Mark Scheme

Q1.

Question	Scheme	Marks
(a)	$f'(4)=6(4)-\frac{7\times 14}{4}=-\frac{1}{2}$	B1
	$m_T=-\frac{1}{2}\Rightarrow m_N=\frac{-1}{-1/2}$	M1
	$y-12=2(x-4)$ or $y=mx+c\Rightarrow y=2x+c\Rightarrow 12=2(4)+c\Rightarrow c=...$	M1 (A1 on ePen)
	$y=2x+4$	A1
		(4)
(b)	$...\frac{(2x-1)(3x+2)}{2\sqrt{x}}=...\frac{6x^2+x-2}{2\sqrt{x}}=...3x^{\frac{3}{2}}+\frac{1}{2}x^{\frac{1}{2}}-x^{-\frac{1}{2}}$	M1
	$f(x)=\frac{6x^2}{2}-\frac{6}{5}x^{\frac{5}{2}}-\frac{1}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}(+c)$ or e.g. $f(x)=\frac{6x^2}{2}-\frac{3}{5/2}x^{\frac{5}{2}}-\frac{1/2}{3/2}x^{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{1/2}(+c)$ or e.g. $f(x)=\frac{6x^2}{2}-\left(\frac{3}{5/2}x^{\frac{5}{2}}+\frac{1/2}{3/2}x^{\frac{3}{2}}-\frac{x^{\frac{1}{2}}}{1/2}\right)(+c)$	M1A1A1
	$12=\frac{6(4)^2}{2}-\frac{6}{5}(4)^{\frac{5}{2}}-\frac{1}{3}(4)^{\frac{3}{2}}+2(4)^{\frac{1}{2}}+c\Rightarrow c=...$	M1
	$(f(x)=)3x^2-\frac{6}{5}x^{\frac{5}{2}}-\frac{1}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+\frac{16}{15}$	A1
		(6)
		Total 10

Notes:

(a)

B1: Correct gradient at P (may be implied)

M1: Attempts to use the perpendicular gradient rule to find the normal gradient.

Look for e.g. $m_N = \frac{-1}{m_T}$ or e.g. $m_T \times m_N = -1 \Rightarrow -\frac{1}{2}m_N = -1 \Rightarrow m_N = ...$

May be implied by their normal gradient.

M1(A1 on Epen): Attempts the equation of the normal using a "changed" gradient with $x = 4$ and $y = 12$ correctly placed.

Alternatively uses $y = mx + c$ with a "changed" gradient with $x = 4$ and $y = 12$ to find a value for c.

A1: Correct equation in the required form

(b) **Allow work for part (b) seen in part (a) to score in (b) provided it is seen or used in (b)**

M1: Expands the numerator of the fraction and attempts to split.

Score for one correct index achieved from correct work e.g. $\frac{...x^2}{\sqrt{x}} \to ...x^{\frac{3}{2}}$ or $\frac{...x}{\sqrt{x}} \to ...x^{\frac{1}{2}}$ or $\frac{...}{\sqrt{x}} \to ...x^{-\frac{1}{2}}$

M1: Attempts to integrate a fractional power. E.g. $...x^{\frac{3}{2}} \to ...x^{\frac{5}{2}}$ or $x^{\frac{1}{2}} \to ...x^{\frac{3}{2}}$ or $x^{-\frac{1}{2}} \to ...x^{\frac{1}{2}}$ etc.

Do not allow this mark for an attempt to integrate the $\sqrt{x}$ in the denominator.

A1: Any 2 correct terms simplified or unsimplified.

A1: All correct simplified or unsimplified. The "+ c" is not required here.

M1: Uses $x = 4$ and $y = 12$ following an attempt to increase at least one power x by 1 to find a value for their c.

Their c may not be fully evaluated but must be a numerical expression.

A1: Correct simplified form. The "$\mathrm{f}(x) =$" is not required just look for the correct expression.

Accept equivalent simplified forms and allow $(\mathrm{f}(x) =)3x^2 - \left(\frac{6}{5}x^{\frac{5}{2}} + \frac{1}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}}\right) + \frac{16}{15}$

Apply isw if applicable unless they multiply all terms by e.g. 15

Q2.

Question Number	Scheme	Marks
(a)	Attempts $\frac{dy}{dx} = 4x$ at $x = 2$	M1
	At $x = 2$ gradient of tangent = 8	A1
		(2)
(b)	$(y_Q =)\ 2(2+h)^2 + 5$	B1
	Gradient $PQ = \frac{\text{their } y_Q - 13}{2+h-2}$	M1
	$\left(= \frac{8h+2h^2}{h}\right) = 8+2h$	A1
		(3)
(c)	States as $h \to 0$ Gradient $PQ \to 8$ = Gradient of tangent	B1
		(1)
		(6 marks)

(a)

M1 Attempts to find the value of $\frac{dy}{dx} = ax,\quad a > 0$ at $x = 2$

A1 For 8. No need to state this is the gradient.

(b)

B1 $(y_Q =)\ 2(2+h)^2 + 5$

M1 Attempts $\pm\frac{y_Q - y_P}{x_Q - x_P}$ condoning slips, but must be a genuine attempt at y_Q

A1 Gradient is $8+2h$ (with no errors seen)

(c)

B1 States as $h \to 0$ Gradient $PQ \to 8$ = Gradient of tangent (oe)

There should be reference to "limit" or "as h tends to 0" (words or symbols) and linked to part (a) (so same gradient, or showing the answers agree). But be generous with the explanation beyond these constraints.

Q3.

Question Number	Scheme	Marks
(a)	Attempts $\left(\frac{dy}{dx}=\right)$ $2x+3$ at $x=3$	M1
	At $x=3$ gradient of tangent $=9$	A1
		(2)
(b)	$(y_Q =)$ $(3+h)^2+3(3+h)-2$	B1
	Gradient $PQ=\frac{(3+h)^2+3(3+h)-2-16}{3+h-3}$ $=\frac{9h+h^2}{h}=9+h$	M1 A1
		(3)
(c)	States as $h\to 0$ Gradient $PQ\to 9$ = Gradient of tangent	B1
		(1)
		(6 marks)

(a)

M1 Attempts to find the value of $\left(\frac{dy}{dx}=\right)$ $ax+3,\quad a>0$ at $x=3$. Look for 3 to be substituted into the expression and proceeding to a value.

A1 9 (Answer only scores both marks)

(b)

B1 $(y_Q =)$ $(3+h)^2+3(3+h)-2$ (seen or implied)

M1 Attempts $\pm\frac{y_Q-16}{x_Q-3}$ condoning slips, but must be a genuine attempt at y_Q. Condone lack of brackets if implied by later working.

A1 $9+h$ with no errors seen and not originating from methods in calculus. This expression may immediately follow a correct simplified expression of $y_Q=h^2+9h+16$

(c)

B1 States as $h\to 0$ Gradient $PQ\to 9$ = Gradient of tangent (oe)

They must have achieved $9+h$ in (b) and 9 in (a)

There should be reference to "limit" or "as h tends to 0" (words or symbols) and linked to part (a) (so same gradient, or showing the answers agree). But be generous with the explanation beyond these contraints.

Online Maths Teaching

Q4.

