## Mark Scheme

Q1.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $(\overrightarrow{AB}=)\,3\mathbf{i}+9\mathbf{j}+3\mathbf{k}$ | B1 | 1.1b |
| | | (1) | |

**Notes for (a)**

**B1:** Correct vector. Allow $3\mathbf{i}+9\mathbf{j}+3\mathbf{k}$ or $\begin{pmatrix}3\\9\\3\end{pmatrix}$ but **not** $\begin{pmatrix}3\mathbf{i}\\9\mathbf{j}\\3\mathbf{k}\end{pmatrix}$ and **not** $(3, 9, 3)$

Condone $\begin{matrix}3\\9\\3\end{matrix}$ for $\begin{pmatrix}3\\9\\3\end{pmatrix}$

Do **not** apply isw here but award for e.g. $3\mathbf{i}+9\mathbf{j}+3\mathbf{k}=\begin{pmatrix}3\mathbf{i}\\9\mathbf{j}\\3\mathbf{k}\end{pmatrix}$

E.g. if they obtain $\overrightarrow{AB}=3\mathbf{i}+9\mathbf{j}+3\mathbf{k}$ and then say $\overrightarrow{AB}=\mathbf{i}+3\mathbf{j}+\mathbf{k}$ then award B0

If part (a) is **not attempted** and the correct $\overrightarrow{AB}$ is seen in part (b) the B1 can be awarded there.

**General Guidance for part (b):**

**As with most vector questions we will see a variety of approaches (correct and incorrect).**

**In general, the marks are awarded as follows:**

- **M1 for a correct complete strategy to find at least one position for *P* (May be implied by at least 2 correct components)**
- **A1 for one correct position for *P***
- **dM1 for a correct complete strategy to find both positions for *P* (May be implied by at least 2 correct components for both positions)**
- **A1 both correct positions for *P* and no others**

**Various examples are shown below.**
**Other methods will be seen but the above marking principles should be applied.**
**You can condone slips in their algebra/processing as long as the intention is clear.**
**The examples given below give the detail to look for depending on the approach.**
**If you see a response and you are not sure if it deserves credit use Review.**

**Note that adding vectors when they should be subtracting will generally score M0 but use review if necessary.**

| | | | |
|---|---|---|---|
| (b) | Examples: $\overrightarrow{OP}=\overrightarrow{OA}+2\overrightarrow{AB}=2\mathbf{i}-3\mathbf{j}+5\mathbf{k}+2(3\mathbf{i}+9\mathbf{j}+3\mathbf{k})=...$ or $\overrightarrow{OP}=\overrightarrow{OB}+\overrightarrow{AB}=5\mathbf{i}+6\mathbf{j}+8\mathbf{k}+(3\mathbf{i}+9\mathbf{j}+3\mathbf{k})=...$ or $\overrightarrow{OP}=\overrightarrow{OA}+\frac{2}{3}\overrightarrow{AB}=2\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\frac{2}{3}(3\mathbf{i}+9\mathbf{j}+3\mathbf{k})=...$ or $\overrightarrow{OP}=\overrightarrow{OB}+\frac{1}{3}\overrightarrow{BA}=5\mathbf{i}+6\mathbf{j}+8\mathbf{k}-\frac{1}{3}(3\mathbf{i}+9\mathbf{j}+3\mathbf{k})=...$ | M1 | 3.1a |
| | $8\mathbf{i}+15\mathbf{j}+11\mathbf{k}$ or $4\mathbf{i}+3\mathbf{j}+7\mathbf{k}$ | A1 | 1.1b |
| | Examples: $\overrightarrow{OP}=\overrightarrow{OA}+2\overrightarrow{AB}=2\mathbf{i}-3\mathbf{j}+5\mathbf{k}+2(3\mathbf{i}+9\mathbf{j}+3\mathbf{k})=...$ or $\overrightarrow{OP}=\overrightarrow{OB}+\overrightarrow{AB}=5\mathbf{i}+6\mathbf{j}+8\mathbf{k}+(3\mathbf{i}+9\mathbf{j}+3\mathbf{k})=...$ **and** $\overrightarrow{OP}=\overrightarrow{OA}+\frac{2}{3}\overrightarrow{AB}=2\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\frac{2}{3}(3\mathbf{i}+9\mathbf{j}+3\mathbf{k})=...$ or $\overrightarrow{OP}=\overrightarrow{OB}+\frac{1}{3}\overrightarrow{BA}=5\mathbf{i}+6\mathbf{j}+8\mathbf{k}-\frac{1}{3}(3\mathbf{i}+9\mathbf{j}+3\mathbf{k})=...$ | dM1 | 3.1a |
| | $8\mathbf{i}+15\mathbf{j}+11\mathbf{k}$ and $4\mathbf{i}+3\mathbf{j}+7\mathbf{k}$ | A1 | 2.2a |
| | | **(4)** | |
| | | | **(5 marks)** |

**Notes for (b)**

**Note that sight of at least one correct position for *P* implies M1A1**

**M1:** Attempts at least one correct strategy for finding *P*

**A1:** One correct position vector or allow coordinates for <u>this</u> mark e.g. (8, 15, 11) or (4, 3, 7) or $x = \ldots, y = \ldots, z = \ldots$

If given as a vector, allow e.g. $8\mathbf{i}+15\mathbf{j}+11\mathbf{k}$, $\begin{pmatrix}8\\15\\11\end{pmatrix}$ but not $\begin{pmatrix}8\mathbf{i}\\15\mathbf{j}\\11\mathbf{k}\end{pmatrix}$

**dM1:** Attempts two correct strategies for finding *P*

**A1:** Both correct position vectors

Must both be vectors so e.g. $8\mathbf{i}+15\mathbf{j}+11\mathbf{k}$, $\begin{pmatrix}8\\15\\11\end{pmatrix}$ and $4\mathbf{i}+3\mathbf{j}+7\mathbf{k}$, $\begin{pmatrix}4\\3\\7\end{pmatrix}$ but not e.g. $\begin{pmatrix}8\mathbf{i}\\15\mathbf{j}\\11\mathbf{k}\end{pmatrix}$

Condone e.g. $\begin{matrix}8\\15\\11\end{matrix}$ for $\begin{pmatrix}8\\15\\11\end{pmatrix}$

**Alternative 1 using vector equation of $l$:**

$$\mathbf{r}=\overrightarrow{OA}+\lambda\overrightarrow{AB}=\begin{pmatrix}2\\-3\\5\end{pmatrix}+\lambda\begin{pmatrix}3\\9\\3\end{pmatrix}\quad\left(\text{or e.g.}\begin{pmatrix}2\\-3\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\3\\1\end{pmatrix}\right)$$

$$\left|\overrightarrow{AP}\right|=2\left|\overrightarrow{BP}\right|\Rightarrow\left|\begin{pmatrix}3\lambda\\9\lambda\\3\lambda\end{pmatrix}\right|=2\left|\begin{pmatrix}3\lambda+2-5\\9\lambda-3-6\\3\lambda+5-8\end{pmatrix}\right|\Rightarrow 9\lambda^2+81\lambda^2+9\lambda^2=4\left[(3\lambda-3)^2+(9\lambda-9)^2+(3\lambda-3)^2\right]$$

$$\Rightarrow\lambda=2,\frac{2}{3}\Rightarrow\mathbf{r}=\begin{pmatrix}2\\-3\\5\end{pmatrix}+2\begin{pmatrix}3\\9\\3\end{pmatrix}\quad\text{or}\quad\begin{pmatrix}2\\-3\\5\end{pmatrix}+\frac{2}{3}\begin{pmatrix}3\\9\\3\end{pmatrix}$$

**M1:** Forms the vector equation of line $l$ using their $\overrightarrow{AB}$ from part (a) or by starting again, forms the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$ then uses $\left|\overrightarrow{AP}\right|=2\left|\overrightarrow{BP}\right|$ and Pythagoras to produce a quadratic equation in "$\lambda$" which they then solve to find "$\lambda$" and use correctly to find at least one position for $P$.
Note if they use **i** + 3**j** + **k** for the direction, they should get $\lambda=6,2$
If all other work is correct, condone not squaring the "2" when applying Pythagoras

**A1:** See main scheme

**dM1:** As the first M and finds both positions for $P$

**A1:** See main scheme

**Alternative 2 using $P$ as $\begin{pmatrix}x\\y\\z\end{pmatrix}$ and that $\overrightarrow{AP}$ and $\overrightarrow{BP}$ are parallel:**

$$\overrightarrow{OP}=\begin{pmatrix}x\\y\\z\end{pmatrix}\Rightarrow\overrightarrow{AP}=\begin{pmatrix}x-2\\y+3\\z-5\end{pmatrix},\ \overrightarrow{BP}=\begin{pmatrix}x-5\\y-6\\z-8\end{pmatrix}$$

$$\left|\overrightarrow{AP}\right|=2\left|\overrightarrow{BP}\right|\Rightarrow\left|\begin{pmatrix}x-2\\y+3\\z-5\end{pmatrix}\right|=2\left|\begin{pmatrix}x-5\\y-6\\z-8\end{pmatrix}\right|\Rightarrow\begin{matrix}(x-2)^2=4(x-5)^2\\(y+3)^2=4(y-6)^2\\(z-5)^2=4(z-8)^2\end{matrix}$$

$$\begin{matrix}(x-2)^2=4(x-5)^2\Rightarrow x=4,\ 8\\(y+3)^2=4(y-6)^2\Rightarrow y=3,\ 15\\(z-5)^2=4(z-8)^2\Rightarrow z=7,\ 11\end{matrix}\Rightarrow\begin{pmatrix}4\\3\\7\end{pmatrix},\begin{pmatrix}8\\15\\11\end{pmatrix}$$

| | |
|---|---|
| **M1:** | Sets $P$ as a general point, forms $\overrightarrow{AP}$ and $\overrightarrow{BP}$ (either way round) then uses $\left\|\overrightarrow{AP}\right\| = 2\left\|\overrightarrow{BP}\right\|$ then squares components and equates to produce quadratic equations in $x$ and $y$ and $z$ which they then solve to find at least one position for $P$. It is not just for finding values which are not then used to form a point (or vector). If all other work is correct, condone not squaring the "2" when squaring. |
| **A1:** | See main scheme |
| **dM1:** | As the first M and finds both positions for $P$. |
| **A1:** | See main scheme |

Note that if the modulus is not used, this method can lead to one correct position for $P$ e.g.

$$\overrightarrow{AP} = 2\overrightarrow{BP} \Rightarrow \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix} = 2\begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix} \Rightarrow \begin{matrix} x=8 \\ y=15 \\ z=11 \end{matrix} \text{ and scores M1 A1}$$

But it is possible to find the other position without squaring e.g.

$$\left|\overrightarrow{AP}\right| = 2\left|\overrightarrow{BP}\right| \Rightarrow \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix} = 2\begin{pmatrix} 5-x \\ 6-y \\ 8-z \end{pmatrix} \Rightarrow \begin{matrix} x=4 \\ y=3 \\ z=7 \end{matrix} \text{ and scores dM1 then A1 as main scheme.}$$

This requires at least 2 correct equations for $x$, $y$ or $z$ for the dM1

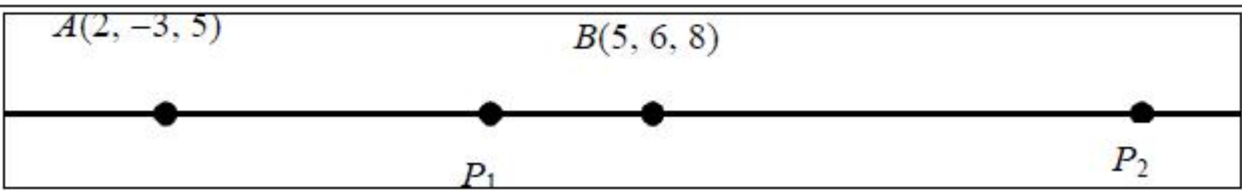
**e.g. Alternative 3 using $P$ as $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and eliminating 2 of the variables:**

$$\overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \overrightarrow{AP} = \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix}, \ \overrightarrow{BP} = \begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix}$$

$$\overrightarrow{AP} = \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix} = \lambda\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \Rightarrow \overrightarrow{AP} = \begin{pmatrix} x-2 \\ 3x-6 \\ x-2 \end{pmatrix}, \ \overrightarrow{BP} = \begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix} = \mu\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \Rightarrow \overrightarrow{BP} = \begin{pmatrix} x-5 \\ 3x-15 \\ x-5 \end{pmatrix}$$

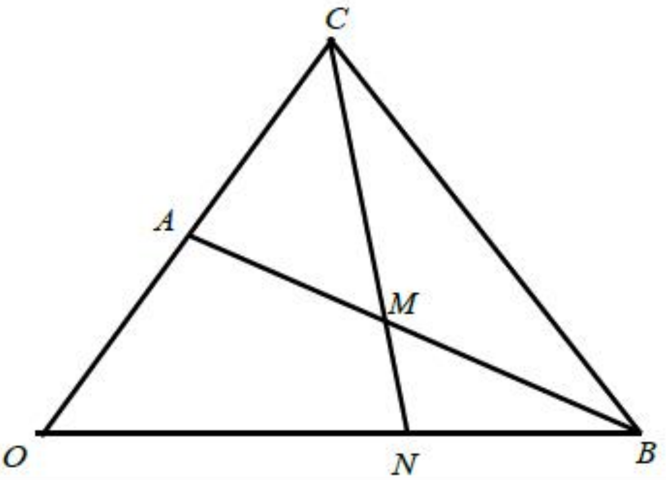
$$\left|\overrightarrow{AP}\right| = 2\left|\overrightarrow{BP}\right| \Rightarrow (x-2)^2 + (3x-6)^2 + (x-2)^2 = 4\left[(x-5)^2 + (3x-15)^2 + (x-5)^2\right] \Rightarrow x = 4, 8$$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}\left(\text{or } \overrightarrow{OB} + \overrightarrow{BP}\right) = \begin{pmatrix} x \\ 3x-9 \\ x+3 \end{pmatrix} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 7 \end{pmatrix} \text{ or } \begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$$



