Trig Exam Questions

Q1.

Given that θ is measured in radians, prove, from first principles, that the derivative of sin θ is cos θ

You may assume the formula for sin($A \pm B$) and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

(Total for question = 5 marks)

Q2.

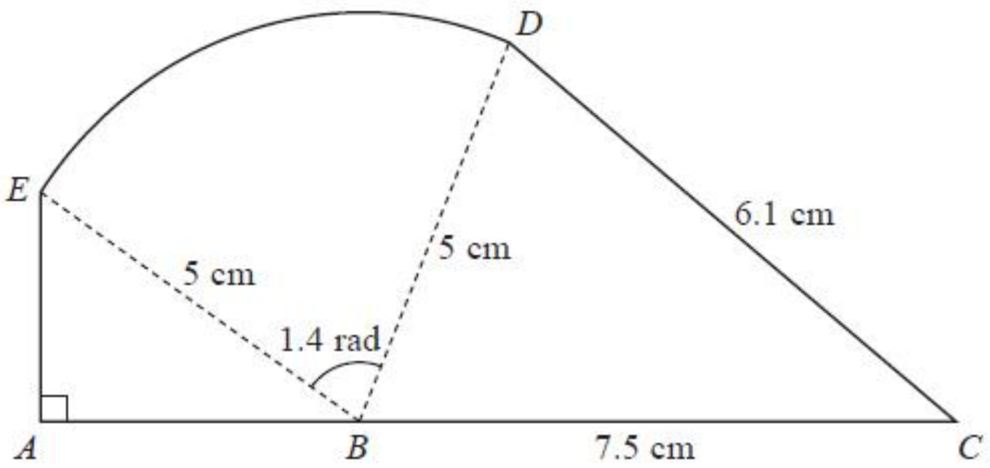

Figure 2

The shape *ABCDEA*, as shown in Figure 2, consists of a right-angled triangle *EAB* and a triangle *DBC* joined to a sector *BDE* of a circle with radius 5 cm and centre *B*.

The points *A*, *B* and *C* lie on a straight line with *BC* = 7.5 cm.

Angle *EAB* = $\frac{\pi}{2}$ radians, angle *EBD* = 1.4 radians and *CD* = 6.1 cm.

(a) Find, in cm^2, the area of the sector *BDE*.

(2)

(b) Find the size of the angle *DBC*, giving your answer in radians to 3 decimal places.

(2)

(c) Find, in cm^2, the area of the shape *ABCDEA*, giving your answer to 3 significant figures.

(5)

(Total 9 marks)

Q3.

(i) Solve, for $0 \leq \theta < 180°$, the equation

$$\frac{\sin 2\theta}{(4\sin 2\theta - 1)} = 1$$

giving your answers to 1 decimal place.

(3)

(ii) Solve, for $0 \leq x < 2\pi$, the equation

$$5\sin^2 x - 2\cos x - 5 = 0$$

giving your answers to 2 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

(Total 8 marks)

Q4.

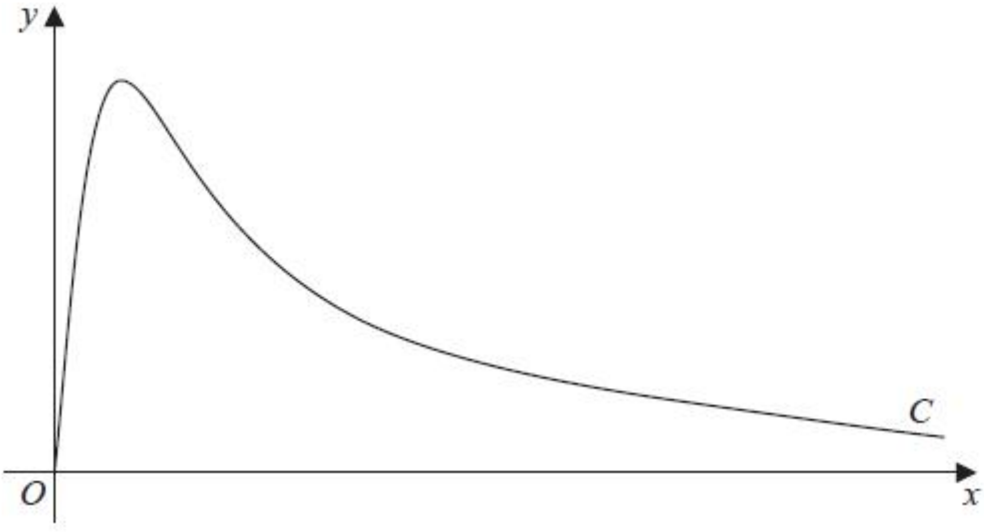

Figure 2

Figure 2 shows a sketch of the curve *C* with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point *P* lies on *C* and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point *P*.