Question	Scheme	Marks
(a)	$f'(x)=2x^{-\frac{1}{2}}+Ax^{-2}+3 \Rightarrow f''(x)=...x^{-\frac{1}{2}-1}+-...x^{-2-1}$	M1
	$\Rightarrow f''(x)=2\times -\frac{1}{2}x^{-\frac{3}{2}}+-2Ax^{-3}=-x^{-\frac{3}{2}}-2Ax^{-3}$	A1
	$f''(4)=0 \Rightarrow -4^{-\frac{3}{2}}-2A\times 4^{-3}=0 \Rightarrow A=...$	dM1
	$-\frac{1}{8}-\frac{2A}{64}=0 \Rightarrow A=-4$	A1
		(4)
(b)	$f(x)=\int 2x^{-\frac{1}{2}}+Ax^{-2}+3\,dx=\frac{2x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+\frac{Ax^{-2+1}}{-2+1}+3x(+c)$	M1
	$=4x^{\frac{1}{2}}-\frac{A}{x}+3x(+c)$	A1ft
	$f(12)=8\sqrt{3} \Rightarrow 4\sqrt{12}-\frac{A}{12}+36+c=8\sqrt{3} \Rightarrow c=...$	dM1
	$c=8\sqrt{3}-4\sqrt{12}-36-\frac{4}{12}=-\frac{109}{3}$ or follow through $c=\frac{A}{12}-36$	A1ft
	So $(f(x))=4x^{\frac{1}{2}}+\frac{4}{x}+3x-\frac{109}{3}$ oe	A1
		(5)
		(9 marks)

Notes: Mark parts (a) and (b) together

Online Maths

(a)

M1: Correct method of differentiation, at least one **correct** power reduced by 1.
The indices or coefficients do not have to be processed/simplified.

A1: Correct differentiation, need not be simplified, but the indices and coefficients must be processed correctly.

dM1: Sets $f''(4)=0$ and proceeds to find a value for A. It is dependent upon the previous M1

Don't be overly concerned with the mechanics of the solution here

A1: $A=-4$

(b)

M1: Attempts to integrate $f'(x)$. Look for a **correct** power increased by 1 on at least one term. The indices and coefficients do not have to be processed/simplified. Constant of integration is not needed for this mark.

A1ft: Correct integration, either with A or follow through their value of A. The indices and coefficients must be processed correctly. Constant of integration is not needed for this mark.

dM1: Sets $f(12)=8\sqrt{3}$ and proceeds to find a value for a constant of integration "c" using their value of A. It is dependent upon the previous M1

A1ft: Correct value for c found. Follow through their value of A, so $c=\frac{A}{12}-36$.

Accept awrt 1 dp

A1: Correct answer, $4x^{\frac{1}{2}}+\frac{4}{x}+3x-\frac{109}{3}$, not follow through.

If they go on to multiply by 3 etc it is A0

Q5.

Question	Scheme	Marks
(a)	$\frac{dy}{dx}=\frac{6}{7}x^2+\frac{2}{7}x-\frac{5}{2}$	M1 A1
		(2)
(b)	At $x=-\frac{7}{2}$, $\frac{dy}{dx}=\frac{6}{7}\left(-\frac{7}{2}\right)^2+\frac{2}{7}\left(-\frac{7}{2}\right)-\frac{5}{2}=...(=7)$	M1
	So at B we know $\frac{dy}{dx}="-\frac{1}{7}"$	M1
	hence $\frac{6}{7}x^2+\frac{2}{7}x-\frac{5}{2}=-\frac{1}{7}$	dM1
	$\Rightarrow 12x^2+4x-35=-2\Rightarrow 12x^2+4x-33=0$*	A1*
		(4)
(c)	E.g. $12x^2+4x-33=0\Rightarrow(2x-3)(6x+11)=0\Rightarrow x=...$	M1
	From graph we can see the x coordinate is positive, so $x=\frac{3}{2}$ at B	A1
		(2)
(d)	Equation of l is $y="-\frac{1}{7}"x-1$	M1
	Finds coordinates of A $x=-\frac{7}{2}\Rightarrow y="-\frac{1}{7}"\times-\frac{7}{2}-1=\left(-\frac{1}{2}\right)$	dM1
	Substitutes $x=-\frac{7}{2}, y="-\frac{1}{2}"$ into $y=\frac{2}{7}x^3+\frac{1}{7}x^2-\frac{5}{2}x+k\Rightarrow k=...$	ddM1
	$k=\frac{5}{4}$ CSO	A1
		(4)
		(12 marks)

(a)

M1: Finds $\frac{dy}{dx}$, look for at least two terms correct. They do not need to be simplified.

A1: Correct derivative, need not be simplified. ISW after a correct answer

(b) Marks cannot be retrospectively awarded from work in (d)

M1: Substitutes $-\frac{7}{2}$ into their $\frac{dy}{dx}$ to find the gradient of C at A

M1: Applies perpendicular condition to their gradient to find gradient at B

dM1: Equates $\frac{dy}{dx}$ to the gradient of the normal at B, depends on first M mark and a changed gradient.

A1*: Reaches the given equation with any correct intermediate line shown following $\frac{6}{7}x^2+\frac{2}{7}x-\frac{5}{2}=-\frac{1}{7}$

(c)

M1: Any valid method to solve the quadratic, factorisation, completing square, formula or calculator may be used (implied by one correct answer). This may be awarded for work in (b)

A1: Correct coordinate $x_B = \frac{3}{2}$ given with reason. See scheme. The reason should reference the sketch, e.g.

E.g. cannot be $-\frac{11}{6}$ as that is negative, condone reasons like "because B is positive"

(d) Marks cannot be awarded from work in (b), but allow the transfer of answers. E.g. $y = "-\frac{1}{7}"x - 1$

Explanation of Main method:
Find equation for *l*, then find coordinates for *A* or *B*, then sub coordinates into equation for *C* to find *k*

M1: Uses their gradient of l and intercept -1 to form the equation of l.

The gradient must a result of a changed $\frac{dy}{dx}$ at $x = -\frac{7}{2}$. It cannot be just made up

dM1: Finds the coordinates of either A or B using the equation for l and either $x_A = -\frac{7}{2}$ or $x_B = \frac{3}{2}$

FYI $x = -\frac{7}{2} \Rightarrow\ y = "-\frac{1}{7}" \times -\frac{7}{2} - 1 = \left(-\frac{1}{2}\right)$ and $x = \frac{3}{2} \Rightarrow\ y = "-\frac{1}{7}" \times \frac{3}{2} - 1 = \left(-\frac{17}{14}\right)$

ddM1: A full method to solve for k. This involves substituting the coordinates of A or B in the equation for curve C. E.g. See scheme but can also use $x = \frac{3}{2}$ $y = "-\frac{17}{14}"$ into $y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k \Rightarrow k = ...$

A1: CSO $k = \frac{5}{4}$

Explanation of Alt method:
Use the *x* coordinate for *A* or *B* in the equation for *C* to find the *y* coordinate for *A* or *B* in terms of *k*. Then use the gradient and point *A* or *B* to form an equation for *l* in terms of *k*. Use the fact that the intercept of *l* is - 1 to form and solve an equation in *k*

Alt I (d)	At A $y = \frac{2}{7}\left(-\frac{7}{2}\right)^3 + \frac{1}{7}\left(-\frac{7}{2}\right)^2 - \frac{5}{2}\left(-\frac{7}{2}\right) + k = ...\left(= k - \frac{7}{4}\right)$	**M1**
	Equation of l is $y - \left(k - \frac{7}{4}\right) = -\frac{1}{7}\left(x + \frac{7}{2}\right)$ or y intercept is $"-\frac{1}{7}" \times "\frac{7}{2}" + "k - \frac{7}{4}"$	**dM1**
	$\Rightarrow y = -\frac{1}{7}x + k - \frac{9}{4} \Rightarrow k - \frac{9}{4} = -1 \Rightarrow k = ...$	**ddM1**
	$k = \frac{5}{4}$	**A1**

M1: Substitutes $x = -\frac{7}{2}$ or $x = \frac{3}{2}$ into the equation for C and finds the y coordinate in terms of k.

This cannot be scored if they substitute any value for y (except for the correct value which would mean that we would be using the main method).