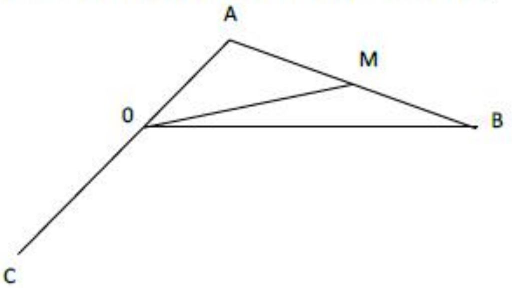

(**Q07 9MA0/02, June 2024**)

**Q2.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| |  | | |
| | $\overrightarrow{OA} = \mathbf{a},\ \overrightarrow{OB} = \mathbf{b}$ | | |
| (a) | $\left\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB} \Rightarrow\right\}\ \overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ | M1 | 3.1a |
| | $\left\{\overrightarrow{CM} = \overrightarrow{CB} + \overrightarrow{BM} = \overrightarrow{CB} + \frac{1}{2}\overrightarrow{BA} \Rightarrow\right\}\ \overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$ | | |
| | $\Rightarrow \overrightarrow{CM} = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ *(needs to be simplified and seen in (a) only)* | A1 | 1.1b |
| | | (2) | |
| (b) | $\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$ | M1 | 1.1b |
| | $\overrightarrow{ON} = 2\mathbf{a} + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$ * | A1* | 2.1 |
| | | (2) | |
| (c) Way 1 | $\left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = ...$ | M1 | 2.2a |
| | $\lambda = \frac{4}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ * | A1* | 2.1 |
| | | (2) | |
| (c) Way 2 | $\overrightarrow{ON} = \mu\mathbf{b} \Rightarrow \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} = \mu\mathbf{b}$ | | |
| | $\mathbf{a}:\ \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = ...\ \left\{\mathbf{b}:\ \frac{1}{2}\lambda = \mu\ \&\ \lambda = \frac{4}{3} \Rightarrow \mu = \frac{2}{3}\right\}$ | M1 | 2.2a |
| | $\lambda = \frac{4}{3}$ or $\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ * | A1* | 2.1 |
| | | (2) | |
| | | | (6 marks) |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (c) Way 3 | $\overrightarrow{OB} = \overrightarrow{ON} + \overrightarrow{NB} \Rightarrow \mathbf{b} = \left(2-\frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} + K\mathbf{b}$ | | |
| | $\mathbf{a}: \left(2-\frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = \ldots \left\{ \mathbf{b}: 1 = \frac{1}{2}\lambda + K \ \& \ \lambda = \frac{4}{3} \Rightarrow K = \frac{1}{3} \right\}$ | M1 | 2.2a |
| | $\lambda = \frac{4}{3}$ or $K = \frac{1}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ or $\overrightarrow{NB} = \frac{1}{3}\mathbf{b} \Rightarrow ON : NB = 2:1$ * | A1 | 2.1 |
| | | **(2)** | |
| (c) Way 4 | $\overrightarrow{ON} = \mu\mathbf{b}$ & $\overrightarrow{CN} = k\overrightarrow{CM} \Rightarrow \overrightarrow{CO} + \overrightarrow{ON} = k\overrightarrow{CM}$ | | |
| | $-2\mathbf{a} + \mu\mathbf{b} = k\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$ | | |
| | $\mathbf{a}: -2 = -\frac{3}{2}k \Rightarrow k = \frac{4}{3}, \quad \mathbf{b}: \ \mu = \frac{1}{2}k \Rightarrow \mu = \frac{1}{2}\left(\frac{4}{3}\right) = \ldots$ | M1 | 2.2a |
| | $\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \left\{ \Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON : NB = 2:1$ * | A1 | 2.1 |
| | | **(2)** | |

| Notes for Question | |
|---|---|
| (a) | |
| M1: | Valid attempt to find $\overrightarrow{CM}$ using a combination of known vectors a and b |
| A1: | A simplified correct answer for $\overrightarrow{CM}$ |
| Note: | Give M1 for $\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$<br>or for $\left\{\overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC} \Rightarrow\right\}$ $\overrightarrow{CM} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) - 2\mathbf{a}$ **only o.e.** |
| (b) | |
| M1: | Uses $\overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$ |
| A1*: | Correct proof |
| Note: | <u>Special Case</u><br>Give SC M1 A0 for the solution $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \lambda\overrightarrow{CM}$<br>$\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$ $\left\{= \left(\frac{1}{2} - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\lambda\right)\mathbf{b}\right\}$ |
| Note: | <u>**Alternative 1:**</u><br>Give M1 A1 for the following alternative solution:<br>$\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \mu\overrightarrow{CM}$<br>$\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \mu\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = \left(\frac{1}{2} - \frac{3}{2}\mu\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\mu\right)\mathbf{b}$<br>$\mu = \lambda - 1 \Rightarrow \quad \overrightarrow{ON} = \left(\frac{1}{2} - \frac{3}{2}(\lambda - 1)\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}(\lambda - 1)\right)\mathbf{b} \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$ |
| (c) | **Way 1, Way 2 and Way 3** |
| M1: | Deduces that $\left(2 - \frac{3}{2}\lambda\right) = 0$ and attempts to find the value of $\lambda$ |
| A1*: | Correct proof |
| (c) | **Way 4** |
| M1: | Complete attempt to find the value of $\mu$ |
| A1*: | Correct proof |

| Notes for Question Continued | |
|---|---|
| Note: | Part (b) and part (c) can be marked together. |
| (a) Special Case | <u>**Special Case where the point *C* is believed to be below the origin *O***</u><br><br>Give Special Case M1 A0 in part (a) for $\left\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} \Rightarrow\right\}$ $\overrightarrow{CM} = 3\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$<br>$\left\{\text{which leads to } \overrightarrow{CM} = \frac{5}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right\}$ |

**Q3.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}, \overrightarrow{OB}$ or $\overrightarrow{AB}$ E.g. $(-24\mathbf{i}-10\mathbf{j})=-2\times(12\mathbf{i}+5\mathbf{j})$ | M1 | 1.1b |
| | Explains that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ (and the stone is travelling in a straight line) the stone passes through the point $O$. | A1 | 2.4 |
| | | **(2)** | |
| (b) | Attempts distance $AB=\sqrt{(12+24)^2+(10+5)^2}$ | M1 | 1.1b |
| | Attempts speed $=\dfrac{\sqrt{(12+24)^2+(10+5)^2}}{4}$ | dM1 | 3.1a |
| | Speed $=$ 9.75 ms$^{-1}$ | A1 | 3.2a |
| | | **(3)** | |
| | | | **(5 marks)** |
| **Alt(a)** | Attempts to find the equation of the line which passes through $A$ and $B$ E.g. $y-5=\dfrac{5+10}{12+24}(x-12)$ $\left(y=\dfrac{5}{12}x\right)$ | M1 | 1.1b |
| | Shows that when $x=0$, $y=0$ and concludes the stone passes through the point $O$. | A1 | 2.4 |

Online Maths

**Notes**

**(a)**

**M1:** Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}$, $\overrightarrow{OB}$ or $\overrightarrow{AB}$ either way around.

E.g. States that $(-24\mathbf{i}-10\mathbf{j})=-2\times(12\mathbf{i}+5\mathbf{j})$

Alternatively, allow an attempt finding the gradient using any two of $AO$, $OB$ or $AB$

Alternatively attempts to find the equation of the line through $A$ and $B$ proceeding as far as $y=...x$ Condone sign slips.

**A1:** States that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ or as $AO$ is parallel to $OB$ (and the stone is travelling in a straight line) the stone passes through the point $O$. Alternatively, shows that the point $(0,0)$ is on the line and concludes (the stone) passes through the point $O$.

**(b)**

**M1:** Attempts to find the distance $AB$ using a correct method.

Condone slips but expect to see an attempt at $\sqrt{a^2+b^2}$ where $a$ or $b$ is correct

**dM1:** Dependent upon the previous mark. Look for an attempt at $\frac{\text{distance } AB}{4}$

**A1:** $9.75\text{ ms}^{-1}$ Requires units

(**Q04 8MA0/01, Oct 2021**)

Online Maths Teaching

**Q4.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $\overrightarrow{PQ}=(3-9)\mathbf{i}+(-5+8)\mathbf{j}$ | M1 | 1.1a |
| | $=-6\mathbf{i}+3\mathbf{j}$ | A1 | 1.1b |
| | | (2) | |
| (b) | Gradient of $PQ=\frac{-5--8}{3-9}\left(=-\frac{1}{2}\right)$ and Gradient of $QR=\frac{18}{9}(=2)$<br>or<br>$\left\|\overrightarrow{PQ}\right\|=\sqrt{"(-6)"^2+"3"^2}\ \left(=3\sqrt{5}\right)$ and<br>$\left\|\overrightarrow{QR}\right\|=\sqrt{9^2+18^2}\ (=9\sqrt{5})$ and $\left\|\overrightarrow{PR}\right\|=\sqrt{"3"^2+"21"^2}\ (=15\sqrt{2})$ | M1 | 3.1a |
| | e.g. shows that $-\frac{1}{2}\times 2=-1$ and deduces angle $PQR=90°$ *<br>or<br>e.g. shows $\left\|\overrightarrow{PQ}\right\|^2+\left\|\overrightarrow{QR}\right\|^2=\left\|\overrightarrow{PR}\right\|^2$ and deduces angle $PQR=90°$ * | A1* | 2.4 |
| | | (2) | |
| (c) | Attempts to find the length $PQ$ and at least one of $QR$ or $PS$ using Pythagoras' Theorem correctly<br>e.g. $\left\|\overrightarrow{PQ}\right\|=\sqrt{"(-6)"^2+"3"^2}$ **and**<br>**either** $\left\|\overrightarrow{QR}\right\|=\sqrt{9^2+18^2}$ **or** $\left\|\overrightarrow{PS}\right\|=\sqrt{"27"^2+"54"^2}$ | M1 | 2.1 |
| | $\left\|\overrightarrow{PQ}\right\|="\sqrt{45}"\ \left(=3\sqrt{5}\right)$ **and**<br>**either** $\left\|\overrightarrow{QR}\right\|=\sqrt{405}\ \left(=9\sqrt{5}\right)$ **or** $\left\|\overrightarrow{PS}\right\|=27\sqrt{5}$ | A1ft | 1.1b |
| | e.g. Area $=\frac{1}{2}\times\left("9\sqrt{5}"+"27\sqrt{5}"\right)\times"\sqrt{45}"$ or $\frac{1}{2}\times 4\times"9\sqrt{5}"\times"3\sqrt{5}"$ | dM1 | 3.1a |
| | $=270$ | A1 | 1.1b |
| | | (4) | |
| | | **(8 marks)** | |

| Notes |
|---|
| **Note that work seen must be used in the relevant part. If there is a lack of labelling of parts then award the marks to the parts which leads to the highest total overall.** |
| (a) |
| M1: Attempts subtraction either way round (does not need to be evaluated). It cannot be awarded for adding the two vectors. If no method shown it may be implied by one correct component or sight of $\mp 6\mathbf{i} \pm 3\mathbf{j}$. |
| A1: Correct answer. Allow $-6\mathbf{i}+3\mathbf{j}$ or $\begin{pmatrix}-6\\3\end{pmatrix}$ but do not allow $\begin{pmatrix}-6\mathbf{i}\\3\mathbf{j}\end{pmatrix}$ isw once a correct answer is seen. |
| **(b) Condone lack of labelling / poor notation for lengths/angles provided the intention is clear** |
| M1: Attempts to find the gradient of the line *PQ* and the gradient of the line *QR*. If they find the reciprocals of **both** they must be labelled e.g. $\frac{dx}{dy}$ o.e. (but not gradient or *m*) |
| Do not allow sign slips for this mark. Alternatively they may find the lengths *PQ*, *QR* and *PR* or $PQ^2$, $QR^2$ and $PR^2$ |

Be aware of Further Maths methods such as attempting the dot product

$$\begin{pmatrix}-6\\3\end{pmatrix}\begin{pmatrix}9\\18\end{pmatrix}=(-6\times9)+(3\times18)$$

A1*: **Correct working** and conclusion that angle $PQR = 90°$

- Using gradients or their reciprocals they need to state or show that the product is equal to -1 o.e. or refer to the values being negative reciprocals of each other
- Using Pythagoras' Theorem they must state or show that $\left|\overrightarrow{PQ}\right|^2+\left|\overrightarrow{QR}\right|^2=\left|\overrightarrow{PR}\right|^2$
- Using the cosine rule and finding angle $PQR = 90°$
- Using the scalar dot product they must show that $\begin{pmatrix}-6\\3\end{pmatrix}\begin{pmatrix}9\\18\end{pmatrix}=0$

In all cases there must be some sort of minimal conclusion that angle $PQR = 90°$ e.g. "hence right angle" or if they start with a preamble it is acceptable to state "hence proven", "QED" or a tick. Use of e.g. cosine rule resulting in $90°$ is sufficient.

**(c) Condone lack of labelling / poor notation for lengths provided the intention is clear**

M1: Correct use of Pythagoras' Theorem to find the length of $PQ$ and at least one of $QR$ or $PS$. Must be used or seen in (c) to score this mark. Condone working using rounded or truncated values.

A1ft: Correct length of $PQ$ and at least one of $QR$ or $PS$. Follow through on their vectors for $PQ$ and $PS$ but $QR$ must be $\sqrt{405}$ or equivalent. Lengths do not need to be simplified but they must be exact. Must be used or seen in (c) to score this mark.

dM1: Correct method to find the area of the trapezium. It is dependent on the first method mark and the method to find any lengths must be correct.

