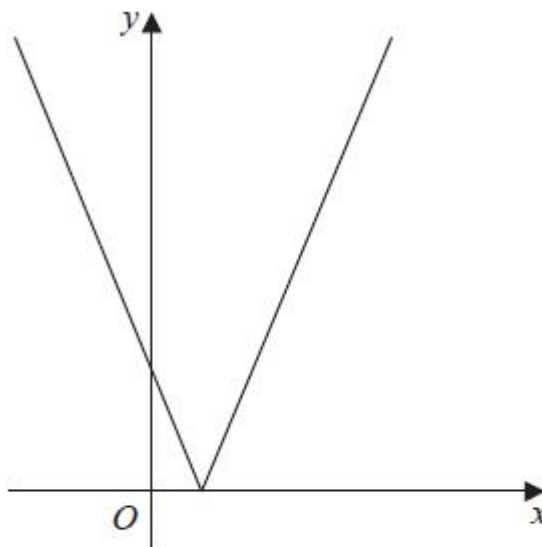


## Exam Questions – Modulus Functions

Q1.



**Figure 4**

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where  $k$  is a positive constant.

(a) Sketch the graph with equation  $y = f(x)$  where

$$f(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(b) Find, in terms of  $k$ , the set of values of  $x$  for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

(4)

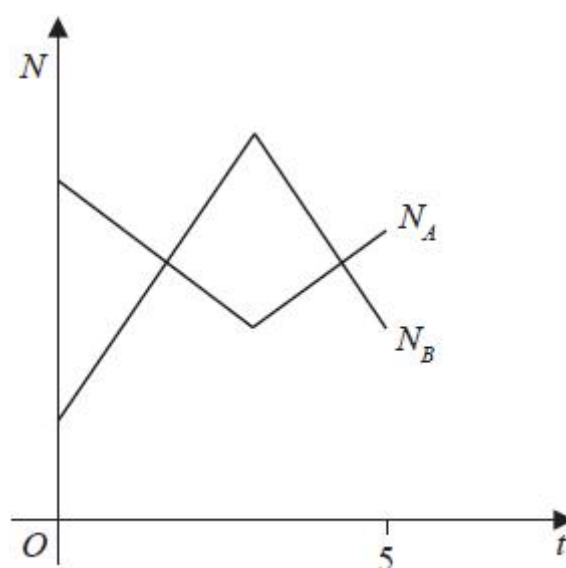
(c) Find, in terms of  $k$ , the coordinates of the minimum point of the graph with equation

$$y = 3 - 5f\left(\frac{1}{2}x\right)$$

(2)

**(Total for question = 10 marks)**

**Q2.**



**Figure 2**

The number of subscribers to two different music streaming companies is being monitored.

The number of subscribers,  $N_A$ , in thousands, to **company A** is modelled by the equation

$$N_A = |t - 3| + 4 \quad t \geq 0$$

where  $t$  is the time in years since monitoring began.

The number of subscribers,  $N_B$ , in thousands, to **company B** is modelled by the equation

$$N_B = 8 - |2t - 6| \quad t \geq 0$$

where  $t$  is the time in years since monitoring began.

Figure 2 shows a sketch of the graph of  $N_A$  and the graph of  $N_B$  over a 5-year period.

**Use the equations of the models to answer parts (a), (b), (c) and (d).**

(a) Find the initial difference between the number of subscribers to **company A** and the number of subscribers to **company B**.

When  $t = T$  **company A** reduced its subscription prices and the number of subscribers increased.

(b) Suggest a value for  $T$ , giving a reason for your answer.

(2)

(c) Find the range of values of  $t$  for which  $N_A > N_B$  giving your answer in set notation.