FYI at B the y coordinate is $k - \frac{69}{28}$

dM1: Uses their gradient for normal at A (or tangent at B) and their y coordinate to find an equation of the line l or to find an expression for the intercept. For use of B expect $y - \left(k - \frac{69}{28}\right) = -\frac{1}{7}\left(x - \frac{3}{2}\right)$

ddM1: Sets their intercept to -1 and solves for k.

A1: $k = \frac{5}{4}$

Explanation of Alt II (d)
Find the equation for *l* (as in main method) but then equate with the equation for *C*. Use the fact that the equation formed has a root of either 3/2 or - 7/2 to set up and solve an equation in *k*.

M1: For an attempt at the equation for l. Score for $y = "-\frac{1}{7}"x - 1$

dM1: Equate equation for l with equation for C and use the fact that a root of this equation is known.

For example $"-\frac{1}{7}"x - 1 = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k \Rightarrow 4x^3 + 2x^2 - 33x + 14k + 14 = 0$

Set $g\left(\pm\frac{3}{2}\right) = 0$ or $g\left(\pm\frac{7}{2}\right) = 0$ where $g(x) = 4x^3 + 2x^2 - 33x + 14k + 14$ to form an equation in k

ddM1: As above but sets $g\left(\frac{3}{2}\right) = 0 \Rightarrow k = ...$ $\quad g\left(-\frac{7}{2}\right) = 0 \Rightarrow k = ...$ which must lead to a value for k

A1: $k = \frac{5}{4}$

Q6.

Question	Scheme	Marks
(a)	$y=(x-2)(x^2-8x+16) \Rightarrow y=x^3-8x^2+16x-2x^2+16x-32 \Rightarrow$ $y=x^3 \pm ...x^2 \pm ...x \pm 32$	M1
	$=x^3-10x^2+32x-32$	A1
	$\frac{dy}{dx}=3x^2-20x+32$ *	M1A1*
		(4)
(b)	$x=6 \Rightarrow y=(6-2)(6-4)^2=16$	B1
	$\frac{dy}{dx}=3(6)^2-20(6)+32=20$	B1
	$y-"16"="20"(x-6)$	M1
	$y=20x-104$	A1
		(4)
(c)	$3x^2-20x+32="20" \Rightarrow 3x^2-20x+12=0$	M1
	$3x^2-20x+12=0 \Rightarrow (3x-2)(x-6)=0 \Rightarrow x=...$	dM1
	$\alpha=\frac{2}{3}$	A1
		(3)
		(11 marks)

Notes

(a)

M1 Attempts to multiply out the three brackets, condoning slips in their working.
Usually $y=(x-2)(x^2-8x+16) \Rightarrow y=x^3-8x^2+16x-2x^2+16x-32 \Rightarrow x^3 \pm ...x^2 \pm ...x \pm 32$
Score for expressions of the form $x^3 \pm ...x^2 \pm ...x \pm 32$. Middle terms do not need to be collected.

A1 $x^3-10x^2+32x-32$ If they do not collect terms together until after differentiating, A1 can be awarded by subsequent work. You would have to see the individual differentiated terms collected rather than implied by the final answer.
They must have attempted to multiply out the brackets for this mark.

M1 $x^n \to x^{n-1}$ correct on one term so either $...x^3 \to ...x^2$ $...x^2 \to ...x$ $Ax \to A$ $B \to 0$

A1* Correct proof with no errors including omission of brackets. At some point they should have had $y=...$ and their final line should finish with $\frac{dy}{dx}=3x^2-20x+32$ including the $\frac{dy}{dx}$ but the terms on the rhs can be in any order.

Alternative method: Product rule - Note the order of marking

2nd M1 $\left(\frac{dy}{dx}=\right)$ $(x-2)\times A(x-4)\pm B(x-4)^2$ applies the product rule. Look for this form or equivalent.

1st A1 $\left(\frac{dy}{dx}=\right)$ $(x-2)\times 2(x-4)+(x-4)^2$

1st M1 $\left(\frac{dy}{dx}=\right)$ $2x^2-8x-4x+16+x^2-8x+16\Rightarrow 3x^2-20x+32$ attempts to multiply out and collect terms to form a 3TQ

2nd A1* $\frac{dy}{dx}=3x^2-20x+32$* with no errors

M1 $y=(x-2)(x^2-8x+16)\Rightarrow y=x^3\pm...x^2\pm...x\pm 32$ (does not require middle terms to score M1)

A1 $y=x^3-10x^2+32x-32$ oe

M1 $\int(3x^2-20x+32)dx=x^3-10x^2+32x+C$ look for correct index on one term

A1* deduce that $C=-32$ and conclude $\frac{dy}{dx}=3x^2-20x+32$ with no errors seen

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(b)

B1 16 is identified as the y coordinate. Beware that $\frac{d^2y}{dx^2}=16$ when $x=6$

B1 20 is identified as the gradient. Eg $\frac{dy}{dx}=20$, $m=20$, $g=20$ or may be used within their equation for the tangent.

M1 Correct straight line method $y-"16"="20"(x-6)$ using:

- their value of y from substituting in $x=6$ into $y=(x-2)(x^2-8x+16)$ or $y=...$ from (a)
- their gradient found from substituting $x=6$ into $\frac{dy}{dx}=3x^2-20x+32$. This cannot be a changed gradient (eg gradient of a normal)

If they use $y=mx+c$ they must proceed as far as $c=...$

A1 $y=20x-104$ cao

(c)

M1 Equates $3x^2-20x+32$ with their 20 and collects terms to obtain a 3TQ. Condone slips in their rearrangement.

dM1 Attempts to solve their 3TQ (see general guidance for solving quadratics) . If they just state the roots then you may need to check these on a calculator. It is dependent on the previous method mark.

A1 $\alpha=\frac{2}{3}$ (allow $x=...$) Ignore sight of 6. Answer on its own scores full marks.

(Note that values of 4, $\frac{8}{3}$ imply they have solved $3x^2-20x+32=0$ which is 0 marks)

Q7.

Question Number	Scheme	Marks
(a)(i)	$4+12x-3x^2 = a \pm 3(x+c)^2$ or $a+b(x\pm 2)^2$	M1
	Two of $16-3(x-2)^2$ or two of $a=16, b=-3, c=-2$	A1
	$16-3(x-2)^2$	A1
(ii)	Coordinates $M=(2,16)$	B1ft B1ft
		(5)
(b)	States or implies that l_2 has equation $y=\text{"}8\text{"}x+k$	M1
	Sets $4+12x-3x^2 = \text{"}8x\text{"}+k$ and proceeds to 3TQ	dM1
	Correct 3TQ $3x^2-4x+k-4=0$	A1
	Attempts to use $b^2-4ac=0$ to find k	ddM1
	$k=\frac{16}{3} \Rightarrow y=8x+\frac{16}{3}$	A1
		(5)
		(10 marks)

(a)(i)

M1 For attempting to complete the square. Look for $b=\pm 3$ or $c=\pm 2$

A1 Two correct constants or two correct integers from $16-3(x-2)^2$

A1 $16-3(x-2)^2$ $(16-3(2-x)^2$ scores M1A1A0)

Alternative by comparing coefficients:

$$a+b(x+c)^2 = a+b(x^2+2xc+c^2) = bx^2+2bcx+a+bc^2$$

$$bx^2+2bcx+a+bc^2 \equiv 4+12x-3x^2$$

$$b=-3$$

$$2bc=12 \Rightarrow c=-2$$

$$a-12=4 \Rightarrow a=16$$

Score M1 for expanding $a+b(x+c)^2$ and compare x^2 coefficients to find a value for b

(NB this can be deduced directly and would score the M mark for $b=\pm 3$ as above)

A1: Continues the process and compares x coefficients to find both $b=-3$ and $c=-2$

A1: $a=16$

(a)(ii)

B1ft Either $x=2$ or $y=16$ but follow through on their $\mathbf{16}-3(x-\mathbf{2})^2$ where $a \neq 0$

B1ft Both $x=2$ and $y=16$ but follow through on their $(-c, a)$ from $a+b(x+c)^2$ where $b \neq \pm 1$

For correct or correct ft coordinates the wrong way round e.g. (16, 2) score SC B1 B0 but apply isw if the correct or correct ft answers are seen as $x = \ldots, y = \ldots$

(b)

M1 States or implies that l_2 has equation $y = "8"x+k,\ k \neq 0$

Follow through on their $y = "\frac{a}{c}"x+k$ or on $y = \left(\frac{y\text{ coordinate of their } M}{x\text{ coordinate of their } M}\right)x+k$

dM1 Sets $4+12x-3x^2 = "8x"+k$ and proceeds to 3TQ

A1 Correct 3TQ $3x^2-4x+k-4=0$ (The "$=0$" may be implied by subsequent work)

ddM1 Attempts to use $b^2-4ac=0$ to find k.

