Questions

Q1.

Given that θ is measured in radians, prove, from first principles, that the derivative of sin θ is cos θ

You may assume the formula for sin($A \pm B$) and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

(Total for question = 5 marks)

Q2.

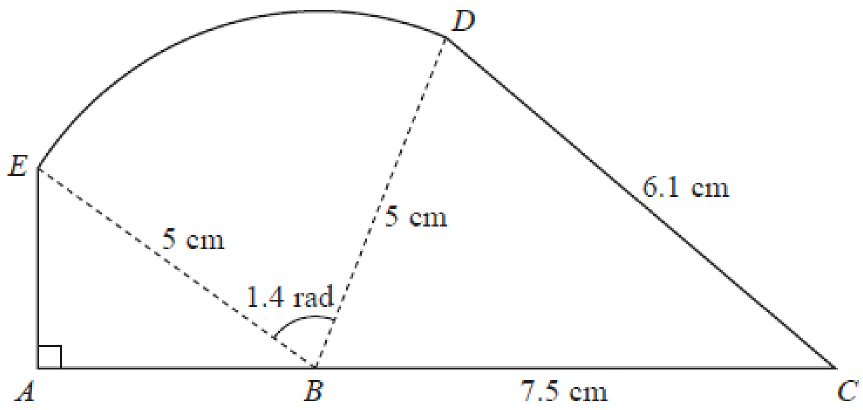


Figure 2

The shape *ABCDEA*, as shown in Figure 2, consists of a right-angled triangle *EAB* and a triangle *DBC* joined to a sector *BDE* of a circle with radius 5 cm and centre *B*.

The points *A*, *B* and *C* lie on a straight line with *BC* = 7.5 cm.

Angle *EAB* = $\pi/2$ radians, angle *EBD* = 1.4 radians and *CD* = 6.1 cm.

(a) Find, in cm^2, the area of the sector *BDE*.

(2)

(b) Find the size of the angle *DBC*, giving your answer in radians to 3 decimal places.

(2)

(c) Find, in cm^2, the area of the shape *ABCDEA*, giving your answer to 3 significant figures.

(5)

(Total 9 marks)

Q3.

(i) Solve, for $0 \le \theta < 180°$, the equation

$$\frac{\sin 2\theta}{(4\sin 2\theta - 1)} = 1$$

giving your answers to 1 decimal place.

(3)

(ii) Solve, for $0 \le x < 2\pi$, the equation

$$5\sin^2 x - 2\cos x - 5 = 0$$

giving your answers to 2 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

(Total 8 marks)

Q4.

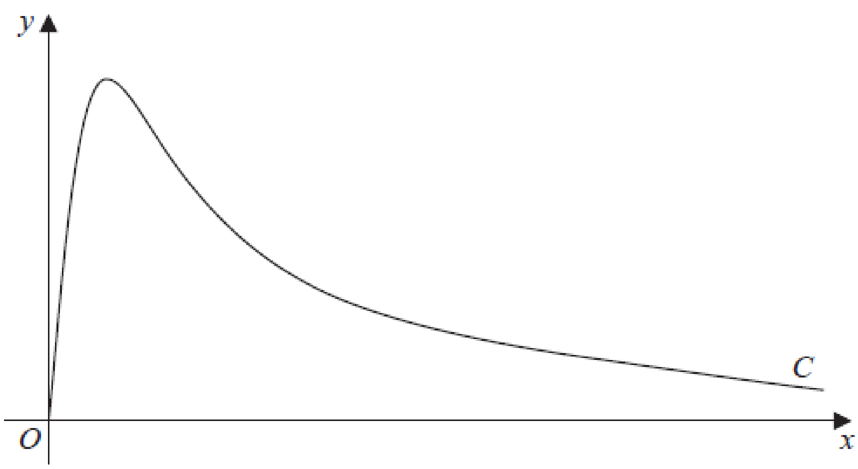


Figure 2

Figure 2 shows a sketch of the curve *C* with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point *P* lies on *C* and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{dy}{dx}$ at the point *P*.

Give your answer as a simplified surd.

(4)

The point *Q* lies on the curve *C*, where $\frac{dy}{dx} = 0$

(b) Find the exact coordinates of the point *Q*.

(2)

(Total for question = 6 marks)

Q5.

(i) Find, using calculus, the *x* coordinate of the turning point of the curve with equation

$$y = e^{3x}\cos 4x, \quad \frac{\pi}{4} \leqslant x < \frac{\pi}{2}$$

Give your answer to 4 decimal places.

(5)

(ii) Given $x = \sin^2 2y, \quad 0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of *y*.

Write your answer in the form

$$\frac{dy}{dx} = p\,\text{cosec}(qy), \qquad 0 < y < \frac{\pi}{4}$$

where *p* and *q* are constants to be determined.

(5)

(Total for question = 10 marks)

Q6.

The curve *C* has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \quad 0 \leqslant t \leqslant \pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of *t*.

(2)

The point *P* lies on *C* where $t = \frac{2\pi}{3}$

The line *l* is the normal to *C* at *P*.

(b) Show that an equation for *l* is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line *l* intersects the curve *C* again at the point *Q*.

(c) Find the exact coordinates of *Q*.

You must show clearly how you obtained your answers.

(6)

(Total for question = 13 marks)

Q7.

(i) Given that

$$x = \sec^2 2y, \quad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{1}{4x\sqrt{(x-1)}}$$

(4)

(ii) Given that

$$y = (x^2 + x3)\ln 2x$$

find the exact value of $\frac{dy}{dx}$ at $x = \frac{e}{2}$, giving your answer in its simplest form.

(5)

(iii) Given that

$$f(x) = \frac{3\cos x}{(x+1)^{\frac{1}{3}}}, \qquad x \neq -1$$

show that

$$f'(x) = \frac{g(x)}{(x+1)^{\frac{4}{3}}}, \qquad x \neq -1$$

where g(x) is an expression to be found.

(3)

(Total 12 marks)

Q8. A curve C has parametric equations

$$x = \sin^2 t, \quad y = 2\tan t, \quad 0 \leqslant t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t.

(4)

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(b) Find the x-coordinate of P.

(6)

(Total 10 marks)

Q9.

Given that

$$x = \sec^2 3y, \qquad 0 < y < \frac{\pi}{6}$$

(a) find $\frac{dx}{dy}$ in terms of *y*.

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(c) Find an expression for $\frac{d^2y}{dx^2}$ in terms of *x*. Give your answer in its simplest form.

(4)

(Total 10 marks)

Q10.

A curve has parametric equations

$$x = \tan^2 t, \qquad y = \sin t, \qquad 0 < t < \frac{\pi}{2}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of *t*. You need not simplify your answer.

(3)

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

Give your answer in the form $y = ax + b$, where *a* and *b* are constants to be determined.

(5)

(c) Find a cartesian equation of the curve in the form $y^2 = f(x)$.

(4)

(Total 12 marks)

Q11.

Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$$

(3)

(Total for question = 3 marks)

Q12.

(a) Prove that

$$\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta, \qquad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

does not have any real solutions.

(1)

(Total for question = 5 marks)

Q13.

The depth of water, *D* metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)^\circ \qquad 0 \leqslant t < 24$$

where *t* is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total for question = 5 marks)

Q14.

(a) Express $\dfrac{2}{P(P-2)}$ in partial fractions.

(3)

A team of biologists is studying a population of a particular species of animal.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{2}P(P-2)\cos 2t, \quad t \geqslant 0$$

where *P* is the population in thousands, and *t* is the time measured in years since the start of the study.

Given that *P* = 3 when *t* = 0,

(b) solve this differential equation to show that

$$P = \frac{6}{3 - e^{\frac{1}{2}\sin 2t}}$$

(7)

(c) find the time taken for the population to reach 4000 for the first time.
Give your answer in years to 3 significant figures.

(3)

(Total for question = 13 marks)

Q15.

On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where a and b are positive constants.

(a) Find a complete equation for the model.

(3)

(b) Use the model to determine the height of the carriage above the ground when $t = 40$

(1)

In an alternative model, the vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is given by

$$H = 29\cos(9t + \alpha)° + \beta \qquad 0 \leq \alpha < 360°$$

where α and β are constants.

(c) Find a complete equation for the alternative model.

(2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

(1)

(Total for question = 7 marks)

Q16.

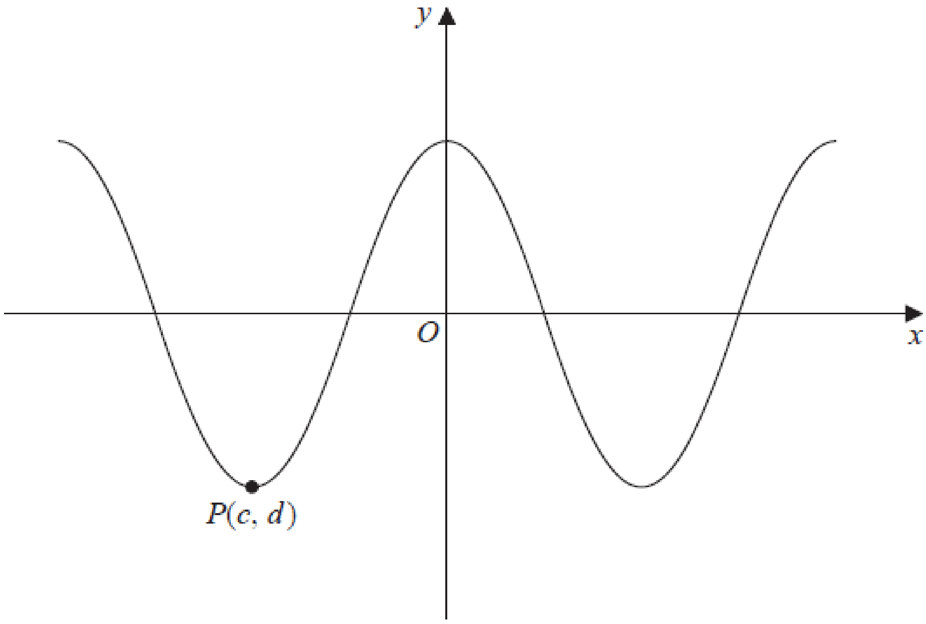


Figure 3

Figure 3 shows part of the curve with equation $y = 3\cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which
a minimum occurs.

(a) State the value of c and the value of d.

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which

transforms the curve with equation $y = 3\cos x°$ to the curve with equation

(i) $y = 3\cos\left(\frac{x°}{4}\right)$

(ii) $y = 3\cos(x - 36)°$

(2)

(c) Solve, for $450° \leq \theta < 720°$

$$3 \cos \theta = 8 \tan \theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(5)

(Total for question = 8 marks)

Q17.

Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos \theta = 2 \sin \theta$$

The attempts of two of the students are shown below.

Student *A*	Student *B*
$\cos \theta = 2 \sin \theta$	$\cos \theta = 2 \sin \theta$
$\tan \theta = 2$	$\cos^2 \theta = 4 \sin^2 \theta$
$\theta = 63.4°$	$1 - \sin^2 \theta = 4\sin^2 \theta$
	$\sin^2 \theta = \frac{1}{5}$
	$\sin \theta = \pm\frac{1}{\sqrt{5}}$
	$\theta = \pm 26.6°$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos \theta = 2 \sin \theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

(Total for question = 3 marks)

Q18.

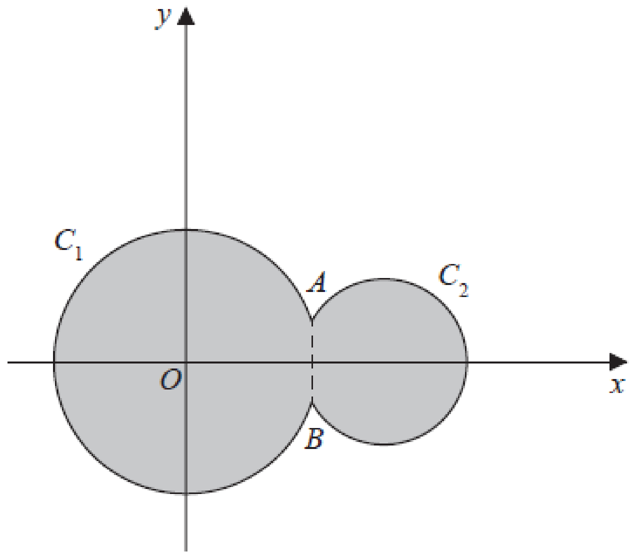


Figure 3

Circle C_1 has equation $x^2 + y^2 = 100$

Circle C_2 has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin.

