{dx} - 3y$ or $3x\frac{dy}{dx} + 3y$
	Note	$2x - 3y - 3x\frac{dy}{dx} - 8y\frac{dy}{dx} \to 2x - 3y = 3x\frac{dy}{dx} + 8y\frac{dy}{dx}$ will get 1st A1 (implied) as the $"=0"$ can be implied by the rearrangement of their equation.
	dM1	**dependent on the FIRST method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are *at least two terms* in $\frac{dy}{dx}$. i.e. $... + (-3x - 8y)\frac{dy}{dx} = ...$ or $... = (3x + 8y)\frac{dy}{dx}$. (Allow combining in 1 variable).
	A1	$\frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ or equivalent.
	Note	cso If the candidate's solution is not completely correct, then do not give this mark.
	Note	You cannot recover work for part (a) in part (b).
(b)	M1	Sets their numerator of their $\frac{dy}{dx}$ equal to zero (or the denominator of their $\frac{dx}{dy}$ equal to zero) o.e.
	Note	1st M1 can also be gained by setting $\frac{dy}{dx}$ equal to zero in their "$2x - 3y - 3x\frac{dy}{dx} - 8y\frac{dy}{dx} = 0$"
	Note	**If their numerator involves one variable only then only the 1st M1 mark is possible in part (b).**
	Note	**If their numerator is a constant then no marks are available in part (b)**
	Note	**If their numerator is in the form** $\pm ax^2 \pm by = 0$ **or** $\pm ax \pm by^2 = 0$ **then the first 3 marks are possible in part (b).**
	Note	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y} = 0$ is not sufficient for M1.
	A1ft	Either • Sets $2x - 3y$ to zero and obtains either $y = \frac{2}{3}x$ or $x = \frac{3}{2}y$ • the follow through result of making either y or x the subject from setting their numerator of their $\frac{dy}{dx}$ equal to zero
	dM1	**dependent on the first method mark being awarded.** Substitutes *either* their $y = \frac{2}{3}x$ *or* their $x = \frac{3}{2}y$ into the original equation to give an equation in one variable only.
	A1	Obtains either $x = \frac{24}{5}$ or $-\frac{24}{5}$ or $y = \frac{16}{5}$ or $-\frac{16}{5}$, (or equivalent) *by correct solution only*. i.e. You can allow for example $x = \frac{48}{10}$ or 4.8, etc.
	Note	$x = \sqrt{\frac{576}{25}}$ (not simplified) or $y = \sqrt{\frac{256}{25}}$ (not simplified) is not sufficient for A1.

(b) ctd	ddM1	**dependent on both previous method marks being awarded in this part.** **<u>Method 1</u>** **Either:** • substitutes their x into their $y=\frac{2}{3}x$ **or** substitutes their y into their $x=\frac{3}{2}y$, **or** • substitutes *the other of* their $y=\frac{2}{3}x$ *or* their $x=\frac{3}{2}y$ into the original equation, **and achieves either:** • exactly two sets of two coordinates **or** • exactly two distinct values for x and exactly two distinct values for y. **<u>Method 2</u>** **Either:** • substitutes their first x-value, x_1 into $x^2-3xy-4y^2+64=0$ to obtain one y-value, y_1 and substitutes their second x-value, x_2 into $x^2-3xy-4y^2+64=0$ to obtain 1 y-value y_2 **or** • substitutes their first y-value, y_1 into $x^2-3xy-4y^2+64=0$ to obtain one x-value x_1 and substitutes their second y-value, y_2 into $x^2-3xy-4y^2+64=0$ to obtain one x-value x_2.
	Note	Three or more sets of coordinates given (without identification of two sets of coordinates) is ddM0.
	A1	Both $\left(\frac{24}{5},\frac{16}{5}\right)$ **and** $\left(-\frac{24}{5},-\frac{16}{5}\right)$, **only by cso.** Note that decimal equivalents are fine.
	Note	Also allow $x=\frac{24}{5}$, $y=\frac{16}{5}$ **and** $x=-\frac{24}{5}$, $y=-\frac{16}{5}$ all seen in their working to part (b).
	Note	Allow $x=\pm\frac{24}{5}$, $y=\pm\frac{16}{5}$ for 3rd A1.
	Note	$x=\pm\frac{24}{5}$, $y=\pm\frac{16}{5}$ followed by eg. $\left(\frac{16}{5},\frac{24}{5}\right)$ **and** $\left(-\frac{16}{5},-\frac{24}{5}\right)$ (eg. coordinates stated the wrong way round) is 3rd A0.
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{dy}{dx}$) to gain all 6 marks in part (b).
	Note	Decimal equivalents to fractions are fine in part (b). i.e. $(4.8, 3.2)$ **and** $(-4.8, -3.2)$.
	Note	$\left(\frac{24}{5},\frac{16}{5}\right)$ **and** $\left(-\frac{24}{5},-\frac{16}{5}\right)$ from no working is M0A0M0A0M0A0.
	Note	Candidates could potentially lose the final 2 marks for setting both their numerator and denominator to zero.
	Note	No credit in this part can be gained by only setting the denominator to zero.