This may be achieved by calculating $\frac{1}{2}\times4\left|QR\right|\times\left|PQ\right|$

Alternatively, they may find the area of a rectangle + triangle so look for:

e.g. $|PQ|\times|QR|+\frac{1}{2}\times(|PS|-|QR|)\times|PQ|=\text{"}\sqrt{45}\text{"}\times\text{"}9\sqrt{5}\text{"}+\frac{1}{2}\times\text{"}18\sqrt{5}\text{"}\times\text{"}\sqrt{45}\text{"}$

Note that there are other combinations of lengths to find the area of a rectangle and either add or subtract triangles as appropriate. Condone working using rounded values.

A1: 270

**Alt (c) "Shoelace method" or other methods using position vectors**

M1: Correct method to find either the position vector of $R$ or the position vector of $S$. May be seen as coordinates. Check any diagram drawn.

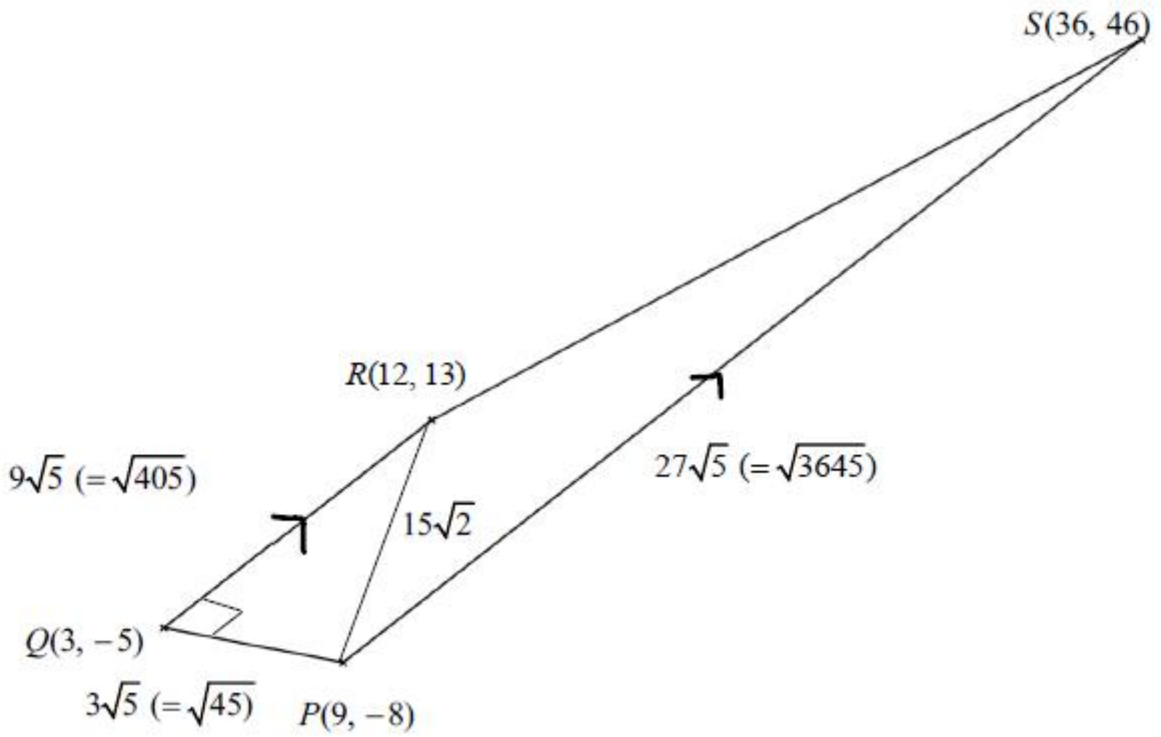
A1: $R$ has position vector $12\mathbf{i}+13\mathbf{j}$ and $S$ has position vector $36\mathbf{i}+46\mathbf{j}$ (or equivalent). May be seen as coordinates. Check any diagram drawn.

dM1: Correct method to find the area of the trapezium via the "shoelace" method:

$$\frac{1}{2}\begin{vmatrix} 9 & -8 \\ 3 & -5 \\ 12 & 13 \\ 36 & 46 \\ 9 & -8 \end{vmatrix} = \frac{1}{2}\left|(9\times(-5)+3\times 13+12\times 46+36\times(-8))-(3\times(-8)+12\times(-5)+36\times 13+9\times 46)\right|$$

$$=\frac{1}{2}|258-798|$$

A1: 270




(**Q03 8MA0/01, June 2024**)

**Q5.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Attempts to add $\overrightarrow{PQ} = 2\mathbf{i}+8\mathbf{j}-2\mathbf{k}$ and $\overrightarrow{QR} = 6\mathbf{i}+6\mathbf{k}$ | M1 | 1.1b |
| | $\left(\overrightarrow{PR} =\right)\ 8\mathbf{i}+8\mathbf{j}+4\mathbf{k}$ | A1 | 1.1b |
| | | **(2)** | |
| (b) | Attempts to show the triangle is isosceles (or right-angled)<br>e.g. Attempts $\left\|\overrightarrow{PQ}\right\| = \sqrt{2^2+8^2+(-2)^2}$ and $\left\|\overrightarrow{QR}\right\| = \sqrt{6^2+6^2}$ | M1 | 3.1a |
| | Shows e.g. $\left\|\overrightarrow{PQ}\right\| = \left\|\overrightarrow{QR}\right\| = \sqrt{72}\ (= 6\sqrt{2})$ | A1 | 1.1b |
| | Attempts to show the triangle is isosceles **and** right-angled<br>e.g. attempts to find the lengths of all three sides AND e.g. attempts to compare lengths via use of "$a^2+b^2=c^2$" | M1 | 1.1b |
| | e.g. Shows that $\left\|\overrightarrow{PQ}\right\|^2 + \left\|\overrightarrow{QR}\right\|^2 = \left\|\overrightarrow{PR}\right\|^2$ as $72+72=144$<br>so $PQR$ is a right-angled triangle | A1 | 2.1 |
| | | **(4)** | |
| | | | **(6 marks)** |

**Notes:**

**(a)** If part (a) is not attempted and the correct $\overrightarrow{PR}$ is seen in part (b) then M1A1 can be awarded

M1: Attempts to add $\overrightarrow{PQ} = 2\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{QR} = 6\mathbf{i} + 6\mathbf{k}$ with at least one correct component of $8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$. A typical misread of $\overrightarrow{QR}$ as $6\mathbf{i} + 6\mathbf{j}$ can score for at least one correct component of $8\mathbf{i} + 14\mathbf{j} - 2\mathbf{k}$

A1: Correct <u>vector</u>. Allow $8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$ or $\begin{pmatrix} 8 \\ 8 \\ 4 \end{pmatrix}$ but **not** $\begin{pmatrix} 8\mathbf{i} \\ 8\mathbf{j} \\ 4\mathbf{k} \end{pmatrix}$ and **not** (8, 8, 4)

Condone $\begin{matrix} 8 \\ 8 \\ 4 \end{matrix}$ for $\begin{pmatrix} 8 \\ 8 \\ 4 \end{pmatrix}$ Do not apply isw here but award for e.g. $8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k} = \begin{pmatrix} 8\mathbf{i} \\ 8\mathbf{j} \\ 4\mathbf{k} \end{pmatrix}$

E.g. if they obtain $\overrightarrow{PR} = 8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$ and then say $\overrightarrow{PR} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ then award A0

**(b)** **Note that M1A0M1A1 is not possible. If they have an incorrect vector in part (a) then the maximum score is M1A1M1A0. A misread of $\overrightarrow{QR}$ as $6\mathbf{i} + 6\mathbf{j}$ in (b) can only score a maximum M1A1M1A0**

**They may attempt to show the triangle is either isosceles or right-angled in either order. You will need to look through their solution and award the order which scores most marks. Usually it will be isosceles first. Condone slips to be recovered.**

To show the triangle is isosceles they only need to show **two sides** (or **two angles**) are the same. They do not need to consider the other side to show it is isosceles.

M1: Attempts to show that the triangle is either isosceles (or right-angled). See table below.

A1: Fully shows that the triangle is isosceles (or right-angled). See table below. Allow slips in their method if recovered as long as they proceed to correct lengths or values. **A conclusion that the triangle is isosceles or right-angled is not required for this mark.**

| **Isosceles** | **Requirement for M1 examples** | **Requirement for A1 examples** |
|---|---|---|
| **Using lengths** | Attempts to find the length or length$^2$ of $PQ$ and $QR$: <br> $\left(\lvert\overrightarrow{PQ}\rvert=\right)\sqrt{2^2+8^2+(-2)^2}\ (=\sqrt{72}=6\sqrt{2})$ <br> or seen as <br> e.g. $2\sqrt{1^2+4^2+(-1)^2}\ (=2\sqrt{18})$ <br> $\left(\lvert\overrightarrow{QR}\rvert=\right)\sqrt{6^2+6^2}\ (=\sqrt{72}=6\sqrt{2})$ or may be seen as e.g. $3\sqrt{2^2+2^2}\ (=3\sqrt{8})$ <br><br> May be implied by e.g. $6\sqrt{2}$ <br> Condone missing brackets around $(-2)^2$ provided the intention is clear to square and add implied by e.g. $6\sqrt{2}$ | States or shows that <br> $\lvert\overrightarrow{PQ}\rvert=\lvert\overrightarrow{QR}\rvert\ \left(=\sqrt{72}\,(=6\sqrt{2})\right)$, or equivalent. <br> Accept e.g. $PQ^2=QR^2$ or "both are 72" <br> Condone poor notation and/or labelling of lengths provided they are not clearly referring to the longest length. <br> e.g. achieves $6\sqrt{2}$ for both $PQ$ and $QR$ and states they are the same scores M1A1 <br> **Only stating isosceles without a comparison of *PQ* and *QR* is A0** |
| | | Uses the sine rule with the lengths and angles embedded in the correct places <br> e.g. $\dfrac{\sin\angle QPR}{QR}=\dfrac{\sin\angle PRQ}{PQ}$ <br> Deduces $\sin\angle QPR=\sin\angle PRQ$ so the angles are the same o.e. |
| **Right-angled** | **Requirement for M1 examples** | **Requirement for A1 examples** |

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| | | |
|---|---|---|
| **Using lengths** | Attempts to find all three lengths or lengths$^2$ $\left(\lvert\overrightarrow{PR}\rvert=\right)\sqrt{"8"^2+"8"^2+"4"^2}\ (=12)$ or may be seen as e.g. $4\sqrt{"2"^2+"2"^2+"1"^2}$ or implied by their 12 **AND ATTEMPTS** $\left(PQ^2+QR^2=PR^2\Rightarrow\right)"72"+"72"="144"$ | States or shows that $\lvert PQ\rvert^2+\lvert QR\rvert^2=\lvert PR\rvert^2$, or equivalent. Condone poor notation and/or labelling of lengths. |
| | Attempts to find all three lengths or lengths$^2$ (see above for guidance) **AND ATTEMPTS** the cosine rule correctly to find $\cos PQR=\frac{"72"+"72"-"144"}{2\times"\sqrt{72}"\times"\sqrt{72}"}$ o.e. | States or shows $\cos PQR=0$ |
| **Scalar dot product (Further Maths)** | Attempts e.g. $\begin{pmatrix}2\\8\\-2\end{pmatrix}\bullet\begin{pmatrix}6\\0\\6\end{pmatrix}=(2\times6)+(8\times0)+(-2\times6)$ oe **Must see the calculation for M1** | States or shows that $(2\times6)+(8\times0)+(-2\times6)=0$ oe (they do not need to write anything more for this mark) |
| | $\left(\cos PQR=\right)\frac{(2\times6)+(8\times0)+(-2\times6)}{"\sqrt{72}"\times"\sqrt{72}"}$ oe | $\cos PQR=\frac{(2\times6)+(8\times0)+(-2\times6)}{\sqrt{72}\times\sqrt{72}}=0$ |

M1: Attempts to show that the triangle is both isosceles **and** right-angled. Usually they will have shown the triangle is isosceles and attempt to show that it is right-angled. **Here are some examples but there will be others:**

| Right-angled | Method examples (required for second method mark) |
|---|---|
| **Using lengths** | Attempts to find all three lengths or lengths$^2$ **AND ATTEMPTS** $\left(PQ^2+QR^2=PR^2\Rightarrow\right)"72"+"72"="144"$ |
| | Attempts to find all three lengths or lengths$^2$ (see above for guidance) **AND ATTEMPTS** the cosine rule in an attempt to find $\cos PQR=\frac{"72"+"72"-"144"}{2\times"\sqrt{72}"\times"\sqrt{72}"}$ |
| **Scalar dot product** | Attempts $\begin{pmatrix}2\\8\\-2\end{pmatrix}\bullet\begin{pmatrix}6\\0\\6\end{pmatrix}=(2\times6)+(8\times0)+(-2\times6)$ |
| | $\left(\cos PQR=\right)\frac{(2\times6)+(8\times0)+(-2\times6)}{"\sqrt{72}"\times"\sqrt{72}"}$ oe |

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**Alternatively, having already shown the triangle is right-angled they may:**

- use trigonometry to show that the two base angles are both $45°$
- attempt to find the required lengths or lengths$^2$ of $PQ$ and $PR$ if not already found (Note they may also find the length or length$^2$ of $QR$ and may use the sine rule or cosine rule)

---

**Note** if they have shown either property then we will condone making an assumption of the other property to justify the size of angle $PRQ$ and/or angle $QPR$ and may use trigonometry to show either

$$\left(\sin\theta = \frac{6\sqrt{2}}{12} \Rightarrow\right)\theta = \arcsin\left(\frac{6\sqrt{2}}{12}\right) = 45° \quad \text{(we must see arcsin or } \sin^{-1}\text{)}$$

$$\sin\theta = \frac{6\sqrt{2}}{12} \Rightarrow \sin\theta = \frac{\sqrt{2}}{2}\left(\text{or } \frac{1}{\sqrt{2}}\right) \Rightarrow \theta = 45° \quad \text{(since this is a known angle)}$$

$$\sin 45° = \frac{\sqrt{2}}{2}\left(\text{or } \frac{1}{\sqrt{2}}\right) = \frac{6\sqrt{2}}{12} \quad \text{(This requires all 3 equivalences (can be in any order))}$$

**Using trigonometry (SOHCAHTOA) to show the second property they will e.g.**

- Use triangle $PQR$ and assume right angled
- Split triangle $PQR$ into two right angled congruent triangles using the isosceles property

**If they use triangle *PQR* then it requires**

- For M1: Calculations showing the required property to score M1
- For A1: To either draw a labelled right angled triangle or state the assumption that it is right angled and conclude

**If they split the triangle *PQR* in half** then they have formed two right angled congruent triangles which (provided they had two lengths the same already) will not be an assumption of a right angle. **Then it requires**

- For M1: Calculations to find either at least one of the base angles of 45° OR to find the right angle. It should be clear whether they
  - found a base angle of PQR and doubled it or
  - found half of the right angle and doubled it.
- For A1: Fully correct calculations and conclusion
  Note if they find a 45 degree angle and double it then it needs to be clear whether this is the right angle or if it is the sum of two base angles because there are four angles of size 45 degrees via this method.
  They would have to e.g. mention about the angle sum of a triangle oe or show a clearly labelled diagram and calculations with labels that match the diagram.