A1 $k=\frac{16}{3} \Rightarrow y=8x+\frac{16}{3}$. Condone just $k=\frac{16}{3}$ if $y=8x+k$ was mentioned as the equation for l_2

Alternative for part (b)

M1 Attempts to differentiate $4+12x-3x^2$ ($x^n \to x^{n-1}$ at least once) and sets equal to their 8

dM1 Solves for x and proceeds to find the coordinates of point of contact

A1 Tangent meets curve at $\left(\frac{2}{3}, \frac{32}{3}\right)$ o.e.

ddM1 Substitutes their $\left(\frac{2}{3}, \frac{32}{3}\right)$ in their $y = "8"x+k$ to find k.

A1 $y=8x+\frac{16}{3}$

Q8.

Question Number	Scheme	Marks
(a)	$\frac{2}{5}$ or decimal equivalent	B1
		(1)
(b)	$m_N = -1 \div "\frac{2}{5}"$	M1
	$y+2 = "-\frac{5}{2}"(x-6)$	M1
	$y = -\frac{5}{2}x + 13$	A1
		(3)
(c)	$"-\frac{5}{2}x+13" = "\frac{2}{5}x+\frac{7}{5}" \Rightarrow "\frac{29}{10}x" = "\frac{58}{5}" \Rightarrow x = ...(=4)$ or $"\frac{5}{2}y-\frac{7}{2}" = "-\frac{2}{5}y+\frac{26}{5}" \Rightarrow "\frac{29}{10}y" = "\frac{87}{10}" \Rightarrow y = ...(=3)$	M1
	$x = "4" \Rightarrow y = ...$ or $y = "3" \Rightarrow x = ...$	dM1
	$(4,3)$	A1
		(3)
(d)	$(2,8)$	B1B1
		(2)
		Total 9

(a)

B1: $\frac{2}{5}$ or decimal equivalent. It must be identified so do not just extract it from a rearranged equation into the form $y = \frac{2}{5}x + ...$ and do not allow $\frac{2}{5}x$. Must be seen in (a). Do not be concerned with the working to achieve $\frac{2}{5}$. Do not accept $\frac{-2}{-5}$

(b)

M1: Correct application of the perpendicular gradient rule

M1: Correct straight-line method with their "changed" gradient. Eg $(y+2) = "-\frac{5}{2}"(x-6)$. Allow one sign slip in the brackets. If they use $y = mx + c$ they must proceed as far as $c = ...$

A1: $y = -\frac{5}{2}x + 13$ oe

(c) Coordinates found with no algebraic working scores 0

M1: A correct method to solve for x or y for their l_1 and l_2. Do not be concerned by the mechanics of their rearrangement. Do not penalise if decimals appear in their working.

dM1: Finds the value of the other variable. It is dependent on the previous method mark.

A1: $(4,3)$ or $x=4,\ y=3$ Condone lack of brackets around the coordinates.

(d) Note on EPEN this is M1A1 we are marking this B1B1

B1: One of $x=2$ or $y=8$ May be seen within a pair of coordinates.

B1: $(2,8)$ or $x=2,\ y=8$ Condone lack of brackets around the coordinates.

Special Case: $(8,2)$ or $\begin{pmatrix}2\\8\end{pmatrix}$ score B1B0

Q9.

Question Number	Scheme	Marks
(a)	Substitutes $x=4$ in $\frac{dy}{dx}=3x\sqrt{x}-10x^{-\frac{1}{2}}=3\times4\times2-\frac{10}{2}=19$	M1A1
	Attempts $(y-(-2))="19"\times(x-4)\Rightarrow y=19x-78$	M1A1 cao
		(4)
(b)	$f'(x)=3x^{\frac{3}{2}}-10x^{-\frac{1}{2}}\Rightarrow f(x)=\frac{6}{5}x^{\frac{5}{2}}-20x^{\frac{1}{2}}+c$	M1 A1 A1
	$x=4, f(x)=-2\Rightarrow$	
	$-2=38.4-40+c\Rightarrow c=...(-0.4)$	M1
	$[f(x)=]\ \frac{6}{5}x^{\frac{5}{2}}-20x^{\frac{1}{2}}-0.4$	A1 cso
		(5)
		(9 marks)

(a)

M1 Substitutes $x=4$ into $\frac{dy}{dx}=3x\sqrt{x}-10x^{-\frac{1}{2}}$. Do not award this mark if they attempt to differentiate the expression first (look at the $-10x^{-\frac{1}{2}}$ for evidence of the power decreasing) but do condone an error made on the power of the first x term if they try to write it as a single power of x.

A1 Gradient = 19

M1 Attempts an equation of a tangent using their $f'(4)$ and $(4,-2)$. If they attempt $(y+2)="19"(x-4)$ at least one of the brackets must be correct. If the form $y=mx+c$ is used they must proceed as far as finding c. They must either have shown their gradient and $(4,-2)$ substituted into $y=mx+c$ and rearrange (maybe with errors) to find c or if they show no working then their c must be correct.

A1 $y=19x-78$ cao

(b)

M1 Raises the power of any term by one $x^{-\frac{1}{2}}\to x^{\frac{1}{2}}, x^{\frac{3}{2}}\to x^{\frac{5}{2}}$ Accept eg $x^{\frac{3}{2}}\to x^{\frac{3}{2}+1}$

A1 Any term correct (may be un-simplified) with or without $+c$

A1 Both terms correct (may be un-simplified) with or without $+c$

M1 Substitutes $x=4, y=-2$ into their $f(x)$ containing $+c$ to obtain c. Condone errors in evaluating and rearranging

A1 $[f(x)=]\ \frac{6}{5}x^{\frac{5}{2}}-20x^{\frac{1}{2}}-0.4$ or equivalent including $(y=)$ ... cso

Q10.

General Guidance for marking

Note that some candidates are misreading the $x^{-\frac{1}{2}}$ as $x^{\frac{1}{2}}$ in part (a) and/or part (b)

In the majority of cases it will be clear if this is the case as candidates often write down the expression before they differentiate in part (a) or integrate in part (b)

If it is clear that a candidate thinks the expression is $\frac{1}{4}x^3 - 8x^{\frac{1}{2}}$ in either or both parts then FULL marks should be awarded for correct work – details are below. We are condoning the fact that *P*(4,12) does not lie on $y = \frac{1}{4}x^3 - 8x^{\frac{1}{2}}$

If candidates do not write down the original expression (in (a) or (b)) then evidence of misreading or not can be taken from their derivative or integral.

Note:

If candidates integrate in (a) and differentiate in (b) then the maximum marks available are:

(a) M0A0M1dM1A0 (b) M0A0dM0A0

If parts are not labelled then you should assume that the first attempt is part (a) and the second attempt is part (b).

