Exam Questions – Differentiation (P1)

Q1.

The curve C has equation $y = f(x)$ where $x > 0$

Given that

- $$f'(x) = 6x - \frac{(2x-1)(3x+2)}{2\sqrt{x}}$$
- the point $P(4, 12)$ lies on C

(a) find the equation of the normal to C at P, giving your answer in the form $y = mx + c$ where m and c are integers to be found,

(4)

(b) find $f(x)$, giving each term in simplest form.

(6)

(Total for question = 10 marks)

Q2.

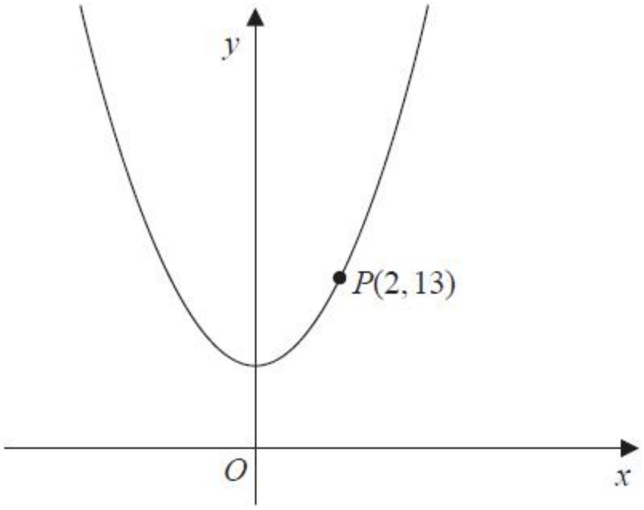

Figure 4

Figure 4 shows part of the curve with equation $y = 2x^2 + 5$

The point $P(2,13)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $2 + h$ also lies on the curve.

(b) Find, in terms of h, the gradient of the line PQ. Give your answer in simplest form.

(3)

(c) Explain briefly the relationship between the answer to (b) and the answer to (a).

(1)

(Total for question = 6 marks)

Q3.

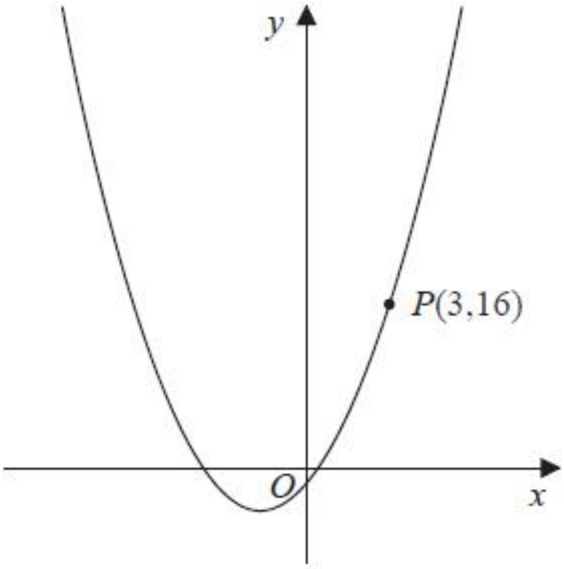

Figure 1

Figure 1 shows part of the curve with equation $y = x^2 + 3x - 2$

The point $P(3,16)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $3 + h$ also lies on the curve.

(b) Find, in terms of h, the gradient of the line PQ. Write your answer in simplest form.

(3)

(c) Explain briefly the relationship between the answer to (b) and the answer to (a).

(1)

(Total for question = 6 marks)

Q4.
The curve C has equation $y = f(x)$, $x > 0$

Given that

- $f'(x) = \frac{2}{\sqrt{x}} + \frac{A}{x^2} + 3$, where A is a constant
- $f''(x) = 0$ when $x = 4$

(a) find the value of A.

(4)

Given also that

- $f(x) = 8\sqrt{3}$, when $x = 12$

(b) find $f(x)$, giving each term in simplest form.

(5)

(Total for question = 9 marks)

Q5.