Q19.

Question	Scheme	Marks	AOs
(a)	Attempts $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4\sec^2 t \tan t}{2\sec^2 t} (= 2\tan t)$	M1 A1	1.1b 1.1b
	At $t = \frac{\pi}{4}$, $\frac{dy}{dx} = 2, x = 3, y = 7$	M1	2.1
	Attempts equation of normal $y - 7 = -\frac{1}{2}(x-3)$	M1	1.1b
	$y = -\frac{1}{2}x + \frac{17}{2}$ *	A1*	2.1
		(5)	
(b)	Attempts to use $\sec^2 t = 1 + \tan^2 t \Rightarrow \frac{y-3}{2} = 1 + \left(\frac{x-1}{2}\right)^2$	M1	3.1a
	$\Rightarrow y - 3 = 2 + \frac{(x-1)^2}{2} \Rightarrow y = \frac{1}{2}(x-1)^2 + 5$ *	A1*	2.1
		(2)	

	(b) Alternative 1:		
	$y = \frac{1}{2}(x-1)^2 + 5 = \frac{1}{2}(2\tan t + 1 - 1)^2 + 5$ $= \frac{1}{2}4\tan^2 t + 5 = 2\left(\sec^2 t - 1\right) + 5$	M1	3.1a
	$= 2\sec^2 t + 3 = y$*	A1	2.1
	(b) Alternative 2:		
	$x = 2\tan t + 1 \Rightarrow t = \tan^{-1}\left(\frac{x-1}{2}\right) \Rightarrow y = 2\sec^2\left(\tan^{-1}\left(\frac{x-1}{2}\right)\right) + 3$ $\Rightarrow y = 2\left(1 + \tan^2\left(\tan^{-1}\left(\frac{x-1}{2}\right)\right)\right) + 3$	M1	3.1a
	$\Rightarrow y = 2\left(1 + \left(\frac{x-1}{2}\right)^2\right) + 3 = \frac{1}{2}(x-1)^2 + 5$*	A1	2.1
	(b) Alternative 3:		
	$\frac{dy}{dx} = 2\tan t = x - 1 \Rightarrow y = \int (x-1)\,dx = \frac{x^2}{2} - x + c$ $(3,7) \to 7 = \frac{3^2}{2} - 3 + c \Rightarrow c = \frac{11}{2}$	M1	3.1a
	$\frac{x^2}{2} - x + \frac{11}{2} = \frac{1}{2}\left(x^2 - 2x\right) + \frac{11}{2} = \frac{1}{2}(x-1)^2 - \frac{1}{2} + \frac{11}{2} = \frac{1}{2}(x-1)^2 + 5$*	A1	2.1

(c)	**Attempts the lower limit for k:** $\frac{1}{2}(x-1)^2+5=-\frac{1}{2}x+k \Rightarrow x^2-x+(11-2k)=0$ $b^2-4ac=1-4(11-2k)=0 \Rightarrow k=...$	M1	2.1
	$(k=)\frac{43}{8}$	A1	1.1b
	Attempts the upper limit for k: $(x,y)_{t=-\frac{\pi}{4}}: t=-\frac{\pi}{4} \Rightarrow x=2\tan\left(-\frac{\pi}{4}\right)+1=-1,\ y=2\sec^2\left(-\frac{\pi}{4}\right)+3=7$ $(-1,\ 7), y=-\frac{1}{2}x+k \Rightarrow 7=\frac{1}{2}+k \Rightarrow k=...$	M1	2.1
	$(k=)\frac{13}{2}$	A1	1.1b
	$\frac{43}{8}<k\leqslant\frac{13}{2}$	A1	2.2a
		(5)	
			(12 marks)
Notes:			