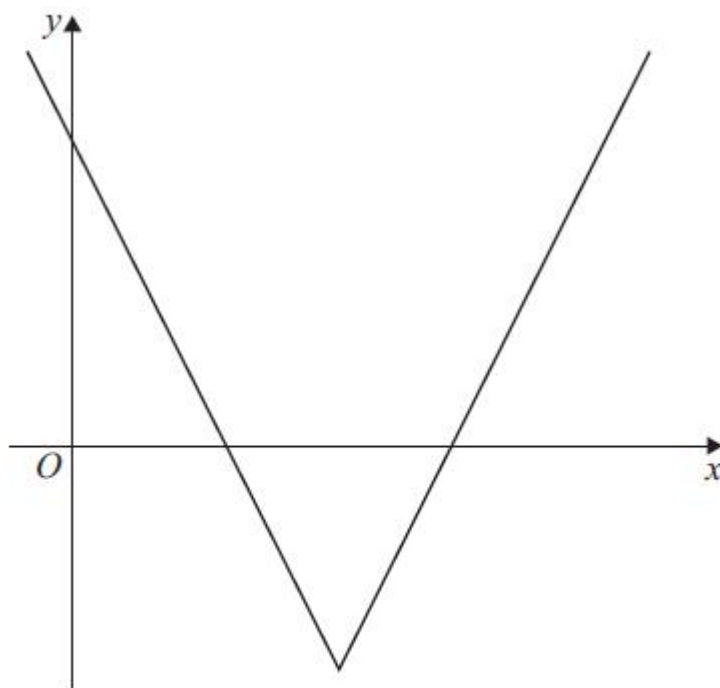
(5)

(d) State a limitation of the model used for **company B**.

(1)

**(Total for question = 10 marks)**

**Q3.**



**Figure 3**

Figure 3 shows a sketch of the graph with equation  $y = f(x)$  where

$$f(x) = 4|x - 3| - 5 \quad x \in \mathbb{R}$$

Given that  $a$  is a constant and  $|a| = 1$

(a) find the possible values of  $f(a)$

The function  $g$  is defined by

$$g(x) = 2x + 17 \quad x \in \mathbb{R}$$

(b) Find the range of  $g \circ f(x)$

(2)

The function  $h$  is defined by

$$h(x) = kx \quad x \in \mathbb{R}$$

where  $k$  is a constant.

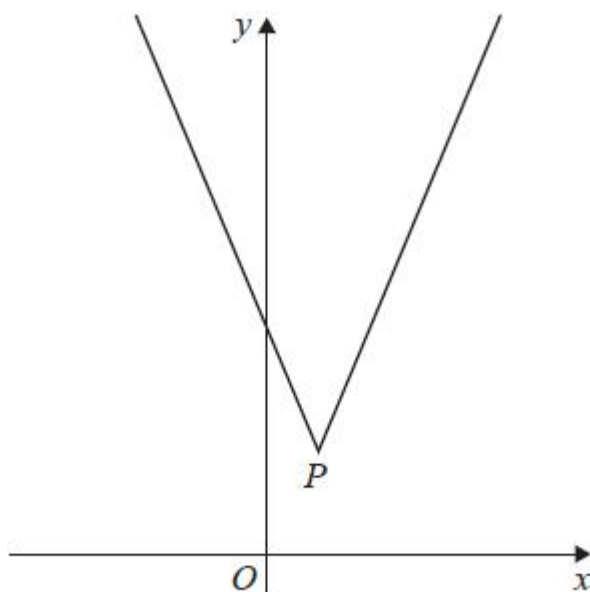
Given that the equation  $f(x) = h(x)$  has no solutions,

(c) find the range of values of  $k$ .

(4)

**(Total for question = 8 marks)**

**Q4.**



**Figure 1**

Figure 1 shows a sketch of the graph with equation

$$y = 3|x - 2| + 5$$

The vertex of the graph is at the point  $P$ , shown in Figure 1.

(a) Find the coordinates of  $P$ .

(2)

(b) Solve the equation

$$16 - 4x = 3|x - 2| + 5$$

(2)

A line  $l$  has equation  $y = kx + 4$  where  $k$  is a constant.

Given that  $l$  intersects  $y = 3|x - 2| + 5$  at 2 distinct points,

(c) find the range of values of  $k$ .

(2)

**(Total for question = 6 marks)**

Q5.

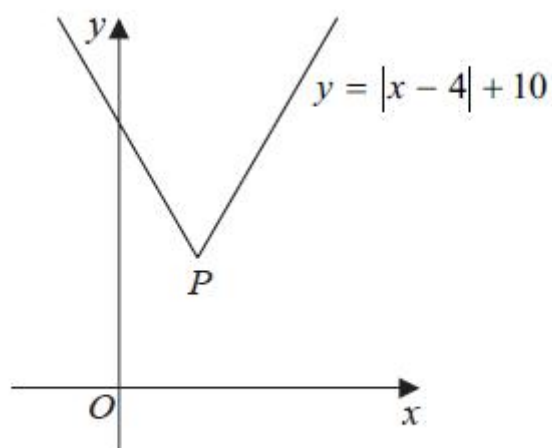


Figure 1

Figure 1 shows a sketch of the graph with equation  $y = |x - 4| + 10$

The point  $P$ , shown in Figure 1, is the vertex of the graph.

