## Mark Scheme

Q1.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $\left\{\frac{dy}{dx}=\right\}2x^2-7x-4$ | M1<br>A1 | 1.1b<br>1.1b |
| | | (2) | |
| (b) | Attempts to solve $\left\{\frac{dy}{dx}=\right\}2x^2-7x-4\ldots0$<br>e.g., $(2x+1)(x-4)=0$ leading to $x=\ldots$ and $x=\ldots$ | M1 | 1.1b |
| | Correct critical values $x=-\frac{1}{2},4$ | A1 | 1.1b |
| | Chooses inside region for their critical values | dM1 | 1.1b |
| | Accept either $-\frac{1}{2}<x<4$ or $-\frac{1}{2}\leqslant x\leqslant 4$ | A1 | 1.1b |
| | | (4) | |
| | | | (6 marks) |

**Notes:**

**(a)**

**M1:** Decreases the power of $x$ by one for at least one of their terms. Look for $x^n \to \ldots x^{n-1}$
Allow for $5 \to 0$

**A1:** $\left\{\frac{dy}{dx}=\right\}2x^2-7x-4$

**(b)**

**M1:** Sets their $\frac{dy}{dx}\ldots0$ where … may be an equality or an inequality and proceeds to find two values for $x$ from a 3TQ using the usual rules. This may be implied by their critical values.

**A1:** Correct critical values $x\ldots-\frac{1}{2},4$

These may come directly from a calculator and might only be seen on a sketch.

**dM1:** Chooses the inside region for their critical values.

**A1:** Accept either $-\frac{1}{2}<x<4$ or $-\frac{1}{2}\leqslant x\leqslant 4$ but not, e.g., $-\frac{1}{2}<x\leqslant 4$

Condone, e.g., $x>-\frac{1}{2},\ x<4$ or $x>-\frac{1}{2}$ and $x<4$ or $\left\{x:x>-\frac{1}{2}\right\}\cap\{x:x<4\}$

or $x\in\left(-\frac{1}{2},4\right)$ or $x\in\left[-\frac{1}{2},4\right]$

**Note:** You may see $x<-\frac{1}{2},x<4$ in their initial work before $-\frac{1}{2}<x<4$. Condone this so long as it is clear that the $-\frac{1}{2}<x<4$ is their final answer.

**Q2.**

| Question Number | Scheme | Marks | Notes |
|---|---|---|---|
| (a) | Integrate: $v = \left(t^3 - 2t^2\right)\mathbf{i} + \left(3t^2 - 5t\right)\mathbf{j} + \mathbf{C}$ | M1<br><br>A2 | At least 3 powers going up. Condone errors in constants. Must be two separate component equations if not in vector form.<br>Could be in column vector form.<br>Allow with no "+ **C**"<br>-1 each integration error. i.e. All correct A1A1<br>1 error A1A0, 2 or more errors A0A0<br>Allow with no "+ **C**" |
| | $t = 3: \mathbf{v} = 9\mathbf{i} + 12\mathbf{j} + \mathbf{C} = 11\mathbf{i} + 10\mathbf{j}$<br>$\mathbf{C} = 2\mathbf{i} - 2\mathbf{j}$ | DM1 | Substitute given values to find **C**.<br>Dependent on the previous M mark |
| | $\mathbf{v} = \left(t^3 - 2t^2 + 2\right)\mathbf{i} + \left(3t^2 - 5t - 2\right)\mathbf{j}$ | A1<br>(5) | Correct velocity (any equivalent form) |
| (b) | Parallel to **i** $\Rightarrow 3t^2 - 5t - 2 = 0$ | M1 | Set **j** component of their **v** equal to zero and solve for $t$ Correct answers imply method, but incorrect answers need to show method clearly. |
| | $(3t+1)(t-2) = 0$, $t = 2$ | A1 | Correct only. Ignore $-\frac{1}{3}$ if present. |
| | $\lvert\mathbf{v}\rvert = 8 - 8 + 2 = 2$ (m s$^{-1}$) | DM1<br><br>A1<br>(4) | Substitute their $t$ to find **v**.<br>Dependent on the previous M mark.<br>The **answer must be a scalar** – the Q asks for speed. Results from negative $t$ must be rejected. |
| | | [9] | |
| | A candidate who has no "+C" can score at most M1A2M0A0 M1A0M1A0 | | |