(4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

(Total for question = 8 marks)

Q19.

(i) Given that $y > 0$, find

$$\int \frac{3y - 4}{y(3y + 2)}\,dy$$

(6)

(ii) (a) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4 - x}\right)}\,dx = \lambda \int_0^{\frac{\pi}{3}} \sin^2\theta\,d\theta$$

where λ is a constant to be determined.

(5)

(b) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4 - x}\right)}\,dx$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

(Total for question = 15 marks)

Q20.

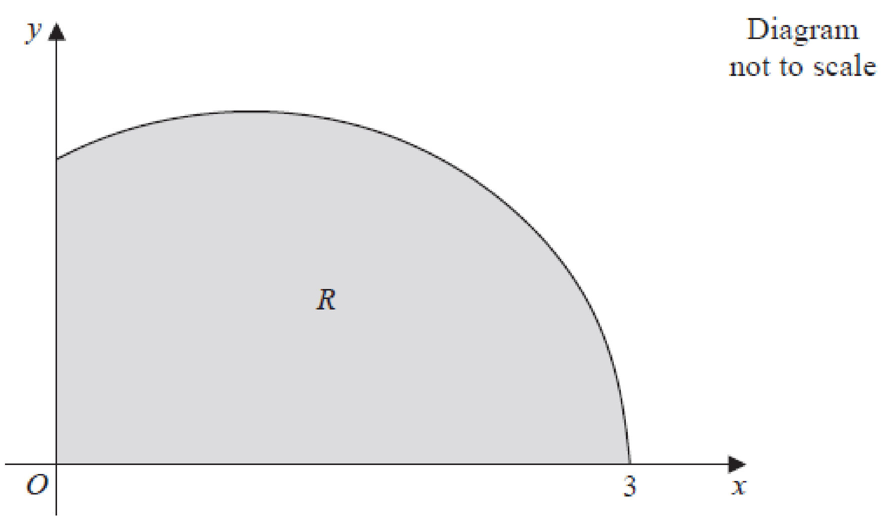


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}, \quad 0 \leqslant x \leqslant 3$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis, and the y-axis.

(a) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\,\mathrm{d}x = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\,\mathrm{d}\theta$$

where k is a constant to be determined.

(5)

(b) Hence find, by integration, the exact area of R.

(3)

(Total for question = 8 marks)

Q21.

Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} x\sin 2x\,\mathrm{d}x$$

(6)

(Total 6 marks)

Q22.

(a) Use integration by parts to find $\int x \sin 3x\, dx$.

(3)

(b) Using your answer to part (a), find $\int x^2 \cos 3x\, dx$.

(3)

(Total 6 marks)

Q23.

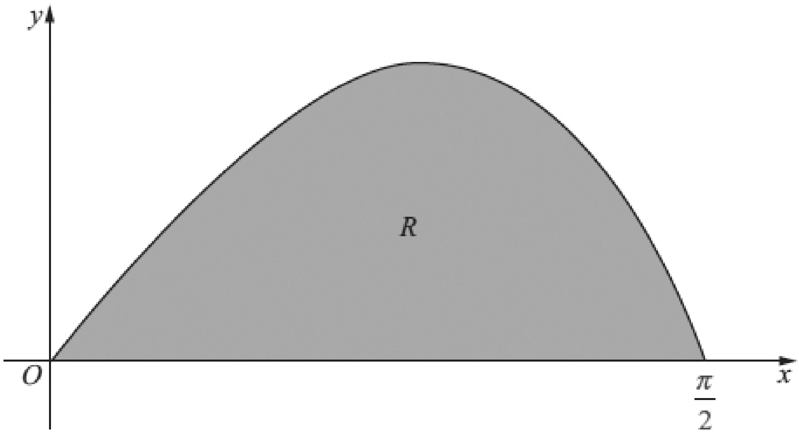


Figure 3

Figure 3 shows a sketch of the curve with equation $y = \frac{2\sin 2x}{(1+\cos x)}$, $0 \leqslant x \leqslant \frac{\pi}{2}$.

The finite region *R*, shown shaded in Figure 3, is bounded by the curve and the *x*-axis.

The table below shows corresponding values of *x* and *y* for $y = \frac{2\sin 2x}{(1+\cos x)}$

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(a) Complete the table above giving the missing value of *y* to 5 decimal places.

(1)

(b) Use the trapezium rule, with all the values of *y* in the completed table, to obtain an estimate for the area of *R*, giving
your answer to 4 decimal places.

(3)

(c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k$$

where *k* is a constant.

(5)

(d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures.

(3)

(Total 12 marks)

Q24.

(a) Find $\int x \cos 2x \, dx$.

(4)

(b) Hence, using the identity $\cos 2x = 2\cos^2 x - 1$, deduce $\int x \cos^2 x \, dx$.

(3)

(Total 7 marks)

Q25.

(i) Find $\int \ln\left(\frac{x}{2}\right) dx$.

(4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx$

(5)

(Total 9 marks)

Q26. $f(\theta) = 4\cos^2\theta - 3\sin^2\theta$

(a) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta.$

(3)

(b) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta f(\theta)\, d\theta.$

(7)

(Total 10 marks)

Q27.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(4)

(b) Hence solve, for $-90° \leq x \leq 180°$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4)

(Total for question = 8 marks)

Q28.

(i) Find

$$\int x\, e^{4x}\, dx$$

(3)

(ii) Find

$$\int \frac{8}{(2x-1)^3}\, dx, \quad x > \tfrac{1}{2}$$

(2)

Given that $y = \frac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y$$

(7)

(Total 12 marks)

Q29.

(a) Express $10 \cos \theta - 3 \sin \theta$ in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$

Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

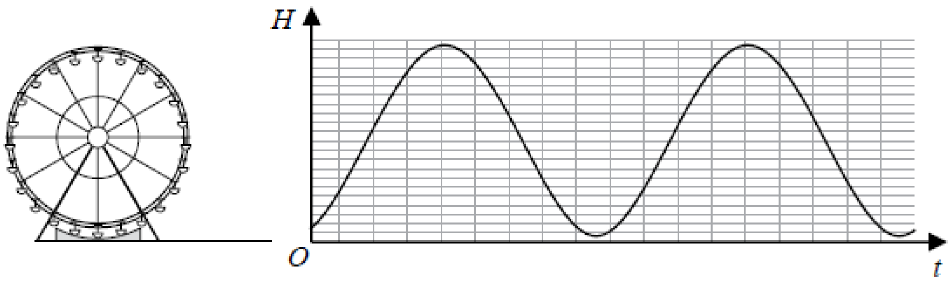


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10 \cos (80t)° + 3 \sin (80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

(Total for question = 9 marks)

Q30.

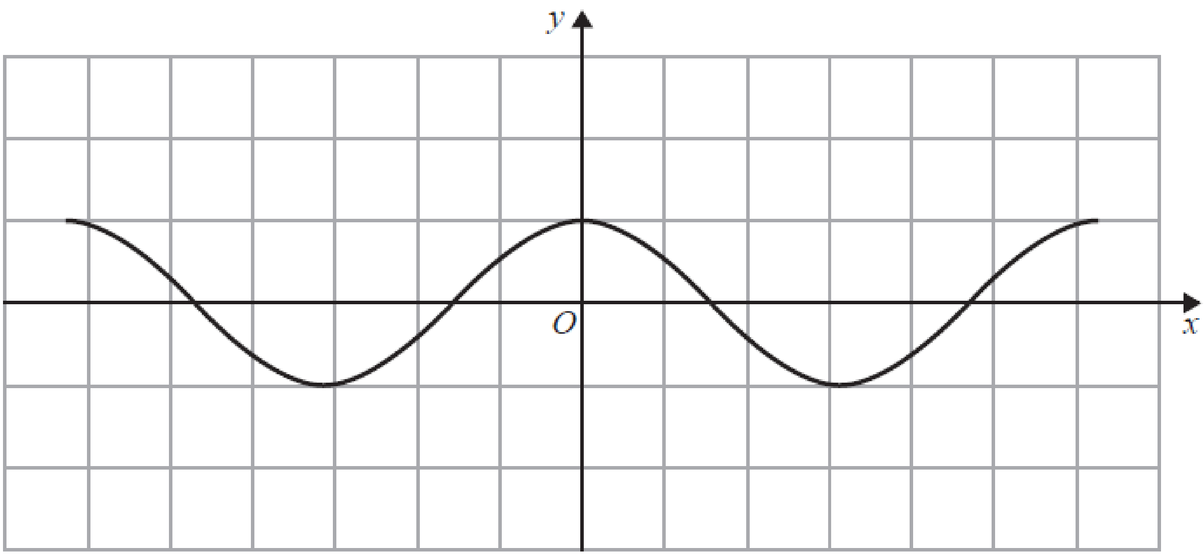


Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians.
Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

Given that the root of the equation is *a*, and that *a* is small,

(b) use the small angle approximation for cos *x* to estimate the value of *a* to 3 decimal places.

(3)

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

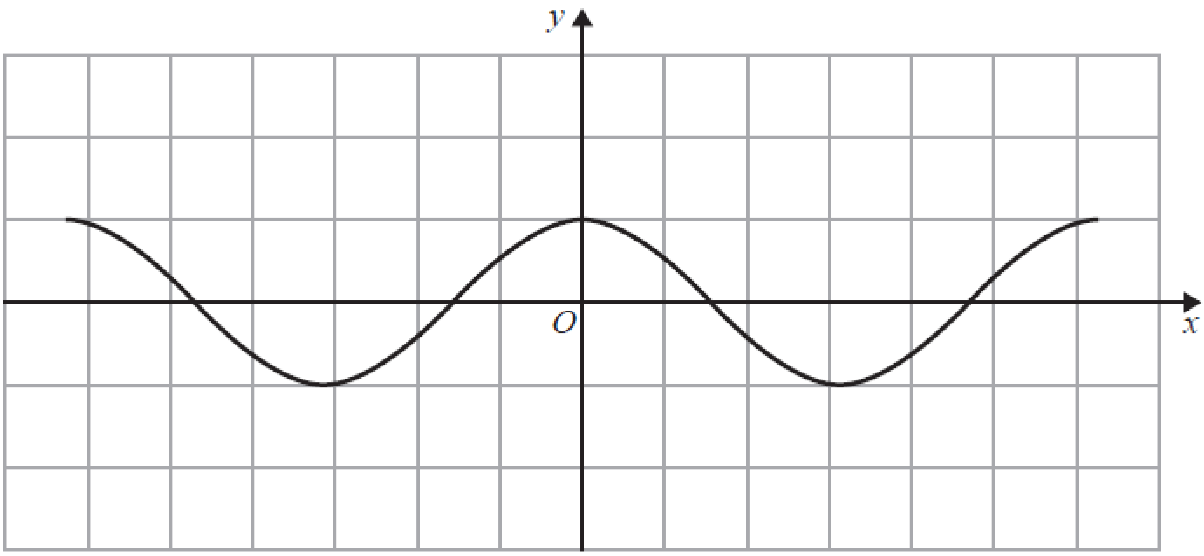


Diagram 1

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(Total for question = 5 marks)

Q31.

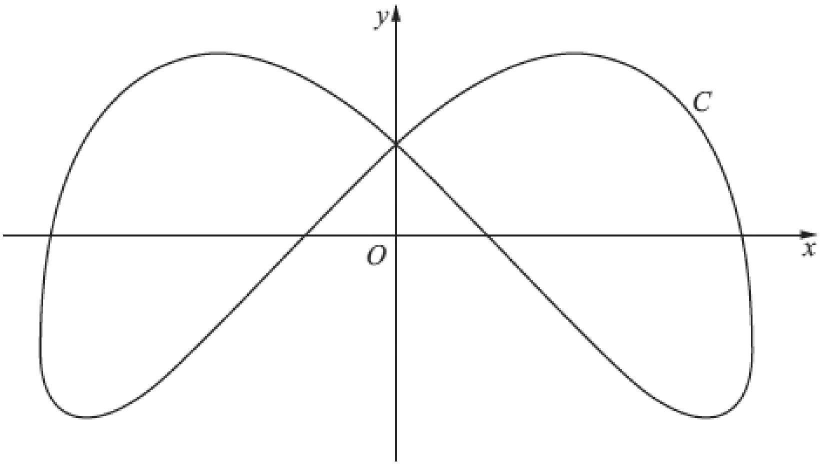


Figure 2

Figure 2 shows a sketch of the curve *C* with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \quad y = 3\cos 2t, \quad 0 \leqslant t < 2\pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(3)

Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$

(5)

(Total 8 marks)

Q32.