(a) State the coordinates of  $P$

(2)

(b) Use algebra to solve

$$|x - 4| + 10 = 3x$$

*(Solutions relying on calculator technology are not acceptable.)*

(2)

**(Total for question = 4 marks)**

**Q6.**

(a) Sketch the graph with equation

$$y = |4x - 3|$$

stating the coordinates of any points where the graph cuts or meets the axes.

(2)

Find the complete set of values of  $x$  for which

(b)

$$|4x - 3| > 2 - 2x$$

(4)

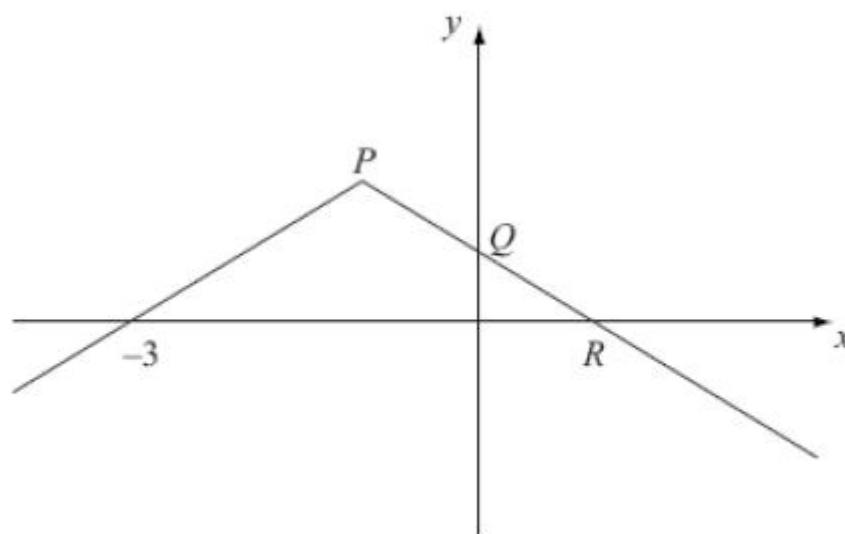
(c)

$$|4x - 3| > \frac{3}{2} - 2x$$

(2)

**(Total 8 marks)**

Q7.



**Figure 1**

Figure 1 shows the graph of  $y = f(x)$ ,  $x \in \mathbb{R}$ ,

The graph consists of two line segments that meet at the point  $P$ .

The graph cuts the  $y$ -axis at the point  $Q$  and the  $x$ -axis at the points  $(-3, 0)$  and  $R$ .

Sketch, on separate diagrams, the graphs of

(a)  $y = |f(x)|$ ,

(2)

(b)  $y = f(-x)$ .

(2)

Given that  $f(x) = 2 - |x + 1|$ ,

(c) find the coordinates of the points  $P$ ,  $Q$  and  $R$ ,

(3)

(d) solve  $f(x) = \frac{1}{2}x$ .

(5)

**(Total 12 marks)**

**Q8.**

The function  $f$  is defined by

$$f: x \mapsto |2x - 5|, \quad x \in \mathbb{R}$$

- (a) Sketch the graph with equation  $y = f(x)$ , showing the coordinates of the points where the graph cuts or meets the axes.

**(2)**

- (b) Solve  $f(x) = 15 + x$ .

**(3)**

The function  $g$  is defined by

$$g: x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 5$$

(c) Find  $fg(2)$ .

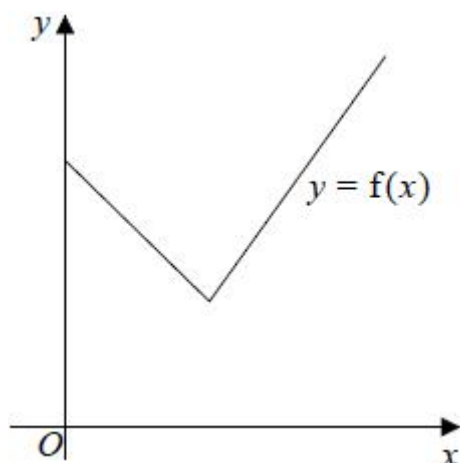
**(2)**

(d) Find the range of  $g$ .