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**Q3.**

| Question Number | Scheme | Marks | Notes |
|---|---|---|---|
| (a) | $\mathbf{a} = \frac{d\mathbf{v}}{dt} = 6t\mathbf{i} + (4 - 2t)\mathbf{j}$ | M1 | Differentiate **v** to obtain **a**. |
| | | A1 | Accept column vector or **i** and **j** components dealt with separately. |
| | When $t = 1$, $\mathbf{a} = 6\mathbf{i} + 2\mathbf{j}$ | DM1 | Substitute $t = 1$ into their **a**. Dependent on 1st M1 |
| | $\lvert\mathbf{a}\rvert = \sqrt{6^2 + 2^2} = \sqrt{40} = 6.32$ (m s$^{-2}$) | DM1 | Use of Pythagoras to find the magnitude of their **a**. Allow with their t. Dependent on 1st M1 |
| | | A1 | Accept awrt 6.32, 6.3 or exact equivalents. |
| | | (5) | |
| (b) | $\mathbf{r} = \int (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}\, dt$ | M1 | Integrate **v** to obtain **r** |
| | $= (t^3 - t + C)\mathbf{i} + (2t^2 - \frac{1}{3}t^3 + D)\mathbf{j}$ | A1 | Condone $C$, $D$ missing |
| | $t = 0, \mathbf{r} = \mathbf{i} \Rightarrow C = 1, D = 0$ | DM1 | Use $t = 0$, $\mathbf{r} = \mathbf{i}$ to find $C$ & $D$ |
| | When $t = 3, \mathbf{r} = 25\mathbf{i} + 9\mathbf{j}$ (m) | DM1 | Substitute $t = 3$ with their $C$ & $D$ to find **r**. Dependent on both previous Ms. |
| | | A1 | cao. Must be a vector. |
| | | (5) | |
| | | **10** | |

**Q4.**

| Question | Scheme | Marks | AO |
|---|---|---|---|
| | | | |
| (a) | Differentiate **v** | M1 | 1.1a |
| | $(\mathbf{a} =) 6\mathbf{i} - \frac{15}{2}t^{\frac{1}{2}}\mathbf{j}$ | A1 | 1.1b |
| | $= 6\mathbf{i} - 15\mathbf{j}$ (m s$^{-2}$) | A1 | 1.1b |
| | | **(3)** | |
| (b) | Integrate **v** | M1 | 1.1a |
| | $(\mathbf{r} =)(\mathbf{r}_0) + 3t^2\mathbf{i} - 2t^{\frac{5}{2}}\mathbf{j}$ | A1 | 1.1b |
| | $= (-20\mathbf{i} + 20\mathbf{j}) + (48\mathbf{i} - 64\mathbf{j}) = 28\mathbf{i} - 44\mathbf{j}$ (m) | A1 | 2.2a |
| | | **(3)** | |
| | | **(6)** | |

| Marks | | Notes |
|---|---|---|
| | | **N.B.** Accept column vectors throughout and condone missing brackets in working but they must be there in final answers |
| **a** | M1 | Use of $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ with attempt to differentiate (both powers decreasing by 1)<br>M0 if **i**'s and **j**'s omitted and they don't recover |
| | A1 | Correct differentiation in any form |
| | A1 | Correct and simplified.<br>Ignore subsequent working (ISW) if they go on and find the magnitude. |
| **b** | M1 | Use of $\mathbf{r} = \int \mathbf{v}\,dt$ with attempt to integrate (both powers increasing by 1)<br>M0 if **i**'s and **j**'s omitted and they don't recover |
| | A1 | Correct integration in any form. Condone $\mathbf{r}_0$ not present |
| | A1 | Correct and simplified. |