Give your answer as a simplified surd.

(4)

The point *Q* lies on the curve *C*, where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

(b) Find the exact coordinates of the point *Q*.

(2)

(Total for question = 6 marks)

Q5.

(i) Find, using calculus, the *x* coordinate of the turning point of the curve with equation

$$y = e^{3x}\cos 4x, \quad \frac{\pi}{4} \leqslant x < \frac{\pi}{2}$$

Give your answer to 4 decimal places.

(5)

(ii) Given $x = \sin^2 2y, \quad 0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of *y*.

Write your answer in the form

$$\frac{dy}{dx} = p\,\text{cosec}(qy), \qquad 0 < y < \frac{\pi}{4}$$

where *p* and *q* are constants to be determined.

(5)

(Total for question = 10 marks)

Q6.

The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \quad 0 \leqslant t \leqslant \pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(2)

The point P lies on C where $t = \frac{2\pi}{3}$

The line l is the normal to C at P.

(b) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.

You must show clearly how you obtained your answers.

(6)

(Total for question = 13 marks)

Q7.

(i) Given that

$$x = \sec^2 2y, \quad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{1}{4x\sqrt{(x-1)}}$$

(4)

(ii) Given that

$$y = (x^2 + x3)\ln 2x$$

find the exact value of $\frac{dy}{dx}$ at $x = \frac{e}{2}$, giving your answer in its simplest form.

(5)

(iii) Given that

$$f(x) = \frac{3\cos x}{(x+1)^{\frac{1}{3}}}, \qquad x \neq -1$$

show that

$$f'(x) = \frac{g(x)}{(x+1)^{\frac{4}{3}}}, \qquad x \neq -1$$

where g(x) is an expression to be found.

(3)

(Total 12 marks)

Q8.
A curve C has parametric equations

$$x = \sin^2 t, \quad y = 2\tan t, \quad 0 \leqslant t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t.

(4)

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(b) Find the x-coordinate of P.

(6)
(Total 10 marks)

Q9.

Given that

$$x = \sec^2 3y, \qquad 0 < y < \frac{\pi}{6}$$

(a) find $\frac{dx}{dy}$ in terms of *y*.

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(c) Find an expression for $\frac{d^2y}{dx^2}$ in terms of *x*. Give your answer in its simplest form.

(4)

(Total 10 marks)

Q10.

A curve has parametric equations

$$x = \tan^2 t, \qquad y = \sin t, \qquad 0 < t < \frac{\pi}{2}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. You need not simplify your answer.

(3)

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

Give your answer in the form $y = ax + b$, where a and b are constants to be determined.

(5)

(c) Find a cartesian equation of the curve in the form $y^2 = f(x)$.

(4)

(Total 12 marks)

Q11.

Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$$

(3)

(Total for question = 3 marks)

Q12.

(a) Prove that

$$\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta, \qquad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

does not have any real solutions.

(1)

(Total for question = 5 marks)

Q13.

The depth of water, *D* metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)^\circ \qquad 0 \leqslant t < 24$$

where *t* is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total for question = 5 marks)

Q14.

(a) Express $\frac{2}{P(P-2)}$ in partial fractions.

(3)

A team of biologists is studying a population of a particular species of animal.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{2}P(P-2)\cos 2t, \quad t \geqslant 0$$

where P is the population in thousands, and t is the time measured in years since the start of the study.

Given that $P = 3$ when $t = 0$,

(b) solve this differential equation to show that

$$P = \frac{6}{3 - e^{\frac{1}{2}\sin 2t}}$$

(7)

(c) find the time taken for the population to reach 4000 for the first time.
Give your answer in years to 3 significant figures.

(3)

(Total for question = 13 marks)

Q15.

On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where a and b are positive constants.

(a) Find a complete equation for the model.

(3)

(b) Use the model to determine the height of the carriage above the ground when $t = 40$

(1)

In an alternative model, the vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is given by

$$H = 29\cos(9t + \alpha)° + \beta \qquad 0 \leq \alpha < 360°$$

where α and β are constants.

(c) Find a complete equation for the alternative model.

(2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

(1)

(Total for question = 7 marks)

Q16.