(a) <u>Must use parametric differentiation to score any marks in this part and not e.g. Cartesian form</u>

M1: For the key step of attempting $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$. There must be some attempt to differentiate both parameters however poor and divide the right way round so using $\frac{dy}{dx}=\frac{y}{x}$ scores M0.

This may be implied by e.g. $\frac{dx}{dt}=2\sec^2 t,\ \frac{dy}{dt}=4\sec^2 t\tan t,\ t=\frac{\pi}{4} \Rightarrow \frac{dx}{dt}=4,\ \frac{dy}{dt}=8 \Rightarrow \frac{dy}{dx}=2$

A1: $\frac{dy}{dx}=\frac{4\sec^2 t\tan t}{2\sec^2 t}$. Correct expression in any form. May be implied as above.

Condone the confusion with variables as long as the intention is clear e.g.

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{4\sec^2 x\tan x}{2\sec^2 x}(=2\tan x)$ and allow subsequent marks if this is interpreted correctly

M1: For attempting to find the values of x, y and the gradient at $t=\frac{\pi}{4}$ **AND** getting at least two correct.

Follow through on their $\frac{dy}{dx}$ so allow for any two of $x=3,\ y=7,\ \frac{dy}{dx}=2\left(\text{or their }\frac{dy}{dx}\text{ at } t=\frac{\pi}{4}\right)$

Note that the $x=3,\ y=7$ may be seen as e.g. (3, 7) on the diagram. There must be a non-trivial $\frac{dy}{dx}$ for this mark e.g. they must have a $\frac{dy}{dx}$ to substitute into.

M1: For a correct attempt at the normal equation using their x and y at $t=\frac{\pi}{4}$ with the negative reciprocal of their $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ having made some attempt at $\frac{dy}{dx}$ and all correctly placed.

For attempts using $y=mx+c$ they must reach as far as a value for c using their x and y at $t=\frac{\pi}{4}$ with the negative reciprocal of their $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ all correctly placed.

A1*: Proceeds with a clear argument to the given answer with no errors.

(b)

M1: Attempts to use $\sec^2 t = 1 + \tan^2 t$ oe to obtain an equation involving y and $(x-1)^2$

E.g. as above or e.g. $y = 2\sec^2 t + 3 = 2\left(1+\tan^2 t\right) + 3 = 2\left(1+\left(\frac{x-1}{2}\right)^2\right) + 3$ for M1 and then

$y = \frac{1}{2}(x-1)^2 + 5$* for A1

A1*: Proceeds with a clear argument to the given answer with no errors

Alternative 1:

M1: Uses the given result, substitutes for x and attempts to use $\sec^2 t = 1 + \tan^2 t$ oe

A1: Proceeds with a clear argument to the y parameter and makes a (minimal) conclusion e.g. "$= y$" QED, hence proven etc.

Alternative 2:

M1: Uses the x parameter to obtain t in terms of arctan, substitutes into y and attempts to use $\sec^2 t = 1 + \tan^2 t$ oe

A1: Proceeds with a clear argument to the given answer with no errors

Alternative 3:

M1: Uses $\frac{dy}{dx}$ from part (a) to express $\frac{dy}{dx}$ in terms of x, integrates and uses (3, 7) to find "c" to reach a Cartesian equation.

A1: Proceeds with a clear argument to the given answer with no errors

Allow the marks for (b) to score anywhere in their solution e.g. if they find the Cartesian equation in part (a)

(c)

M1: A full attempt to find the lower limit for k.

$\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k \Rightarrow x^2 - x + (11-2k) = 0 \Rightarrow b^2 - 4ac = 1 - 4(11-2k) = 0 \Rightarrow k = ...$

Score **M1** for setting $\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k$, rearranging to 3TQ form and attempts $b^2 - 4ac...0$ e.g. $b^2 - 4ac > 0$ or e.g. $b^2 - 4ac < 0$ correctly to find a value for k.