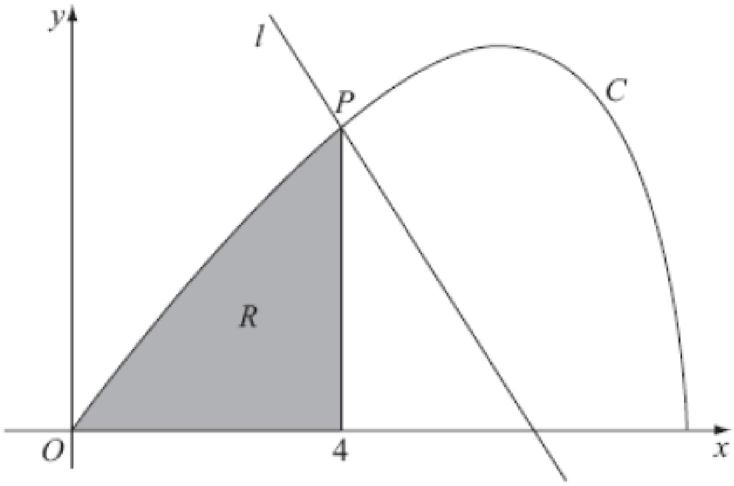


Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leq t \leq \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64 \sin^2 t \cos t \, dt.$$

(4)

(d) Use this integral to find the area of *R*, giving your answer in the form $a + b\sqrt{3}$, where *a* and *b* are constants to be determined.

(4)

(Total 16 marks)

Q33.

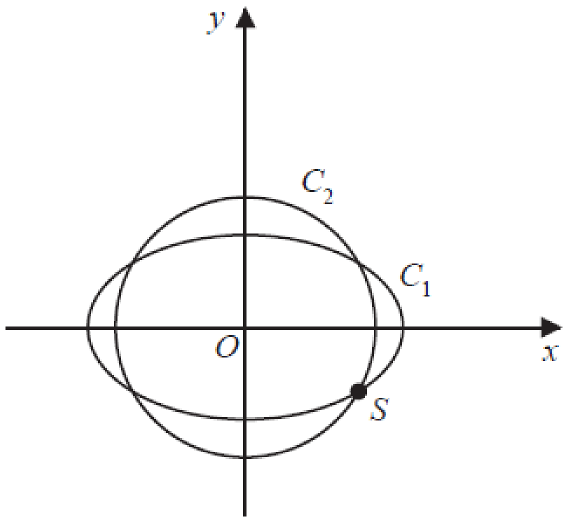


Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \quad y = 4\sqrt{2}\sin t, \quad 0 \leqslant t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in Figure 2.

Given that one of these points, *S*, lies in the 4th quadrant, find the Cartesian coordinates of *S*.

(Total for question = 6 marks)

Q34.

(a) Prove that

$$2\cot 2x + \tan x \equiv \cot x \qquad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $-\pi \leq x < \pi$,

$$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2$$

Give your answers to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(Total for question = 10 marks)

Q35.

(a) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \ n \in \mathbb{Z}$$

(5)

(b) Hence solve, for $0 \leqslant \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}$$

Give your answers to 3 decimal places.

(4)

(Total for question = 9 marks)

Q36.

(a) Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

(Total for question = 9 marks)

Q37.

(a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + a)$, where R and a are constants, $R > 0$ and $0 < a$ 90° Give the exact value of R and give the value of a to 2 decimal places.

(3)

(b) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15$$

Give your answers to one decimal place.

(5)

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15$$

Give your answer to one decimal place.

(2)

(Total for question = 10 marks)

Q38.

(a) Express $2 \sin \theta - 4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α to 3 decimal places.

(3)

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(b) (i) the maximum value of $H(\theta)$,

(ii) the smallest value of θ, for $0 \leq \theta < \pi$, at which this maximum value occurs.

(3)

Find

(c) (i) the minimum value of $H(\theta)$,

(ii) the largest value of θ, for $0 \leq \theta < \pi$, at which this minimum value occurs.

(3)

(Total 9 marks)

Q39.

(a) Write $5\cos \theta - 2\sin \theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 \leq \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(b) Show that the equation

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5\cos 2x - 2\sin 2x = c$$

where c is a positive constant to be determined.

(2)

(c) Hence or otherwise, solve, for $0 \leq x < \pi$,

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total for question = 9 marks)

Q40.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos\theta} + \tan\theta \equiv \frac{\cos\theta}{1-\sin\theta} \qquad \theta \neq (2n+1)90° \quad n \in \mathbb{Z}$$

(3)

Given that $\cos 2x \neq 0$

(b) solve for $0 < x < 90°$

$$\frac{1}{\cos 2x} + \tan 2x = 3\cos 2x$$

giving your answers to one decimal place.

(5)

(Total for question = 8 marks)

Q41.

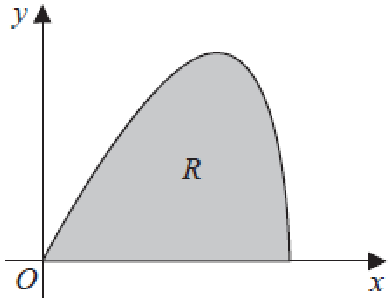


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6\sin t \qquad y = 5\sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region *R*, shown shaded in Figure 3, is bounded by the curve and the *x*-axis.

(a) (i) Show that the area of *R* is given by $\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t \, dt$

(3)

(ii) Hence show, by algebraic integration, that the area of *R* is exactly 20

(3)

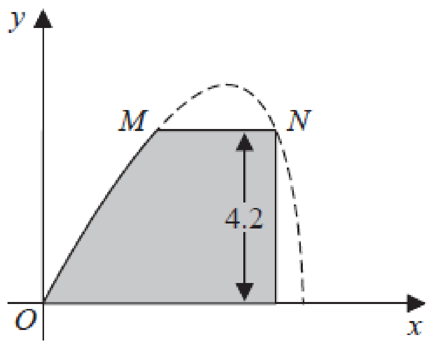


Figure 4

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4.
Using the model and given that

- *x* and *y* are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width *MN* along the top of the dam

(b) calculate the width of the walkway.

(5)

(Total for question = 11 marks)

Q42.

(a) Show that

$$\text{cosec } 2x + \cot 2x = \cot x, \quad x \neq 90n°, \quad n \in \mathbb{R}.$$

(5)

(b) Hence, or otherwise, solve, for $0 \leq \theta < 180°$,

$$\text{cosec } (4\theta + 10°) + \cot (4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(Total 10 marks)

Q43.

(i) Solve, for $0 \leq \theta < 360°$, the equation

$$9\sin(\theta + 60°) = 4$$

giving your answers to 1 decimal place.
You must show each step of your working.

(4)

(ii) Solve, for $-\pi \leq x < \pi$, the equation

$$2\tan x - 3\sin x = 0$$

giving your answers to 2 decimal places where appropriate.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

(Total 9 marks)

Q44.

(a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for 0 ≤ *x* ≤ 360°, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

(Total for question = 7 marks)

Q45.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta(8\cos\theta + 23\sin^2\theta) = 8\sin 2\theta(1 + \tan^2\theta)$$

may be written as

$$\sin 2\theta(A\cos^2\theta + B\cos\theta + C) = 0$$

where A, B and C are constants to be found.

(3)

(b) Hence, solve for $360° \leqslant x \leqslant 540°$

$$2\tan x(8\cos x + 23\sin^2 x) = 8\sin 2x(1 + \tan^2 x) \qquad x \in \mathbb{R} \qquad x \neq 450°$$

(4)

(Total for question = 7 marks)

Q46.

(a) Express 2 cos θ + 8 sin θ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The first three terms of an arithmetic sequence are

$$\cos x \qquad \cos x + \sin x \qquad \cos x + 2\sin x \qquad x \neq n\pi$$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

(b) (i) find the exact maximum value of S_9

(ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

(3)

(Total for question = 6 marks)

Q47.

(a) Express sin$x$ + 2 cos$x$ in the form $R\sin(x + \alpha)$ where R and α are constants, $R > 0$

and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The temperature, θ °C , inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2\cos\left(\frac{\pi t}{12} - 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day,

(1)

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3)

(Total for question = 7 marks)

Q48.

Figure 1

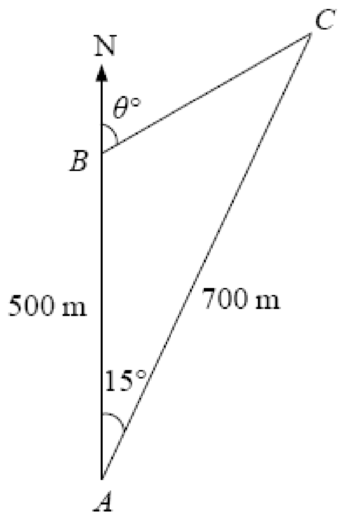


Figure 1 shows 3 yachts *A*, *B* and *C* which are assumed to be in the same horizontal plane. Yacht *B* is 500 m due north of yacht *A* and yacht *C* is 700 m from A. The bearing of *C* from *A* is 015°.

(a) Calculate the distance between yacht *B* and yacht *C*, in metres to 3 significant figures.

(3)

The bearing of yacht *C* from yacht *B* is θ°, as shown in Figure 1.

(b) Calculate the value of θ.

(4)

(Total 7 marks)

Q49.

(i) Use an appropriate double angle formula to show that

$$\text{cosec}2x = \lambda \text{ cosec } x \text{ sec } x,$$

and state the value of the constant λ.

(3)

(ii) Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta$$

You must show all your working. Give your answers in terms of π.

(6)

(Total 9 marks)

Q50.

(a) Solve for $0 \leq x < 360°$, giving your answers in degrees to 1 decimal place,

$$3\sin(x + 45°) = 2$$

(4)

(b) Find, for $0 \leq x < 2\pi$, all the solutions of

$$2\sin^2 x + 2 = 7\cos x$$

giving your answers in radians.

You must show clearly how you obtained your answers.

(6)

(Total 10 marks)

Q51.

(a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \text{cosec}^2\theta$.

(2)

(b) Solve, for $0 \leq \theta < 180°$, the equation

$$2 \cot^2 \theta - 9 \operatorname{cosec} \theta = 3,$$

giving your answers to 1 decimal place.

(6)

(Total 8 marks)

Q52. (a) Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, f (t), of a warehouse is modelled using the equation

$$f(t) = 10 + 3 \cos(15t)° + 4 \sin(15t)°,$$

where t is the time in hours from midday and $0 \leqslant t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(d) Find the value of t when this minimum temperature occurs.

(3)

(Total 12 marks)

Q53. (a) Use the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$

(2)

The curves C_1 and C_2 have equations

$$C_1: \quad y = 3\sin 2x$$

$$C_2: \quad y = 4\sin^2 x - 2\cos 2x$$

(b) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

(3)

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

(d) Hence find, for $0 \leq x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place.