**Q5.**

| Q | Scheme | Marks | Notes |
|---|---|---|---|
| a | Horizontal motion: $x = 3t$ | B1 | |
| | Vertical motion: $y = 4t - \frac{g}{2}t^2$ | M1 | Correct use of *suvat*. Condone sign error(s) |
| | | A1 | |
| | $\left(y = 4\times\frac{x}{3} - \frac{g}{2}\times\frac{x^2}{9}\right)$, $\lambda = -\left(\frac{4\lambda}{3} - \frac{g\lambda^2}{18}\right)$ | M1 | Use $y = -x$ and form an equation in one variable |
| | , $\frac{7\lambda}{3} = \frac{g\lambda^2}{18}$ | M1 | solve for $\lambda$ |
| | $\lambda = \frac{42}{g}$ or 4.3 (4.29) | A1 (6) | Not $\frac{30}{7}$ |
| alta | Horizontal motion: $x = 3t$ | B1 | |
| | Vertical motion: $y = 4t - \frac{g}{2}t^2$ | M1 | Correct use of *suvat*. Condone sign error(s) |
| | | A1 | |
| | $\Rightarrow -3t = 4t - \frac{1}{2}gt^2$, $\left(t = \frac{14}{g}\right)$ | M1 | Use $y = -x$ and form an equation in one variable |
| | $\lambda = 3t$ | M1 | Solve for $\lambda$ |
| | $\lambda = 4.3$ (4.29) | A1 (6) | |
| b | At A: $v \rightarrow 3$ (m s$^{-1}$) | B1 | |
| | $v\uparrow$ $4 - g\times\frac{14}{g}$ | M1 | Complete method using *suvat* to find $v\uparrow$ with their $t$ or $\lambda$ |
| | $= -10$ (m s$^{-1}$) | A1 | Accept +10 with direction confirmed by diagram |
| | Speed $= \sqrt{(\text{their } 10)^2 + (3)^2}$ | DM1 | Dependent on the first M1 in (b) |
| | $= \sqrt{109}$ ( m s$^{-1}$) | A1 | (10.4) Allow for $v\uparrow = 10$ |
| | $\tan^{-1}\left(\frac{\text{their } 10}{3}\right)$ or $\tan^{-1}\left(\frac{3}{\text{their } 10}\right)$ | DM1 | Use trig to find a relevant angle. Dependent on the first M1 in (b) |
| | Direction $= 73.3°$ below the horizontal | A1 | (1.28 radians) Accept direction $3\mathbf{i} - 10\mathbf{j}$ Do not accept a bearing |
| | | (7) | |
| Alt b | Loss in GPE : $mg\lambda = 42m$ | B1 | |
| | Gain in KE : $\frac{1}{2}mv^2 - \frac{1}{2}m\times 25$ | M1 | Terms must be dimensionally correct. Condone sign error. |
| | | A1 | |
| | Solve for $v$: $42 = \frac{1}{2}v^2 - \frac{25}{2}$ | M1 | |
| | $v = \sqrt{109}$ | A1 | |
| | $v\cos\theta = 3$ | M1 | Use trig. to find a relevant angle |
| | $\theta = 73.3°$ below the horizontal | A1 (7) | Accept correct angle marked correctly on a diagram. |
| | | [13] | |

Q6.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Multiply out and differentiate *wrt* to time (or use of product rule i.e. must have two terms with correct structure) | M1 | 1.1a |
| | $v = 2t^3 - 3t^2 + t$ | A1 | 1.1b |
| | $2t^3 - 3t^2 + t = 0$ and solve: $t(2t-1)(t-1) = 0$ | DM1 | 1.1b |
| | $t = 0$ or $t = \frac{1}{2}$ or $t = 1$; any two | A1 | 1.1b |
| | All three | A1 | 1.1b |
| | | (5) | |
| (b) | Find $x$ when $t = 0, \frac{1}{2}, 1$ and $2$ : $(0, \frac{1}{32}, 0, 2)$ | M1 | 2.1 |
| | Distance = $\frac{1}{32} + \frac{1}{32} + 2$ | M1 | 2.1 |
| | $2\frac{1}{16}$ (m) oe or 2.06 or better | A1 | 1.1b |
| | | (3) | |
| (c) | $x = \frac{1}{2}t^2(t-1)^2$ | M1 | 3.1a |
| | $\frac{1}{2}$ perfect square so $x \geq 0$ i.e. never negative | A1 cso | 2.4 |
| | | (2) | |
| | | | (10 marks) |

**Notes:**

(a)

**M1:** Must have 3 terms and at least two powers going down by 1

**A1:** A correct expression

**DM1:** Dependent on first M, for equating to zero and attempting to solve a <u>cubic</u>

**A1:** Any two of the three values (Two correct answers can imply a correct method)

**A1:** The third value

(b)

**M1:** For attempting to find the values of $x$ (at least two) at their $t$ values found in (a) or at $t=2$ or equivalent e.g. they may integrate their $v$ and sub in at least two of their $t$ values

**M1:** Using a correct strategy to combine their distances (must have at least 3 distances)

**A1:** $2\frac{1}{16}$ (m) oe or 2.06 or better

(c)

**M1:** Identify strategy to solve the problem such as:

(i) writing $x$ as $\frac{1}{2} \times$ perfect square

(ii) or using $x$ values identified in (b).

(iii) or using calculus i.e. identifying min points on $x$–$t$ graph.

(iv) or using $x$-$t$ graph.