Scheme – No Misread

Question	Scheme	Marks
(a)	$y = \frac{1}{4}x^3 - 8x^{-\frac{1}{2}} \Rightarrow \left(\frac{dy}{dx} =\right) \frac{1}{4} \times 3x^2 - 8 \times -\frac{1}{2}x^{-\frac{3}{2}}$	M1 A1
	$\left.\frac{dy}{dx}\right\rvert_{x=4} = \ldots \left(\frac{25}{2}\right)$	M1
	$y - 12 = "\frac{25}{2}"(x-4)$	dM1
	$25x - 2y - 76 = 0$ oe e.g. $-25x + 2y + 76 = 0$	A1
		(5)
(b)	$(\text{f}(x) =) \int \frac{1}{4}x^3 - 8x^{-\frac{1}{2}}\,dx = \frac{1}{4}\frac{x^4}{4} - \frac{8x^{\frac{1}{2}}}{\frac{1}{2}} (+c)$	M1 A1
	$\text{f}(4) = 12 \Rightarrow 16 - 32 + c = 12 \Rightarrow c = \ldots(28)$	dM1 A1
	So $(\text{f}(x) =) \frac{x^4}{16} - 16\sqrt{x} + "28"$	A1ft
		(5)
		(10 marks)

Notes:

(a)

M1: Correct method of differentiation, at least one power reduced by 1.

Award for $\frac{1}{4}x^3 \to ...x^2$ or $-8x^{-\frac{1}{2}} \to ...x^{-\frac{3}{2}}$

A1: Correct differentiation, need not be simplified.

M1: Finds $\frac{dy}{dx}$ at $x = 4$. Requires the substitution of $x = 4$ into a "changed" function to find a value.

Note that some candidates are using the "12" in some way to find the gradient e.g.

$$12 = \frac{1}{4}\times 3(4)^2 - 8\times -\frac{1}{2}(4)^{-\frac{3}{2}} + c \Rightarrow c = 2 \Rightarrow m = 2$$ and this scores M0

dM1: Depends on previous M. Correct method for the tangent **not** the normal. Uses their $\frac{dy}{dx}$ at $x = 4$ with $y = 12$ and $x = 4$ correctly placed. If using $y = mx + c$ they must proceed as far as finding a value for c.

A1: Correct equation in the required form including "= 0" or any non-zero integer multiple of it.

(b)

M1: Attempts to integrate f(x), look for power increased by 1 on at least one term.

Award for $\frac{1}{4}x^3 \to ...x^4$ or $-\frac{1}{8}x^{-\frac{1}{2}} \to ...x^{\frac{1}{2}}$

A1: Correct integration, need not be simplified. (no need for constant for this mark).

dM1: Depends on first M. Sets f (4) = 12 and proceeds to find a value for c - must have a constant of integration to score this mark.

A1: Correct value for c found.

A1ft: Correct answer following through their c only i.e. the algebraic part must be correct.

Accept with fractional index rather than square root. Allow $0.0625x^4$ for $\frac{x^4}{16}$. **Depends on the previous method mark.**

The "f (x) =" is **not** required so just look for the correct expression. Apply isw if necessary, e.g. correct work leading to $(f(x)=)\frac{x^4}{16} - 16x^{\frac{1}{2}} + 28$ followed by $(f(x)=)x^4 - 256x^{\frac{1}{2}} + 448$ can score full marks in (b).

Condone poor notation e.g. leaving the final answer as $(f(x)=)\int \frac{x^4}{16} - 16x^{\frac{1}{2}} + 28$

Scheme – Misread in (a) and/or (b)

Question	Scheme	Marks
(a)	$y=\frac{1}{4}x^3-8x^{\frac{1}{2}} \Rightarrow \left(\frac{dy}{dx}=\right)\frac{1}{4}\times 3x^2-8\times\frac{1}{2}x^{-\frac{1}{2}}$	M1 A1
	$\left.\frac{dy}{dx}\right\|_{x=4} = ...(10)$	M1
	$y-12="10"(x-4)$	dM1
	$10x-y-28=0$ oe e.g. $-10x+y+28=0$	A1
		(5)
(b)	$(f(x)=)\int \frac{1}{4}x^3-8x^{\frac{1}{2}}\,dx=\frac{1}{4}\frac{x^4}{4}-\frac{8x^{\frac{3}{2}}}{\frac{3}{2}}(+c)$	M1 A1
	$f(4)=12 \Rightarrow 16-\frac{128}{3}+c=12 \Rightarrow c=...\left(\frac{116}{3}\right)$	dM1 A1
	So $(f(x)=)\frac{x^4}{16}-\frac{16}{3}x^{\frac{3}{2}}+"\frac{116}{3}"$	A1ft
		(5)
		(10 marks)

Notes:

(a)

M1: Correct method of differentiation, at least one power reduced by 1.

Award for $\frac{1}{4}x^3 \to ...x^2$ or $-8x^{\frac{1}{2}} \to ...x^{-\frac{1}{2}}$

A1: Correct differentiation, need not be simplified. Ignore any spurious "= 0".

M1: Finds $\frac{dy}{dx}$ at $x = 4$. Requires the substitution of $x = 4$ into a "changed" function to find a value.

Note that some candidates are using the "12" in some way to find the gradient e.g.

$$12=\frac{1}{4}\times 3(4)^2-8\times-\frac{1}{2}(4)^{-\frac{1}{2}}+c \Rightarrow c=2 \Rightarrow m=2$$ and this scores M0

dM1: Depends on previous M. Correct method for the tangent **not** the normal. Uses their $\frac{dy}{dx}$ at $x = 4$ with $y = 12$ and $x = 4$ correctly placed. If using $y = mx + c$ they must proceed as far as finding a value for c.

A1: Correct equation in the required form including "= 0" or any non-zero integer multiple of it.

(b)

M1: Attempts to integrate f (x), look for power increased by 1 on at least one term.

Award for $\frac{1}{4}x^3 \to ...x^4$ or $-\frac{1}{8}x^{\frac{1}{2}} \to ...x^{\frac{3}{2}}$

A1: Correct integration, need not be simplified. (no need for constant for this mark).

dM1: Depends on first M. Sets f (4) = 12 and proceeds to find a value for c - must have a constant of integration to score this mark.

A1: Correct value for c found.

A1ft: Correct answer following through their c only i.e. the algebraic part must be correct. Accept with fractional index rather than square root. Allow $0.0625x^4$ for $\frac{x^4}{16}$ and equivalent mixed fractions for $\frac{16}{3}$ and $\frac{116}{3}$. **Depends on the previous method mark.**

The "f (x) =" is **not** required so just look for the correct (or correct ft) expression. Apply isw if necessary e.g. correct work leading to $(f(x)=)\frac{x^4}{16}-\frac{16}{3}x^{\frac{3}{2}}+\frac{116}{3}$ followed by $(f(x)=)3x^4-256x^{\frac{3}{2}}+1856$ can score full marks in (b).

Condone poor notation e.g. leaving the final answer as $(f(x)=)\int\frac{x^4}{16}-\frac{16}{3}x^{\frac{3}{2}}+\frac{116}{3}$

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Q11.

Question Number	Scheme	Marks
	$y=2x^{\frac{5}{2}}-4x+3$	
(a)	$\left\{\frac{dy}{dx}=\right\}5x^{\frac{3}{2}}-4$	M1, A1
		(2)
(b)	$5x^{\frac{3}{2}}-4=16\Rightarrow x^{\frac{3}{2}}=4$	M1, A1
	$\Rightarrow x=4^{\frac{2}{3}}=2^{\frac{4}{3}} \qquad \Rightarrow k=\frac{4}{3}$	A1
		(3)
		(5 marks)

Note: Allow benefit of doubt if some fractional indices look like they are inline but it is clear that powers are intended.

