Questions

Q1.

(a) Given that *k* is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) \mathrm{d}x$$

simplifying your answer.

(3)

(b) Hence find the value of *k* such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx \right) \mathrm{d}x = 8$$

(3)

(Total for question = 6 marks)

Q2.

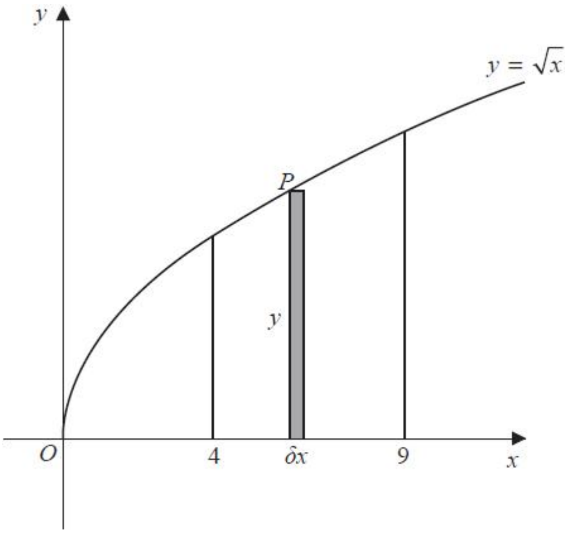

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \sqrt{x}$

The point P(x, y) lies on the curve.

The rectangle, shown shaded on Figure 3, has height y and width δx.

Calculate

$$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\,\delta x$$

(Total for question = 3 marks)

Q3.

(a) Express $\frac{5}{(x-1)(3x+2)}$ in partial fractions.

(3)

(b) Hence find $\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x$, where $x > 1$.

(3)

(c) Find the particular solution of the differential equation

$(x - 1)(3x + 2)\frac{\mathrm{d}y}{\mathrm{d}x} = 5y, \quad x > 1,$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = \mathrm{f}(x)$.

(6)

(Total 12 marks)

Q4.

(a) Express $\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x$ as an integral.

(1)

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x = \ln k$$

where *k* is a constant to be found.

(2)

(Total for question = 3 marks)

Q5.

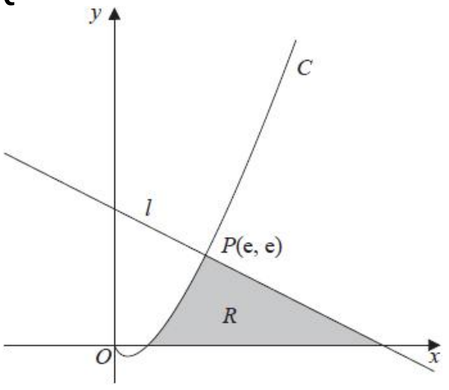

Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = x \ln x$, $x > 0$

The line l is the normal to C at the point P(e, e)

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the x-axis.

Show that the exact area of R is $Ae^2 + B$ where A and B are rational numbers to be found.

(10)

(Total for question = 10 marks)

Q6.
In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

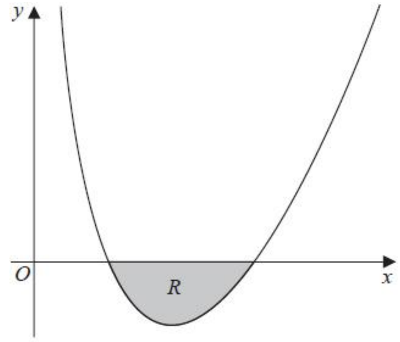

Figure 3

Figure 3 shows a sketch of part of a curve with equation

$$y = \frac{(x-2)(x-4)}{4\sqrt{x}} \qquad x > 0$$

The region *R*, shown shaded in Figure 3, is bounded by the curve and the *x*-axis.

Find the exact area of *R*, writing your answer in the form $a\sqrt{2} + b$, where *a* and *b* are constants to be found.

(Total for question = 6 marks)

Q7.

The rate of decay of the mass of a particular substance is modelled by the differential equation

$$\frac{dx}{dt} = -\frac{5}{2}x, \qquad t \geqslant 0$$

where x is the mass of the substance measured in grams and t is the time measured in days.

Given that $x = 60$ when $t = 0$,

(a) solve the differential equation, giving x in terms of t. You should show all steps in your working and give your answer in its simplest form.

(4)

(b) Find the time taken for the mass of the substance to decay from 60 grams to 20 grams.

Give your answer to the nearest minute.

(3)

(Total for question = 7 marks)

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Q8.

(i) Given that $y > 0$, find

$$\int \frac{3y-4}{y(3y+2)}\,dy$$

(6)

(ii) (a) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,dx = \lambda \int_0^{\frac{\pi}{3}} \sin^2\theta\,d\theta$$

where λ is a constant to be determined.