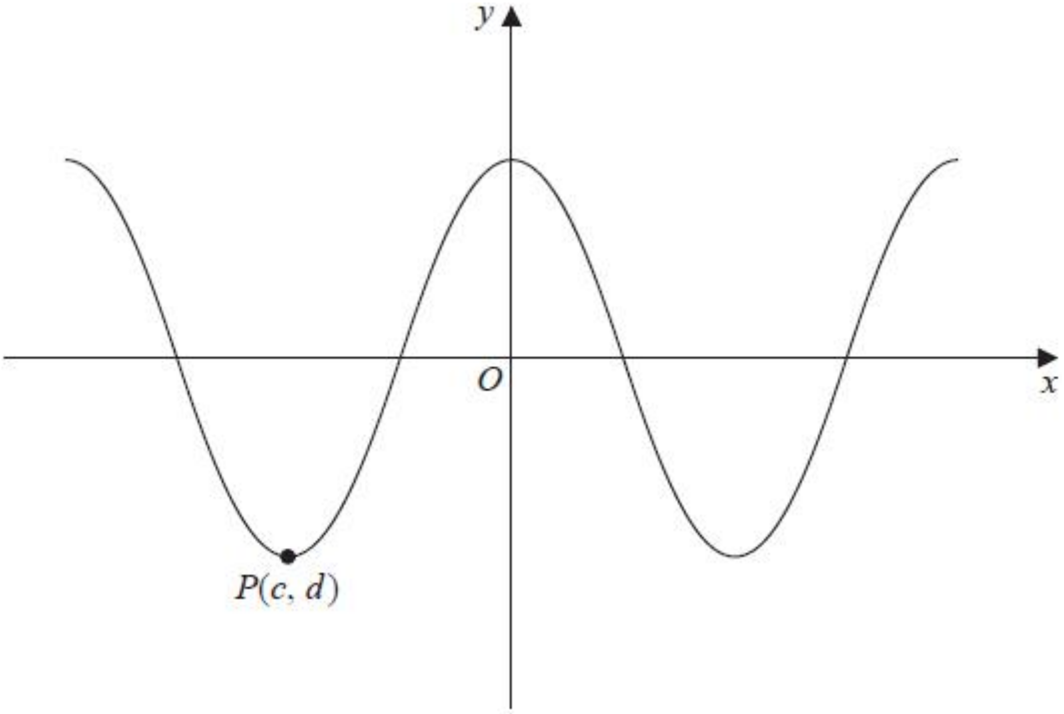

Figure 3

Figure 3 shows part of the curve with equation $y = 3\cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which a minimum occurs.

(a) State the value of c and the value of d.

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3\cos x°$ to the curve with equation

(i) $y = 3\cos\left(\frac{x°}{4}\right)$

(ii) $y = 3\cos(x - 36)°$

(2)

(c) Solve, for $450° \leq \theta < 720°$

$$3 \cos \theta = 8 \tan \theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(5)

(Total for question = 8 marks)

Q17.

Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos\theta = 2\sin\theta$$

The attempts of two of the students are shown below.

Student *A*

$$\cos\theta = 2\sin\theta$$
$$\tan\theta = 2$$
$$\theta = 63.4°$$

Student *B*

$$\cos\theta = 2\sin\theta$$
$$\cos^2\theta = 4\sin^2\theta$$
$$1 - \sin^2\theta = 4\sin^2\theta$$
$$\sin^2\theta = \frac{1}{5}$$
$$\sin\theta = \pm\frac{1}{\sqrt{5}}$$
$$\theta = \pm 26.6°$$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

(Total for question = 3 marks)

Q18.

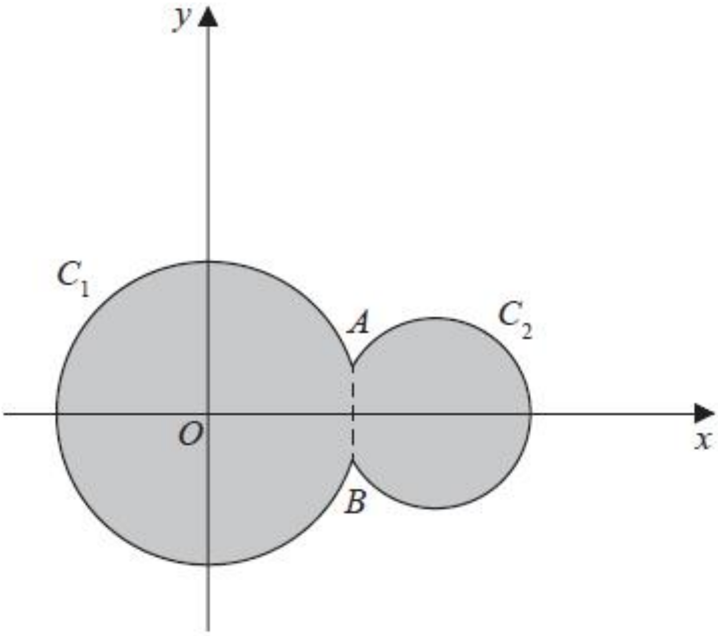

Figure 3

Circle C_1 has equation $x^2 + y^2 = 100$

Circle C_2 has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin.