(a)

M1: For decreasing a correct power by one including $3\to 0$

A1: For $5x^{\frac{3}{2}}-4$

(b)

M1: Sets their $\frac{dy}{dx}=\alpha x^{\frac{3}{2}}+\beta=16$ and proceeds to $x^{\frac{3}{2}}=\delta$ or $\frac{dy}{dx}=\alpha\left(2^k\right)^{\frac{3}{2}}+\beta=16$ and proceeds with correct index work to $\left(2^k\right)^{\frac{3}{2}}=q$ or $2^{pk}=q$ (oe)

A1: Achieves $x^{\frac{3}{2}}=4$ or $\left(2^k\right)^{\frac{3}{2}}=4$ or $2^{3k}=16$ (oe in form $2^{pk}=q$)

A1: $k=\frac{4}{3}$ (oe) but accept $x=2^{\frac{4}{3}}$ as long as no incorrect statement for k follows. Note if a decimal is given it must be recurring, not rounded.

They may use logs to solve, do not be concerned about use of rounded values in working. If $x^{\frac{3}{2}}=4$ is achieved and they reach $k=\frac{4}{3}$ from log work even with incorrect rounding shown, assume full calculator accuracy was kept in working and award the mark.

Q12.

Question Number	Scheme	Marks
(a)	$\dfrac{2(4)^2+a\times4+b}{4\sqrt{4}}=7\Rightarrow 32+4a+b=56\Rightarrow 4a+b=24$ *	M1A1*
		(2)
(b)	$4a+b=24,\ a+b=-9\Rightarrow a=11,\ b=-20$	M1A1
	$\dfrac{x^{\frac{3}{2}}}{2}+\dfrac{11x^{\frac{1}{2}}}{4}-5x^{-\frac{1}{2}}$	M1
	$\displaystyle\int \dfrac{x^{\frac{3}{2}}}{2}+\dfrac{11x^{\frac{1}{2}}}{4}-5x^{-\frac{1}{2}}\,dx\Rightarrow \text{Two of } \dfrac{x^{\frac{5}{2}}}{5},\ \dfrac{"11"x^{\frac{3}{2}}}{6},\ \dfrac{"-20"}{2}x^{\frac{1}{2}}$	dM1A1ft
	$\dfrac{(4)^{\frac{5}{2}}}{5}+\dfrac{11(4)^{\frac{3}{2}}}{6}-10(4)^{\frac{1}{2}}+c=-5\Rightarrow c=...$	M1
	$(\mathrm{f}(x)=)\ \dfrac{1}{5}x^{\frac{5}{2}}+\dfrac{11}{6}x^{\frac{3}{2}}-10x^{\frac{1}{2}}-\dfrac{91}{15}$	A1
		(7)
(c)	$(7,-5)$	B1
		(1)
		(10 marks)

Mark (a) and (b) together

(a)

M1 Attempts to substitute $x=4$ into $\mathrm{f}'(x)$ and sets equal to 7. Condone slips or incorrect manipulation prior to substituting in $x=4$ provided the intention is clear.

A1* Rearranges and achieves the given answer with no errors and at least one intermediate stage of working seen.

(b)

M1 Attempts to solve the two equations in a and b simultaneously to find a value for a or b. You do not need to be concerned with the mechanics of the rearrangement. Implied by any value for a or b

A1 $a=11,\ b=-20$ (correct values for a and b scores M1A1)

M1 Attempts to split the fraction and achieves at least one term with a correct index i.e. $x^{\frac{3}{2}},\ x^{\frac{1}{2}}$ or $x^{-\frac{1}{2}}$ (for the relevant term). Do not allow for e.g. $\dfrac{...}{x^{\frac{1}{2}}}$ but the required index $x^{-\frac{1}{2}}$ may be implied by further work. You do not need to be concerned by the coefficient of the term, which may be in terms of a or b.

Do not allow to be scored for e.g. $\dfrac{2x^2}{4\sqrt{x}}\to ...x^{\frac{1}{2}}$

dM1 Increases the power by one on one of the terms in x with a correct index. It is dependent on the previous method mark. $x^{\frac{3}{2}}\to x^{\frac{5}{2}},\ x^{\frac{1}{2}}\to x^{\frac{3}{2}}$ or $x^{-\frac{1}{2}}\to x^{\frac{1}{2}}$. The index does not need to be processed for this mark e.g. $x^{\frac{3}{2}}\to x^{\frac{3}{2}+1}$.

Do not award this mark for increasing the power by one on the denominator term.

If either *a* or *b* are not numerical then A0ftM0A0 is scored for the remaining marks in part (b)

Condone using numerical values for both *a* and *b* which appear with no evidence of working.

A1ft **Two correct terms of** $\frac{1}{5}x^{\frac{5}{2}}$ or $\frac{\text{"11"}}{6}x^{\frac{3}{2}}$ or $\frac{\text{"}-20\text{"}}{2}x^{\frac{1}{2}}$ The coefficients of the terms do not need to be simplified, but the index must be processed (follow through their *a* and/or *b* which must be numerical but both must have been found)

M1 Either states e.g. $x = 4, y = -5$ and states a value for *c* or attempts to substitute $(4, -5)$ into their changed expression with a constant of integration and proceeds to find a value for *c*. They must have a constant of integration in their expression to score this mark.
You do not need to check the mechanics of their rearrangement.

A1 $(\mathrm{f}(x) =)\, \frac{1}{5}x^{\frac{5}{2}} + \frac{11}{6}x^{\frac{3}{2}} - 10x^{\frac{1}{2}} - \frac{91}{15}$ or exact simplified equivalent. isw once a correct answer is seen.
Must have the correct value for *c* substituted in.
Condone $-6.0\dot{6}$ for $-\frac{91}{15}$ but not e.g. -6.07 or $-6.066....$
Accept $(\mathrm{f}(x) =)\, \frac{1}{30}\left(6x^{\frac{5}{2}} + 55x^{\frac{3}{2}} - 300x^{\frac{1}{2}} - 182\right)$

(c) **Check by the question or at the start of their work in the main body of the text**

B1 $(7, -5)$ May be written as $x = 7,\ \ y = -5$. Condone missing brackets.

Q13.

Question Number	Scheme	Marks
ai	$f'(4)=\dfrac{4(4)^2+10-7(4)^{\frac{1}{2}}}{4(4)^{\frac{1}{2}}}=\dfrac{15}{2}$	B1
ii	$"\dfrac{15}{2}"\to-\dfrac{2}{15}$	M1
	$y+1="-\dfrac{2}{15}"(x-4)$	M1
	$2x+15y+7=0$	A1
		(4)
b	$\dfrac{4x^2+10-7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}}=\pm...x^{\frac{3}{2}}\pm...x^{-\frac{1}{2}}\pm...$	M1
	Two of the terms of $x^{\frac{3}{2}}+\dfrac{5}{2}x^{-\frac{1}{2}}-\dfrac{7}{4}$	A1
	$\int\left(x^{\frac{3}{2}}+\dfrac{5}{2}x^{-\frac{1}{2}}-\dfrac{7}{4}\right)\mathrm{d}x=\dfrac{2}{5}x^{\frac{5}{2}}+5x^{\frac{1}{2}}-\dfrac{7}{4}x\ (+c)$	dM1A1ft
	$\dfrac{2}{5}(4)^{\frac{5}{2}}+5(4)^{\frac{1}{2}}-\dfrac{7}{4}(4)+c=-1\Rightarrow c=...$	ddM1
	$(f(x)=)\ \dfrac{2}{5}x^{\frac{5}{2}}+5x^{\frac{1}{2}}-\dfrac{7}{4}x-\dfrac{84}{5}$	A1
		(6)
		(10 marks)

Mark (a) and (b) together so do not be concerned with labelling of the parts

(a)

(i)

B1: $\dfrac{15}{2}$ oe stated (as the gradient of f(x) at *P*).

