Questions

Q1.

$$\mathbf{A} = \begin{pmatrix} -2 & 1 & -3 \\ k & 1 & 3 \\ 2 & -1 & k \end{pmatrix}, \text{ where } k \text{ is a constant}$$

Given that the matrix **A** is singular, find the possible values of k.

(4)

(Total for question = 4 marks)

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Q2.

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$$

(a) Find $\mathbf{A}^{-1}$

(2)

The transformation represented by the matrix **B** followed by the transformation represented by the matrix **A** is equivalent to the transformation represented by the matrix **P**.

(b) Find **B**, giving your answer in its simplest form.

(3)

(Total for question = 5 marks)

Q3.

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$$

Given that **M** = (**A** + **B**)(2**A** − **B**),

(a) calculate the matrix **M**,

(6)

(b) find the matrix **C** such that **MC** = **A**.

(4)

(Total 10 marks)

Q4.

(i) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix},$$

(a) find **AB**.
(b) Explain why **AB** ≠ **BA**.

(4)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 2k & -2 \\ 3 & k \end{pmatrix},$$ where k is a real number

find $\mathbf{C}^{-1}$, giving your answer in terms of k.

(3)

(Total 7 marks)

Q5.

(i)

A = $\begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix}$, where k is a constant

Given that

$$\mathbf{B} = \mathbf{A} + 3\mathbf{I}$$

where **I** is the 2 × 2 identity matrix, find

(a) **B** in terms of k,

(2)

(b) the value of k for which **B** is singular.

(2)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 2 & -1 & 5 \end{pmatrix}$$

and

$$\mathbf{E} = \mathbf{CD}$$

find **E**.

(2)

(Total 6 marks)

Q6.

$$\mathbf{A} = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$$

and **I** is the 2 × 2 identity matrix.

(a) Prove that

$$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I}$$

(2)

(b) Hence show that

$$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})$$

(2)

The transformation represented by **A** maps the point P onto the point Q.

Given that Q has coordinates $(2k + 8, -2k - 5)$, where k is a constant,

(c) find, in terms of k, the coordinates of P.

(4)

(Total 8 marks)

Q7.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$$

(a) Show that **A** is non-singular.

(2)

(b) Find **B** such that $\mathbf{BA}^2 = \mathbf{A}$.

(4)

(Total 6 marks)

Q8.

A system of three equations is defined by

$$kx + 3y - z = 3$$
$$3x - y + z = -k$$
$$-16x - ky - kz = k$$

where k is a positive constant.

Given that there is no unique solution to all three equations,

(a) show that $k = 2$

(2)

Using $k = 2$

(b) determine whether the three equations are consistent, justifying your answer.

(3)

(c) Interpret the answer to part (b) geometrically.

(1)

(Total for question = 6 marks)

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Q9.

$$\mathbf{A} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation *U* represented by the matrix **A**.

(3)

The transformation *V*, represented by the 2 × 2 matrix **B**, is a reflection in the line $y = -x$

(b) Write down the matrix **B**.

(1)

Given that *U* followed by *V* is the transformation *T*, which is represented by the matrix **C**,

(c) find the matrix **C**.

(2)

(d) Show that there is a real number *k* for which the point (1, *k*) is invariant under *T*.

(4)

(Total for question = 10 marks)

Q10.

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & -3 \\ 4 & -2 & 1 \\ 3 & 5 & -2 \end{pmatrix}$$

(a) Find $\mathbf{M}^{-1}$ giving each element in exact form.

(2)

(b) Solve the simultaneous equations

$$2x + y - 3z = -4$$

$$4x - 2y + z = 9$$

$$3x + 5y - 2z = 5$$

(2)

(c) Interpret the answer to part (b) geometrically.

(1)

(Total for question = 5 marks)

Q11.

Tyler invested a total of £5000 across three different accounts; a savings account, a property bond account and a share dealing account.

Tyler invested £400 more in the property bond account than in the savings account.

