**Questions**

**Q1.**

**In this question you must show detailed reasoning.**

$f(x) = x^3 - 21x^2 + Ax - 91$ where $A$ is a real constant

The roots of $f(x) = 0$ are

$\alpha$, $\alpha + 3\beta$ and $\alpha + 6\beta$

where $\alpha$ and $\beta$ are real constants.

Use algebra to determine the value of each of these roots.

**(7)**

**(Total for question = 7 marks)**

**Q2.**

The cubic equation

$z^3 - 3z^2 + z + 5 = 0$

has roots $\alpha$, $\beta$ and $\gamma$.

Without solving the equation, find the cubic equation whose roots are $(2\alpha + 1)$, $(2\beta + 1)$ and $(2\gamma + 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where $p$, $q$ and $r$ are integers to be found.

**(5)**

**(Total for question = 5 marks)**

**Q3.**

$f(z) = z^3 + z^2 + pz + q$

where $p$ and $q$ are real constants.

The equation $f(z) = 0$ has roots $z_1$, $z_2$ and $z_3$

When plotted on an Argand diagram, the points representing $z_1$, $z_2$ and $z_3$ form the vertices of a triangle of area 35

Given that $z_1 = 3$, find the values of $p$ and $q$.

**(7)**

**(Total for question = 7 marks)**

**Q4.**

The cubic equation

$x^3 + 3x^2 - 8x + 6 = 0$

has roots $\alpha$, $\beta$ and $\gamma$.

Without solving the equation, find the cubic equation whose roots are $(\alpha - 1)$, $(\beta - 1)$ and $(\gamma - 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where $p$, $q$ and $r$ are integers to be found.

**(5)**

**(Total for question = 5 marks)**

**Q5.**

$f(z) = z^3 - 8z^2 + pz - 24$

where $p$ is a real constant.

Given that the equation $f(z) = 0$ has distinct roots

$$\alpha, \beta \text{ and } \left(\alpha + \frac{12}{\alpha} - \beta\right)$$

(a) solve completely the equation $f(z) = 0$

**(6)**

(b) Hence find the value of $p$.

**(2)**

**(Total for question = 8 marks)**

**Q6.**

The roots of the equation

$x^3 - 2x^2 + 4x - 5 = 0$

are $p$, $q$ and $r$.

Without solving the equation, find the value of

(i) $\frac{2}{p} + \frac{2}{q} + \frac{2}{r}$

(ii) $(p - 4)(q - 4)(r - 4)$

(iii) $p^3 + q^3 + r^3$

**(Total for question = 8 marks)**

**Q7.**

The roots of the equation

$x^3 - 8x^2 + 28x - 32 = 0$

are $\alpha$, $\beta$ and $\gamma$

Without solving the equation, find the value of

(i) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$

(ii) $(\alpha + 2)(\beta + 2)(\gamma + 2)$

(iii) $\alpha^2 + \beta^2 + \gamma^2$

**(8)**

**(Total for question = 8 marks)**

**Q8.**

The cubic equation

$3x^3 + x^2 - 4x + 1 = 0$

has roots $\alpha$, $\beta$, and $\gamma$.

Without solving the cubic equation,

(a) determine the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$

**(3)**

(b) find a cubic equation that has roots $\dfrac{1}{\alpha}$, $\dfrac{1}{\beta}$ and $\dfrac{1}{\gamma}$, giving your answer in the form

$x^3 + ax^2 + bx + c = 0$, where $a$, $b$ and $c$ are integers to be determined.

**(3)**

**(Total for question = 6 marks)**

**Q9.**

The cubic equation

$$9x^3 - 5x^2 + 4x + 7 = 0$$

has roots $\alpha$, $\beta$ and $\gamma$.

Without solving the equation, find the cubic equation whose roots are $(3\alpha - 2)$, $(3\beta - 2)$ and $(3\gamma - 2)$, giving your answer in the form $aw^3 + bw^2 + cw + d = 0$, where $a$, $b$, $c$ and $d$ are integers to be determined.

**(Total for question = 5 marks)**

**Q10.**

The roots of the quartic equation

$3x^4 + 5x^3 - 7x + 6 = 0$

are $\alpha$, $\beta$, $\gamma$ and $\delta$

Making your method clear and without solving the equation, determine the exact value of

(i) $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$

**(3)**

(ii) $\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma} + \frac{2}{\delta}$

**(3)**

(iii) $(3 - \alpha)(3 - \beta)(3 - \gamma)(3 - \delta)$

**(3)**

**(Total for question = 9 marks)**

**Q11.**

The cubic equation

$4x^3 + px^2 - 14x + q = 0$

where $p$ and $q$ are real positive constants, has roots $\alpha$, $\beta$ and $\gamma$

Given that $\alpha^2 + \beta^2 + \gamma^2 = 16$

(a) show that $p = 12$

**(3)**

Given that $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{14}{3}$

(b) determine the value of $q$

**(3)**

Without solving the cubic equation,

(c) determine the value of $(\alpha - 1)(\beta - 1)(\gamma - 1)$

**(4)**

**(Total for question = 10 marks)**

**Q12.**

The cubic equation

$2x^3 - 3x^2 + 5x + 7 = 0$

has roots $\alpha$, $\beta$ and $\gamma$.

Without solving the equation, determine the exact value of

(i) $\alpha^2 + \beta^2 + \gamma^2$

**(3)**

(ii) $\frac{3}{\alpha} + \frac{3}{\beta} + \frac{3}{\gamma}$

**(3)**

(iii) $(5 - \alpha)(5 - \beta)(5 - \gamma)$

**(3)**

**(Total for question = 9 marks)**

**Q13.**

The quartic equation

$2x^4 + Ax^3 - Ax^2 - 5x + 6 = 0$

where $A$ is a real constant, has roots $\alpha$, $\beta$, $\gamma$ and $\delta$

(a) Determine the value of

$$\frac{3}{\alpha} + \frac{3}{\beta} + \frac{3}{\gamma} + \frac{3}{\delta}$$

**(3)**

Given that $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = -\frac{3}{4}$

(b) determine the possible values of $A$

**(5)**

**(Total for question = 8 marks)**

**Q14.**

$f(z) = z^3 + pz^2 + qz - 15$

where $p$ and $q$ are real constants.

Given that the equation $f(z) = 0$ has roots

$\alpha, \frac{5}{\alpha}$ and $\left(\alpha + \frac{5}{\alpha} - 1\right)$

(a) solve completely the equation $f(z) = 0$

**(5)**

(b) Hence find the value of $p$.

**(2)**

**(Total for question = 7 marks)**