(ii)

M1 Finds the negative reciprocal of their gradient in part (i). If they do not have a gradient in (i) then only allow $-\dfrac{2}{15}$ or an attempt at $-\dfrac{1}{f'(4)}$

M1 Attempts to find the equation of the normal using a changed gradient to that found in (i) and $(4,-1)$ with the coordinates in the correct positions. If they do not have a gradient in (i) then allow any gradient $\neq\dfrac{15}{2}$. If they use $y=mx+c$ they must proceed as far as $c=...$

A1: $2x+15y+7=0$ or any multiple of this where all the coefficients are integers and all terms are on the same side of the equation. e.g. $30y+14+4x=0$ scores A1

(b)

M1: Splits into **three** separate terms with at least one term with the correct index. The index does not need to be processed. e.g. $x^{2-\frac{1}{2}}$

A1: Two of $x^{\frac{3}{2}} + \frac{5}{2}x^{-\frac{1}{2}} - \frac{7}{4}$ (unsimplified but the indices must be processed). May appear as a list of terms on different lines. May be implied by a correctly integrated expression.

dM1: Attempts to increase the power by one on at least one term. It is dependent on the previous method mark. The index does not need to be processed for this mark. It cannot be scored for attempting to integrate individual terms on the numerator or denominator.

A1ft: $\frac{2}{5}x^{\frac{5}{2}} + 5x^{\frac{1}{2}} - \frac{7}{4}x$ or exact unsimplified equivalent (indices processed). e.g. $\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{\frac{5}{2}x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{7}{4}x$

Follow through on an expression of the form $Ax^{\frac{3}{2}} + Bx^{-\frac{1}{2}} + C \rightarrow \frac{Ax^{\frac{5}{2}}}{\frac{5}{2}} + \frac{Bx^{\frac{1}{2}}}{\frac{1}{2}} + Cx$ where A, B and C are all non-zero.
Do not be concerned with the presence or omission of the constant of integration.
Accept x^1 for x.
Ignore any spurious notation including the presence of the integral sign.

ddM1: Attempts to substitute $x = 4$ into their changed expression (condone one slip in substituting in), sets equal to -1 and attempts to find c.
This mark cannot be scored if they do not have a constant of integration.
It is dependent on both of the previous method marks.
Do not be concerned by the mechanics of the rearrangement and condone arithmetical slips, but they must achieve a value.

The substitution may be implied by a correct value for c for their integrated expression if no working is shown (you may need to check this), or allow for $\pm\frac{84}{5}$

A1: $(\text{f}(x) =)\ \frac{2}{5}x^{\frac{5}{2}} + 5x^{\frac{1}{2}} - \frac{7}{4}x - \frac{84}{5}$ or any equivalent expression e.g. $0.4x^{\frac{5}{2}} + 5x^{\frac{1}{2}} - 1.75x - 16.8$
Accept x^1 for x. Condone equivalent fractions for the coefficients provided both numerator and denominator are integers. e.g. $(\text{f}(x) =)\ \frac{x^{\frac{5}{2}}}{2.5} + 5x^{\frac{1}{2}} - \frac{7}{4}x - \frac{84}{5}$ is A0
Withhold the final mark if there is still integration notation around the answer or if it is set equal to 0

Q14.

Question Number	Scheme	Marks
(a)	$P=\left(-\frac{1}{2},0\right)$	B1
		(1)
(b)	$f(x)=(x-4)(2x+1)^2 \Rightarrow f(x)=ax^3+bx^2+cx+d$	M1
	$=4x^3-12x^2-15x-4$ oe	A1
	$f'(x)=12x^2-24x-15$	dM1 A1
		(4)
(c)	Attempts $f'(2.5)=12\times2.5^2-24\times2.5-15=0$	M1A1
	Finds y coordinate for $x=2.5$ $y=-54$	A1
		(3)
(d)	$a=-\frac{1}{2},(+)4$	B1, B1
		(2)
		(10 marks)

(a)

B1 $P=\left(-\frac{1}{2},0\right)$ or exact equivalent. Accept coordinates written separately.

If both $(4,0)$ and $\left(-\frac{1}{2},0\right)$ are given, then P must be identified as $\left(-\frac{1}{2},0\right)$

(b)

M1 Attempts to multiply out to form a 4 - term cubic. The terms do not have to be collected.
Look for a multiplied-out expression that would simplify to ax^3+bx^2+cx+d

A1 $4x^3-12x^2-15x-4$. The terms do not have to be collected for this mark.

dM1 Reduces the power by one in all terms. The indices must be processed.
Look for $ax^3+bx^2+cx+d \to px^2+qx+r$ with no zero constants

A1 cso $f'(x)=12x^2-24x-15$ which must be fully simplified.
Cannot be awarded, for instance, from incorrect e.g. $f(x)=4x^3-12x^2-15x\underline{\underline{+4}}$

Note: May be attempted by the product rule.

M1A1: $f(x)=(x-4)(2x+1)^2 \Rightarrow f'(x)=1\times(2x+1)^2+A(x-4)(2x+1)$

All 4 marks $f(x)=(x-4)(2x+1)^2 \Rightarrow f'(x)=1\times(2x+1)^2+4(x-4)(2x+1)$

Note that the terms do not need to be multiplied out under this method

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(c)

M1 Attempts $f'(2.5)$ for their $f'(x)$

A1 Shows $f'(2.5)=12\times2.5^2-24\times2.5-15=0$ with either embedded values shown or $f'(2.5)=75-60-15=0$
For this to be scored $f'(x)$ must be correct

A1 CSO Finds y coordinate for $x=2.5 \Rightarrow$ Equation of tangent $y=-54$ but allow $k=-54$

An alternative method would be to

M1 Attempts to solve their $f'(x)=0$

A1 For $f'(x)=0 \Rightarrow x=(-0.5), 2.5$ For this to be scored $f'(x)$ must be correct

A1 CSO Finds y coordinate for $x=2.5 \Rightarrow$ Equation of tangent $y=-54$ but allow $k=-54$

(d)

B1 For one of $-\frac{1}{2}, (+)\,4$. Alternatively score for both $a=+\frac{1}{2}, -4$
Implied by $y=f\left(x-\frac{1}{2}\right)$ or $y=f(x+4)$ for this mark only

B1 For both $a=-\frac{1}{2}, (+)\,4$ and no others. Cannot be $x=\ldots$ but allow just the values $-\frac{1}{2}, (+)\,4$

Q15.