  Note that use of tan is unlikely to score because if they just use two equal lengths for the two shorter sides of their right angled triangle, then $\tan(A)=1$, so the angles will always be 45°, 45° and 90° – it is the use of the length of 12 (or 6) which is going to lead to showing the triangle is right angled via these routes.

A1: Fully shows that the triangle is isosceles and right-angled **and concludes** that the triangle is **both isosceles and right-angled**. These conclusions may appear at separate stages of their solution. Condone poor notation and/or labelling of lengths provided the intention is clear. Condone slips if recovered. If they have a preamble then they must have a minimal conclusion e.g. proven, tick, QED

**Note if they have an incorrect vector in part (a) then the maximum score is M1A1M1A0**

(**Q10 9MA0/01, June 2025**)

**Q6.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Attempts $\overrightarrow{OM}$ and $\overrightarrow{ON}$ and subtracts to find $\pm\overrightarrow{MN}$<br>E.g. $\overrightarrow{OM} = 4\mathbf{i} + \frac{3}{2}\mathbf{j}$<br>$\overrightarrow{ON} = 3\mathbf{j} + \mathbf{k}$<br>$\left(\overrightarrow{MN}\right) = (3\mathbf{j}+\mathbf{k}) - \left(4\mathbf{i} + \frac{3}{2}\mathbf{j}\right) = -4\mathbf{i} + \frac{3}{2}\mathbf{j} + \mathbf{k}$ | M1 | 1.1b |
| | Uses the given information to find either $\pm\overrightarrow{MP}$ or $\pm\overrightarrow{PN}$<br>Either $\overrightarrow{MP} = \frac{3}{4}\overrightarrow{MN} = \frac{3}{4}\left(-4\mathbf{i} + \frac{3}{2}\mathbf{j} + \mathbf{k}\right) = -3\mathbf{i} + \frac{9}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}$<br>Or $\overrightarrow{PN} = \frac{1}{4}\overrightarrow{MN} = \frac{1}{4}\left(-4\mathbf{i} + \frac{3}{2}\mathbf{j} + \mathbf{k}\right) = -\mathbf{i} + \frac{3}{8}\mathbf{j} + \frac{1}{4}\mathbf{k}$ | M1<br>A1 | 3.1a<br>1.1b |
| | $\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = \left(4\mathbf{i} + \frac{3}{2}\mathbf{j}\right) + \left(-3\mathbf{i} + \frac{9}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}\right) = \mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}$ *<br>or e.g.<br>$\overrightarrow{OP} = \overrightarrow{ON} + \overrightarrow{NP} = (3\mathbf{j}+\mathbf{k}) + \left(\mathbf{i} - \frac{3}{8}\mathbf{j} - \frac{1}{4}\mathbf{k}\right) = \mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}$ * | A1* | 2.1 |
| | | **(4)** | |
| (b) | $\overrightarrow{OQ} = \frac{8}{7}\left(\mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}\right) = \ldots$ | M1 | 3.1a |
| | Coordinates $Q = \left(\frac{8}{7}, 3, \frac{6}{7}\right)$ | A1 | 3.2a |
| | | **(2)** | |
| | | | **(6 marks)** |

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**Notes:**

(a)

**M1:** Attempts to find $\overrightarrow{MN}$ (ignore labelling for this mark)

**M1:** For the key step in using $\overrightarrow{MP} = 3\overrightarrow{PN}$ in a correct attempt to find $\pm\overrightarrow{MP}$ or $\pm\overrightarrow{PN}$

**A1:** Correct $\pm\overrightarrow{MP}$ or $\pm\overrightarrow{PN}$

**A1*:** Completes proof showing all key steps to obtain $\mathbf{i}+\frac{21}{8}\mathbf{j}+\frac{3}{4}\mathbf{k}$ or e.g. $\begin{pmatrix} 1 \\ \frac{21}{8} \\ \frac{3}{4} \end{pmatrix}$ but not $\begin{pmatrix} \mathbf{i} \\ \frac{21}{8}\mathbf{j} \\ \frac{3}{4}\mathbf{k} \end{pmatrix}$

(b)

**M1:** Attempts $\overrightarrow{OQ} = \frac{8}{7}\left(\mathbf{i}+\frac{21}{8}\mathbf{j}+\frac{3}{4}\mathbf{k}\right) = \ldots$ May be implied by a correct vector or point.

**A1:** Deduces that $Q = \left(\frac{8}{7}, 3, \frac{6}{7}\right)$.

Do not allow as a vector unless the correct coordinates are seen first then isw.

(**Q12 9MA0/02/M, June 2025**)

**Q7.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $\left(\overrightarrow{AD}=\overrightarrow{AB}-\overrightarrow{DB}=\right)\ 2\mathbf{a}+3\mathbf{b}-(-4\mathbf{a}+k\mathbf{b})\ \ \left(=6\mathbf{a}+(3-k)\mathbf{b}\right)$ | M1 | 1.1b |
| | $\frac{15}{6}=2.5\Rightarrow\frac{-5}{3-k}=2.5\rightarrow k=\ldots$ | dM1 | 1.1b |
| | $k=5$ * | A1* | 2.1 |
| | | (3) | |

**Notes**

(a) **Note: Condone the use of column vectors throughout this question. There may be working on the diagram that can be awarded marks.**

M1: Attempts either $\left(\overrightarrow{AD}=\overrightarrow{AB}-\overrightarrow{DB}=\right)\ 2\mathbf{a}+3\mathbf{b}-(-4\mathbf{a}+k\mathbf{b})$ or $\left(\overrightarrow{DA}=\right)\ -4\mathbf{a}+k\mathbf{b}-(2\mathbf{a}+3\mathbf{b})$ or $\left(\overrightarrow{DB}=\overrightarrow{DA}+\overrightarrow{AB}=\right)\ \alpha(15\mathbf{a}-5\mathbf{b})+2\mathbf{a}+3\mathbf{b}$

Allow subtraction either way round and may be implied by one correct component or by e.g. $2\mathbf{a}+3\mathbf{b}=\overrightarrow{AD}-4\mathbf{a}+k\mathbf{b}$

For reference: $\overrightarrow{AD}=6\mathbf{a}+(3-k)\mathbf{b}$ and $\overrightarrow{DA}=-6\mathbf{a}+(k-3)\mathbf{b}$

Allow e.g. $\left(\overrightarrow{AD}=\right)\begin{pmatrix}6\mathbf{a}\\(3-k)\mathbf{b}\end{pmatrix}$ or $\left(\overrightarrow{AD}=\right)\begin{pmatrix}6\\3-k\end{pmatrix}$ including without the brackets $\begin{matrix}6\\3-k\end{matrix}$

Condone the use of gradients or ratios for $\overrightarrow{AD}$ e.g. $\frac{6}{3-k}$ or "$6:3-k$" to imply this mark (either way round).

There are alternatives using e.g. $\overrightarrow{DC}$ but, in these cases, we require two expressions for the same vector, of which one expression must use $\overrightarrow{DB}$.

dM1: A full method to solve the problem. Some possible approaches:

- attempts to find a scale factor and uses it to find $k$
- sets up equivalent fractions e.g. $\frac{15}{6}=-\frac{5}{3-k}$ or e.g. $\frac{2--4}{3-k}=\frac{15}{-5}$ and solves for $k$
- sets up equivalent ratios e.g. $6:15=3-k:-5$ and solves for $k$
- sets up simultaneous equations and solves for $k$ e.g. $2\mathbf{a}+3\mathbf{b}+4\mathbf{a}-k\mathbf{b}=\alpha(15\mathbf{a}-5\mathbf{b})$ o.e. or $\left(\overrightarrow{DB}=\overrightarrow{DA}+\overrightarrow{AB}=\right)\ \alpha(-15\mathbf{a}+5\mathbf{b})+2\mathbf{a}+3\mathbf{b}=-4\mathbf{a}+k\mathbf{b}$ leading to $6=15\alpha$ $\left(\alpha=\frac{2}{5}\right)$ and $3-k=-5\alpha$ hence $k=\ldots$

Note that their coefficient might be the reciprocal, from e.g. $\beta(2\mathbf{a}+3\mathbf{b}+4\mathbf{a}-k\mathbf{b})=15\mathbf{a}-5\mathbf{b}$ $(\beta=2.5)$ and directions might be reversed in which case $\alpha=-0.4$ or $\beta=-2.5$ can be used.

A1*: Arrives at $k = 5$ via a correct method. Usually this will be following:

- A correct expression for $\overrightarrow{AD}$ (e.g. $2\mathbf{a}+3\mathbf{b}-(-4\mathbf{a}+k\mathbf{b})$) or $\overrightarrow{DA}$ or $\overrightarrow{BD}$ or $\overrightarrow{DB}$ which may be mislabelled.
- Correct scale factor stated (±2.5 or ±0.4) or implied (e.g., by $\frac{-5}{3-k}=\frac{15}{6}$ or $3(3-k)=-6$)
- A correct intermediate equation that leads to $k = 5$

  An example minimal response might look like:

  e.g. $\alpha(15\mathbf{a}-5\mathbf{b})=6\mathbf{a}+(3-k)\mathbf{b} \rightarrow 6=15\alpha \rightarrow \alpha=\frac{2}{5} \rightarrow -5\left(\frac{2}{5}\right)=3-k \rightarrow k=5$

  or e.g. $6\mathbf{a}+(3-k)\mathbf{b} \rightarrow \frac{15}{6}=-\frac{5}{3-k} \rightarrow k=5$

  Condone missing/invisible brackets if recovered.

**Alternative by verification:**

M1: Sets $k = 5$, substitutes into $\overrightarrow{DB}$ and attempts $\left(\overrightarrow{AD}=\overrightarrow{AB}-\overrightarrow{DB}=\right)$ $2\mathbf{a}+3\mathbf{b}-(-4\mathbf{a}+5\mathbf{b})$ o.e.

dM1: Attempts to compare $\overrightarrow{AD}$ o.e. with $\overrightarrow{BC}$ (usually $6\mathbf{a}-2\mathbf{b}=\alpha(15\mathbf{a}-5\mathbf{b})$) and finds $\alpha$

A1*: Requires:

- Correct $\overrightarrow{AD}$ o.e. e.g. $\overrightarrow{DA}$
- Correct scale factor $\alpha=\pm\frac{2}{5}$ or $\beta=\pm2.5$ (sign dependent on their approach)
- Conclusion referencing the lines being parallel e.g. "hence $\overrightarrow{AD}$ and $\overrightarrow{BC}$ are parallel."