(5)

(b) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

(Total for question = 15 marks)

Q9.

(a) Find

$$\int 10x(x^{\frac{1}{2}} - 2)\,dx$$

giving each term in its simplest form.

(4)

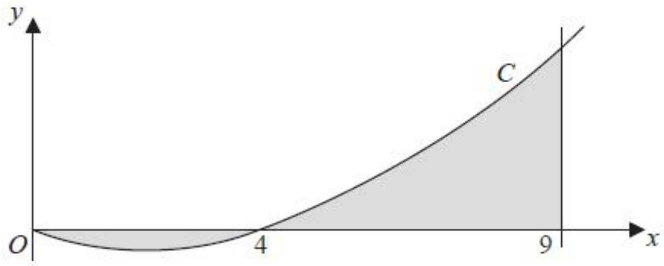

Figure 2

Figure 2 shows a sketch of part of the curve *C* with equation

$$y = 10x(x^{\frac{1}{2}} - 2), \qquad x \geqslant 0$$

The curve *C* starts at the origin and crosses the *x*-axis at the point (4, 0).

The area, shown shaded in Figure 2, consists of two finite regions and is bounded by the curve *C*, the *x*-axis and the line $x = 9$

(b) Use your answer from part (a) to find the total area of the shaded regions.

(5)

(Total for question = 9 marks)

Q10.

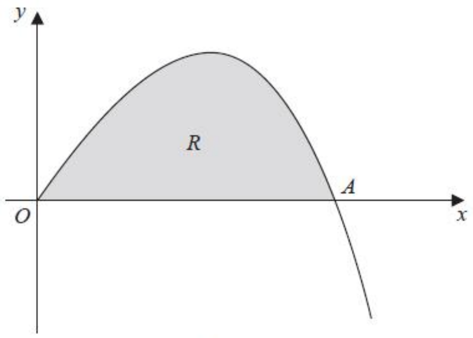

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}, \ x \geqslant 0$

The curve meets the *x*-axis at the origin *O* and cuts the *x*-axis at the point *A*.

(a) Find, in terms of ln2, the *x* coordinate of the point *A*.

(2)

(b) Find

$$\int xe^{\frac{1}{2}x}\,dx$$

(3)

The finite region *R*, shown shaded in Figure 1, is bounded by the *x*-axis and the curve with equation

$$y = 4x - xe^{\frac{1}{2}x}, \ x \geqslant 0$$

(c) Find, by integration, the exact value for the area of *R*.
Give your answer in terms of ln2

(3)

(Total for question = 8 marks)

Q11.

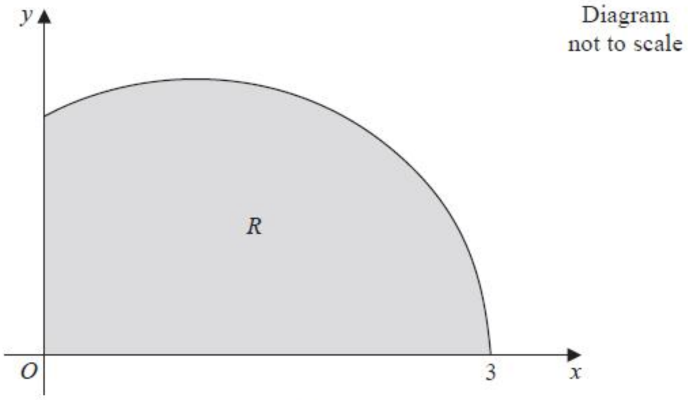

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}$, $0 \leqslant x \leqslant 3$

The finite region *R*, shown shaded in Figure 2, is bounded by the curve, the *x*-axis, and the *y*-axis.

(a) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\,\mathrm{d}x = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\,\mathrm{d}\theta$$

where *k* is a constant to be determined.

(5)

(b) Hence find, by integration, the exact area of *R*.

(3)

(Total for question = 8 marks)

Q12.

$$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x}, \quad x > 0$$

Given that $y = 37$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(7)

(Total 7 marks)

Q13.

Using the substitution $u = 2 + \sqrt{(2x + 1)}$, or other suitable substitutions, find the exact value of

$$\int_0^4 \frac{1}{2 + \sqrt{(2x+1)}}\,dx$$

giving your answer in the form $A + 2\ln B$, where A is an integer and B is a positive constant.

(8)

(Total 8 marks)

Q14.