**A1 cso :** Fully correct explanation to show that $x \geq 0$ i.e. never negative

**Q7.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Put $t = 2$ in **v** and use Pythagoras: $\sqrt{12^2 + (-6\sqrt{2})^2}$ | M1 | 3.1a |
| | $\sqrt{216}, 6\sqrt{6}$ or 15 or better $(\text{m s}^{-1})$ | A1 | 1.1b |
| | | (2) | |
| (b) | Differentiate **v** wrt $t$ to obtain **a** | M1 | 3.4 |
| | $6t\mathbf{i} - 3t^{-\frac{1}{2}}\mathbf{j}$ oe $(\text{m s}^{-2})$ isw | A1 | 1.1b |
| | | (2) | |
| (c) | Integrate **v** wrt $t$ to obtain **r** | M1 | 3.4 |
| | $\mathbf{r} = t^3\mathbf{i} - 4t^{\frac{3}{2}}\mathbf{j}\ (+\mathbf{C})$ | A1 | 1.1b |
| | $(\mathbf{i} - 4\mathbf{j}) = 4^3\mathbf{i} - 4 \times 4^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ | M1 | 3.1a |
| | $(-62\mathbf{i} + 24\mathbf{j})$ (m) isw e.g. if they go on to find the distance. | A1 | 1.1b |
| | | (4) | |
| | | | **(8 marks)** |

| **Notes: Accept column vectors throughout apart from the answer to (b).** | | |
|---|---|---|
| a | M1 | Need square root but -ve sign not required. Allow **i**'s and/or **j**'s to go missing from their **v** at $t = 2$, provided they have applied Pythagoras correctly. |
| | A1 | cao<br>N.B. Correct answer with no working can score 2 marks. |
| b | M1 | Both powers decreasing by 1. Allow a column vector.<br>M0 if **i** or **j** is missing but allow recovery in (b). |
| | A1 | cao. Do not accept a column vector. |
| c | M1 | Both powers increasing by 1<br>M0 if **i** or **j** is missing but allow recovery. |
| | A1 | (**r** = ) not required |
| | M1 | Putting $\mathbf{r} = (\mathbf{i} - 4\mathbf{j})$ and $t = 4$ into their displacement **vector** expression which must have **C** (allow $C$) to give an equation in **C** only, seen or implied.<br>Must have attempted to integrate **v** for this mark to be available.<br>N.B. **C** does not need to be found and this is a method mark, so allow slips. |
| | A1 | cao |

**Q8.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (i)(a) | Integrate **a** wrt $t$ to obtain velocity | M1 | 3.4 |
| | $\mathbf{v} = (t - 2t^2)\mathbf{i} + \left(3t - \frac{1}{3}t^3\right)\mathbf{j}\ (+\mathbf{C})$ | A1 | 1.1b |
| | $8\mathbf{i} - \frac{28}{3}\mathbf{j}\ (\text{m s}^{-1})$ | A1 | 1.1b |
| | | (3) | |
| (i)(b) | Equate **i** component of **v** to zero | M1 | 3.1a |
| | $t - 2t^2 + 36 = 0$ | A1ft | 1.1b |
| | $t = 4.5$ (ignore an incorrect second solution) | A1 | 1.1b |
| | | (3) | |
| (ii) | Differentiate **r** wrt to $t$ to obtain velocity | M1 | 3.4 |
| | $\mathbf{v} = (2t - 1)\mathbf{i} + 3\mathbf{j}$ | A1 | 1.1b |
| | Use magnitude to give an equation in $t$ only | M1 | 2.1 |
| | $(2t-1)^2 + 3^2 = 5^2$ | A1 | 1.1b |
| | Solve problem by solving this equation for $t$ | M1 | 3.1a |
| | $t = 2.5$ | A1 | 1.1b |
| | | (6) | |
| | | **(12 marks)** | |

| Notes: Accept column vectors throughout | | |
|---|---|---|
| (i)(a) | M1 | At least 3 terms with powers increasing by 1 (but M0 if clearly just multiplying by $t$) |
| | A1 | Correct expression |
| | A1 | Accept $8\mathbf{i} - 9.3\mathbf{j}$ or better. Isw if speed found. |
| (i)(b) | M1 | Must have an equation in $t$ only (Must have integrated to find a velocity vector) |
| | A1 ft | Correct equation follow through on their **v** but must be a 3 term quadratic |
| | A1 | cao |
| (ii) | M1 | At least 2 terms with powers decreasing by 1 (but M0 if clearly just dividing by $t$) |
| | A1 | Correct expression |
| | M1 | Use magnitude to give an equation in $t$ only, must have differentiated to find a velocity (M0 if they use $\sqrt{x^2 - y^2}$ ) |
| | A1 | Correct equation $\sqrt{(2t-1)^2 + 3^2} = 5$ |
| | M1 | Solve a 3 term quadratic for $t$ which has come from differentiating and using a magnitude. This M mark can be implied by a correct answer with no working. |
| | A1 | 2.5 |