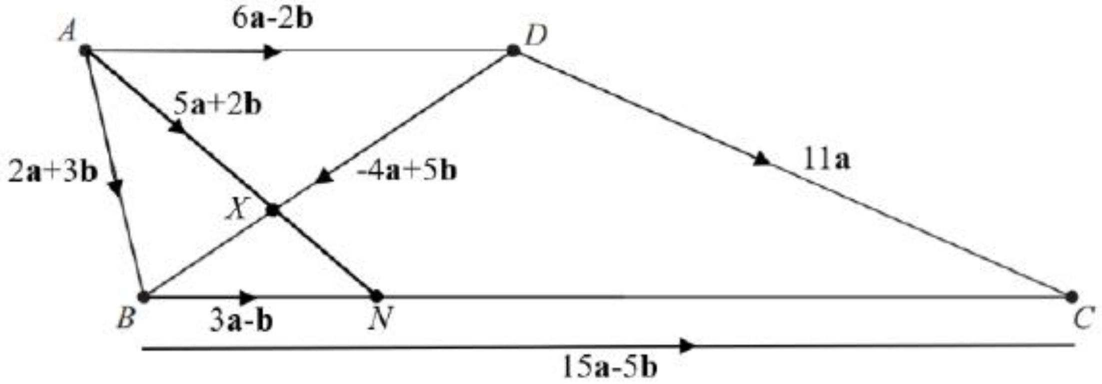
| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (b) | e.g., $\left(\overrightarrow{BN}=\right)\ \frac{1}{5}\overrightarrow{BC}\ (=3\mathbf{a}-\mathbf{b})$ | B1 | 2.2a |
| | e.g., $\left(\overrightarrow{BX}=\lambda\overrightarrow{BD}=\right)\ \lambda(4\mathbf{a}-5\mathbf{b})$ | M1 | 2.1 |
| | e.g., $\left(\overrightarrow{BX}=\lambda\overrightarrow{BD}=\right)\ \lambda(4\mathbf{a}-5\mathbf{b})$ **and** e.g. $\left(\overrightarrow{BX}=\overrightarrow{BA}+\mu\overrightarrow{AN}=\right)\ (-2\mathbf{a}-3\mathbf{b})+\mu(2\mathbf{a}+3\mathbf{b}+"3\mathbf{a}-\mathbf{b}")$ | dM1 | 3.1a |
| | $4\lambda=-2+5\mu$, $-5\lambda=-3+2\mu$ $\Rightarrow \lambda=\ldots\left(\frac{1}{3}\right)$ or $\mu=\ldots\left(\frac{2}{3}\right)$ | ddM1 | 1.1b |
| | $1:2$ | A1 | 2.2a |
| | | **(5)** | |
| | | **(8 marks)** | |

| Notes | |
|---|---|
| **(b)** | **Note: Condone the use of column vectors throughout this question.** **There may be working on the diagram that can be awarded marks.** |
| B1: | Deduces a correct interpretation of the ratio $BN:NC=1:4$ that enables a start to a solution. i.e., progresses to a correct statement that is at least as far as $\left(\overrightarrow{BN}=\right)\ \frac{1}{5}\overrightarrow{BC}$ (or e.g. $3\mathbf{a}-\mathbf{b}$) or $\left(\overrightarrow{CN}=\right)\ \frac{4}{5}\overrightarrow{CB}$ (or e.g. $4\mathbf{b}-12\mathbf{a}$). May be embedded in e.g. $\left(\overrightarrow{AN}=\right)\ 2\mathbf{a}+3\mathbf{b}+\frac{1}{5}\overrightarrow{BC}$ Allow e.g. $\frac{1}{5}\begin{pmatrix}15\\-5\end{pmatrix}$ or $\begin{pmatrix}3\\-1\end{pmatrix}$ or $\begin{pmatrix}3a\\-b\end{pmatrix}$ for this mark. |
| M1: | For the key step in attempting a valid expression for $\overrightarrow{BX}$ **(or** $\overrightarrow{AX}$ **or** $\overrightarrow{DX}$ **or** $\overrightarrow{CX}$ **or** $\overrightarrow{NX}$) in terms of **a** and **b**. See diagram/notes on the next page for helpful vectors. Condone slips provided their intention is clear. **Note that they might be using $\lambda$ and $\mu$ the other way round or alternative variables.** |
| **Note:** | May be seen as a single expression such as $\overrightarrow{AB}=\overrightarrow{AX}+\overrightarrow{XB}$ $\left(\text{i.e. } \overrightarrow{AB}=p\overrightarrow{AN}+q\overrightarrow{DB}\right)$ or $\overrightarrow{BN}=\overrightarrow{BX}+\overrightarrow{XN}$ $\left(\text{i.e. } \overrightarrow{BN}=p\overrightarrow{BD}+q\overrightarrow{AN}\right)$ either of which scores M1dM1 simultaneously. |
| dM1: | For the key step in attempting a **second** valid expression for their $\overrightarrow{BX}$ **(or** $\overrightarrow{AX}$ **or** $\overrightarrow{DX}$ **or** $\overrightarrow{CX}$ **or** $\overrightarrow{NX}$) in terms of **a** and **b** which enables the problem to be solved, i.e., it must not be parallel in approach to the first. See diagram/notes on the next page for helpful vectors. One expression should involve $"\lambda"(4\mathbf{a}-5\mathbf{b})$ and the other should involve $"\mu"("5\mathbf{a}+2\mathbf{b}")$ Condone slips provided their intention is clear. **They must use different parameters in their approaches, e.g., $\lambda$ and $\mu$.** If using e.g. $\overrightarrow{DX}=-6\mathbf{a}+2\mathbf{b}+\mu(5\mathbf{a}+2\mathbf{b})$ and $\overrightarrow{XB}=\lambda(-4\mathbf{a}+5\mathbf{b})$ this mark is not scored until they set $\overrightarrow{DX}+\overrightarrow{XB}=\overrightarrow{DB}$ as $-6\mathbf{a}+2\mathbf{b}+\mu(5\mathbf{a}+2\mathbf{b})+\lambda(-4\mathbf{a}+5\mathbf{b})=-4\mathbf{a}+5\mathbf{b}$ Dependent on the previous method mark. |
| ddM1: | Compares coefficients of **a** and **b** to create simultaneous equations in their parameters and attempts to solve (which may be by calculator) leading to a value for one of their parameters. Condone slips provided the intention is clear. This mark may be implied by a **correct** value for e.g. $\lambda$ following their two **correct** expressions for e.g. $\overrightarrow{BX}$ Dependent on both previous method marks. |
| A1: | $1:2$ o.e. Must follow a correct value for their parameter. The correct ratio seen does **not** imply full marks. Candidates must show detailed reasoning. |

Allow equivalent ratios e.g. $\frac{1}{3}:\frac{2}{3}$ and ISW (e.g. $1:3$) but they must be the correct way round.

There may be attempts using similar triangles. Send to review.

**Helpful Diagram:**




Note: Some examples of valid expressions for the M and dM marks for part (b) are:
In each expression they may use different parameters and e.g. $1-\lambda$ might just be e.g. $\phi$.

- $\overrightarrow{BX}=\lambda\overrightarrow{BD}=\lambda(4\mathbf{a}-5\mathbf{b})$
- $\overrightarrow{BX}=\overrightarrow{BA}+\mu\overrightarrow{AN}=(-2\mathbf{a}-3\mathbf{b})+\mu(2\mathbf{a}+3\mathbf{b}+\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{BX}=\overrightarrow{BN}+(1-\mu)\overrightarrow{NA}=(\text{"}3\mathbf{a}-\mathbf{b}\text{"})+(1-\mu)(-2\mathbf{a}-3\mathbf{b}+\text{"}-3\mathbf{a}+\mathbf{b}\text{"})$
- $\overrightarrow{DX}=(1-\lambda)\overrightarrow{DB}=(1-\lambda)(-4\mathbf{a}+5\mathbf{b})$
- $\overrightarrow{DX}=\overrightarrow{DA}+\mu\overrightarrow{AN}=(\text{"}-6\mathbf{a}+2\mathbf{b}\text{"})+\mu(2\mathbf{a}+3\mathbf{b}+\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{DX}=\overrightarrow{DN}+(1-\mu)\overrightarrow{NA}=(\text{"}-\mathbf{a}+4\mathbf{b}\text{"})+(1-\mu)(-2\mathbf{a}-3\mathbf{b}+\text{"}-3\mathbf{a}+\mathbf{b}\text{"})$
- $\overrightarrow{AX}=\mu\overrightarrow{AN}=\mu(2\mathbf{a}+3\mathbf{b}+\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{AX}=\overrightarrow{AB}+\lambda\overrightarrow{BD}=(2\mathbf{a}+3\mathbf{b})+\lambda(4\mathbf{a}-5\mathbf{b})$
- $\overrightarrow{AX}=\overrightarrow{AD}+(1-\lambda)\overrightarrow{DB}=(\text{"}6\mathbf{a}-2\mathbf{b}\text{"})+(1-\lambda)(-4\mathbf{a}+5\mathbf{b})$
- $\overrightarrow{XN}=\mu\overrightarrow{AN}=\mu(2\mathbf{a}+3\mathbf{b}+\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{XN}=\lambda\overrightarrow{DB}+\overrightarrow{BN}=\lambda(-4\mathbf{a}+5\mathbf{b})+(\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{XN}=(1-\lambda)\overrightarrow{BD}+\overrightarrow{DN}=(1-\lambda)(4\mathbf{a}-5\mathbf{b})+(\text{"}-\mathbf{a}+4\mathbf{b}\text{"})$

or alternatives using $C$ or $N$ as starting points, but these are unlikely.

(**Q14 9MA0/02, June 2025**)

**Q8.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $\left(\lvert\overrightarrow{OB}\rvert=\right)\sqrt{(...)^2+(\pm2)^2}$ or $\left(\lvert\overrightarrow{OB}\rvert=\right)\sqrt{(\pm6)^2+(...)^2}$<br>or<br>$\left(\lvert\overrightarrow{OB}\rvert^2=\right)(...)^2+(\pm2)^2$ or $\left(\lvert\overrightarrow{OB}\rvert^2=\right)(\pm6)^2+(...)^2$ | M1 | 1.1b |
| | $\left(\lvert\overrightarrow{OB}\rvert=\right)\sqrt{(\pm2)^2+(\pm6)^2}$ | dM1 | 1.1b |
| | $\sqrt{40}$ or $2\sqrt{10}$ | A1 | 1.1b |
| | | **(3)** | |
| (b) | $\lvert\overrightarrow{OA}\rvert=\sqrt{73}$ or $\lvert\overrightarrow{AB}\rvert=\sqrt{29}$ | B1 | 1.1b |
| | $40=73+29-2\sqrt{73}\sqrt{29}\cos OAB \Rightarrow \cos OAB=\dfrac{73+29-40}{2\sqrt{73}\sqrt{29}}$<br>$\Rightarrow OAB=\cos^{-1}\left(\dfrac{73+29-40}{2\sqrt{73}\sqrt{29}}\right)$<br>or<br>$\cos OAB=\dfrac{73+29-40}{2\sqrt{73}\sqrt{29}}\Rightarrow OAB=...$ | M1 | 3.1a |
| | $OAB=47.6°$ | A1 | 1.1b |
| | | **(3)** | |
| | | **(6 marks)** | |

**Notes:**

**Note that marks in (a) can be scored in (b) as long as they are not contradictory.**

**Note that they are not asked for $\overrightarrow{OB}$ in (a) but $\lvert\overrightarrow{OB}\rvert$. As such, all they need are the magnitudes of the components of $\overrightarrow{OB}$ to find $\lvert\overrightarrow{OB}\rvert$ so you can ignore if $\overrightarrow{OB}$ is correct or not in both parts and full marks can be awarded even if there are sign errors in their $\overrightarrow{OB}$ if they write it as a vector.**

**(a)**

**M1:** Attempts $\lvert\overrightarrow{OB}\rvert$ or $\lvert\overrightarrow{OB}\rvert^2$ with one component correct and the other component non-zero.

Allow $\sqrt{(\pm2)^2+(...)^2}$ or $\sqrt{(\pm6)^2+(...)^2}$ or $(...)^2+(\pm2)^2$ or $(\pm6)^2+(...)^2$

and condone e.g. $-2^2$ or $-6^2$

But it must clearly not be an attempt at e.g. $\lvert\overrightarrow{AB}\rvert$ e.g. $\sqrt{5^2+2^2}$

**dM1:** Complete and correct method for $\lvert\overrightarrow{OB}\rvert$ i.e. $\lvert\overrightarrow{OB}\rvert=\sqrt{(\pm2)^2+(\pm6)^2}$

**A1:** $\sqrt{40}$ or $2\sqrt{10}$ only but isw if they then use decimals.

| | |
|---|---|
| **Beware in (b) that assuming $OAB$ is right angled can give answers that look approximately correct e.g.** $\sin OAB = \frac{OB}{AB} = \frac{\sqrt{40}}{\sqrt{73}} \Rightarrow OAB = \sin^{-1}\frac{\sqrt{40}}{\sqrt{73}} = 47.75...$ **but is an incorrect method.** | |

**In (b) mark the method that is most successful.**

**(b)** **Way 1: Cosine rule**

**B1:** Finds either of $\left|\overrightarrow{OA}\right| = \sqrt{73}$ or $\left|\overrightarrow{AB}\right| = \sqrt{29}$ allow for sight of these values even if not associated with a vector. They may be seen on a diagram or embedded in an attempt at the cosine rule. May be implied by decimal values (see diagram)

**M1:** A complete and correct method for finding angle $OAB$ with their $OA$, $OB$ and $AB$.
Correct attempt at the cosine rule leading to a value for angle $OAB$ using arccos.
Following the correct use of the cosine rule, if a value for angle $OAB$ is just written down or there is no evidence of arccos, you may need to check.
Following the correct use of the cosine rule, sufficient evidence could be e.g.
$\cos OAB = k \Rightarrow OAB = \cos^{-1} k = ...$

**A1:** awrt $47.6°$ Condone omission of degrees symbol.

**(b)** **Way 2: Right angled triangles**

**B1:** Finds any of $\tan^{-1}\left(\frac{8}{3}\right) = 69.4°$ , $\tan^{-1}\left(\frac{2}{5}\right) = 21.8°$ , $\tan^{-1}\left(\frac{3}{8}\right) = 20.6°$ , $\tan^{-1}\left(\frac{5}{2}\right) = 68.2°$
May be implied.

**M1:** A complete and correct method for finding angle $OAB$.
e.g. attempts $\tan^{-1}\left(\frac{5}{2}\right) - \tan^{-1}\left(\frac{3}{8}\right)$ or $\tan^{-1}\left(\frac{8}{3}\right) - \tan^{-1}\left(\frac{2}{5}\right)$ or $90° - \tan^{-1}\left(\frac{3}{8}\right) - \tan^{-1}\left(\frac{2}{5}\right)$
leading to a value for angle $OAB$.