(4)

(Total 12 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ $\Rightarrow \sin(\theta+h)=\sin\theta\cos h+\cos\theta\sin h$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$	A1	1.1b
	$=\dfrac{\sin h}{h}\cos\theta+\left(\dfrac{\cos h-1}{h}\right)\sin\theta$	M1	2.1
	Uses $h\to 0,\ \dfrac{\sin h}{h}\to 1$ and $\dfrac{\cos h-1}{h}\to 0$ Hence the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$gradient of the curve $\Rightarrow \dfrac{dy}{d\theta}=\cos\theta$ *	A1*	2.5
		(5 marks)	

Notes:

B1: States or implies that the gradient of the chord is $\dfrac{\sin(\theta+h)-\sin\theta}{h}$ or similar such as $\dfrac{\sin(\theta+\delta\theta)-\sin\theta}{\theta+\delta\theta-\theta}$ for a small h or $\delta\theta$

M1: Uses the compound angle identity for $\sin(A + B)$ with $A=\theta,\ B=h$ or $\delta\theta$

A1: Obtains $\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$ or equivalent

M1: Writes their expression in terms of $\dfrac{\sin h}{h}$ and $\dfrac{\cos h-1}{h}$

A1*: Uses correct language to explain that $\dfrac{dy}{d\theta}=\cos\theta$

For this method they should use all of the given statements $h\to 0,\ \dfrac{\sin h}{h}\to 1$, $\dfrac{\cos h-1}{h}\to 0$ meaning that the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$

and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{dy}{d\theta}=\cos\theta$

Question	Scheme	Marks	AOs
alt	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Sets $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\dfrac{\sin\left(\theta+\frac{h}{2}+\frac{h}{2}\right)-\sin\left(\theta+\frac{h}{2}-\frac{h}{2}\right)}{h}$ and uses the compound angle identity for $\sin(A+B)$ and $\sin(A-B)$ with $A=\theta+\frac{h}{2}$, $B=\frac{h}{2}$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=$ $\dfrac{\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)+\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]-\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)-\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]}{h}$	A1	1.1b
	$=\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\cos\left(\theta+\frac{h}{2}\right)$	M1	2.1
	Uses $h\to 0,\ \frac{h}{2}\to 0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to 1$ and $\cos\left(\theta+\frac{h}{2}\right)\to\cos\theta$ Therefore the $\lim_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{dy}{d\theta}=\cos\theta$ *	A1*	2.5
		(5 marks)	

Additional notes:

A1*: Uses correct language to explain that $\dfrac{dy}{d\theta}=\cos\theta$. For this method they should use the (adapted) given statement $h\to 0,\ \frac{h}{2}\to 0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to 1$ with $\cos\left(\theta+\frac{h}{2}\right)\to\cos\theta$ meaning that the $\lim_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{dy}{d\theta}=\cos\theta$

(**Q10 9MA0/01, Specimen papers**)

Q2.

Question Number	Scheme		Marks
(a)	Area $BDE = \frac{1}{2}(5)^2(1.4)$	M1: Use of the correct formula or method for the area of the sector	M1A1
	$= 17.5\ (\text{cm}^2)$	A1: 17.5 oe	
			[2]
(b)	**Parts (b) and (c) can be marked together**		
	$6.1^2 = 5^2 + 7.5^2 - (2 \times 5 \times 7.5 \cos DBC)$ or $\cos DBC = \frac{5^2 + 7.5^2 - 6.1^2}{2 \times 5 \times 7.5}$ (or equivalent)		M1
	M1: A correct statement involving the angle DBC		
	Angle $DBC = 0.943201...$	awrt 0.943	A1
	Note that work for (b) may be seen on the diagram or in part (c)		
			[2]
(c)	**Note that candidates may work in degrees in (c)** (Angle $DBC = 54.04....\text{degrees}$)		
	Area $CBD = \frac{1}{2}5(7.5)\sin(0.943)$		
	Angle $EBA = \pi - 1.4 - "0.943"$ **(Maybe seen on the diagram)**	Area $CBD = \frac{1}{2}5(7.5)\sin(\text{their } 0.943)$ or awrt 15.2. (Note area of $CBD = 15.177...$) A correct method for the area of triangle CBD which can be implied by awrt 15.2	M1
	$\pi - 1.4 - "\text{their } 0.943"$ A value for angle EBA of awrt 0.8 (from 0.7985926536... or 0.7983916536...) or value for angle EBA of $(1.74159... - \text{their angle } DBC)$ would imply this mark.		M1
	$AB = 5\cos(\pi - 1.4 - "0.943")$ or $AE = 5\sin(\pi - 1.4 - "0.943")$	$AB = 5\cos(\pi - 1.4 - \text{their } 0.943)$ $AB = 5\cos(0.79859...) = 3.488577938...$ Allow M1 for $AB = \text{awrt } 3.49$ Or $AE = 5\sin(\pi - 1.4 - \text{their } 0.943)$ $AE = 5\sin(0.79859...) = 3.581874365688...$ Allow M1 for $AE = \text{awrt } 3.58$ **It must be clear that $\pi - 1.4 - "0.943"$ is being used for angle EBA. Note that some candidates use the sin rule here but it must be used correctly – do not allow mixing of degrees and radians.**	M1
	Area $EAB = \frac{1}{2}5\cos(\pi - 1.4 - "0.943") \times 5\sin(\pi - 1.4 - "0.943")$		
	This is dependent on the previous M1 and there must be no other errors in finding the area of triangle EAB		dM1
	Allow M1 for area $EAB = \text{awrt } 6.2$		
	Area $ABCDE = 15.17... + 17.5 + 6.24... = 38.92...$		
		awrt 38.9	A1cso
			[5]
	Note that a sign error in (b) can give the obtuse angle (2.198....) and could lead to the correct answer in (c) – this would lose the final mark in (c)		**Total 9**

(**Q15 6664/01, June 2014**)

Q3.

Question Number	Scheme		Marks
(i)	$\dfrac{\sin 2\theta}{(4\sin 2\theta - 1)} = 1;\ \ 0 \leqslant \theta < 180^{\circ}$		
	$\sin 2\theta = \dfrac{1}{3}$	$\sin 2\theta = k$ where $-1 < k < 1$ **Must be 2θ and not θ.**	M1
	$\{2\theta = \{19.4712..., 160.5288...\}\}$		
	$\theta = \{9.7356..., 80.2644...\}$	A1: Either awrt 9.7 or awrt 80.3 A1: Both awrt 9.7 and awrt 80.3	A1 A1
	Do not penalise poor accuracy more than once e.g. 9.8 and 80.2 from correct work could score M1A1A0		
	If <u>both</u> answers are correct in radians award A1A0 otherwise A0A0 **Correct answers are 0.2 and 1.4**		
	Extra solutions in range in an otherwise fully correct solution deduct the last A1		
			[3]
(ii)	$5\sin^2 x - 2\cos x - 5 = 0,\ \ 0 \leq x < 2\pi.$		
	$5(1-\cos^2 x) - 2\cos x - 5 = 0$	Applies $\sin^2 x = 1 - \cos^2 x$	M1
	$5\cos^2 x + 2\cos x = 0$ $\cos x(5\cos x + 2) = 0$ $\Rightarrow \cos x = ...$	Cancelling out $\cos x$ or a valid attempt at solving the quadratic in cos*x* **and** giving cos*x* = ... **Dependent on the previous method mark.**	dM1
	awrt 1.98 or awrt 4.3(0)	Degrees: 113.58, 246.42	A1
	Both 1.98 and 4.3(0)	or their α and their $2\pi - \alpha$, where $\alpha \neq \dfrac{\pi}{2}$. If working in degrees allow 360 – their α	A1ft
	awrt 1.57 or $\dfrac{\pi}{2}$ and 4.71 or $\dfrac{3\pi}{2}$ **or** 90° and 270°	**These answers only but ignore other answers <u>outside</u> the range**	B1
			[5]
	NB: $x = \text{awrt}\left\{1.98,\ 4.3(0),\ 1.57 \text{ or } \dfrac{\pi}{2},\ 4.71 \text{ or } \dfrac{3\pi}{2}\right\}$		**8**
	Answers in degrees: 113.58, 246.42, 90, 270 **Could score M1M1A0A1ftB1 (4/5)**		

(**Q16 6664/01/R, June 2014**)

Q4.

Question Number	Scheme	Notes	Marks
	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \quad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 1**	$\frac{dx}{dt} = 4\sec^2 t, \ \frac{dy}{dt} = 10\sqrt{3}\cos 2t$	**Either both** x and y are differentiated correctly with respect to t **or** their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **or** applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
	$\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t} \quad \left\{= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t\right\}$	Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\left\{\text{At } P\left(4\sqrt{3}, \frac{15}{2}\right), \ t = \frac{\pi}{3}\right\}$		
	$\frac{dy}{dx} = \frac{10\sqrt{3}\cos\left(\frac{2\pi}{3}\right)}{4\sec^2\left(\frac{\pi}{3}\right)}$	**dependent on the previous M mark** *Some evidence* of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]
(b)	$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$		
	So $x = 4\tan\left(\frac{\pi}{4}\right), \ y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4\tan\left(\frac{\pi}{4}\right)$ **or** $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ **or** $x = 4$ **or** $y = 5\sqrt{3}$ **or** $y =$ awrt 8.7	M1
	Coordinates are $\left(4, 5\sqrt{3}\right)$	$\left(4, 5\sqrt{3}\right)$ **or** $x = 4, y = 5\sqrt{3}$	A1
			[2]
			6

		Question Notes
(a)	**1st A1**	Correct $\frac{dy}{dx}$. E.g. $\frac{10\sqrt{3}\cos 2t}{4\sec^2 t}$ or $\frac{5}{2}\sqrt{3}\cos 2t\cos^2 t$ or $\frac{5}{2}\sqrt{3}\cos^2 t(\cos^2 t - \sin^2 t)$ or any equivalent form.
	Note	Give the final A0 for a final answer of $-\frac{10}{32}\sqrt{3}$ **without reference to** $-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$
	Note	Give the final A0 for more than one value stated for $\frac{dy}{dx}$
(b)	**Note**	Also allow M1 for either $x = 4\tan(45)$ or $y = 5\sqrt{3}\sin(2(45))$
	Note	M1 can be gained by ignoring previous working in part (a) and/or part (b)
	Note	Give A0 for stating more than one set of coordinates for Q.
	Note	Writing $x = 4, y = 5\sqrt{3}$ followed by $\left(5\sqrt{3}, 4\right)$ is A0.

Question Number	Scheme	Notes	Marks
	$x = 4\tan t,\quad y = 5\sqrt{3}\sin 2t,\qquad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 2**	$\tan t = \frac{x}{4} \Rightarrow \sin t = \frac{x}{\sqrt{(x^2+16)}}$, $\cos t = \frac{4}{\sqrt{(x^2+16)}} \Rightarrow y = \frac{40\sqrt{3}x}{x^2+16}$		
	$\left\{\begin{matrix} u = 40\sqrt{3}x & v = x^2+16 \\ \frac{du}{dx} = 40\sqrt{3} & \frac{dv}{dx} = 2x \end{matrix}\right\}$		
	$\frac{dy}{dx} = \frac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2}\ \left\{= \frac{40\sqrt{3}(16-x^2)}{(x^2+16)^2}\right\}$	$\frac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified	A1
	$\frac{dy}{dx} = \frac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	**dependent on the previous M mark** *Some evidence* of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]
(a) **Way 3**	$y = 5\sqrt{3}\sin\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)$		
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{2}{1+\left(\frac{x}{4}\right)^2}\right)\left(\frac{1}{4}\right)$	$\frac{dy}{dx} = \pm A\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{1}{1+x^2}\right)$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified.	A1
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}(\sqrt{3})\right)\left(\frac{2}{1+3}\right)\left(\frac{1}{4}\right)\ \left\{= 5\sqrt{3}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\right\}$	**dependent on the previous M mark** *Some evidence* of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]

(**Q33 6666/01, June 2016**)

Q5.