After one year

- the savings account had increased in value by 1.5%
- the property bond account had increased in value by 3.5%
- the share dealing account had **decreased** in value by 2.5%

- the total value across Tyler's three accounts had increased by £79

Form and solve a matrix equation to find out how much money was invested by Tyler in each account.

(7)

(Total for question = 7 marks)

Q12.

(i)

$$\mathbf{A} = \begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}$$

where a and b are non-zero constants.
Given that the matrix **A** is self-inverse,
(a) determine the value of b and the possible values for a.

(5)

The matrix **A** represents a linear transformation M.
Using the smaller value of a from part (a),
(b) show that the invariant points of the linear transformation M form a line, stating the equation of this line.

(3)

(ii)

$$\mathbf{P} = \begin{pmatrix} p & 2p \\ -1 & 3p \end{pmatrix}$$

where p is a positive constant.
The matrix **P** represents a linear transformation U.
The triangle T has vertices at the points with coordinates (1, 2), (3, 2) and (2, 5).
The area of the image of T under the linear transformation U is 15
(a) Determine the value of p.

(4)

The transformation V consists of a stretch scale factor 3 parallel to the x-axis with the y-axis invariant followed by a stretch scale factor –2 parallel to the y-axis with the x-axis invariant. The transformation V is represented by the matrix **Q**.
(b) Write down the matrix **Q**.

(2)

Given that U followed by V is the transformation W, which is represented by the matrix **R**,
(c) find the matrix **R**.

(2)

(Total for question = 16 marks)

Q13.

$$\mathbf{M} = \begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}$$

(a) Show that the matrix **M** is non-singular.

(2)

The transformation T of the plane is represented by the matrix **M**.

The triangle R is transformed to the triangle S by the transformation T.

Given that the area of S is 63 square units,

(b) find the area of R.

(2)

(c) Show that the line $y = 2x$ is invariant under the transformation T.

(2)

(Total for question = 6 marks)

Q14.

The population of chimpanzees in a particular country consists of juveniles and adults. Juvenile chimpanzees do not reproduce.

In a study, the numbers of juvenile and adult chimpanzees were estimated at the start of each year. A model for the population satisfies the matrix system

$$\begin{pmatrix} J_{n+1} \\ A_{n+1} \end{pmatrix} = \begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix} \begin{pmatrix} J_n \\ A_n \end{pmatrix} \qquad n = 0, 1, 2, \ldots$$

where a is a constant, and J_n and A_n are the respective numbers of juvenile and adult chimpanzees n years after the start of the study.

(a) Interpret the meaning of the constant a in the context of the model.

(1)

At the start of the study, the total number of chimpanzees in the country was estimated to be 64 000

According to the model, after one year the number of juvenile chimpanzees is 15 360 and the number of adult chimpanzees is 43 008

(b) (i) Find, in terms of a

$$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1}$$

(3)

(ii) Hence, or otherwise, find the value of a.

(3)

(iii) Calculate the change in the number of juvenile chimpanzees in the first year of the study, according to this model.

(2)

Given that the number of juvenile chimpanzees is known to be in decline in the country,

(c) comment on the short-term suitability of this model.

(1)

A study of the population revealed that adult chimpanzees stop reproducing at the age of 40 years.

(d) Refine the matrix system for the model to reflect this information, giving a reason for your answer.

(*There is no need to estimate any unknown values for the refined model, but any known values should be made clear.*)

(2)

(Total for question = 12 marks)

Q15.

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 4 \\ k & 2 & -2 \\ 4 & 1 & -2 \end{pmatrix} \qquad \mathbf{N} = \begin{pmatrix} k-7 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}$$

where k is a constant.

(a) Determine, in simplest form in terms of k, the matrix **M N**.

(2)

(b) Given that $k = 5$

(i) write down **M N**

(ii) hence write down $\mathbf{M}^{-1}$

(2)

(c) Solve the simultaneous equations

$$\begin{aligned} 2x + y + 4z &= 2 \\ 5x + 2y - 2z &= 3 \\ 4x + y - 2z &= -1 \end{aligned}$$

(2)

(d) Interpret the answer to part (c) geometrically.