**A1:** awrt $47.6°$ Condone omission of degrees symbol.

**(b)** **Way 3: Scalar product**

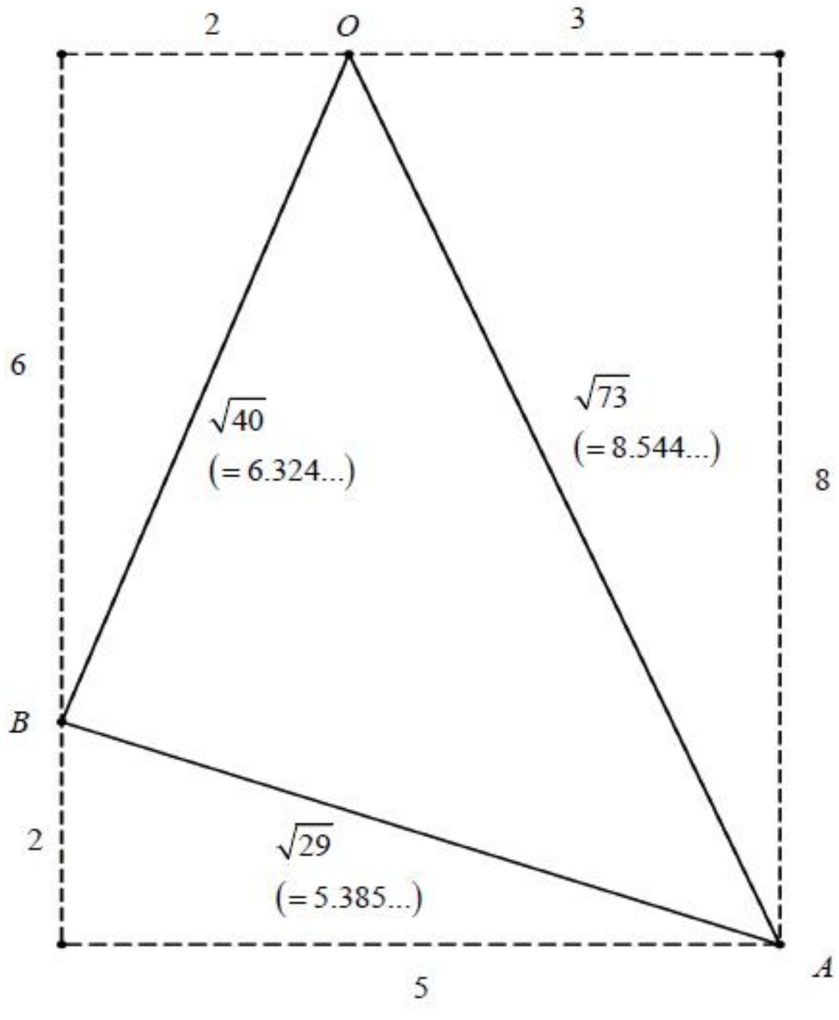
**B1:** Finds $\overrightarrow{AO} \bullet \overrightarrow{AB} = \begin{pmatrix} -3 \\ 8 \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 2 \end{pmatrix} = 15 + 16 = 31$
Allow $\pm\overrightarrow{AO} \bullet \pm\overrightarrow{AB} = \pm 31$

**M1:** A complete and correct method for finding angle $OAB$ with their $OA$, $OB$ and $AB$.
e.g. $31 = \left|\overrightarrow{AO}\right|\left|\overrightarrow{AB}\right| \cos OAB = \sqrt{3^2 + 8^2}\sqrt{5^2 + 2^2} \cos OAB \Rightarrow \cos OAB = \frac{31}{\sqrt{73}\sqrt{29}} \Rightarrow OAB = ...$
If e.g. $\overrightarrow{OA} \bullet \overrightarrow{AB}$ is attempted then they need to find e.g. $OAB = 180° - \cos^{-1}\frac{-31}{\sqrt{73}\sqrt{29}}$
Following the correct use of the scalar product, if a value for angle $OAB$ is just written down or there is no evidence of arccos, you may need to check.

**A1:** awrt $47.6°$ Condone omission of degrees symbol.

There may be other methods for finding angle $OAB$.

**For reference:**




(**Q03 8MA0/01, June 2025**)

**Q9.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| | $\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OC} = a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$, $a < 0$<br>$\overrightarrow{AB} = \overrightarrow{BD}$, $\lvert\overrightarrow{AB}\rvert = 4$ | | |
| (a) | E.g. $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB}$<br>or $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB} = \overrightarrow{OB} + \overrightarrow{OB} - \overrightarrow{OA} = 2\overrightarrow{OB} - \overrightarrow{OA}$<br>or $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AB}$ | | |
| | $= \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \left\{ = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -5 \\ 7 \end{pmatrix} \right\}$<br>or $= \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + 2\left(\begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}\right) \left\{ = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + 2\begin{pmatrix} 2 \\ -5 \\ 7 \end{pmatrix} \right\}$ | M1 | 3.1a |
| | $= \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$ or $6\mathbf{i} - 7\mathbf{j} + 10\mathbf{k}$ | A1 | 1.1b |
| | | (2) | |
| (b) | $(a-2)^2 + (5-3)^2 + (-2 - -4)^2$ | M1 | 1.1b |
| | $\left\{\lvert\overrightarrow{AC}\rvert = 4 \Rightarrow\right\} (a-2)^2 + (5-3)^2 + (-2 - -4)^2 = (4)^2$<br>$\Rightarrow (a-2)^2 = 8 \Rightarrow a = \ldots$ or $\Rightarrow a^2 - 4a - 4 = 0 \Rightarrow a = \ldots$ | dM1 | 2.1 |
| | (as $a < 0 \Rightarrow$) $a = 2 - 2\sqrt{2}$ (or $a = 2 - \sqrt{8}$) | A1 | 1.1b |
| | | (3) | |
| | | | (5 marks) |

| Notes for Question | |
|---|---|
| (a) | |
| M1: | Complete *applied* strategy to find a vector expression for $\overrightarrow{OD}$ |
| A1: | See scheme |
| Note: | Give M0 for subtracting the wrong way wrong to give e.g. $(4\mathbf{i}-2\mathbf{j}+3\mathbf{k})+(2\mathbf{i}+3\mathbf{j}-4\mathbf{k})\ -(4\mathbf{i}-2\mathbf{j}+3\mathbf{k})=(4\mathbf{i}-2\mathbf{j}+3\mathbf{k})+(-2\mathbf{i}+5\mathbf{j}-7\mathbf{k})=(2\mathbf{i}+3\mathbf{j}-4\mathbf{k})$ |
| Note: | Writing e.g. $\overrightarrow{OD}=\overrightarrow{OB}+\overrightarrow{AB}$ or $\overrightarrow{OD}=2\overrightarrow{OB}-\overrightarrow{OA}$ with no other work is M0 |
| Note: | Finding *coordinates*, i.e. $(6,-7,10)$ without reference to the correct position vectors is A0 |
| Note: | Allow M1A1 for writing down $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$ with no working |
| Note: | M1 can be implied for at least two correct components in their position vector of $D$ |
| (b) | |
| M1: | Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OC}$, then squares and adds each of the 3 components<br>**Note:** Ignore labelling |
| dM1: | Complete method of *correctly* applying Pythagoras' Theorem on $\lvert\overrightarrow{AC}\rvert=4$ and using a correct method of solving their resulting quadratic equation to find at least one of $a=\ldots$ |
| Note: | Condone at least one of either awrt 4.8 or awrt $-0.83$ for the dM mark |
| A1: | Obtains **only one** exact value, $a=2-2\sqrt{2}$ |
| Note: | Writing $a=2\pm2\sqrt{2}$, without evidence of rejecting $a=2+2\sqrt{2}$ is A0 |
| Note: | Allow exact alternatives such as $2-\sqrt{8}$ or $\dfrac{4-\sqrt{32}}{2}$ for A1, and isw can be applied |
| Note: | Writing $a=-0.828...$, without reference to a correct exact value is A0 |

(**Q02 9MA0/02, June 2018**)

**Q10.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar | M1 | 1.1b |
| | $\overrightarrow{AB} = -9\mathbf{i} + 3\mathbf{j}$ | A1 | 1.1b |
| | | (2) | |
| (b) | Finds length using 'Pythagoras' $\lvert AB \rvert = \sqrt{(-9)^2 + (3)^2}$ | M1 | 1.1b |
| | $\lvert AB \rvert = 3\sqrt{10}$ | A1ft | 1.1b |
| | | (2) | |
| | | | **(4 marks)** |

**Notes**

**(a)**

**M1:** Attempts subtraction either way around.

This may be implied by one correct component $\overrightarrow{AB} = \pm 9\mathbf{i} \pm 3\mathbf{j}$

There must be some attempt to write in vector form.

**A1:** cao (allow column vector notation but not the coordinate)

Correct notation should be used. Accept $-9\mathrm{i} + 3\mathrm{j}$ or $\begin{pmatrix} -9 \\ 3 \end{pmatrix}$ but not $\begin{pmatrix} -9\mathrm{i} \\ 3\mathrm{j} \end{pmatrix}$

**(b)**

**M1:** Correct use of Pythagoras theorem or modulus formula using their answer to (a)

Note that $\lvert AB \rvert = \sqrt{(9)^2 + (3)^2}$ is also correct.

Condone missing brackets in the expression $\lvert AB \rvert = \sqrt{-9^2 + (3)^2}$

Also allow a restart usually accompanied by a diagram.

**A1ft:** $\lvert AB \rvert = 3\sqrt{10}$ ft from their answer to (a) as long as it has both an **i** and **j** component.

It must be simplified, if appropriate. Note that $\pm 3\sqrt{10}$ would be M1 A0

*Note that, in cases where there is no working, the correct answer implies M1A1 in each part of this question*

(**Q03 8MA0/01, June 2018**)

**Q11.**

| Question | Scheme | Marks | AOs |
| --- | --- | --- | --- |
| (a) | Attempts two of the relevant vectors<br>$\pm\overrightarrow{AB} = \pm(-4\mathbf{i}+7\mathbf{j}+\mathbf{k})$<br>$\pm\overrightarrow{AC} = \pm(-20\mathbf{i}+(p+3)\mathbf{j}+5\mathbf{k})$<br>$\pm\overrightarrow{BC} = \pm(-16\mathbf{i}+(p-4)\mathbf{j}+4\mathbf{k})$ | M1 | 3.1a |
| | Uses two of the three vectors in such a way as to find the value of $p$. E.g. $p+3=5\times 7$ | dM1 | 2.1 |
| | $p=32$ | A1 | 1.1b |
| | | (3) | |
| | **(a) Alternative:** | | |
| | $r_{AB} = 4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k})$ | M1 | 3.1a |
| | $4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k}) = -16\mathbf{i}+p\mathbf{j}+10\mathbf{k} \Rightarrow \lambda=5$<br>$4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k}) = -16\mathbf{i}+p\mathbf{j}+10\mathbf{k} \Rightarrow p=35-3$ | dM1 | 2.1 |
| | $p=32$ | A1 | 1.1b |
| (b) | Deduces that $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts<br>$\overrightarrow{CD} = 16\mathbf{i}+(4\lambda-"32")\mathbf{j}+(6\lambda-10)\mathbf{k}$ | M1 | 3.1a |
| | Correct attempt at $\lambda$ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$<br>$\overrightarrow{CD} = 16\mathbf{i}+(4\lambda-"32")\mathbf{j}+(6\lambda-10)\mathbf{k}$<br>$\overrightarrow{OA} = 4\mathbf{i}-3\mathbf{j}+5\mathbf{k}$<br>$4\lambda-32=-12 \Rightarrow \lambda = \ldots$ OR $6\lambda-10=20 \Rightarrow \lambda=\ldots$ | dM1 | 1.1b |
| | $\left\|\overrightarrow{OD}\right\| = 5\times\sqrt{4^2+6^2} = 10\sqrt{13}$ | A1 | 1.1b |
| | | (3) | |

| | | | |
|---|---|---|---|
| | **(b) Alternative:** | | |
| | Deduces that $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k}$ and attempts $\overrightarrow{OD} = \overrightarrow{OC} + \mu\overrightarrow{OA} = -16\mathbf{i} + 32\mathbf{j} + 10\mathbf{k} + \mu(4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ | M1 | 3.1a |
| | Correct attempt at $\lambda$ or $\mu$ using the fact that $\lambda\overrightarrow{OB} = \overrightarrow{OC} + \mu\overrightarrow{OA}$ E.g. $-16 + 4\mu = 0 \Rightarrow \mu = 4$ | dM1 | 1.1b |
| | $\lvert\overrightarrow{OD}\rvert = 5 \times \sqrt{4^2 + 6^2} = 10\sqrt{13}$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(6 marks)** |
| **Notes:** | | | |

(a)

**M1**: Attempts two of the three relevant vectors by **subtracting** either way around. See scheme.

Allow equivalent work e.g. $\pm\overrightarrow{AB} = \pm(\overrightarrow{OB} + \overrightarrow{AO})$

If no working is shown, method can be implied by 2 correct components.

**dM1**: For the key step in using the fact that if the vectors are parallel, they will be multiples of each other (where the multiple is something other than 1) to find $p$.