(4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

(Total for question = 8 marks)

Q19.

(i) Given that $y > 0$, find

$$\int \frac{3y - 4}{y(3y + 2)} \, dy$$

(6)

(ii) (a) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x = \lambda \int_0^{\frac{\pi}{3}} \sin^2\theta \,\mathrm{d}\theta$$

where λ is a constant to be determined.

(5)

(b) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

(Total for question = 15 marks)

Q20.

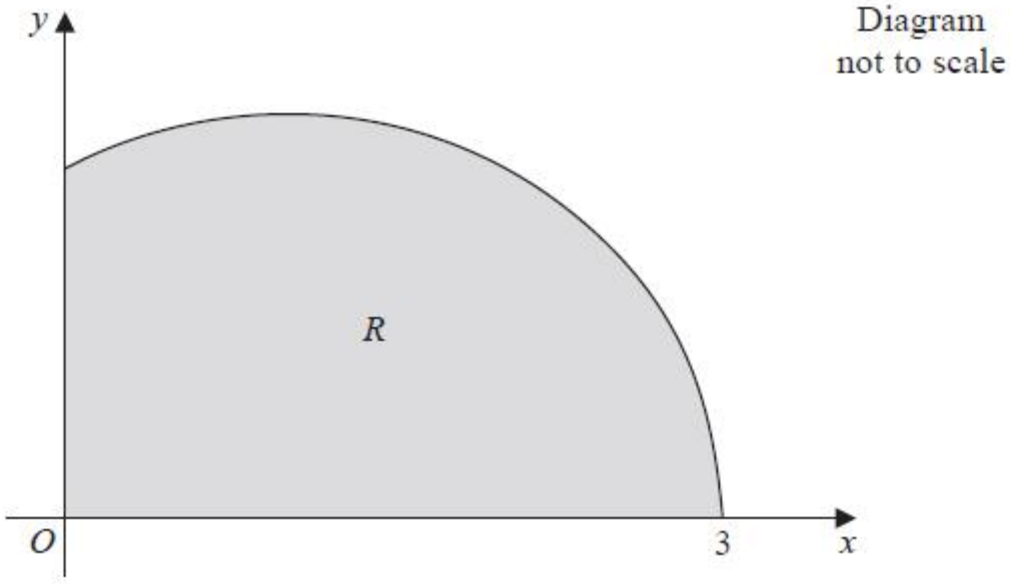

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3 - x)(x + 1)}, \quad 0 \leqslant x \leqslant 3$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis, and the y-axis.

(a) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3 - x)(x + 1)}\,\mathrm{d}x = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\,\mathrm{d}\theta$$

where k is a constant to be determined.

(5)

(b) Hence find, by integration, the exact area of *R*.

(3)

(Total for question = 8 marks)

Q21.

Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$$

(6)

(Total 6 marks)

Q22.

(a) Use integration by parts to find $\int x \sin 3x dx.$

(3)

(b) Using your answer to part (a), find $\int x^2 \cos 3x \, dx$.

(3)

(Total 6 marks)

Q23.

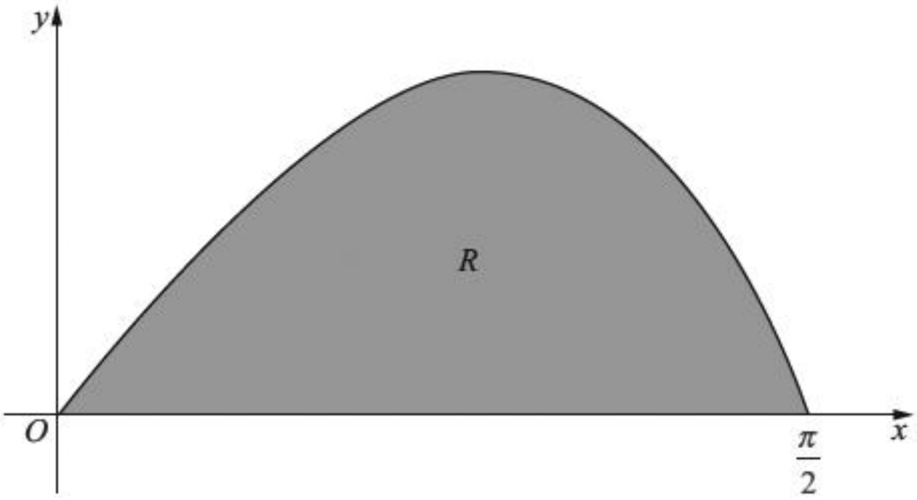

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \dfrac{2\sin 2x}{(1+\cos x)},\ 0 \leqslant x \leqslant \dfrac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \dfrac{2\sin 2x}{(1+\cos x)}$

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(a) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving
your answer to 4 decimal places.