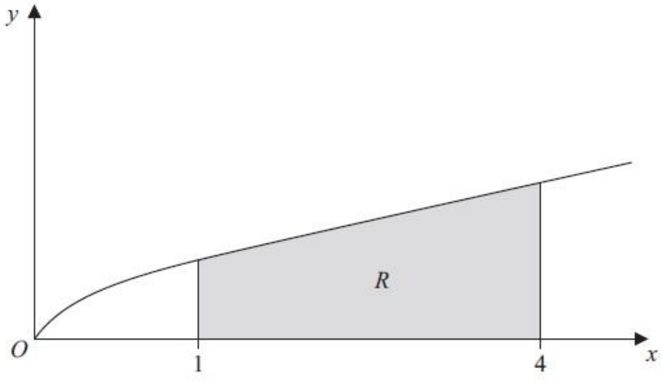

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \frac{x}{1 + \sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places.

x	1	2	3	4
y	0.5	0.8284		1.3333

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R, giving your answer to 3 decimal places.

(3)

(c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R.

(8)

(Total 12 marks)

Q15.

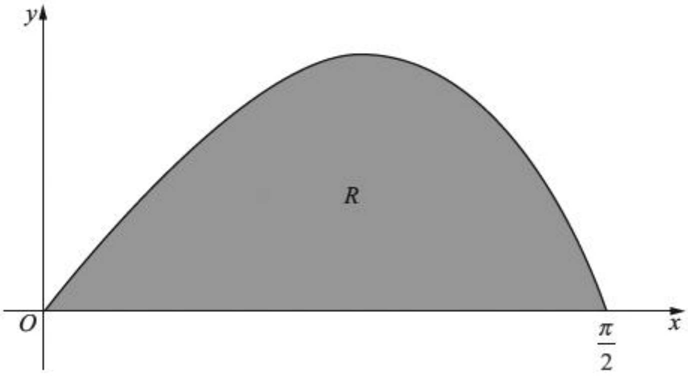

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \frac{2\sin 2x}{(1+\cos x)}$, $0 \leqslant x \leqslant \frac{\pi}{2}$.

The finite region *R*, shown shaded in Figure 3, is bounded by the curve and the *x*-axis.

The table below shows corresponding values of *x* and *y* for $y = \frac{2\sin 2x}{(1+\cos x)}$

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(a) Complete the table above giving the missing value of *y* to 5 decimal places.

(1)

(b) Use the trapezium rule, with all the values of *y* in the completed table, to obtain an estimate for the area of *R*, giving
your answer to 4 decimal places.

(3)

(c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1 + \cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k$$

where k is a constant.

(5)

(d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures.

(3)

(Total 12 marks)

Q16.

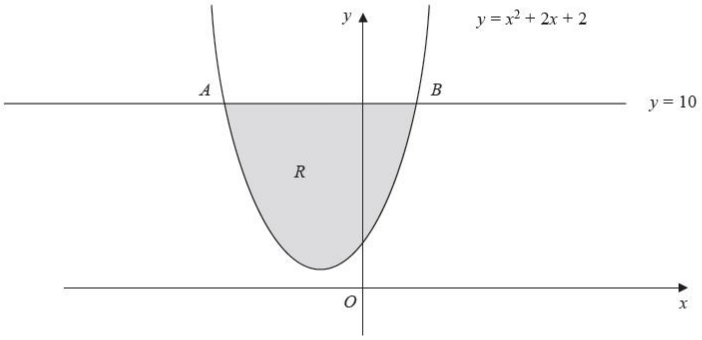

Figure 1

The line with equation $y = 10$ cuts the curve with equation $y = x^2 + 2x + 2$ at the points A and B as shown in Figure 1. The figure is not drawn to scale.

(a) Find by calculation the x-coordinate of A and the x-coordinate of B.

(2)

The shaded region R is bounded by the line with equation $y = 10$ and the curve as shown in Figure 1.

(b) Use calculus to find the exact area of R.

(7)

(Total 9 marks)

Q17.

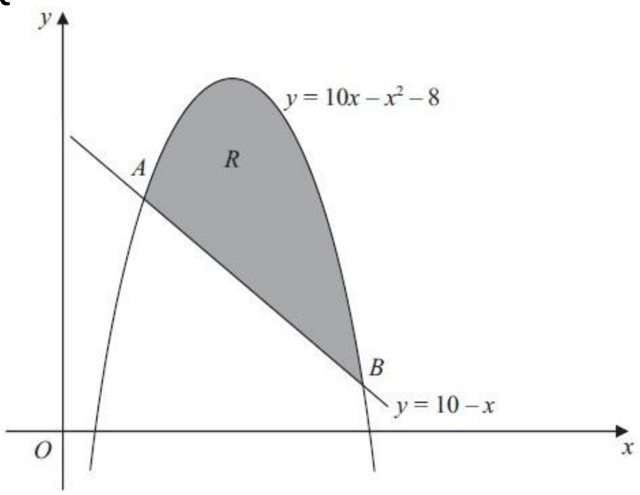

Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$

The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

(7)

(Total 12 marks)