Question	Scheme	Marks
(i)	$y = e^{3x}\cos 4x \Rightarrow \left(\frac{dy}{dx}\right) = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$	M1A1
	Sets $\cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x = 0 \Rightarrow 3\cos 4x - 4\sin 4x = 0$	M1
	$\Rightarrow x = \frac{1}{4}\arctan\frac{3}{4}$	M1
	$\Rightarrow x =$ awrt 0.9463 4dp	A1
		(5)
(ii)	$x = \sin^2 2y \Rightarrow \frac{dx}{dy} = 2\sin 2y \times 2\cos 2y$	M1A1
	Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dx}{dy} = 2\sin 4y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5) (10 marks)
(ii) Alt I	$x = \sin^2 2y \Rightarrow x = \frac{1}{2} - \frac{1}{2}\cos 4y$	2nd M1
	$\frac{dx}{dy} = 2\sin 4y$	1st M1 A1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5)
(ii) Alt II	$x^{\frac{1}{2}} = \sin 2y \Rightarrow \frac{1}{2}x^{-\frac{1}{2}} = 2\cos 2y\frac{dy}{dx}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$ AND $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5)
(ii) Alt III	$x^{\frac{1}{2}} = \sin 2y \Rightarrow 2y = \text{invsin}\, x^{\frac{1}{2}} \Rightarrow 2\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} \times \frac{1}{2}x^{-\frac{1}{2}}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$, $\sqrt{1-x} = \cos 2y$ and $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5)

(i)

M1 Uses the product rule $uv' + vu'$ to achieve $\left(\frac{dy}{dx}\right) = Ae^{3x}\cos 4x \pm Be^{3x}\sin 4x \quad A, B \neq 0$

The product rule if stated must be correct

A1 Correct (unsimplified) $\frac{dy}{dx} = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$

M1 Sets/implies their $\frac{dy}{dx}=0$ factorises/cancels)by e^{3x} to form a trig equation in just $\sin 4x$ and $\cos 4x$

M1 Uses the identity $\frac{\sin 4x}{\cos 4x}\equiv\tan 4x$, moves from $\tan 4x = C, C\neq 0$ using correct order of operations to $x=...$ Accept $x=$ awrt 0.16 (radians) $x=$ awrt 9.22 (degrees) for this mark.
If a candidate elects to pursue a more difficult method using $R\cos(\theta+\alpha)$, for example, the minimum expectation will be that they get (1) the identity correct, and (2) the values of R and α correct to 2dp. So for the correct equation you would only accept $5\cos(4x+\text{awrt } 0.93)$ or $5\sin(4x-\text{awrt } 0.64)$ before using the correct order of operations to $x=...$
Similarly candidates who square $3\cos 4x-4\sin 4x=0$ then use a Pythagorean identity should proceed from either $\sin 4x=\frac{3}{5}$ or $\cos 4x=\frac{4}{5}$ before using the correct order of operations ...

A1 $\Rightarrow x=$ awrt 0.9463.
Ignore any answers outside the domain. Withhold mark for additional answers inside the domain

(ii)
M1 Uses chain rule (or product rule) to achieve $\pm P\sin 2y\cos 2y$ as a derivative.
There is no need for lhs to be seen/ correct
If the product rule is used look for $\frac{dy}{dx}=\pm A\sin 2y\cos 2y\pm B\sin 2y\cos 2y$, (with $\frac{dy}{dx}$ struck through)

A1 Both lhs and rhs correct (unsimplified) . $\frac{dx}{dy}=2\sin 2y\times 2\cos 2y=(4\sin 2y\cos 2y)$ or
$1=2\sin 2y\times 2\cos 2y\frac{dy}{dx}$

M1 Uses $\sin 4y=2\sin 2y\cos 2y$ in their expression.
You may just see a statement such as $4\sin 2y\cos 2y=2\sin 4y$ which is fine.
Candidates who write $\frac{dy}{dx}=A\sin 2x\cos 2x$ can score this for $\frac{dy}{dx}=\frac{A}{2}\sin 4x$ (with $\frac{dy}{dx}$ struck through)

M1 Uses $\frac{dy}{dx}=1\Big/\frac{dx}{dy}$ **for their expression in** y. Concentrate on the trig identity rather than the coefficient in awarding this. Eg $\frac{dx}{dy}=2\sin 4y\Rightarrow\frac{dy}{dx}=2\text{cosec}4y$ is condoned for the M1
If $\frac{dx}{dy}=a+b$ do not allow $\frac{dy}{dx}=\frac{1}{a}+\frac{1}{b}$

A1 $\frac{dy}{dx}=\frac{1}{2}\text{cosec}4y$ If a candidate then proceeds to write down incorrect values of p and q then do not withhold the mark.

NB: See the three alternatives which may be less common but mark in exactly the same way. If you are uncertain as how to mark these please consult your team leader.

In **Alt I** the second M is for writing $x=\sin^2 2y\Rightarrow x=\pm\frac{1}{2}\pm\frac{1}{2}\cos 4y$ from $\cos 4y=\pm 1\pm 2\sin^2 2y$

In **Alt II** the first M is for writing $x^{\frac{1}{2}}=\sin 2y$ and differentiating both sides to $Px^{-\frac{1}{2}}=Q\cos 2y\frac{dy}{dx}$ oe

In **Alt 111** the first M is for writing $2y=\text{invsin}\left(x^{0.5}\right)$ oe and differentiating to $M\frac{dy}{dx}=N\frac{1}{\sqrt{1-\left(x^{0.5}\right)^2}}\times x^{-0.5}$

(**Q24 6665/01, June 2016**)

Q6.

Question	Scheme	Marks	AOs
(a)	Attempts $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$	M1	1.1b
	$\frac{dy}{dx}=\frac{\sqrt{3}\sin 2t}{\sin t}\quad \left(=2\sqrt{3}\cos t\right)$	A1	1.1b
		(2)	
(b)	Substitutes $t=\frac{2\pi}{3}$ in $\frac{dy}{dx}=\frac{\sqrt{3}\sin 2t}{\sin t}=\left(-\sqrt{3}\right)$	M1	2.1
	Uses gradient of normal $=-\frac{1}{dy/dx}=\left(\frac{1}{\sqrt{3}}\right)$	M1	2.1
	Coordinates of $P=\left(-1,-\frac{\sqrt{3}}{2}\right)$	B1	1.1b
	Correct form of normal $y+\frac{\sqrt{3}}{2}=\frac{1}{\sqrt{3}}(x+1)$	M1	2.1
	Completes proof $\Rightarrow 2x-2\sqrt{3}y-1=0$ *	A1*	1.1b
		(5)	
(c)	Substitutes $x=2\cos t$ and $y=\sqrt{3}\cos 2t$ into $2x-2\sqrt{3}y-1=0$	M1	3.1a
	Uses the identity $\cos 2t=2\cos^2 t-1$ to produce a quadratic in $\cos t$	M1	3.1a
	$\Rightarrow 12\cos^2 t-4\cos t-5=0$	A1	1.1b
	Finds $\cos t=\frac{5}{6},\ \cancel{-\frac{1}{2}}$	M1	2.4
	Substitutes their $\cos t=\frac{5}{6}$ into $x=2\cos t,\ y=\sqrt{3}\cos 2t,$	M1	1.1b
	$Q=\left(\frac{5}{3},\frac{7}{18}\sqrt{3}\right)$	A1	1.1b
		(6)	
			(13 marks)

Notes:

(a)

M1: Attempts $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ and achieves a form $k\frac{\sin 2t}{\sin t}$ Alternatively candidates may apply the double angle identity for $\cos 2t$ and achieve a form $k\frac{\sin t \cos t}{\sin t}$

A1: Scored for a correct answer, either $\frac{\sqrt{3}\sin 2t}{\sin t}$ or $2\sqrt{3}\cos t$

(b)

M1: For substituting $t = \frac{2\pi}{3}$ in their $\frac{dy}{dx}$ which must be in terms of t

M1: Uses the gradient of the normal is the negative reciprocal of the value of $\frac{dy}{dx}$. This may be seen in the equation of l.

B1: States or uses (in their tangent or normal) that $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$

M1: Uses their numerical value of $-1\Big/\frac{dy}{dx}$ with their $\left(-1, -\frac{\sqrt{3}}{2}\right)$ to form an equation of the normal at P

A1*: This is a proof and all aspects need to be correct. Correct answer only $2x - 2\sqrt{3}y - 1 = 0$

(c)

M1: For substituting $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$ to produce an equation in t. Alternatively candidates could use $\cos 2t = 2\cos^2 t - 1$ to set up an equation of the form $y = Ax^2 + B$.

M1: Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic equation in $\cos t$
In the alternative method it is for combining their $y = Ax^2 + B$ with $2x - 2\sqrt{3}y - 1 = 0$ to get an equation in just one variable

A1: For the correct quadratic equation $12\cos^2 t - 4\cos t - 5 = 0$
Alternatively the equations in x and y are $3x^2 - 2x - 5 = 0$ $12\sqrt{3}y^2 + 4y - 7\sqrt{3} = 0$

M1: Solves the quadratic equation in $\cos t$ (or x or y) and rejects the value corresponding to P.

M1: Substitutes their $\cos t = \frac{5}{6}$ or their $t = \arccos\left(\frac{5}{6}\right)$ in $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$
If a value of x or y has been found it is for finding the other coordinate.

A1: $Q = \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$. Allow $x = \frac{5}{3}, y = \frac{7}{18}\sqrt{3}$ but do not allow decimal equivalents.

(**Q13 9MA0/01, Specimen papers**)

Q7.

Question Number	Scheme	Marks
(i)	$\frac{dx}{dy} = 4\sec^2 2y \tan 2y$	B1
	Use $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$	M1
	Uses $\tan^2 2y = \sec^2 2y - 1$ and $\sec 2y = \sqrt{x}$ to get $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of just x	M1
	$\frac{dy}{dx} = \frac{1}{4x(x-1)^{\frac{1}{2}}}$ (conclusion stated with no errors previously)	A1*
		(4)
(ii)	$\frac{dy}{dx} = (x^2 + x^3) \times \frac{2}{2x} + (2x + 3x^2)\ln 2x$	M1 A1 A1
	When $x = \frac{e}{2}$, $\frac{dy}{dx} = 3(\frac{e}{2}) + 4(\frac{e}{2})^2 = 3(\frac{e}{2}) + e^2$	dM1 A1
		(5)
(iii)	$f'(x) = \frac{(x+1)^{\frac{1}{3}}(-3\sin x) - 3\cos x(\frac{1}{3}(x+1)^{-\frac{2}{3}})}{(x+1)^{\frac{2}{3}}}$	M1 A1
	$f'(x) = \frac{-3(x+1)(\sin x) - \cos x}{(x+1)^{\frac{4}{3}}}$	A1
		(3)
		12 marks

(i)

B1 $\frac{dx}{dy} = 4\sec^2 2y \tan 2y$ or equivalent such as $\frac{dx}{dy} = 4\frac{\sin 2y \cos 2y}{\cos^4 2y}$

Accept $\frac{dx}{dy} = 2\sec 2y \tan 2y \times \sec 2y + 2\sec 2y \tan 2y \times \sec 2y$, $1 = 4\sec^2 2y \tan 2y \frac{dy}{dx}$

M1 Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to get an expression for $\frac{dy}{dx}$ in terms of y.

It may be scored following the award of the next M1 if $\frac{dx}{dy}$ has been written in terms of x.

Follow through on their expression but condone errors on the coefficient.

For example $\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sec^2 2y \tan 2y}$ is OK as is $\frac{dy}{dx} = \frac{2}{\sec^2 2y \tan 2y}$

Do not accept y' s going to x' s. So for example $\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sec^2 2x \tan 2x}$ is M0

M1 Uses $\tan^2 2y = \sec^2 2y - 1$ **and** $x = \sec^2 2y$ to get their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of just x

$\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dx}{dy} = 2x\sqrt{(\sec^2 2y - 1)} = 2x\sqrt{x-1}$ is incorrect but scores M1

$\frac{dx}{dy} = 2\sec 2y \tan 2y \Rightarrow \frac{dx}{dy} = 2\sec 2y\sqrt{(\sec^2 2y - 1)} = 2\sqrt{x}\sqrt{x-1}$ is incorrect but scores M1

The stating and use $1 + \tan^2 x = \sec^2 x$ is unlikely to score this mark.

Accept $1 + \tan^2 2y = \sec^2 2y \Rightarrow 1 + \tan^2 2y = x \Rightarrow \tan 2y = \sqrt{x-1}$. So $\frac{dy}{dx} = \frac{1}{4\sec^2 2y \tan 2y} = \frac{1}{4x\sqrt{x-1}}$

Condone examples where the candidate adapts something to get the given answer

Eg. $\frac{dy}{dx} = \frac{1}{4\sec^2 2y \tan^2 2y} = \frac{1}{4\sec^2 2y\left(\sec^2 2y - 1\right)} = \frac{1}{4x\sqrt{(x-1)}}$

A1* Completely correct solution. This is a 'show that' question and it is a requirement that all elements are seen.

(ii)

M1 Uses the product rule to differentiate $(x^2 + x^3)\ln 2x$. If the rule is stated it must be correct. It may be implied by their $u = .., u' = .., v = .., v' = ..$ followed by $vu' + uv'$. If the rule is neither stated nor implied only accept expressions of the form $\ln 2x \times (ax + bx^2) + (x^2 + x^3) \times \frac{C}{x}$

It is acceptable to multiply out the expression to get $x^2 \ln 2x + x^3 \ln 2x$ but the product rule must be applied to both terms

A1 One term correct (unsimplified). Either $(x^2 + x^3) \times \frac{2}{2x}$ or $(2x + 3x^2)\ln 2x$

If they have multiplied out before differentiating the equivalent would be two of the four terms correct.