(3)

(c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\, dx = 4\ln(1+\cos x) - 4\cos x + k$$

where k is a constant.

(5)

(d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures.

(3)

(Total 12 marks)

Q24.

(a) Find $\int x \cos 2x \, dx$.

(4)

(b) Hence, using the identity $\cos 2x = 2\cos^2 x - 1$, deduce $\int x \cos^2 x \, dx$.

(3)

(Total 7 marks)

Q25.

(i) Find $\int \ln\left(\frac{x}{2}\right) dx$.

(4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx$

(5)

(Total 9 marks)

Q26.

$f(\theta) = 4\cos^2\theta - 3\sin^2\theta$

(a) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta.$

(3)

(b) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta f(\theta) \, d\theta.$

(7)

(Total 10 marks)

Q27.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(4)

(b) Hence solve, for $-90° \leq x \leq 180°$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4)

(Total for question = 8 marks)

Q28.

(i) Find

$$\int x\, e^{4x}\, dx$$

(3)

(ii) Find

$$\int \frac{8}{(2x-1)^3}\,dx, \quad x > \tfrac{1}{2}$$

(2)

Given that $y = \frac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y$$

(7)

(Total 12 marks)

Q29.

(a) Express $10 \cos \theta - 3 \sin \theta$ in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$

Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

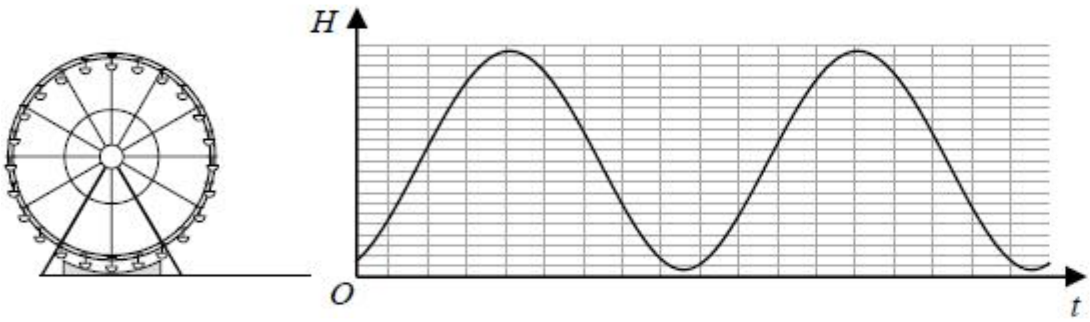

Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10 \cos (80t)° + 3 \sin (80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

(Total for question = 9 marks)

Q30.