A1: $k = \frac{43}{8}$ oe. Look for this value e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

An alternative method using calculus for lower limit:

$$y = \frac{1}{2}(x-1)^2 + 5 \Rightarrow \frac{dy}{dx} = x - 1,\ x - 1 = -\frac{1}{2} \Rightarrow x = \frac{1}{2}$$

$$x = \frac{1}{2} \Rightarrow y = \frac{1}{2}\left(\frac{1}{2} - 1\right)^2 + 5 = \frac{41}{8}$$

$$y = -\frac{1}{2}x + k \Rightarrow \frac{41}{8} = -\frac{1}{4} + k \Rightarrow k = ...$$

Score **M1** for $\frac{dy}{dx} =$ "a linear expression in x", sets $= -\frac{1}{2}$, solves a linear equation to find x and then substitutes into the given result in (b) to find y and then uses $y = -\frac{1}{2}x + k$ to find a value for k. **A1**: $k = \frac{43}{8}$ oe. Look for this value e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

An alternative method using parameters for lower limit:

$$y=-\frac{1}{2}x+k \Rightarrow 2\sec^2 t+3=-\frac{1}{2}(2\tan t+1)+k$$

$$\Rightarrow 2\left(1+\tan^2 t\right)+3=-\frac{1}{2}(2\tan t+1)+k \Rightarrow 2\tan^2 t+\tan t+5.5-k=0$$

$$b^2-4ac=0 \Rightarrow 1-4\times 2(5.5-k)=0 \Rightarrow k=\frac{43}{8}$$

Score **M1** for substituting parametric form of x and y into $y=-\frac{1}{2}x+k$, uses $\sec^2 t=1+\tan^2 t$ rearranges to 3TQ form and attempts $b^2-4ac...0$ or e.g. $b^2-4ac>0$ or $b^2-4ac<0$ correctly to find a value for k.

A1: $k=\frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k>\frac{43}{8}$, $k<\frac{43}{8}$

M1: A full attempt to find the **upper** limit for k. This requires an attempt to find the value of x and the value of y using $t=-\frac{\pi}{4}$, the substitution of these values into $y=-\frac{1}{2}x+k$ and solves for k.

A1: $k=\frac{13}{2}$. Look for this value e.g. may appear in an inequality.

A1: Deduces the correct range for k: $\frac{43}{8}<k\leqslant\frac{13}{2}$

Allow equivalent notation e.g. $\left(k\leqslant\frac{13}{2} \text{ and } k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2}\cap k>\frac{43}{8}\right)$, $\left(\frac{43}{8},\frac{13}{2}\right]$

But not e.g. $\left(k\leqslant\frac{13}{2},\ k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2}\cup k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2} \text{ or } k>\frac{43}{8}\right)$ and do not allow if in terms of x.

Allow equivalent exact values for $\frac{43}{8}$, $\frac{13}{2}$

There may be other methods for finding the upper limit which are valid. If you are in any doubt if a method deserves credit then use Review.

Q20.