A1 A completely correct (unsimplified) expression $\frac{dy}{dx} = (x^2 + x^3) \times \frac{2}{2x} + (2x + 3x^2)\ln 2x$

dM1 Fully substitutes $x = \frac{e}{2}$ (dependent on previous M mark) into their expression for $\frac{dy}{dx} = ...$ Implied by awrt 11.5

A1 $\frac{dy}{dx} = 3(\frac{e}{2}) + e^2$ Accept equivalent simplified forms such as $\frac{dy}{dx} = 1.5e + e^2$, $\frac{dy}{dx} = e(1.5 + e)$, $\frac{dy}{dx} = \frac{e(2e+3)}{2}$

(iii)

M1 Uses quotient rule with $u = 3\cos x$, $v = (x+1)^{\frac{1}{3}}$, $u' = \pm A\sin x$ and $v' = B(x+1)^{-\frac{2}{3}}$.

If the rule is quoted it must be correct. It may be implied by their $u = 3\cos x$, $v = (x+1)^{\frac{1}{3}}$, $u' = \pm A\sin x$ and $v' = B(x+1)^{-\frac{2}{3}}$ followed by $\frac{vu' - uv'}{v^2}$

Additionally this could be scored by using the product rule with $u = 3\cos x$, $v = (x+1)^{-\frac{1}{3}}$ $u' = \pm A\sin x$ and $v' = B(x+1)^{-\frac{4}{3}}$. If the rule is quoted it must be correct. It may be implied by their $u = 3\cos x$, $v = (x+1)^{-\frac{1}{3}}$ $u' = \pm A\sin x$ and $v' = B(x+1)^{-\frac{4}{3}}$ followed by $vu' + uv'$

If it is not quoted nor implied only accept either of the two expressions

1) Using quotient form $\frac{(x+1)^{\frac{1}{3}} \times \pm A\sin x - 3\cos x \times B(x+1)^{-\frac{2}{3}}}{\left((x+1)^{\frac{1}{3}}\right)^2}$ or $\frac{(x+1)^{\frac{1}{3}} \times \pm A\sin x - 3\cos x \times B(x+1)^{-\frac{2}{3}}}{(x+1)^{\frac{1}{9}}}$

2) Using product form $(x+1)^{-\frac{1}{3}} \times \pm A\sin x + 3\cos x \times B(x+1)^{-\frac{4}{3}}$

A1 A correct gradient. Accept $f'(x) = \frac{(x+1)^{\frac{1}{3}}(-3\sin x) - 3\cos x(\frac{1}{3}(x+1)^{-\frac{2}{3}})}{\left((x+1)^{\frac{1}{3}}\right)^2}$

$$\text{or } f'(x) = (x+1)^{-\frac{1}{3}} \times -3\sin x + 3\cos x \times -\frac{1}{3}(x+1)^{-\frac{4}{3}}$$

A1 $f'(x) = \frac{-3(x+1)(\sin x) - \cos x}{(x+1)^{\frac{4}{3}}}$ oe. or a statement that $g(x) = -3(x+1)(\sin x) - \cos x$ oe.

(**Q23 6665/01/R, June 2014**)

Q8.

Question Number	Scheme	Marks
	(a) $\frac{dx}{dt} = 2\sin t \cos t,\ \frac{dy}{dt} = 2\sec^2 t$	B1 B1
	$\frac{dy}{dx} = \frac{\sec^2 t}{\sin t \cos t} \left(= \frac{1}{\sin t \cos^3 t} \right)$ or equivalent	M1 A1 **(4)**
	(b) At $t = \frac{\pi}{3},\ x = \frac{3}{4},\ y = 2\sqrt{3}$	B1
	$\frac{dy}{dx} = \frac{\sec^2 \frac{\pi}{3}}{\sin\frac{\pi}{3}\cos\frac{\pi}{3}} = \frac{16}{\sqrt{3}}$	M1 A1
	$y - 2\sqrt{3} = \frac{16}{\sqrt{3}}\left(x - \frac{3}{4}\right)$	M1
	$y = 0 \Rightarrow x = \frac{3}{8}$	M1 A1 **(6)**
		[10]

(**Q31 6666/01, June 2010**)

Q9.

Question Number	Scheme	Marks
(a)	$\frac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y = \left(6\sec^2 3y \tan 3y\right)$ $\left(\text{oe } \frac{6\sin 3y}{\cos^3 3y}\right)$	M1A1 (2)
(b)	Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to obtain $\frac{dy}{dx} = \frac{1}{6\sec^2 3y \tan 3y}$	M1
	$\tan^2 3y = \sec^2 3y - 1 = x - 1$	B1
	Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in just x.	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$ CSO	A1* (4)
(c)	$\frac{d^2y}{dx^2} - \frac{0-[6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$	M1A1
	$\frac{d^2y}{dx^2} = \frac{6-9x}{36x^2(x-1)^{\frac{3}{2}}} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$	dM1A1
		(4)
		(10 marks)
Alt 1 to (a)	$x = (\cos 3y)^{-2} \Rightarrow \frac{dx}{dy} = -2(\cos 3y)^{-3} \times -3\sin 3y$	M1A1
Alt 2 to (a)	$x = \sec 3y \times \sec 3y \Rightarrow \frac{dx}{dy} = \sec 3y \times 3\sec 3y \tan 3y + \sec 3y \times 3\sec 3y \tan 3y$	M1A1
Alt 1 To (c)	$\frac{d^2y}{dx^2} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$	M1A1
	$= \frac{1}{6}x^{-2}(x-1)^{-\frac{3}{2}}[x(-\frac{1}{2}) + (-1)(x-1)]$	dM1
	$= \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ oe	A1
		(4)

Notes for Question

(a)

M1 Uses the chain rule to get $A\sec 3y\sec 3y\tan 3y = \left(A\sec^2 3y\tan 3y\right)$.

There is no need to get the lhs of the expression. Alternatively could use the chain rule on $(\cos 3y)^{-2} \Rightarrow A(\cos 3y)^{-3}\sin 3y$

or the quotient rule on $\dfrac{1}{(\cos 3y)^2} \Rightarrow \dfrac{\pm A\cos 3y\sin 3y}{(\cos 3y)^4}$

A1 $\dfrac{dx}{dy} = 2\times 3\sec 3y\sec 3y\tan 3y$ or equivalent. There is no need to simplify the rhs but both sides must be correct.

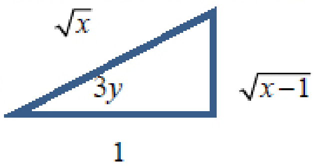
(b)

M1 Uses $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$ to get an expression for $\dfrac{dy}{dx}$. Follow through on their $\dfrac{dx}{dy}$

Allow slips on the coefficient but not trig expression.

B1 Writes $\tan^2 3y = \sec^2 3y - 1$ or an equivalent such as $\tan 3y = \sqrt{\sec^2 3y - 1}$ and uses $x = \sec^2 3y$ to obtain either $\tan^2 3y = x - 1$ or $\tan 3y = (x-1)^{\frac{1}{2}}$

All elements **must be present.**

Accept 

$\cos 3y = \dfrac{1}{\sqrt{x}} \Rightarrow \tan 3y = \sqrt{x-1}$

If the differential was in terms of $\sin 3y, \cos 3y$ it is awarded for $\sin 3y = \dfrac{\sqrt{x-1}}{\sqrt{x}}$

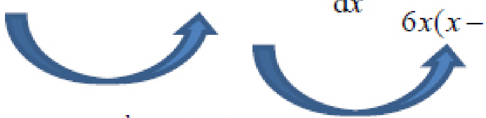
M1 Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ or equivalent to get $\dfrac{dy}{dx}$ in just x. Allow slips on the signs in $\tan^2 3y = \sec^2 3y - 1$.

It may be implied- see below

A1* CSO. This is a given solution and you must be convinced that all steps are shown.

Note that the two method marks may occur the other way around

Eg. $\dfrac{dx}{dy} = 6\sec^2 3y\tan 3y = 6x(x-1)^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{6x(x-1)^{\frac{1}{2}}}$



Scores the 2nd method

Scores the 1st method

The above solution will score M1, B0, M1, A0

Notes for Question Continued

Example 1- Scores 0 marks in part (b)

$$\frac{\mathrm{d}x}{\mathrm{d}y}=6\sec^2 3y\tan 3y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{6\sec^2 3x\tan 3x}=\frac{1}{6\sec^2 3x\sqrt{\sec^2 3x-1}}=\frac{1}{6x(x-1)^{\frac{1}{2}}}$$

Example 2- Scores M1B1M1A0

$$\frac{\mathrm{d}x}{\mathrm{d}y}=2\sec^2 3y\tan 3y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sec^2 3y\tan 3y}=\frac{1}{2\sec^2 3y\sqrt{\sec^2 3y-1}}=\frac{1}{2x(x-1)^{\frac{1}{2}}}$$

(c) **Using Quotient and Product Rules**

M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u=1$ and $v=6x(x-1)^{\frac{1}{2}}$ **and** achieving

$u'=0$ and $v'=A(x-1)^{\frac{1}{2}}+Bx(x-1)^{-\frac{1}{2}}$.

If the formulae are quoted, **both** must be correct. If they are not quoted nor implied by their working allow expressions of the form

$$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)=\frac{0-[A(x-1)^{\frac{1}{2}}+Bx(x-1)^{-\frac{1}{2}}]}{\left(6x(x-1)^{\frac{1}{2}}\right)^2} \quad \text{or} \quad \left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)=\frac{0-A(x-1)^{\frac{1}{2}}\pm Bx(x-1)^{-\frac{1}{2}}}{Cx^2(x-1)}$$

A1 Correct un simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{0-[6(x-1)^{\frac{1}{2}}+3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$ oe

dM1 Multiply numerator and denominator by $(x-1)^{\frac{1}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{1}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe

Notes for Question Continued

(c) Using Product and Chain Rules

M1 Writes $\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = Ax^{-1}(x-1)^{-\frac{1}{2}}$ and uses the product rule with u or $v = Ax^{-1}$ and v or $u = (x-1)^{-\frac{1}{2}}$. If any rule is quoted it must be correct.

If the rules are not quoted nor implied then award if you see an expression of the form

$(x-1)^{-\frac{3}{2}} \times Bx^{-1} \pm C(x-1)^{-\frac{1}{2}} \times x^{-2}$

A1 $\frac{d^2y}{dx^2} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$

dM1 Factorises out / uses a common denominator of $x^{-2}(x-1)^{-\frac{3}{2}}$ producing a linear factor/numerator which must be simplified by collecting like terms. Need a single fraction.

A1 Correct simplified expression $\frac{d^2y}{dx^2} == \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ *oe*

(c) **Using Quotient and Chain rules Rules**

M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u = (x-1)^{-\frac{1}{2}}$ and $v = 6x$ **and** achieving

$u' = A(x-1)^{-\frac{3}{2}}$ and $v' = B$.

If the formulae is quoted, it must be correct. If it is not quoted nor implied by their working allow an expression of the form

$$\left(\frac{d^2y}{dx^2}\right) = \frac{Cx(x-1)^{-\frac{3}{2}} - D(x-1)^{-\frac{1}{2}}}{Ex^2}$$

A1 Correct un simplified expression $\frac{d^2y}{dx^2} = \frac{6x \times -\frac{1}{2}(x-1)^{-\frac{3}{2}} - (x-1)^{-\frac{1}{2}} \times 6}{(6x)^2}$

dM1 Multiply numerator and denominator by $(x-1)^{\frac{3}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{3}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{d^2y}{dx^2} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe $\frac{d^2y}{dx^2} = \frac{(2-3x)x^{-2}(x-1)^{-\frac{3}{2}}}{12}$

Notes for Question Continued	
(c)	**Using just the chain rule**
M1	Writes $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = \frac{1}{(36x^3-36x^2)^{\frac{1}{2}}} = (36x^3-36x^2)^{-\frac{1}{2}}$ and proceeds by the chain rule to $A(36x^3-36x^2)^{-\frac{3}{2}}(Bx^2-Cx)$.
M1	Would automatically follow under this method if the first M has been scored

(**Q25 6665/01, June 2013**)

Q10.