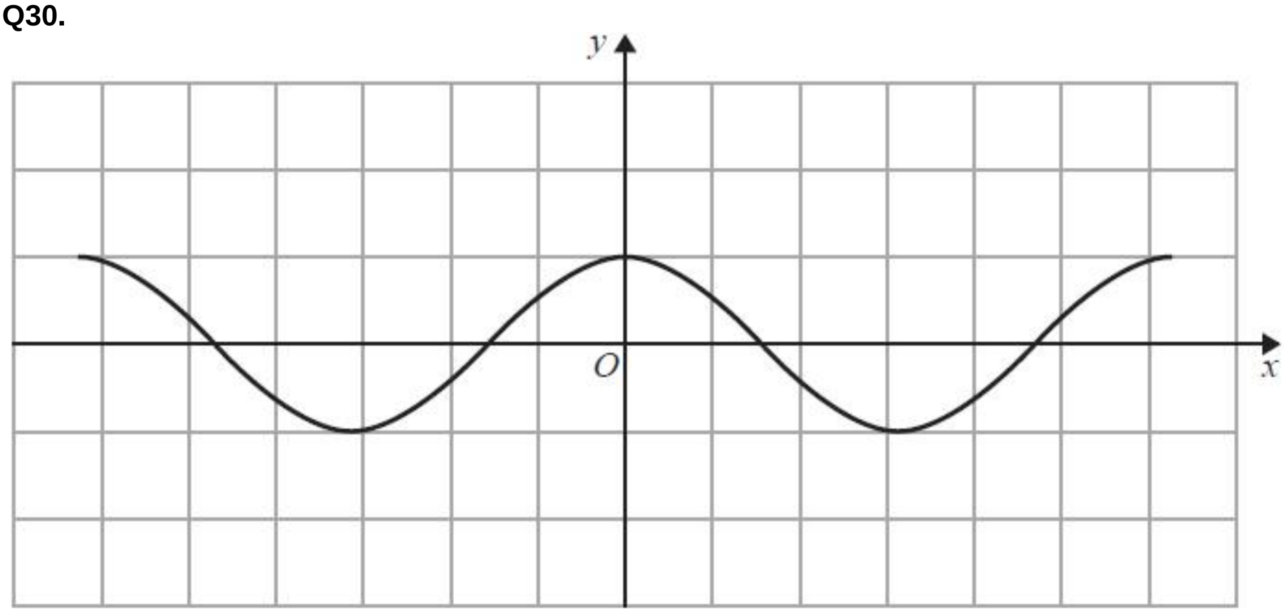

Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians.

(a) Use Figure 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

Given that the root of the equation is a, and that a is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of a to 3 decimal places.

(3)

(Total for question = 5 marks)

Q31.

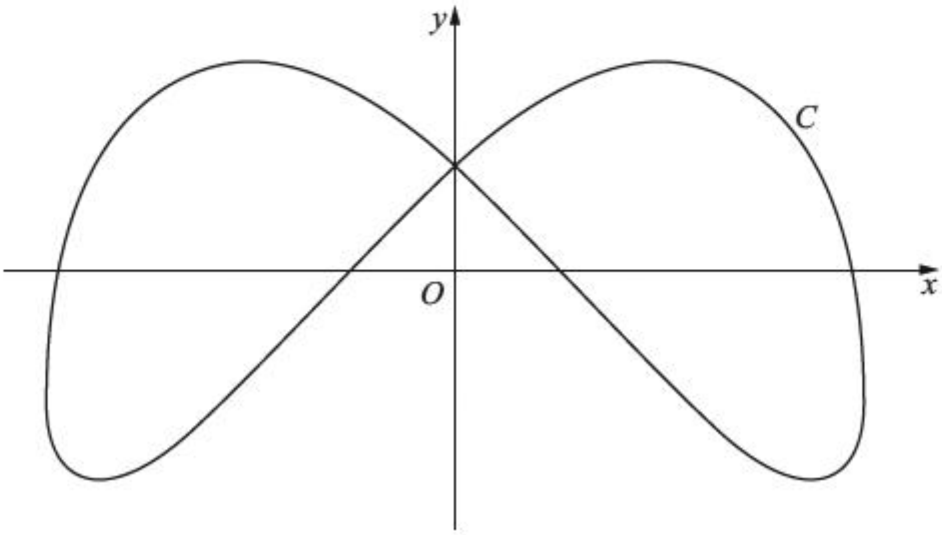

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \qquad y = 3\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(3)

Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$

(5)

(Total 8 marks)

Q32.

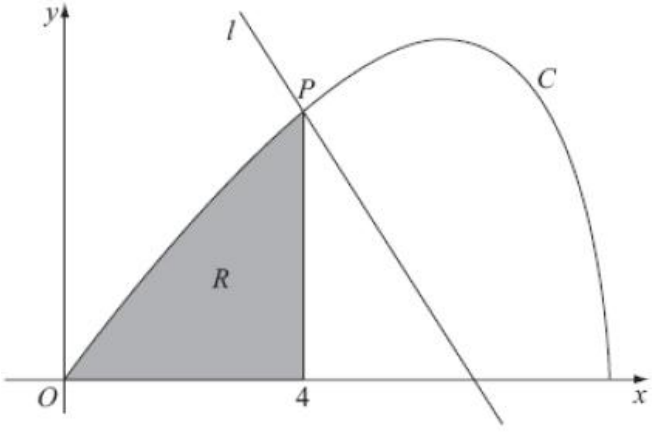

Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leq t \leq \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64 \sin^2 t \cos t \, dt$.

(4)

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

(Total 16 marks)

Q33.

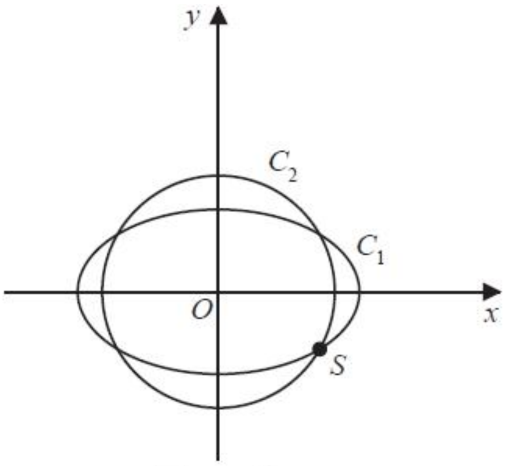

Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \quad y = 4\sqrt{2}\sin t, \quad 0 \leqslant t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in Figure 2.