**(3)**

**(Total 10 marks)**

Q9.



**Figure 2**

Figure 2 shows a sketch of part of the graph  $y = f(x)$ , where

$$f(x) = 2|3 - x| + 5, \quad x \geq 0$$

(a) State the range of  $f$

(1)

(b) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation  $f(x) = k$ , where  $k$  is a constant, has two distinct roots,

(c) state the set of possible values for  $k$ .

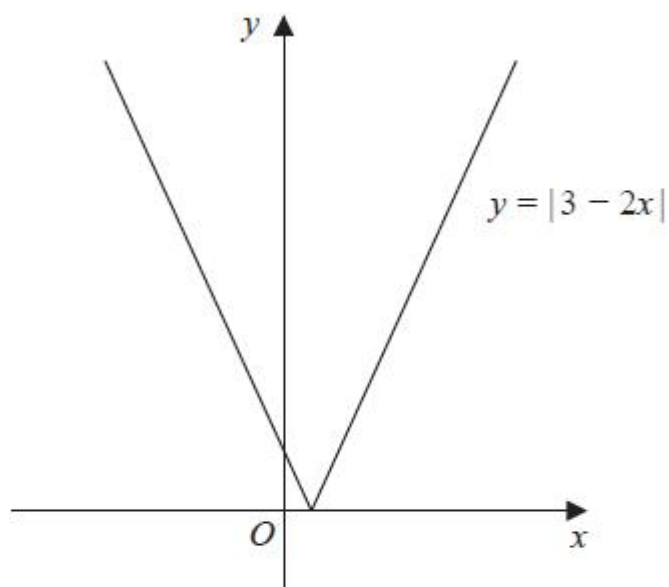
(2)

**(Total for question = 6 marks)**

**Q10.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**



**Figure 1**

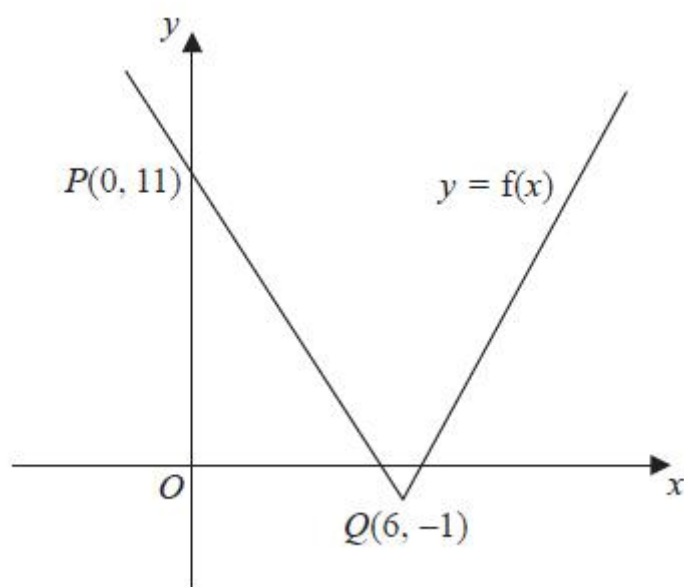
Figure 1 shows a sketch of the graph with equation  $y = |3 - 2x|$

Solve

$$|3 - 2x| = 7 + x$$

**(Total for question = 4 marks)**

Q11.



**Figure 1**

Figure 1 shows part of the graph with equation  $y = f(x)$ ,  $x \in \mathbb{R}$ .

The graph consists of two line segments that meet at the point  $Q(6, -1)$ .

The graph crosses the  $y$ -axis at the point  $P(0, 11)$ .

Sketch, on separate diagrams, the graphs of

(a)  $y = |f(x)|$

(2)

(b)  $y = 2f(-x) + 3$

(3)

On each diagram, show the coordinates of the points corresponding to  $P$  and  $Q$ .

Given that  $f(x) = a|x - b| - 1$ , where  $a$  and  $b$  are constants,

(c) state the value of  $a$  and the value of  $b$ .

(2)

**(Total 7 marks)**