Question Number	Scheme		Marks
(a)	$x = \tan^2 t$, $\quad y = \sin t$		
	$\frac{\mathrm{d}x}{\mathrm{d}t} = 2(\tan t)\sec^2 t$, $\frac{\mathrm{d}y}{\mathrm{d}t} = \cos t$	Correct $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$	B1
	$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos t}{2\tan t\sec^2 t}$ $\left(= \frac{\cos^4 t}{2\sin t}\right)$	$\frac{\pm\cos t}{\text{their } \frac{\mathrm{d}x}{\mathrm{d}t}}$	M1
		$\frac{+\cos t}{\text{their } \frac{\mathrm{d}x}{\mathrm{d}t}}$	A1 $\sqrt{}$
			[3]
(b)	When $t = \frac{\pi}{4}$, $\quad x = 1$, $y = \frac{1}{\sqrt{2}}$ (need values)	The point $\left(1, \frac{1}{\sqrt{2}}\right)$ or $(1$, awrt $0.71)$ **These coordinates can be implied.** ($y = \sin\left(\frac{\pi}{4}\right)$ is not sufficient for B1)	B1, B1
	When $t = \frac{\pi}{4}$, $\text{m}(\mathbf{T}) = \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos\frac{\pi}{4}}{2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4}}$		

	$= \dfrac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2} = \dfrac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{2}}\right)} = \dfrac{\frac{1}{\sqrt{2}}}{2.(1)(2)} = \dfrac{1}{4\sqrt{2}} = \dfrac{\sqrt{2}}{8}$	any of the five underlined expressions or awrt 0.18	B1 aef
	T: $y - \frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(x-1)$	Finding an equation of a tangent with ***their point*** and ***their tangent gradient*** or finds c by using $y = (\underline{\text{their gradient}})x + "\underline{c}"$.	M1 $\sqrt{\ }$ aef
	T: $y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}$ or $y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}$	Correct simplified EXACT equation of tangent	A1 aef **cso**
	or $\frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(1) + c \Rightarrow c = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}} = \frac{3}{4\sqrt{2}}$		
	Hence **T:** $y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}$ or $y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}$		**[5]**

Note: The x and y coordinates must be the right way round.

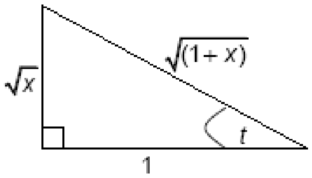
A candidate who incorrectly differentiates $\tan^2 t$ to give $\frac{dx}{dt} = 2\sec^2 t$ or $\frac{dx}{dt} = \sec^4 t$ is then able to fluke the correct answer in part (b). Such candidates can potentially get: (a) B0M1A1 $\sqrt{\ }$ (b) B1B1B1M1A0 **cso**.
Note: cso means "correct solution only".
Note: part (a) not fully correct implies candidate can

(c) **Way 1**	$x = \tan^2 t = \dfrac{\sin^2 t}{\cos^2 t}$ $\quad y = \sin t$		
	$x = \dfrac{\sin^2 t}{1-\sin^2 t}$	Uses $\cos^2 t = 1 - \sin^2 t$	M1
	$x = \dfrac{y^2}{1-y^2}$	Eliminates *'t' to* write an equation involving *x* and *y*.	M1
	$x(1-y^2) = y^2 \Rightarrow x - xy^2 = y^2$		
	$x = y^2 + xy^2 \Rightarrow x = y^2(1+x)$	Rearranging and factorising with an attempt to make y^2 the subject.	ddM1
	$y^2 = \dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1
			[4]
Aliter (c) **Way 2**	$1 + \cot^2 t = \operatorname{cosec}^2 t$	Uses $1 + \cot^2 t = \operatorname{cosec}^2 t$	M1
	$= \dfrac{1}{\sin^2 t}$	Uses $\operatorname{cosec}^2 t = \dfrac{1}{\sin^2 t}$	M1 implied
	Hence, $1 + \dfrac{1}{x} = \dfrac{1}{y^2}$	Eliminates *'t' to* write an equation involving *x* and *y*.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Aliter (c) **Way 3**	$x = \tan^2 t \qquad y = \sin t$		
	$1 + \tan^2 t = \sec^2 t$	Uses $1 + \tan^2 t = \sec^2 t$	M1
	$= \dfrac{1}{\cos^2 t}$	Uses $\sec^2 t = \dfrac{1}{\cos^2 t}$	M1
	$= \dfrac{1}{1 - \sin^2 t}$		
	Hence, $1 + x = \dfrac{1}{1 - y^2}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]
Aliter (c) **Way 4**	$y^2 = \sin^2 t = 1 - \cos^2 t$	Uses $\sin^2 t = 1 - \cos^2 t$	M1
	$= 1 - \dfrac{1}{\sec^2 t}$	Uses $\cos^2 t = \dfrac{1}{\sec^2 t}$	M1
	$= 1 - \dfrac{1}{(1 + \tan^2 t)}$	then uses $\sec^2 t = 1 + \tan^2 t$	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Aliter (c) **Way 5**	$x = \tan^2 t \qquad y = \sin t$ $x = \tan^2 t \;\Rightarrow\; \tan t = \sqrt{x}$ 	Draws a right-angled triangle and places both $\sqrt{x}$ and 1 on the triangle	M1
		Uses Pythagoras to deduce the hypotenuse	M1
	Hence, $y = \sin t = \dfrac{\sqrt{x}}{\sqrt{1+x}}$	Eliminates 't' to write an equation involving x and y.	ddM1
	Hence, $y^2 = \dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1 **[4]**
			12 marks

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

There are so many ways that a candidate can proceed with part (c). If a candidate produces a correct solution then please award all four marks. If they use a method commensurate with the five ways as detailed on the mark scheme then award the marks appropriately. If you are unsure of how to apply the scheme please escalate your response up to your team leader.

(**Q33 6666/01, June 2007**)

Q11.

Question	Scheme	Marks	AOs
	Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1 - \dfrac{(4\theta)^2}{2}$ in $\dfrac{1-\cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
	Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1 - \dfrac{(4\theta)^2}{2} \rightarrow \dfrac{1-\left(1-\dfrac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
	$= \dfrac{4}{3}$ oe	A1	1.1b
		(3)	
			(3 marks)

M1: Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2}$ in the given expression.

See below for description of marking of $\cos 4\theta$

M1: Attempts to substitute both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2}$

$$\rightarrow \frac{1 - \left(1 - \frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$$ and attempts to simplify.

Condone missing bracket on the 4θ so $\cos 4\theta \approx 1 - \frac{4\theta^2}{2}$ would score the method

Expect to see it simplified to a single term which could be in terms of θ

Look for an answer of k but condone $k\theta$ following a slip

A1: Uses both identities and simplifies to $\frac{4}{3}$ or exact equivalent with no incorrect lines BUT allow recovery on missing bracket for $\cos 4\theta \approx 1 - \frac{4\theta^2}{2}$.

Eg. $\frac{1 - \left(1 - \frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta} = \frac{8\theta^2}{6\theta} = \frac{4}{3}$ is M1 M1 A0

Condone awrt 1.33.

Alt: $\frac{1 - \cos 4\theta}{2\theta \sin 3\theta} = \frac{1 - \left(1 - 2\sin^2 2\theta\right)}{2\theta \sin 3\theta} = \frac{2\sin^2 2\theta}{2\theta \sin 3\theta} = \frac{2 \times (2\theta)^2}{2\theta \times 3\theta} = \frac{4}{3}$

M1 For an attempt at $\sin 3\theta \approx 3\theta$ or the identity $\cos 4\theta = 1 - 2\sin^2 2\theta$ with $\sin 2\theta \approx 2\theta$

M1 For both of the above and attempts to simplify to a single term.

A1 $\frac{4}{3}$ oe

(**Q01 9MA0/01, June 2018**)

Q12.

Question	Scheme	Marks	AOs
(a)	$\tan\theta+\cot\theta \equiv \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$	M1	2.1
	$\equiv \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$	A1	1.1b
	$\equiv \frac{1}{\frac{1}{2}\sin 2\theta}$	M1	2.1
	$\equiv 2\text{cosec}2\theta$ *	A1*	1.1b
		(4)	
(b)	States $\tan\theta+\cot\theta=1 \Rightarrow \sin 2\theta = 2$ AND no real solutions as $-1 \leqslant \sin 2\theta \leqslant 1$	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Writes $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

A1: Achieves a correct intermediate answer of $\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$

M1: Uses the double angle formula $\sin 2\theta = 2\sin\theta\cos\theta$

A1*: Completes proof with no errors. This is a given answer.

Note: There are many alternative methods. For example

$\tan\theta+\cot\theta \equiv \tan\theta + \frac{1}{\tan\theta} \equiv \frac{\tan^2\theta+1}{\tan\theta} \equiv \frac{\sec^2\theta}{\tan\theta} \equiv \frac{1}{\cos^2\theta \times \frac{\sin\theta}{\cos\theta}} \equiv \frac{1}{\cos\theta\times\sin\theta}$ then as the main scheme.

(b)

B1: Scored for sight of $\sin 2\theta = 2$ and a reason as to why this equation has no real solutions.
Possible reasons could be $-1 \leqslant \sin 2\theta \leqslant 1$........and therefore $\sin 2\theta \neq 2$
or $\sin 2\theta = 2 \Rightarrow 2\theta = \arcsin 2$ which has no answers as $-1 \leqslant \sin 2\theta \leqslant 1$

(**Q09 9MA0/01, Specimen papers**)

Q13.

Question	Scheme	Marks	AOs
(a)	$D = 5 + 2\sin(30 \times 6.5)° = \text{awrt } 4.48\text{m}$ with units	B1	3.4
		(1)	
(b)	$3.8 = 5 + 2\sin(30t)° \Rightarrow \sin(30t)° = -0.6$	M1 A1	1.1b 1.1b
	$t = 10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

Notes:

(a)

B1: Scored for using the model ie. substituting $t = 6.5$ into $D = 5 + 2\sin(30t)°$ and stating $D = awrt\ 4.48\text{m}$. The units must be seen somewhere in (a). So allow when $D = 4.482.. = 4.5\text{ m}$

Allow the mark for a correct answer without any working.

(b)

M1: For using $D = 3.8$ and proceeding to $\sin(30t)° = k,\quad |k| \leq 1$

A1: $\sin(30t)° = -0.6$ This may be implied by any correct answer for t such as $t = 7.2$

If the A1 implied, the calculation must be performed in degrees.

dM1: For finding **the first value** of t for their $\sin(30t)° = k$ after $t = 8.5$.

You may well see other values as well which is not an issue for this dM mark

(Note that $\sin(30t)° = -0.6 \Rightarrow 30t = 216.9°$ as well but this gives $t = 7.2$)

For the correct $\sin(30t)° = -0.6 \Rightarrow 30t = 323.1 \Rightarrow t = \text{awrt } 10.8$

For the incorrect $\sin(30t)° = +0.6 \Rightarrow 30t = 396.9 \Rightarrow t = \text{awrt } 13.2$

So award this mark if you see $30t = \text{invsin } their -0.6$ to give the first value of t where $30\,t > 255$

A1: Allow 10:46 a.m. (12 hour clock notation) or 10:46 (24 hour clock notation) oe

Allow 10:47 a.m. (12 hour clock notation) or 10:47 (24 hour clock notation) oe

DO NOT allow 646 minutes or 10 hours 46 minutes.

(**Q08 9MA0/01, June 2018**)

Q14.