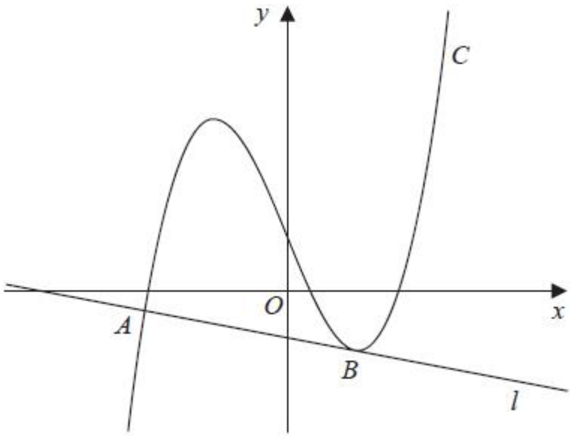

Figure 5

Figure 5 shows a sketch of the curve *C* with equation

$$y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k$$

where *k* is a constant.

(a) Find $\frac{dy}{dx}$

(2)

The line *l*, shown in Figure 5, is the normal to *C* at the point *A* with *x* coordinate $-\frac{7}{2}$

Given that *l* is also a tangent to *C* at the point *B*,

(b) show that the *x* coordinate of the point *B* is a solution of the equation

$$12x^2 + 4x - 33 = 0$$

(4)

(c) Hence find the *x* coordinate of *B*, justifying your answer.

(2)

Given that the *y* intercept of *l* is −1

(d) find the value of *k*.

(4)

(Total for question = 12 marks)

Q6.

The curve C has equation

$$y = (x - 2)(x - 4)^2$$

(a) Show that

$$\frac{dy}{dx} = 3x^2 - 20x + 32$$

(4)

The line l_1 is the tangent to C at the point where $x = 6$

(b) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

The line l_2 is the tangent to C at the point where $x = \alpha$

Given that l_1 and l_2 are parallel and distinct,

(c) find the value of α

(3)

(Total for question = 11 marks)

Q7.

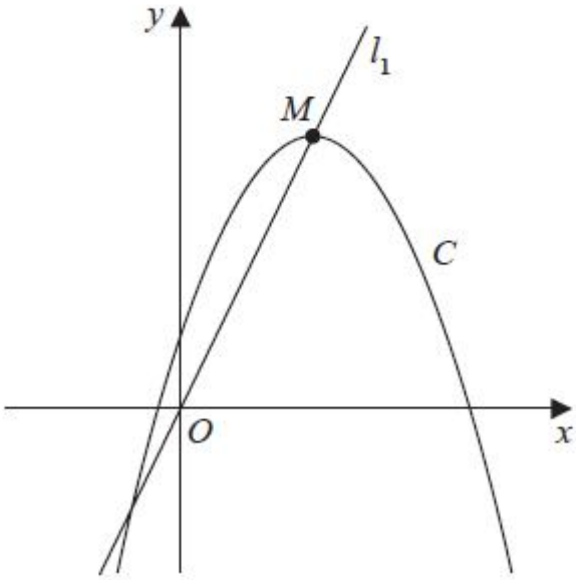

Figure 4

Figure 4 shows a sketch of the curve *C* with equation

$$y = 4 + 12x - 3x^2$$

The point *M* is the maximum turning point on *C*.

(a) (i) Write $4 + 12x - 3x^2$ in the form

$$a + b(x + c)^2$$

where *a*, *b* and *c* are constants to be found.

(ii) Hence, or otherwise, state the coordinates of *M*.

(5)

The line l_1 passes through *O* and *M*, as shown in Figure 4.

A line l_2 touches *C* and is parallel to l_1

(b) Find an equation for l_2

(5)

(Total for question = 10 marks)

Q8.

The line l_1 has equation

$$2x - 5y + 7 = 0$$

(a) Find the gradient of l_1

(1)

Given that

- the point A has coordinates (6, –2)
- the line l_2 passes through A and is perpendicular to l_1

(b) find the equation of l_2 giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

The lines l_1 and l_2 intersect at the point M.