(1)

(Total for question = 7 marks)

Q16.

(i) In each of the following cases, find a 2 × 2 matrix that represents

(a) a reflection in the line $y = -x$,
(b) a rotation of 135° anticlockwise about (0, 0),
(c) a reflection in the line $y = -x$ followed by a rotation of 135° anticlockwise about (0, 0).

(4)

(ii) The triangle T has vertices at the points $(1, k)$, $(3, 0)$ and $(11, 0)$, where k is a constant.

Triangle T is transformed onto the triangle T' by the matrix

$$\begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix}$$

Given that the area of triangle T' is 364 square units, find the value of k.

(6)

(Total 10 marks)

Q17.

The transformation *U*, represented by the 2 × 2 matrix **P**, is a rotation through 90º anticlockwise about the origin.

(a) Write down the matrix **P**.

(1)

The transformation *V*, represented by the 2 × 2 matrix **Q**, is a reflection in the line $y = -x$.

(b) Write down the matrix **Q**.

(1)

Given that *U* followed by *V* is transformation *T*, which is represented by the matrix **R**,

(c) express **R** in terms of **P** and **Q**,

(1)

(d) find the matrix **R**,

(2)

(e) give a full geometrical description of *T* as a single transformation.

(2)

(Total 7 marks)

Q18.

$$\mathbf{M} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}$$

(a) Find det **M**.

(1)

The transformation represented by **M** maps the point $S(2a - 7, a - 1)$, where a is a constant, onto the point $S'(25, -14)$.

(b) Find the value of a.

(3)

The point R has coordinates (6, 0).
Given that O is the origin,

(c) find the area of triangle ORS.

(2)

Triangle ORS is mapped onto triangle $OR'S'$ by the transformation represented by **M**.

(d) Find the area of triangle $OR'S'$.

(2)

Given that

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(e) describe fully the single geometrical transformation represented by **A**.

(2)

The transformation represented by **A** followed by the transformation represented by **B** is equivalent to the transformation represented by **M**.

(f) Find **B**.

(4)

(Total 14 marks)

Q19.

$$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$$

(a) Find det **A**.

(1)

(b) Find $\mathbf{A}^{-1}$.

(2)

The triangle *R* is transformed to the triangle *S* by the matrix **A**.
Given that the area of triangle *S* is 72 square units,

(c) find the area of triangle *R*.

(2)

The triangle *S* has vertices at the points (0, 4), (8, 16) and (12, 4).

(d) Find the coordinates of the vertices of *R*.

(4)

(Total 9 marks)

Q20.

(i)

$$\mathbf{P} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

(a) Describe fully the single geometrical transformation *P* represented by the matrix **P**.

(2)

(b) State the equation of **one** invariant line under the transformation *P*.

(1)

(ii)

$$\mathbf{Q} = \begin{pmatrix} \cos 2\theta & 0 \\ 1 & \tan 2\theta \end{pmatrix} \qquad \text{where } 0 \leqslant \theta < 360°$$

The matrix **Q** represents the transformation *Q*.
Triangle *T* is transformed to triangle *T '* by the transformation *Q*.
Given that

- the coordinates of the vertices of *T* are (2, 3), (3, 6) and (8, 3)
- the area of *T'* is 4.5

determine the possible values of θ

(*Solutions relying entirely on calculator technology are not acceptable*)

(7)

(Total for question = 10 marks)

Q21.

$$\mathbf{M} = \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$$

(a) Show that **M** is non-singular.

(2)

The hexagon *R* is transformed to the hexagon *S* by the transformation represented by the matrix **M**.

Given that the area of hexagon *R* is 5 square units,

(b) find the area of hexagon *S*.

(1)

The matrix **M** represents an enlargement, with centre (0, 0) and scale factor k, where $k > 0$, followed by a rotation anti-clockwise through an angle θ about (0, 0).

(c) Find the value of k.

(2)

(d) Find the value of θ.

(2)

(Total for question = 7 marks)