Question Number	Scheme	Marks
	$\frac{4x^2+9}{2\sqrt{x}} = \frac{4x^2}{2\sqrt{x}} + \frac{9}{2\sqrt{x}} = 2x^{\frac{3}{2}} + \frac{9}{2}x^{-\frac{1}{2}}$	M1 A1
	$\left(\frac{dy}{dx}=\right)\ 3x^{\frac{1}{2}} - \frac{9}{4}x^{-\frac{3}{2}}$	M1 A1
	$\left(\frac{dy}{dx}=\right)\ 3x^{\frac{1}{2}} - \frac{9}{4}x^{-\frac{3}{2}} = 0 \Rightarrow x^2 = \frac{3}{4} \Rightarrow x = \frac{\sqrt{3}}{2}$	M1 A1
		(6) **(6 marks)**
Alt(I)	Quotient rule	
	$u = 4x^2+9, u' = 8x, v = 2\sqrt{x}, v' = x^{-\frac{1}{2}}$	M1A1
	$\left(\frac{dy}{dx}=\right)\ \frac{2\sqrt{x}\times 8x - (4x^2+9)\times x^{-\frac{1}{2}}}{4x}$	M1A1
	$\left(\frac{dy}{dx}=\right)\ \frac{2\sqrt{x}\times 8x - (4x^2+9)\times x^{-\frac{1}{2}}}{4x} = 0 \Rightarrow x^2 = \frac{3}{4} \Rightarrow x = \frac{\sqrt{3}}{2}$	M1A1
Alt(II)	Product rule	
	$u = 4x^2+9, u' = 8x, v = \frac{1}{2}x^{-\frac{1}{2}}, v' = \frac{1}{4}x^{-\frac{3}{2}}$	M1A1
	$\left(\frac{dy}{dx}=\right)\ (4x^2+9)\times\frac{1}{4}x^{-\frac{3}{2}} + 8x\times\frac{1}{2}x^{-\frac{1}{2}}$	M1A1
	$\left(\frac{dy}{dx}=\right)\ \frac{2\sqrt{x}\times 8x - (4x^2+9)\times x^{-\frac{1}{2}}}{4x} = 0 \Rightarrow x^2 = \frac{3}{4} \Rightarrow x = \frac{\sqrt{3}}{2}$	M1A1

M1 Attempts to divide by $2\sqrt{x}$. Award for one correct term (including $\frac{9}{2x^{\frac{1}{2}}}$). Allow if they combine the two terms with a common denominator of 2, but the indices must have been processed.

If they use the quotient rule it is for selecting u and v and attempting to differentiate. Look for $u = 4x^2 + 9, u' = ...x, v = 2\sqrt{x}, v' = ...x^{-\frac{1}{2}}$. If they use the product rule then it is for $u = 4x^2 + 9, u' = 8x, v = \frac{1}{2}x^{-\frac{1}{2}}, v' = \frac{1}{4}x^{-\frac{3}{2}}$

A1 $2x^{\frac{3}{2}} + \frac{9}{2}x^{-\frac{1}{2}}$ which may be left unsimplified but the indices must be processed. Using the product or quotient rule it is for having correct u' and v'.

M1 Attempts to differentiate the expression written as a sum. Award for one power decreasing by one on one of their terms following through their sum and the indices must have been processed. Cannot be awarded if they just differentiate top and bottom of the fraction.

Using the quotient rule look for expressions of the form $\left(\frac{dy}{dx} =\right) \frac{2\sqrt{x} \times ...x \pm (4x^2 + 9) \times ...x^{-\frac{1}{2}}}{4x}$

Using the product rule look for expressions of the form $\left(\frac{dy}{dx} =\right) (4x^2 + 9) \times ...x^{-\frac{3}{2}} + ...x \times \frac{1}{2}x^{-\frac{1}{2}}$

A1 $\left(\frac{dy}{dx} =\right) 3x^{\frac{1}{2}} - \frac{9}{4}x^{-\frac{3}{2}}$ which may be left unsimplified but the indices must be processed.

Using the quotient rule award for $\left(\frac{dy}{dx} =\right) \frac{2\sqrt{x} \times 8x - (4x^2 + 9) \times x^{-\frac{1}{2}}}{4x}$.

Using the product rule award for $\left(\frac{dy}{dx} =\right) (4x^2 + 9) \times \frac{1}{4}x^{-\frac{3}{2}} + 8x \times \frac{1}{2}x^{-\frac{1}{2}}$

M1 Sets $\frac{dy}{dx} = 0$ and proceeds to $x^{\pm 2} = ...$ or $x^{\pm 4} = ...$ following a derivative in the form $\left(\frac{dy}{dx} =\right) Ax^{\frac{1}{2}} - Bx^{-\frac{3}{2}}, \quad A, B > 0$ (which may be unsimplified).

Using the quotient rule or product rule look to proceed from one of the forms above to $x^{\pm 2} = ...$ or $x^{\pm 4} = ...$

Rounded versions of the answer do not imply this mark. Do not be too concerned by the mechanics of their arrangement but we must see some attempt to carry out algebraic manipulation proceeding to the required form.

A1 $(x =) \ \frac{\sqrt{3}}{2}$ or exact equivalent cso (Note that a correct exact answer can imply the final M1A1 but a rounded answer with no working such as awrt 0.87 is M0A0)

Withhold the final mark if $-\frac{\sqrt{3}}{2}$ is not rejected.

Q16.

Question Number	Scheme	Marks
(a)	$\mathrm{f}'(8)=\dfrac{32}{3\times 8^2}+3-2\sqrt[3]{8}\quad\left(=-\dfrac{5}{6}\right)$	M1
	$y-2=\text{"}-\dfrac{5}{6}\text{"}(x-8)$	dM1
	$y=-\dfrac{5}{6}x+\dfrac{26}{3}$	A1
		(3)
(b)	$\mathrm{f}'(x)=\dfrac{32}{3x^2}+3-2\sqrt[3]{x}=\ldots x^{-2}+3+\ldots x^{\frac{1}{3}}$	
	$x^{-2}\to x^{-1},\ 3\to 3x,\ x^{\frac{1}{3}}\to x^{\frac{4}{3}}$	M1
	$\mathrm{f}(x)=\int\dfrac{32}{3}x^{-2}+3-2x^{\frac{1}{3}}\mathrm{d}x=-\dfrac{32}{3}x^{-1}+3x-\dfrac{3}{2}x^{\frac{4}{3}}+c$	A1A1
	$2=-\dfrac{32}{3}\times 8^{-1}+3\times 8-\dfrac{3}{2}\times 8^{\frac{4}{3}}+c\Rightarrow c=\ldots$	dM1
	$(\mathrm{f}(x)=)\ -\dfrac{32}{3}x^{-1}+3x-\dfrac{3}{2}x^{\frac{4}{3}}+\dfrac{10}{3}$	A1
		(5)
		(8 marks)

Ignore labelling of parts (a) and (b)

(a)

M1 Substitutes $x=8$ to find a value for $f'(8)$. Condone slips in their substitution and $-\frac{5}{6}$ seen will imply this mark.

dM1 It is for the method of finding a line passing though $(8,2)$ using their value for $f'(8)$. Score for $(y-2)="-\frac{5}{6}"(x-8)$ with both brackets correct. If they use $y=mx+c$ they must proceed as far as $c=\ldots$ It is dependent on the previous method mark.

A1 $y=-\frac{5}{6}x+\frac{26}{3}$

(b)

M1 Integrates by raising the power on one of the terms (ie $x^{-2}\to x^{-1}, 3\to 3x, x^{\frac{1}{3}}\to x^{\frac{4}{3}}$)

A1 Two terms correct of $-\frac{32}{3}x^{-1}$, $+3x$ or $-\frac{3}{2}x^{\frac{4}{3}}$ seen (or unsimplified equivalents). The indices must be processed.

A1 $-\frac{32}{3}x^{-1}+3x-\frac{3}{2}x^{\frac{4}{3}}(+c)$ seen or unsimplified equivalent. Condone the lack of $+c$ for this mark. $-10.7x^{-1}$ is not a correct term but allow $-10.\dot{6}x^{-1}$.

dM1 Substitutes $x=8, y=2$ into their f(x) and proceeds to find c. It is dependent on the previous method mark and condone slips in their rearrangement to find c.

A1 $(f(x)=)\ -\frac{32}{3}x^{-1}+3x-\frac{3}{2}x^{\frac{4}{3}}+\frac{10}{3}$ or simplified equivalent. isw after a correct answer Eg $-\frac{32}{3x}+3x-\frac{3}{2}x^{\frac{4}{3}}+\frac{10}{3}$ or $-\frac{10.\dot{6}}{x}+3x-1.5x^{\frac{4}{3}}+3.\dot{3}$ but do not accept rounded decimals for the coefficients.