Question Number	Scheme		Marks
	Note: You can mark parts (a) and (b) together.		
(a)	$x = 4t + 3,\ \ y = 4t + 8 + \dfrac{5}{2t}$		
	$\dfrac{dx}{dt} = 4,\ \ \dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	Both $\dfrac{dx}{dt} = 4$ or $\dfrac{dt}{dx} = \dfrac{1}{4}$ and $\dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	B1
	So, $\dfrac{dy}{dx} = \dfrac{4 - \frac{5}{2}t^{-2}}{4}\ \left\{= 1 - \dfrac{5}{8}t^{-2} = 1 - \dfrac{5}{8t^2}\right\}$	Candidate's $\dfrac{dy}{dt}$ divided by a candidate's $\dfrac{dx}{dt}$	M1 o.e.
	$\{\text{When } t = 2,\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 cao	A1
			[3]
	Way 2: Cartesian Method		
	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$, simplified or un-simplifed.	B1
		$\dfrac{dy}{dx} = \pm\lambda \pm \dfrac{\mu}{(x-3)^2},\ \ \lambda \neq 0, \mu \neq 0$	M1
	$\{\text{When } t = 2, x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 cao	A1
			[3]
	Way 3: Cartesian Method		
	$\dfrac{dy}{dx} = \dfrac{(2x+2)(x-3) - (x^2+2x-5)}{(x-3)^2}$	Correct expression for $\dfrac{dy}{dx}$, simplified or un-simplified.	B1
	$\left\{= \dfrac{x^2 - 6x - 1}{(x-3)^2}\right\}$	$\dfrac{dy}{dx} = \dfrac{f'(x)(x-3) - 1f(x)}{(x-3)^2}$, where $f(x) = \text{their } "x^2 + ax + b",\ g(x) = x - 3$	M1
	$\{\text{When } t = 2, x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 cao	A1
			[3]
(b)	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\frac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \dfrac{10}{x-3}$		
	$y = \dfrac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ *or* $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ *or* $y = \dfrac{(x+5)(x-3) + 10}{x-3}$ *or* $y = \dfrac{(x+5)(x-3)}{x-3} + \dfrac{10}{x-3}$	See notes	dM1
	$\Rightarrow y = \dfrac{x^2 + 2x - 5}{x-3},\ \ \{a = 2 \text{ and } b = -5\}$	Correct algebra leading to $y = \dfrac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$	A1 cso
			[3] 6

Question Number	Scheme		Marks
(b)	**Alternative Method 1 of Equating Coefficients**		
	$y = \frac{x^2+ax+b}{x-3} \Rightarrow y(x-3) = x^2+ax+b$ $y(x-3) = (4t+3)^2 + 2(4t+3) - 5 = 16t^2+32t+10$ $x^2+ax+b = (4t+3)^2 + a(4t+3) + b$		
	$(4t+3)^2 + a(4t+3) + b = 16t^2+32t+10$	Correct method of obtaining an equation in only t, a and b	M1
	t: $24+4a = 32 \Rightarrow a = 2$ constant: $9+3a+b = 10 \Rightarrow b = -5$	Equates their coefficients in t and finds both $a = ...$ and $b = ...$	dM1
		$a = 2$ and $b = -5$	A1
			[3]
(b)	**Alternative Method 2 of Equating Coefficients**		
	$\left\{t = \frac{x-3}{4} \Rightarrow\right\} y = 4\left(\frac{x-3}{4}\right) + 8 + \frac{5}{2\left(\frac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x-3+8+\frac{10}{x-3} \Rightarrow y = x+5+\frac{10}{(x-3)}$ $y(x-3) = (x+5)(x-3)+10 \Rightarrow x^2+ax+b = (x+5)(x-3)+10$		dM1
	$\Rightarrow y = \frac{x^2+2x-5}{x-3}$ or equating coefficients to give $a=2$ and $b=-5$	Correct algebra leading to $y = \frac{x^2+2x-5}{x-3}$ or $a=2$ and $b=-5$	A1 cso
			[3]

		Question Notes
(a)	B1	$\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$ or $\frac{dy}{dt} = \frac{8t^2-5}{2t^2}$ or $\frac{dy}{dt} = 4 - 5(2t)^{-2}(2)$, etc.
	Note	$\frac{dy}{dt}$ can be simplified or un-simplified.
	Note	**You can imply the B1 mark by later working.**
	M1	Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$ or $\frac{dy}{dt}$ multiplied by a candidate's $\frac{dt}{dx}$
	Note	M1 can be also be obtained by substituting $t = 2$ into both their $\frac{dy}{dt}$ and their $\frac{dx}{dt}$ and then dividing their values the correct way round.
	A1	$\frac{27}{32}$ or 0.84375 **cao**
(b)	M1	Eliminates t to achieve an equation in only x and y.
	dM1	**dependent on the first method mark being awarded.** **Either: (ignoring sign slips or constant slips, noting that k can be 1)** • Combining all three parts of their $x-3+8+\left(\frac{10}{x-3}\right)$ to form a single fraction with a common denominator of $\pm k(x-3)$. Accept three separate fractions with the same denominator. • Combining both parts of their $x+5+\left(\frac{10}{x-3}\right)$, (where $x+5$ is their $4\left(\frac{x-3}{4}\right)+8$), to form a single fraction with a common denominator of $\pm k(x-3)$. Accept two separate fractions with the same denominator. • Multiplies both sides of their $y = x-3+8+\left(\frac{10}{x-3}\right)$ or their $y = x+5+\left(\frac{10}{x-3}\right)$ by $\pm k(x-3)$. Note that all terms in their equation must be multiplied by $\pm k(x-3)$.
	Note	Condone "invisible" brackets for dM1.
	A1	Correct algebra with no incorrect working leading to $y = \frac{x^2+2x-5}{x-3}$ or $a=2$ and $b=-5$
	Note	**Some examples for the award of dM1 in (b):** **dM0** for $y = x-3+8+\frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3)+8+10}{x-3}$. Should be $...+8(x-3)+...$ **dM0** for $y = x-3+\frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3)+10}{x-3}$. The "8" part has been omitted. **dM0** for $y = x+5+\frac{10}{x-3} \rightarrow y = \frac{x(x-3)+5+10}{x-3}$. Should be $...+5(x-3)+...$ **dM0** for $y = x+5+\frac{10}{x-3} \rightarrow y(x-3) = x(x-3)+5(x-3)+10(x-3)$. Should be just 10.
	Note	$y = x+5+\frac{10}{x-3} \rightarrow y = \frac{x^2+2x-5}{x-3}$ with no intermediate working is dM1A1.