Given that one of these points, *S*, lies in the 4$^{\text{th}}$ quadrant, find the Cartesian coordinates of *S*.

(Total for question = 6 marks)

Q34.

(a) Prove that

$$2\cot 2x + \tan x \equiv \cot x \qquad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $-\pi \leq x < \pi$,

$$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2$$

Give your answers to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(Total for question = 10 marks)

Q35.

(a) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \ n \in \mathbb{Z}$$

(5)

(b) Hence solve, for $0 \leqslant \theta < 2\pi,$

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}$$

Give your answers to 3 decimal places.

(4)

(Total for question = 9 marks)

Q36.

(a) Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

(Total for question = 9 marks)

Q37.

(a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + a)$, where R and a are constants, $R > 0$ and $0 < a$ 90° Give the exact value of R and give the value of a to 2 decimal places.

(3)

(b) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15$$

Give your answers to one decimal place.

(5)

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15$$

Give your answer to one decimal place.

(2)

(Total for question = 10 marks)

Q38.

(a) Express $2 \sin \theta - 4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α to 3 decimal places.

(3)

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(b) (i) the maximum value of $H(\theta)$,

(ii) the smallest value of θ, for $0 \leq \theta < \pi$, at which this maximum value occurs.

(3)

Find

(c) (i) the minimum value of $H(\theta)$,

(ii) the largest value of θ, for $0 \leq \theta < \pi$, at which this minimum value occurs.

(3)

(Total 9 marks)

Q39.

(a) Write $5\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants,

$R > 0$ and $0 \le \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(b) Show that the equation

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5\cos 2x - 2\sin 2x = c$$

where c is a positive constant to be determined.

(2)

(c) Hence or otherwise, solve, for $0 \le x < \pi$,

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total for question = 9 marks)

Q40.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos\theta} + \tan\theta \equiv \frac{\cos\theta}{1 - \sin\theta} \qquad \theta \neq (2n+1)90° \quad n \in \mathbb{Z}$$

(3)

Given that $\cos 2x \neq 0$

(b) solve for $0 < x < 90°$

$$\frac{1}{\cos 2x} + \tan 2x = 3\cos 2x$$

giving your answers to one decimal place.

(5)

(Total for question = 8 marks)

Online Maths Teaching

Q41.

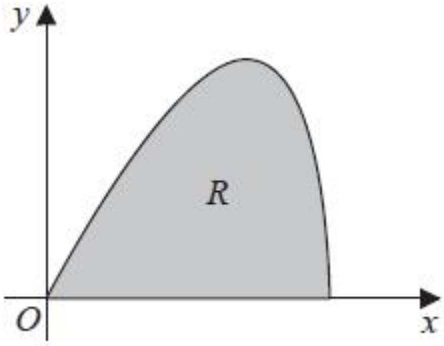

Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6\sin t \qquad y = 5\sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region *R*, shown shaded in Figure 3, is bounded by the curve and the *x*-axis.

(a) (i) Show that the area of *R* is given by $\displaystyle\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t\,\mathrm{d}t$

(3)

(ii) Hence show, by algebraic integration, that the area of *R* is exactly 20

(3)

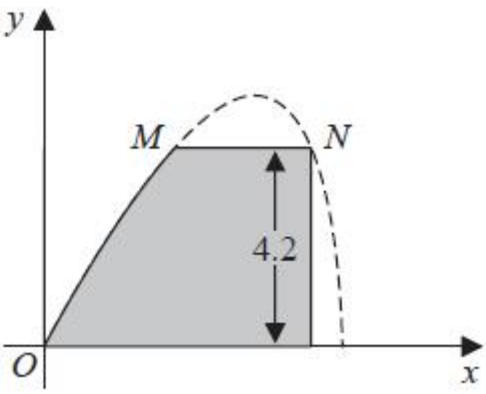

Figure 4

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4.
Using the model and given that

- x and y are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width MN along the top of the dam

(b) calculate the width of the walkway.

(5)

(Total for question = 11 marks)

Q42.

(a) Show that

$$\text{cosec } 2x + \cot 2x = \cot x, \quad x \neq 90n°, \quad n \in \mathbb{R}.$$

(5)

(b) Hence, or otherwise, solve, for $0 \leq \theta < 180°$,

$$\text{cosec } (4\theta + 10°) + \cot (4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(Total 10 marks)

Q43.