E.g. $p + 3 = 5 \times 7$, $p - 4 = \frac{4}{5}(p + 3)$, $p - 4 = 4 \times 7$

**A1**: $p = 32$ (Condone 32j)

**For reference,** $\overrightarrow{BC} = 4\overrightarrow{AB}$, $\overrightarrow{AC} = 5\overrightarrow{AB}$, $\overrightarrow{BC} = \frac{4}{5}\overrightarrow{AC}$, $\overrightarrow{AC} = \frac{5}{4}\overrightarrow{BC}$

**Note that candidates generally only need to use 2 components to find $p$ and if the 3rd component has errors but is not used, full marks can be awarded.**

Alternative:

**M1**: Forms the vector equation using $A$ or $B$ as position and $\pm\overrightarrow{AB}$ as the direction

**dM1**: For the key step in using the fact that $C$ lies on the line to find $p$

**A1**: $p = 32$ (Condone 32j)

**For reference,** $\overrightarrow{BC} = 4\overrightarrow{AB}$, $\overrightarrow{AC} = 5\overrightarrow{AB}$, $\overrightarrow{BC} = \frac{4}{5}\overrightarrow{AC}$, $\overrightarrow{AC} = \frac{5}{4}\overrightarrow{BC}$

**Note that candidates generally only need to use 2 components to find $p$ and if the 3rd component has errors but is not used, full marks can be awarded.**

**There will be other approaches e.g. using "gradients" and "ratios" and the method marks can be implied – if you are unsure if such attempts deserve credit use Review**

(b) Vector approach

**M1**: Deduces that $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - "32")\mathbf{j} + (6\lambda - 10)\mathbf{k}$

**dM1**: Correct attempt at finding $\lambda$ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

E.g. $16\mathbf{i} + (4\lambda - "32")\mathbf{j} + (6\lambda - 10)\mathbf{k} = 4\alpha\mathbf{i} - 3\alpha\mathbf{j} + 5\alpha\mathbf{k} \Rightarrow \alpha = 4 \Rightarrow 4\lambda - "32" = -3 \times "4" \Rightarrow \lambda = ...$

**A1**: $\left|\overrightarrow{OD}\right| = 10\sqrt{13}$

**Alternative:**

**M1**: Deduces that $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k}$ and attempts

$\overrightarrow{OD} = \overrightarrow{OC} + \mu\overrightarrow{OA} = -16\mathbf{i} + 32\mathbf{j} + 10\mathbf{k} + \mu(4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$

**dM1**: Correct attempt at finding $\lambda$ or $\mu$ using the fact that $\lambda\overrightarrow{OB} = \overrightarrow{OC} + \mu\overrightarrow{OA}$

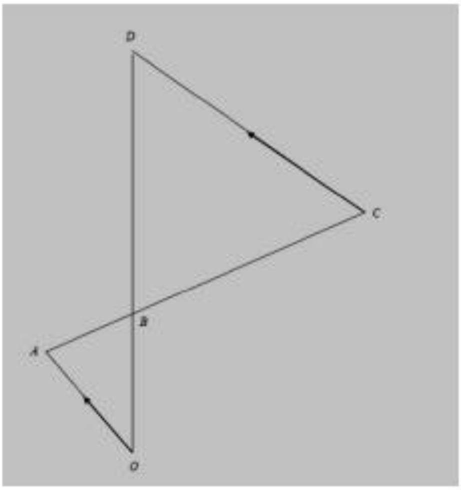
E.g. $(-16 + 4\mu)\mathbf{i} + ("32" - 3\mu)\mathbf{j} + (10 + 5\mu)\mathbf{k} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k} \Rightarrow -16 + 4\mu = 0 \Rightarrow \mu = ...$

May also solve simultaneously using $y$ and $z$ components to find $\lambda$ or $\mu$

**A1**: $\left|\overrightarrow{OD}\right| = 10\sqrt{13}$

Note that the correct vector is 20**j** + 30**k**

(b) Similar triangle approach




**M1**: For the key step in recognising that triangle $BCD$ and triangle $BAO$ are similar with a ratio of lengths of 4:1

**dM1**: States or uses the fact that $\left|\overrightarrow{OD}\right| = 5 \times \left|\overrightarrow{OB}\right|$

Stating this will score M1 dM1 provided there is no evidence of incorrect work

**Note that they may establish this result using the work from (a) but must be used here to score.**

**A1**: $\left|\overrightarrow{OD}\right| = 10\sqrt{13}$

(**Q13 9MA0/02, June 2022**)

**Q12.**

| Question Number | Scheme | Marks | AO's |
|---|---|---|---|
| | Attempts any one of<br>$(\pm\overrightarrow{PQ}=)\pm(\mathbf{q}-\mathbf{p})$, $(\pm\overrightarrow{PR}=)\pm(\mathbf{r}-\mathbf{p})$, $(\pm\overrightarrow{QR}=)\pm(\mathbf{r}-\mathbf{q})$<br>Or e.g.<br>$(\pm\overrightarrow{PQ}=)\pm(\overrightarrow{OQ}-\overrightarrow{OP})$, $(\pm\overrightarrow{PR}=)\pm(\overrightarrow{OR}-\overrightarrow{OP})$, $(\pm\overrightarrow{QR}=)\pm(\overrightarrow{OR}-\overrightarrow{OQ})$ | M1 | 1.1b |
| | Attempts e.g.<br>$\mathbf{r}-\mathbf{q}=2(\mathbf{q}-\mathbf{p})$<br>$\mathbf{r}-\mathbf{p}=3(\mathbf{q}-\mathbf{p})$<br>$\frac{2}{3}(\mathbf{q}-\mathbf{p})=\frac{1}{3}(\mathbf{r}-\mathbf{q})$<br>$\mathbf{q}=\mathbf{p}+\frac{1}{3}(\mathbf{r}-\mathbf{p})$<br>$\mathbf{q}=\mathbf{r}+\frac{2}{3}(\mathbf{p}-\mathbf{r})$ | dM1 | 3.1a |
| | E.g.<br>$\Rightarrow \mathbf{r}-\mathbf{q}=2\mathbf{q}-2\mathbf{p} \Rightarrow 2\mathbf{p}+\mathbf{r}=3\mathbf{q} \Rightarrow \mathbf{q}=\frac{1}{3}(\mathbf{r}+2\mathbf{p})$ * | A1* | 2.1 |
| | | **(3)** | |
| | | | **(3 marks)** |

**Notes:**

**M1:** Attempts any of the relevant vectors by subtracting either way around. This may be implied by sight of any one of $\pm(\mathbf{q}-\mathbf{p})$, $\pm(\mathbf{r}-\mathbf{p})$, $\pm(\mathbf{r}-\mathbf{q})$ ignoring how they are labelled

**dM1:** Uses the given information and writes it correctly in vector form that if rearranged would give the printed answer

**A1*:** Fully correct work leading to the given answer. Allow OQ = … as long as OQ has been defined as **q** earlier.

In the working allow use of P instead of **p** and Q instead of **q** as long as the intention is clear.

(**Q02 9MA0/02, Oct 2020**)

**Q13.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Attempts both $\left\|\overrightarrow{PQ}\right\| = \sqrt{2^2+3^2+(-4)^2}$ and $\left\|\overrightarrow{QR}\right\| = \sqrt{5^2+(-2)^2}$ | M1 | 3.1a |
| | States that $\left\|\overrightarrow{PQ}\right\| = \left\|\overrightarrow{QR}\right\| = \sqrt{29}$ so $PQRS$ is a rhombus | A1 | 2.4 |
| | | (2) | |
| (b) | Attempts BOTH $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = 7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ AND $\overrightarrow{QS} = -\overrightarrow{PQ} + \overrightarrow{PS} = 3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$ | M1 | 3.1a |
| | Correct $\overrightarrow{PR} = 7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ and $\overrightarrow{QS} = 3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$ | A1 | 1.1b |
| | Correct method for area $PQRS$. E.g. $\frac{1}{2}\times\left\|\overrightarrow{PR}\right\|\times\left\|\overrightarrow{QS}\right\|$ | dM1 | 2.1 |
| | $= \sqrt{517}$ | A1 | 1.1b |
| | | (4) | |
| | | | (6 marks) |
| Alt (b) Example using the cosine rule | Attempts $\left\|\overrightarrow{QS}\right\| = \sqrt{3^2+(-3)^2+2^2} = \sqrt{22}$ and so $22 = 29+29-2\sqrt{29}\sqrt{29}\cos SPQ$ | M1 | 3.1a |
| | $\cos PQR = -\frac{18}{29}$ or $\cos SPQ = \frac{18}{29}$ Condone angles in degrees 51.6, 128.4 (1dp) or radians 2.24, 0.901 (3sf) here | A1 | 1.1b |
| | Correct method for area $PQRS$. E.g. $PQ\times QR\sin PQR = \sqrt{2^2+3^2+(-4)^2}\times\sqrt{5^2+(-2)^2}\times\frac{\sqrt{517}}{29}$ | dM1 | 2.1 |
| | $= \sqrt{517}$ | A1 | 1.1b |
| | | (4) | |

**FYI**

$\overrightarrow{QR} = 5\mathbf{i}+0\mathbf{j}-2\mathbf{k}$

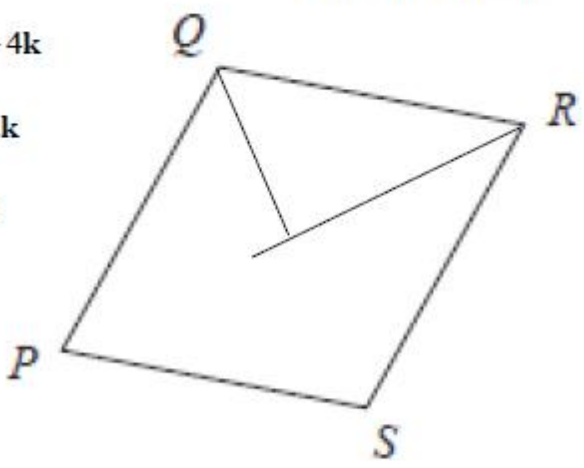
$\overrightarrow{PQ} = 2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$

$\overrightarrow{SQ} = -3\mathbf{i}+3\mathbf{j}-2\mathbf{k}$

$\overrightarrow{MQ} = -1.5\mathbf{i}+1.5\mathbf{j}-1\mathbf{k}$

$M$

$\overrightarrow{PM} = 3.5\mathbf{i}+1.5\mathbf{j}-3\mathbf{k}$




$\overrightarrow{PR} = 7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$

**(a) Do not award marks in part (a) from work in part (b).**

M1: Attempts both $|\overrightarrow{PQ}| = \sqrt{2^2+3^2+(\pm4)^2}$ and $|\overrightarrow{QR}| = \sqrt{5^2+(\pm2)^2}$ or $PQ^2$ and $QR^2$. For this mark only, condone just the correct answers $|\overrightarrow{PQ}| = \sqrt{29}$ and $|\overrightarrow{QR}| = \sqrt{29}$. Alternatively attempts $\overrightarrow{PR} \bullet \overrightarrow{QS}$ or $PM^2, MQ^2$ and $PQ^2$ where $M$ is the mid point of $PR$

A1: Shows that $|\overrightarrow{PQ}| = |\overrightarrow{QR}| = \sqrt{29}$ (with calculations) and states $PQRS$ is a rhombus.

Condone poor notation such as $\overrightarrow{PQ} = \sqrt{29}$ here, So $\overrightarrow{PQ} = \overrightarrow{QR} = \sqrt{29}$ hence rhombus.

Requires both a reason and a conclusion. The reason may be given at the start of their solution.

In the alternatives $\overrightarrow{PR} \bullet \overrightarrow{QS} = (7\mathbf{i}+3\mathbf{j}-6\mathbf{k}) \bullet (3\mathbf{i}-3\mathbf{j}+2\mathbf{k}) = 21-9-12 = 0$ so diagonals cross at $90°$ so $PQRS$ is a rhombus or $PM^2 + MQ^2 = PQ^2 = 23.5+5.5 = 29 \Rightarrow \angle PMQ = 90° \Rightarrow$ Rhombus

(b) **Candidates can transfer answers from (a) to use in part (b) to find the area**

**Look through their complete solution first. The first two marks are for finding the elements that are required to calculate the area. The second set of two marks is for combining these elements correctly. If the method is NOT shown on how to find vector it can be implied by two correct components. Allow as column vectors.**

M1: For a key step in solving the problem. It is scored for attempting to find both key vectors.

Attempts both $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = (7\mathbf{i}+3\mathbf{j}-6\mathbf{k})$ **AND** $\overrightarrow{QS} = -\overrightarrow{PQ} + \overrightarrow{PS} = (3\mathbf{i}-3\mathbf{j}+2\mathbf{k})$

You may see $\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ} + \frac{1}{2}\overrightarrow{QR} = \left(\frac{7}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}-3\mathbf{k}\right)$ **AND** $\overrightarrow{QM} = -\frac{1}{2}\overrightarrow{PQ} + \frac{1}{2}\overrightarrow{PS} = \left(\frac{3}{2}\mathbf{i}-\frac{3}{2}\mathbf{j}+1\mathbf{k}\right)$

A1: Accurately finds both key vectors whose lengths are required to solve the problem.

Score for both $\overrightarrow{PR} = 7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ and $\overrightarrow{QS} = 3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$ (Allow either way around.)

or both $\overrightarrow{PM} = \frac{7}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}-3\mathbf{k}$ and $\overrightarrow{QM} = \frac{3}{2}\mathbf{i}-\frac{3}{2}\mathbf{j}+1\mathbf{k}$ (Allow either way around.)

dM1: Constructs a rigorous method leading to the area $PQRS$. Dependent upon previous M.

E.g. See scheme. Alt: the sum of the area of four right angled triangles e.g. $4\times\frac{1}{2}\times|\overrightarrow{PM}|\times|\overrightarrow{QM}|$,

A1: $\sqrt{517}$

**Alternatives for (b). Two such ways are set out below**

**Alt 1-Examples via cosine rule but you may see use of scalar product via a Further Maths method.**

M1: For a key step in solving the problem. In this case it for an attempt at cos $PQR$ or cos $SPQ$.

Don't be too concerned with the labelling of the angle which may appear as $\theta$.

$$\text{Attempts } \pm\begin{pmatrix}2\\3\\-4\end{pmatrix}\bullet\pm\begin{pmatrix}5\\0\\-2\end{pmatrix}=\sqrt{2^2+3^2+(-4)^2}\times\sqrt{5^2+(-2)^2}\cos PQR$$

A1: Finds the cosine of one of the angles in the Figure.

Look for $\cos\ldots=-\frac{18}{29}$ or $\cos\ldots=\frac{18}{29}$ which may have been achieved via the cosine rule.