Q21.

Question number	Scheme	Marks
(a)	$kr^2 + cxy = 4$ or $kr^2 + c[(x+y)^2 - x^2 - y^2] = 4$	M1
	$\frac{1}{4}\pi x^2 + 2xy = 4$	A1
	$y = \dfrac{4 - \frac{1}{4}\pi x^2}{2x} = \dfrac{16 - \pi x^2}{8x}$ *	B1 cso (3)
(b)	$P = 2x + cy + k\pi r$ where $c = 2$ or 4 and $k = ¼$ or ½	M1
	$P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{4 - \frac{1}{4}\pi x^2}{2x}\right)$ or $P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{16 - \pi x^2}{8x}\right)$ o.e.	A1
	$P = \dfrac{\pi x}{2} + 2x + \dfrac{8}{x} - \dfrac{\pi x}{2}$ so $P = \dfrac{8}{x} + 2x$ *	A1 (3)
(c)	$\left(\dfrac{dP}{dx} =\right) -\dfrac{8}{x^2} + 2$	M1 A1
	$-\dfrac{8}{x^2} + 2 = 0 \Rightarrow x^2 = ..$	M1
	and so $x = 2$ o.e. (ignore extra answer $x = -2$)	A1
	$P = 4 + 4 = 8$ (m)	B1 (5)
(d)	$y = \dfrac{4 - \pi}{4}$, (and so width) = 21 (cm)	M1, A1 (2)
		13
Notes	(a) **M1**: Putting sum of one or two xy terms and one kr^2 term equal to 4 (k and c may be wrong) **A1**: For any correct form of this equation with x for radius (may be unsimplified) **B1** : Making y the subject of their formula to give this printed answer with no errors (b) **M1** : Uses Perimeter formula of the form $2x + cy + k\pi r$ where $c = 2$ or 4 and $k = ¼$ or ½ **A1**: Correct unsimplified formula with y substituted as shown, i.e. $c = 4$, $k = ½$, $r = x$ and $y = \dfrac{16 - \pi x^2}{8x}$ or $y = \left(\dfrac{4 - \frac{1}{4}\pi x^2}{2x}\right)$ **A1**: obtains printed answer with at least one line of correct simplification or expansion before giving printed answer or stating result has been shown or equivalent (c) **M1**: At least one power of x decreased by 1 (Allow $2x$ becomes 2) **A1**: accept any equivalent correct answer **M1**: Setting $\dfrac{dP}{dx}$=0 and finding a value for correct power of x for candidate **A1** : For $x = 2$. (This mark may be given for equivalent and may be implied by correct P) **B1**: 8 (cao) N.B. This may be awarded if seen in part (d) (d) **M1** : Substitute x value found in (c) into equation for y from (a) (or substitute x and P into equation for P from (b)) and evaluate (may see 0.2146 and correct answer implies M1 or need to see substitution if x value was wrong.) **A1** is for 21 or 21cm or 0.21m as this is to nearest cm	