(i) Solve, for $0 \leq \theta < 360°$, the equation

$$9\sin(\theta + 60°) = 4$$

giving your answers to 1 decimal place.
You must show each step of your working.

(4)

(ii) Solve, for $-\pi \leq x < \pi$, the equation

$$2\tan x - 3\sin x = 0$$

giving your answers to 2 decimal places where appropriate.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

(Total 9 marks)

Q44.

(a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \le x \le 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

(Total for question = 7 marks)

Q45.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta(8\cos\theta + 23\sin^2\theta) = 8\sin 2\theta(1 + \tan^2\theta)$$

may be written as

$$\sin 2\theta(A\cos^2\theta + B\cos\theta + C) = 0$$

where A, B and C are constants to be found.

(3)

(b) Hence, solve for $360° \leqslant x \leqslant 540°$

$$2\tan x(8\cos x + 23\sin^2 x) = 8\sin 2x(1 + \tan^2 x) \qquad x \in \mathbb{R} \qquad x \neq 450°$$

(4)

(Total for question = 7 marks)

Q46.

(a) Express 2 cos θ + 8 sin θ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The first three terms of an arithmetic sequence are

$$\cos x \qquad \cos x + \sin x \qquad \cos x + 2\sin x \qquad x \neq n\pi$$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

(b) (i) find the exact maximum value of S_9

(ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

(3)

(Total for question = 6 marks)

Q47.

(a) Express $\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$ where R and α are constants, $R > 0$

and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The temperature, θ °C , inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2\cos\left(\frac{\pi t}{12} - 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day,

(1)

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3)

(Total for question = 7 marks)

Q48.

Figure 1

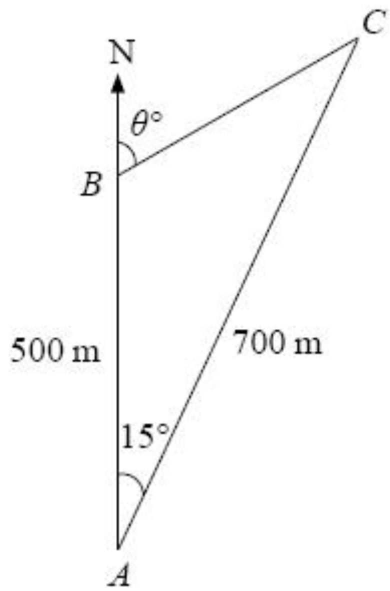

Figure 1 shows 3 yachts *A*, *B* and *C* which are assumed to be in the same horizontal plane. Yacht *B* is 500 m due north of yacht *A* and yacht *C* is 700 m from A. The bearing of *C* from *A* is 015°.

(a) Calculate the distance between yacht *B* and yacht *C*, in metres to 3 significant figures.

(3)

The bearing of yacht *C* from yacht *B* is $\theta°$, as shown in Figure 1.

(b) Calculate the value of θ.

(4)

(Total 7 marks)

Q49.

(i) Use an appropriate double angle formula to show that

$$\operatorname{cosec}2x = \lambda \operatorname{cosec} x \sec x,$$

and state the value of the constant λ.

(3)

(ii) Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta$$

You must show all your working. Give your answers in terms of π.

(6)

(Total 9 marks)

Q50.

(a) Solve for $0 \leq x < 360°$, giving your answers in degrees to 1 decimal place,

$$3\sin(x + 45°) = 2$$

(4)

(b) Find, for $0 \leq x < 2\pi$, all the solutions of

$$2\sin^2 x + 2 = 7\cos x$$

giving your answers in radians.

You must show clearly how you obtained your answers.

(6)

(Total 10 marks)

Q51.

(a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \text{cosec}^2\theta$.

(2)

(b) Solve, for $0 \leq \theta < 180°$, the equation

$$2 \cot^2\theta - 9 \text{ cosec } \theta = 3,$$

giving your answers to 1 decimal place.

(6)

(Total 8 marks)

Q52.

(a) Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, f (t), of a warehouse is modelled using the equation

$$f(t) = 10 + 3 \cos(15t)° + 4 \sin(15t)°,$$

where t is the time in hours from midday and $0 \leqslant t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(d) Find the value of t when this minimum temperature occurs.

(3)

(Total 12 marks)

Q53.

(a) Use the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$

(2)

The curves C_1 and C_2 have equations

$$C_1: \quad y = 3\sin 2x$$

$$C_2: \quad y = 4\sin^2 x - 2\cos 2x$$

(b) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

(3)

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

(d) Hence find, for $0 \leq x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place.

(4)

(Total 12 marks)