Accept rounded answers and the angles in degrees 51.6, 128.4 (1dp) or radians 2.24, 0.901 (3sf) here.

dM1: Constructs a rigorous method leading to the area $PQRS$. Implied by awrt 22.7

$$\text{E.g. } PQ\times QR\sin PQR=\sqrt{2^2+3^2+(-4)^2}\times\sqrt{5^2+(-2)^2}\times\frac{\sqrt{517}}{29}$$

A1: $\sqrt{517}$

**Alt 2-Example via vector product via a Further Maths method.**

M1: For a key step in solving the problem. In this case it for an attempt at $\pm\overrightarrow{PQ}\times\overrightarrow{QR}$

$$\text{E.g. } \overrightarrow{PQ}\times\overrightarrow{QR}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&3&-4\\5&0&-2\end{vmatrix}=(3\times-2-0\times-4)\mathbf{i}-(2\times-2-5\times-4)\mathbf{j}+(2\times0-3\times5)\mathbf{k}$$

A1: E.g. $\overrightarrow{PQ}\times\overrightarrow{QR}=-6\mathbf{i}-16\mathbf{j}-15\mathbf{k}$

dM1: Constructs a rigorous method leading to the area $PQRS$. In this case $\left|\overrightarrow{PQ}\times\overrightarrow{QR}\right|$

A1: $=\sqrt{(-6)^2+(-16)^2+(-15)^2}=\sqrt{517}$

(**Q09 9MA0/01, June 2022**)

**Q14.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $\overrightarrow{QR} = \overrightarrow{PR} - \overrightarrow{PQ} = 13\mathbf{i} - 15\mathbf{j} - (3\mathbf{i} + 5\mathbf{j})$ | M1 | 1.1a |
| | $= 10\mathbf{i} - 20\mathbf{j}$ | A1 | 1.1b |
| | | (2) | |
| (b) | $\left\|\overrightarrow{QR}\right\| = \sqrt{"10"^2 + "(-20)"^2}$ | M1 | 2.5 |
| | $= 10\sqrt{5}$ | A1ft | 1.1b |
| | | (2) | |
| (c) | $\overrightarrow{PS} = \overrightarrow{PQ} + \frac{3}{5}\overrightarrow{QR} = 3\mathbf{i} + 5\mathbf{j} + \frac{3}{5}("10\mathbf{i} - 20\mathbf{j}") = \ldots$ <br> or <br> $\overrightarrow{PS} = \overrightarrow{PR} + \frac{2}{5}\overrightarrow{RQ} = 13\mathbf{i} - 15\mathbf{j} + \frac{2}{5}("-10\mathbf{i} + 20\mathbf{j}") = \ldots$ | M1 | 3.1a |
| | $= 9\mathbf{i} - 7\mathbf{j}$ | A1 | 1.1b |
| | | (2) | |
| | | | (6 marks) |

| Notes |
|---|
| (a) |
| M1: Attempts subtraction either way round. This cannot be awarded for adding the two vectors. If no method shown it may be implied by one correct component. eg $10\mathbf{i}-10\mathbf{j}$ on its own can score M1. |
| A1: Correct answer. Allow $10\mathbf{i}-20\mathbf{j}$ and $\begin{pmatrix}10\\-20\end{pmatrix}$ but not $\begin{pmatrix}10\mathbf{i}\\-20\mathbf{j}\end{pmatrix}$ |
| (b) |
| M1: Correct use of Pythagoras. Attempts to "square and add" before square rooting. The embedded values are sufficient. Follow through on their $\overrightarrow{QR}$ |
| A1ft: $10\sqrt{5}$ following (a) of the form $\pm10\mathbf{i}\pm20\mathbf{j}$ |
| (c) |
| M1: Full attempt at finding a $\overrightarrow{PS}$. They must be attempting $\overrightarrow{PQ}\pm\frac{3}{5}\overrightarrow{QR}$ or $\overrightarrow{PS}=\overrightarrow{PR}\pm\frac{2}{5}\overrightarrow{RQ}$ but condone arithmetical slips after that. |
| Cannot be scored for just stating eg $\overrightarrow{PQ}\pm\frac{3}{5}\overrightarrow{QR}$ |
| Follow through on their $\overrightarrow{QR}$. Terms do not need to be collected for this mark. If no method shown it may be implied by one correct component following through on their $\overrightarrow{QR}$ |

A1: Correct vector as shown. Allow $9\mathbf{i}-7\mathbf{j}$ and $\begin{pmatrix}9\\-7\end{pmatrix}$.

Only withhold the mark for $\begin{pmatrix}9\mathbf{i}\\-7\mathbf{j}\end{pmatrix}$ if the mark has not already been withheld in (a) for $\begin{pmatrix}10\mathbf{i}\\-20\mathbf{j}\end{pmatrix}$

**Alt (c)** (Expressing $\overrightarrow{PS}$ in terms of the given vectors) They must be attempting $\frac{2}{5}\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{PR}$

M1: $(\overrightarrow{PS}=\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{QR}=\overrightarrow{PQ}+\frac{3}{5}(\overrightarrow{PR}-\overrightarrow{PQ}))$

$\Rightarrow \frac{2}{5}\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{PR}=\frac{2}{5}(3\mathbf{i}+5\mathbf{j})+\frac{3}{5}(13\mathbf{i}-15\mathbf{j})=\ldots$

A1: Correct vector as shown. Allow $9\mathbf{i}-7\mathbf{j}$ and $\begin{pmatrix}9\\-7\end{pmatrix}$.

Only withhold the mark for $\begin{pmatrix}9\mathbf{i}\\-7\mathbf{j}\end{pmatrix}$ if the mark has not already been withheld in (a) for $\begin{pmatrix}10\mathbf{i}\\-20\mathbf{j}\end{pmatrix}$

(**Q03 8MA0/01, June 2022**)

**Q15.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| | Attempts $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 2\mathbf{i}+3\mathbf{j}+\mathbf{k}+\mathbf{i}-9\mathbf{j}+3\mathbf{k} = 3\mathbf{i}-6\mathbf{j}+4\mathbf{k}$ | M1 | 3.1a |
| | Attempts to find any one length using 3-d Pythagoras | M1 | 2.1 |
| | Finds all of $\lvert AB\rvert = \sqrt{14}, \quad \lvert AC\rvert = \sqrt{61}, \quad \lvert BC\rvert = \sqrt{91}$ | A1ft | 1.1b |
| | $\cos BAC = \dfrac{14+61-91}{2\sqrt{14}\sqrt{61}}$ | M1 | 2.1 |
| | angle $BAC = 105.9°$ * | A1* | 1.1b |
| | | (5) | |
| | | (5 marks) | |

**Notes:**

**M1:** Attempts to find $\overrightarrow{AC}$ by using $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

**M1:** Attempts to find any one length by use of Pythagoras' Theorem

**A1ft:** Finds all three lengths in the triangle. Follow through on their $\lvert AC\rvert$

**M1:** Attempts to find $BAC$ using $\cos BAC = \dfrac{\lvert AB\rvert^2 + \lvert AC\rvert^2 - \lvert BC\rvert^2}{2\lvert AB\rvert\lvert AC\rvert}$

Allow this to be scored for other methods such as $\cos BAC = \dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{\lvert AB\rvert\lvert AC\rvert}$

**A1*:** This is a show that and all aspects must be correct. Angle $BAC =$ 105.9 °

(**Q07 9MA0/01, Specimen papers** )

**Q16.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (-8\mathbf{i}+9\mathbf{j}) - (10\mathbf{i}-3\mathbf{j})$ | M1 | 1.1b |
| | $= -18\mathbf{i} + 12\mathbf{j}$ | A1 | 1.1b |
| | | (2) | |
| (b) | $\lvert\overrightarrow{AB}\rvert = \sqrt{"18"^2 + "12"^2}\ \{= \sqrt{468}\}$ | M1 | 1.1b |
| | $= 6\sqrt{13}$ | A1 | 1.1b |
| | | (2) | |
| (c) | For the key step in using the fact that $BCA$ forms a straight line in an attempt to find "$p$"<br>$\overrightarrow{AB} = \lambda\overrightarrow{BC} \Rightarrow -18\mathbf{i} + 12\mathbf{j} = 6\lambda\mathbf{i} + \lambda(p-9)\mathbf{j}$ with components equated leading to a value for $\lambda$ and to $p = \ldots$ | M1 | 2.1 |
| | (i) $p = 5$ | A1 | 1.1b |
| | (ii) ratio $= 2:3$ | B1 (A1 on EPEN) | 2.2a |
| | | (3) | |
| | | **(7 marks)** | |

| **Notes:** | |
|---|---|
| **(a)** | **Must be seen in (a)** |
| **M1:** | Attempts subtraction either way round. This cannot be awarded for adding the two vectors. If no method shown it may be implied by one correct component. Allow as coordinates for this mark. Condone missing brackets, e.g., $-8\mathbf{i}+9\mathbf{j}-10\mathbf{i}-3\mathbf{j}$ |
| **A1:** | cao $-18\mathbf{i}+12\mathbf{j}$ o.e. $\begin{pmatrix}-18\\12\end{pmatrix}$ Condone $\begin{matrix}-18\\12\end{matrix}$ Do not allow $\begin{pmatrix}-18\mathbf{i}\\12\mathbf{j}\end{pmatrix}$ or $(-18, 12)$ or $\left(\dfrac{-18}{12}\right)$ for the A1. |
| **(b)** | |
| **M1:** | Attempts to use Pythagoras' theorem on their vector from part (a). Allow restarts. $\lvert\overline{AB}\rvert=\sqrt{"18"^2+"12"^2}\ \{=\sqrt{468}\}$ Note that -18 will commonly be squared as 18 May be implied by awrt 21.6 This will need checking if (a) is incorrect. |
| **A1:** | cao $6\sqrt{13}$ May come from $\begin{pmatrix}\pm 18\\\pm 12\end{pmatrix}$ |
| **(c)** | |
| **M1:** | For the key step in using the fact that *BCA* forms a straight line in an attempt to find "*p*" Condone sign slips. Award, for example, for $\pm\dfrac{p-9}{6}=\pm\dfrac{2}{3}$ leading to $p=\ldots$ It is implied by $p=5$ unless it comes directly from work that is clearly incorrect. |

e.g., award for an attempt to use

- $\overline{AB} = \alpha\overline{AC} \Rightarrow -18\mathbf{i} + 12\mathbf{j} = -12\alpha\mathbf{i} + \alpha(p+3)\mathbf{j}$ with components equated leading to a value for $\alpha$ and to $p = \ldots$
- gradient $BC$ = gradient $BA = -\frac{2}{3}$ e.g., $\frac{p-9}{6} = \frac{9--3}{-8-10}$ leading to $p = \ldots$
- triangles $BCM$ and $BAN$ are similar with lengths in a ratio 1:3. e.g., $p = 9 - \frac{1}{3}\times 12$ **or** $p = -3 + \frac{2}{3}\times 12$
- attempt to find the equation of line $AB$ using both points (FYI line $AB$ has equation $y = -\frac{2}{3}x + \frac{11}{3}$) and then sub in $x = -2$ leading to $p = \ldots$
- $\frac{p+3}{12} = \frac{2}{3}$ **or** $\frac{p+3}{2} = 9 - p$ leading to $p = \ldots$

**A1:** $p = 5$ Correct answer implies both marks, unless it comes directly from work that is clearly incorrect.

**B1:** States ratio = 2: 3 or equivalent such as 1: 1.5 or 22:33
Note that 3:2 is incorrect but condone {Area}$AOB$ : {Area}$AOC$ = 3: 2
This might follow incorrect work or even incorrect $p$
For reference, area $AOC = 22$, area $AOB = 33$ and area $BOC = 11$

(**Q13 8MA0/01, June 2023**)

**Q17.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $\overrightarrow{AB} = (3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}) - (2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k})$ | M1 | 1.1b |
| | $= \mathbf{i} - 8\mathbf{j} + 2\mathbf{k}$ | A1 | 1.1b |
| | | **(2)** | |
| (b) | States $\overrightarrow{OC} = 2 \times \overrightarrow{AB}$ | M1 | 1.1b |
| | Explains that as $OC$ is parallel to $AB$, so $OABC$ is a trapezium. | A1 | 2.4 |
| | | **(2)** | |
| | | | **(4 marks)** |
| **Notes:** | | | |

**(a)**

**M1:** Attempts to subtract either way around. If no method is seen it is implied by two of $\pm 1\mathbf{i} \pm 8\mathbf{j} \pm 2\mathbf{k}$.

**A1:** $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ or $\begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix}$ but not $(1,-8,2)$

**(b)**

**M1:** Compares their $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ with $2\mathbf{i}-16\mathbf{j}+4\mathbf{k}$ by stating **any one of**

- $\overline{OC} = 2\times\overline{AB}$
- $\begin{pmatrix} 2 \\ -16 \\ 4 \end{pmatrix} = 2\times\begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix}$
- $\overline{OC} = \lambda\times\overline{AB}$ or vice versa

This may be awarded if *AB* was subtracted "the wrong way around" or if there was one numerical slip

**A1:** A full explanation as to why *OABC* is a trapezium.

Requires fully correct calculations, so part (a) must be $\overline{AB} = (\mathbf{i}-8\mathbf{j}+2\mathbf{k})$

It requires a reason and minimal conclusion.

Example 1:

$\overline{OC} = 2\times\overline{AB}$, therefore *OC* is parallel to *AB* so *OABC* is a trapezium

Example 2:

A trapezium has one pair of parallel sides. As $\overline{OC} = 2\times\overline{AB}$, they are parallel, so ✓.

Example 3

As $\begin{pmatrix} 2 \\ -16 \\ 4 \end{pmatrix} = 2\times\begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix}$, *OC* and *AB* are parallel, so proven

Example 4

Accept as $\overline{OC} = \lambda\times\overline{AB}$, they are parallel so true

Note: There are two definitions for a trapezium. One stating that it is a shape with one pair of parallel sides, the other with **only one** pair of parallel sides. Any calculations to do with sides *OA* and *CB* in this question may be ignored, even if incorrect.

(**Q03 9MA0/01, Oct 2020**)