Exam Questions Product rule and quotient rule (Differentiation)

Q1.

h(x) = $\frac{2}{x+2}+\frac{4}{x^2+5}-\frac{18}{(x^2+5)(x+2)}$, $\quad x \geq 0$

(a) Show that h(x) = $\frac{2x}{x^2+5}$

(4)

(b) Hence, or otherwise, find h´(x) in its simplest form.

(3)

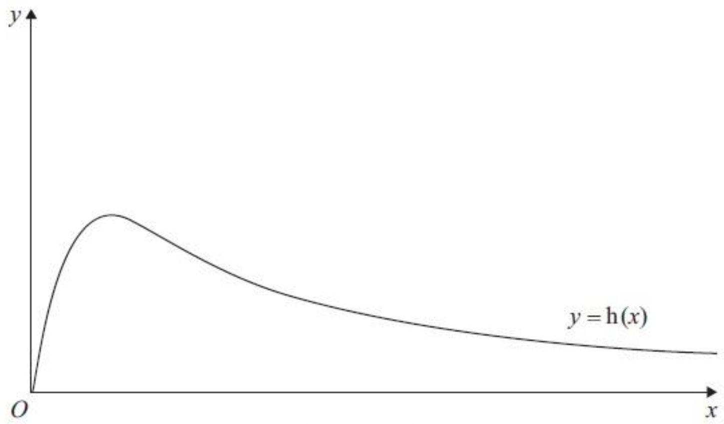

Figure 2

Figure 2 shows a graph of the curve with equation y = h(x) .

(c) Calculate the range of h(x) .

(5)

(Total 12 marks)

Q2.

$$f(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, \qquad x > 2, x \in \mathbb{R}$$

(a) Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x - 2}$$

find the values of the constants A and B.

(4)

(b) Hence or otherwise, using calculus, find an equation of the normal to the curve with equation y = f(x) at the point where x = 3

(5)

(Total for question = 9 marks)

Q3.

$$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\frac{dy}{dx} = \frac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(b) Hence deduce the range of values for x for which $\frac{dy}{dx} < 0$

(1)

(Total for question = 5 marks)

Q4.

The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \ x \in \mathbb{R}$$
$$g : x \mapsto \frac{3}{x} - 4, \ x > 0, \ x \in \mathbb{R}$$

(a) Find the inverse function f^{-1} .

(2)

(b) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve gf (x) = 0 .

(2)

(d) Use calculus to find the coordinates of the stationary point on the graph of y = gf(x).

(5)

(Total 13 marks)

Q5.

The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p.

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

(Total for question = 7 marks)

Q6.

The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(a) the value of w,

(2)

(b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

(Total 7 marks)

Q7.

(i) Differentiate with respect to x

(a) $y = x^3 \ln 2x$

(b) $y = (x + \sin 2x)^3$

(6)

Given that $x = \cot y$,

(ii) show that $\frac{dy}{dx} = \frac{-1}{1+x^2}$

(5)

(Total 11 marks)

Q8.
Differentiate with respect to x

(a) $\ln(x^2 + 3x + 5)$

(2)

(b) $\frac{\cos x}{x^2}$

(3)
(Total 5 marks)

Q9.
The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

(a) Show that

$$\frac{dy}{dx} = \frac{6 \sin 2x + 4 \cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)

(Total 8 marks)

Q10.
A curve C has equation

$$y = x^2 e^x.$$

(a) Find $\frac{dy}{dx}$, using the product rule for differentiation.

(3)

(b) Hence find the coordinates of the turning points of C.

(3)

(c) Find $\frac{d^2y}{dx^2}$.

(2)

(d) Determine the nature of each turning point of the curve C.

(2)

(Total for question = 10 marks)

Q11.

A curve has equation $y = f(x)$, where

$$f(x) = \frac{7xe^{x}}{\sqrt{e^{3x} - 2}} \qquad x > \ln\sqrt[3]{2}$$

(a) Show that

$$f'(x) = \frac{7e^{x}(e^{3x}(2 - x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where A and B are constants to be found.

(5)

(b) Hence show that the x coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

(2)

The equation $x = \frac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots α and β where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for α and β

Diagram 1 shows a plot of part of the curve with equation $y = \frac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation $y = x$

Using Diagram 1 on page 42

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for β

(1)

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(d) (i) the value of x_2

(ii) the value of β

(3)

Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places.

(2)