(c) Using algebra and showing all your working, find the coordinates of M.

(*Solutions relying on calculator technology are not acceptable*.)

(3)

Given that the diagonals of a square $ABCD$ meet at M,

(d) find the coordinates of the point C.

(2)

(Total for question = 9 marks)

Q9.

The curve with equation $y = \mathrm{f}(x)$, $x > 0$, passes through the point $P(4, -2)$.

Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x\sqrt{x} - 10x^{-\frac{1}{2}}$$

(a) find the equation of the tangent to the curve at P, writing your answer in the form $y = mx + c$, where m and c are integers to be found.

(4)

(b) Find $\mathrm{f}(x)$.

(5)

(Total for question = 9 marks)

Q10.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Find the equation of the tangent to the curve with equation

$$y = \frac{1}{4}x^3 - 8x^{-\frac{1}{2}}$$

at the point $P(4, 12)$
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(5)

The curve with equation $y = \mathrm{f}(x)$ also passes through the point $P(4, 12)$

Given that

$$\mathrm{f}'(x) = \frac{1}{4}x^3 - 8x^{-\frac{1}{2}}$$

(b) find $\mathrm{f}(x)$ giving the coefficients in simplest form.

(5)

(Total for question = 10 marks)

Q11.

The curve C has equation

$$y = 2x^{\frac{5}{2}} - 4x + 3$$

(a) Find $\frac{dy}{dx}$ writing your answer in simplest form.

(2)

The point P lies on C.

Given that

- the x coordinate of P is 2^k where k is a constant
- the gradient of C at the point P is 16

(b) find the value of k.

(3)

(Total for question = 5 marks)

Q12.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = \text{f}(x)$, $x > 0$

Given that

- the point $P(4, -5)$ lies on C
- $\text{f}'(x) = \dfrac{2x^2 + ax + b}{4\sqrt{x}}$, where a and b are constants
- the gradient of the tangent to C at P is 7

(a) show that

$$4a + b = 24$$

(2)

Given also that $a + b = -9$

(b) find, in simplest form, $\text{f}(x)$

(7)

Curve C is transformed to the curve with equation $y = \text{f}(x - 3)$

Given that point P is transformed to the point Q,

(c) state the coordinates of Q.

(1)

(Total for question = 10 marks)

Q13.

The curve C has equation $y = f(x)$ where $x > 0$

Given that

- $$f'(x) = \frac{4x^2 + 10 - 7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}}$$
- the point $P(4, -1)$ lies on C

(a) (i) find the value of the gradient of C at P

(ii) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

(b) Find f(x).

(6)

(Total for question = 10 marks)

Q14.

A curve has equation $y = f(x)$, where

$$f(x) = (x - 4)(2x + 1)^2$$

The curve touches the x-axis at the point P and crosses the x-axis at the point Q.

(a) State the coordinates of the point P.

(1)

(b) Find f′(x).

(4)

(c) Hence show that the equation of the tangent to the curve at the point where $x = \frac{5}{2}$ can be expressed in the form $y = k$, where k is a constant to be found.

(3)

The curve with equation $y = f(x + a)$, where a is a constant, passes through the origin O.

(d) State the possible values of a.

(2)

(Total for question = 10 marks)

Q15.

A curve has equation

$$y = \frac{4x^2 + 9}{2\sqrt{x}} \qquad x > 0$$

Find the x coordinate of the point on the curve at which $\frac{dy}{dx} = 0$

(6)

(Total for question = 6 marks)

Q16.

The curve C has equation $y = f(x)$, $x > 0$

Given that

- C passes through the point P (8, 2)
- f ʹ(x) = $\frac{32}{3x^2} + 3 - 2\left(\sqrt[3]{x}\right)$

(a) find the equation of the tangent to C at P. Write your answer in the form $y = mx + c$, where m and c are constants to
be found.

(3)

(b) Find, in simplest form, f(x).

(5)

(Total for question = 8 marks)