Question Number	Scheme		Marks
(a)	$\dfrac{2}{P(P-2)} \equiv \dfrac{A}{P} + \dfrac{B}{(P-2)}$		
	$2 \equiv A(P-2) + BP$	Can be implied.	M1
	$A = -1,\ B = 1$	Either one.	A1
	giving $\dfrac{1}{(P-2)} - \dfrac{1}{P}$	See notes. **cao, aef**	A1
			[3]
(b)	$\dfrac{dP}{dt} = \dfrac{1}{2}P(P-2)\cos 2t$		
	$\displaystyle\int \dfrac{2}{P(P-2)}\,dP = \int \cos 2t\,dt$	can be implied by later working	B1 oe
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t\ \ (+c)$	$\pm\lambda\ln(P-2) \pm \mu\ln P$, $\lambda \neq 0,\ \mu \neq 0$	M1
		$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t$	A1
	$\{t = 0,\ P = 3 \Rightarrow\}\quad \ln 1 - \ln 3 = 0 + c\quad \{\Rightarrow\ c = -\ln 3\ \text{ or }\ \ln(\tfrac{1}{3})\}$	See notes	M1
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t - \ln 3$ $\ln\left(\dfrac{3(P-2)}{P}\right) = \dfrac{1}{2}\sin 2t$		
	$\dfrac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	M1
	$3(P-2) = Pe^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = Pe^{\frac{1}{2}\sin 2t}$ gives $3P - Pe^{\frac{1}{2}\sin 2t} = 6 \Rightarrow P(3 - e^{\frac{1}{2}\sin 2t}) = 6$	A complete method of rearranging to make P the subject. **Must have a constant of integration that need not be evaluated (see note)**	dM1
	$P = \dfrac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	A1 * cso
			[7]
(c)	$\{\text{population} = 4000 \Rightarrow\}\ P = 4$	States $P = 4$ or applies $P = 4$	M1
	$\dfrac{1}{2}\sin 2t = \ln\left(\dfrac{3(4-2)}{4}\right)\ \left\{= \ln\left(\dfrac{3}{2}\right)\right\}$	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, $\lambda \neq 0,\ k > 0$ where λ and k are numerical values and λ can be 1	M1
	$t = 0.4728700467...$	anything that rounds to 0.473 Do not apply isw here	A1
			[3] **13**

Question Number	Scheme	Marks
	Method 2 for Q7(b)	
(b)	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t \ \ (+c)$ — As before for…	**B1M1A1**
	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$	
	$\frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t + c}$ or $\frac{(P-2)}{P} = Ae^{\frac{1}{2}\sin 2t}$ — Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	**3rd M1**
	$(P-2) = APe^{\frac{1}{2}\sin 2t} \Rightarrow P - APe^{\frac{1}{2}\sin 2t} = 2$ $\Rightarrow P(1 - Ae^{\frac{1}{2}\sin 2t}) = 2 \Rightarrow P = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2t})}$ — A complete method of rearranging to make P the subject. Condone sign slips or constant errors. **Must have a constant of integration that need not be evaluated (see note)**	**4th dM1**
	$\{t = 0, P = 3 \Rightarrow\}\ \ 3 = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2(0)})}$ — See notes **(Allocate this mark as the 2nd M1 mark on ePEN).**	**2nd M1**
	$\left\{\Rightarrow 3 = \frac{2}{(1-A)} \Rightarrow A = \frac{1}{3}\right\}$	
	$\Rightarrow P = \frac{2}{\left(1 - \frac{1}{3}e^{\frac{1}{2}\sin 2t}\right)} \Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ * — Correct proof.	**A1** * cso

Question Notes

(a)	**M1**	Forming a correct identity. For example, $2 \equiv A(P-2) + BP$ from $\frac{2}{P(P-2)} = \frac{A}{P} + \frac{B}{(P-2)}$
	Note	A and B are not referred to in question.
	A1	Either one of $A = -1$ or $B = 1$.
	A1	$\frac{1}{(P-2)} - \frac{1}{P}$ or any equivalent form. This answer *cannot* be recovered from part (b).
	Note	M1A1A1 can also be given for a candidate who finds both $A = -1$ and $B = 1$ and $\frac{A}{P} + \frac{B}{(P-2)}$ is seen in their working.
	Note	Candidates can use 'cover-up' rule to write down $\frac{1}{(P-2)} - \frac{1}{P}$, so as to gain all three marks.
	Note	Equating coefficients from $2 \equiv A(P-2) + BP$ gives $A + B = 2, -2A = 2 \Rightarrow A = -1,\ B = 1$

(b)	**B1**	Separates variables as shown on the Mark Scheme. dP and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Eg:** $\int \frac{2}{P^2-2P}\,\mathrm{d}P = \int \cos 2t\,\mathrm{d}t$ or $\int \frac{1}{P(P-2)}\,\mathrm{d}P = \frac{1}{2}\int \cos 2t\,\mathrm{d}t$ o.e. are also fine for B1.
	1st M1	$\pm\lambda\ln(P-2) \pm \mu\ln P,\ \lambda \neq 0,\ \mu \neq 0$. Also allow $\pm\lambda\ln(M(P-2)) \pm \mu\ln NP$; M, N can be 1.
	Note	Condone $2\ln(P-2)+2\ln P$ or $2\ln(P(P-2))$ or $2\ln(P^2-2P)$ or $\ln(P^2-2P)$
	1st A1	Correct result of $\ln(P-2)-\ln P = \frac{1}{2}\sin 2t$ or $2\ln(P-2)-2\ln P = \sin 2t$ o.e. with or without $+c$
	2nd M1	Some evidence of using both $t=0$ and $P=3$ in an integrated equation containing a constant of integration. Eg: c or A, etc.
	3rd M1	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c,\ \lambda,\mu,\beta,K,\delta \neq 0$, applies a fully correct method to eliminate their logarithms.
	4th M1	**dependent on the third method mark being awarded.** A complete method of rearranging to make P the subject. Condone sign slips or constant errors.
	Note	For the 3rd M1 and 4th M1 marks, a candidate needs to have included a constant of integration, in their working. eg. c, A, $\ln A$ or an evaluated constant of integration.
	2nd A1	Correct proof of $P = \frac{6}{(3-e^{\frac{1}{2}\sin 2t})}$. **Note:** This answer is given in the question.
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$ followed by $\frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t} + e^c$ is 3rd M0, 4th M0, 2nd A0.
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c \rightarrow \frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t + c} \rightarrow \frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t} + e^c$ is final M1M0A0
		4th M1 for making *P* the subject **Note there are three type of manipulations here which are considered acceptable for making *P* the subject.** (1) M1 for $\frac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3(P-2) = Pe^{\frac{1}{2}\sin 2t} \Rightarrow 3P-6 = Pe^{\frac{1}{2}\sin 2t} \Rightarrow P(3-e^{\frac{1}{2}\sin 2t}) = 6$ $\Rightarrow P = \frac{6}{(3-e^{\frac{1}{2}\sin 2t})}$ (2) M1 for $\frac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3-\frac{6}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3-e^{\frac{1}{2}\sin 2t} = \frac{6}{P} \Rightarrow\Rightarrow P = \frac{6}{(3-e^{\frac{1}{2}\sin 2t})}$ (3) M1 for $\left\{\ln(P-2)+\ln P = \frac{1}{2}\sin 2t + \ln 3 \Rightarrow\right\} P(P-2) = 3e^{\frac{1}{2}\sin 2t} \Rightarrow P^2-2P = 3e^{\frac{1}{2}\sin 2t}$ $\Rightarrow (P-1)^2 - 1 = 3e^{\frac{1}{2}\sin 2t}$ leading to $P = ..$
(c)	**M1**	States $P=4$ or applies $P=4$
	M1	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, where λ and k are numerical values and λ can be 1
	A1	anything that rounds to 0.473. *(Do not apply isw here)*
	Note	*Do not apply ignore subsequent working for A1.* (Eg: 0.473 followed by 473 years is A0.)
	Note	Use of $P=4000$: Without the mention of $P=4$, $\frac{1}{2}\sin 2t = \ln 2.9985$ or $\sin 2t = 2\ln 2.9985$ or $\sin 2t = 2.1912...$ will usually imply M0M1A0
	Note	Use of Degrees: $t =$ awrt 27.1 will usually imply M1M1A0

(**Q32 6666/01, June 2015**)

Q15.

Question	Scheme	Marks	AOs
(a)	$a = 60$	B1	3.1b
	$2 = "60" - b(-20)^2 \Rightarrow b = ...$	M1	3.4
	$H = 60 - 0.145(t-20)^2$	A1	3.3
		(3)	
(b)	Height = 2 m	B1	3.4
		(1)	
(c)	$\alpha = 180$ or $\beta = 31$	M1	3.4
	$H = 29\cos(9t+180)° + 31$	A1	3.3
		(2)	
(d)	e.g. "The model allows for more than one circuit"	B1	3.5a
		(1)	
			(7 marks)

Notes

(a)

B1: $a = 60$ (may be seen in their final equation of the model or implied by 60 substituted for a in the model)

M1: Attempts to find b by substituting in $t = 0$, $H = 2$ and their a and proceeding to a value for b.
May be seen as two simultaneous equations formed:
$2 = a - b(-20)^2$ and $60 = a - b(20-20)^2$ proceeding to a value for b

A1: $H = 60 - 0.145(t-20)^2$ or equivalent such as $H = -\frac{29}{200}t^2 + 5.8t + 2$ or $H = 60 - \frac{29}{200}(t-20)^2$ isw once a correct equation for the model is seen. Must be in terms of H and t. If they just state $a = 60$, $b = 0.145$ then A0
A correct answer with no working seen scores full marks.

(b)

B1: 2 cao (condone lack of units) This can be scored even if their model in (a) is incorrect (they may have used symmetry to determine this value)

(c)

M1: $(\alpha =)\ 180$ or $(\beta =)\ 31$ Condone $(\alpha =)\ \pi$

A1: $H = 29\cos(9t+180)° + 31$ or equivalent e.g. $H = -29\cos(9t) + 31$ isw once a correct equation for the model is seen. Must be in terms of H and t. If they just state $\alpha = 180$, $\beta = 31$ then A0.
A correct equation with no working seen scores both marks. Does not require the degree symbol.

(d)

B1: Score for a reason which makes reference to any of

- the alternative model allows repetition (allow phrases e.g. "multiple cycles", "repeated circuits", "cyclical", "periodic", "loops around", "the original model can only go up and down once")
- the alternative model after 2 minutes the carriage will be back at the start (e.g."at 2 mins, $H = 2$")
- the original/quadratic model after 40 seconds (or any time after this) will be negative (e.g. "the height will be negative which cannot happen")
- the original model after 2 minutes would not be back at the start

Do not allow vague responses on their own e.g. "the original model is a parabola"
If calculations are used then they must be correct using a correct model (allow rounded or truncated)
Look for a valid reason and ignore reference to anything else as long as it does not contradict

t	0	5	10	15	20	25	30	35	40	45	50	55	60	80	100	120
h	2	27	46	56	60	56	46	27	2	-31	-71	-118	-172	-462	-868	-1390

(**Q13 9MA0/01, June 2023**)

Q16.

Question	Scheme	Marks	AOs
(a)	$(-180°, -3)$	B1	1.1b
		(1)	
(b)	(i) $(-720°, -3)$	B1ft	2.2a
	(ii) $(-144°, -3)$	B1 ft	2.2a
		(2)	
(c)	Attempts to use both $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta = 1$ and solves a quadratic equation in $\sin\theta$ to find at least one value of θ	M1	3.1a
	$3\cos\theta = 8\tan\theta \Rightarrow 3\cos^2\theta = 8\sin\theta$	B1	1.1b
	$3\sin^2\theta + 8\sin\theta - 3 = 0$ $(3\sin\theta - 1)(\sin\theta + 3) = 0$	M1	1.1b
	$\sin\theta = \frac{1}{3}$	A1	2.2a
	awrt $520.5°$ only	A1	2.1
		(5)	
			(8 marks)

(a)

B1: Deduces that $P(-180°, -3)$ or $c = -180^{(o)}, d = -3$

(b)(i)

B1ft: Deduces that $P'(-720°, -3)$ Follow through on their $(c, d) \to (4c, d)$ where d is negative

(b)(ii)

B1ft: Deduces that $P'(-144°, -3)$ Follow through on their $(c, d) \to (c + 36°, d)$ where d is negative

(c)

M1: An overall problem solving mark, condoning slips, for an attempt to

- use $\tan\theta = \frac{\sin\theta}{\cos\theta}$,
- use $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$
- find at least one value of θ from a quadratic equation in $\sin\theta$

B1: Uses the correct identity and multiplies across to give $3\cos\theta = 8\tan\theta \Rightarrow 3\cos^2\theta = 8\sin\theta$ oe

M1: Uses the correct identity $\sin^2\theta + \cos^2\theta = 1$ to form a 3TQ in $\sin\theta$ which they attempt to solve using an appropriate method. It is OK to use a calculator to solve this

A1: $\sin\theta = \frac{1}{3}$ Accept sight of $\frac{1}{3}$. Ignore any reference to the other root even if it is "used"

A1: Full method with all identities correct leading to the answer of awrt $520.5°$ and no other values.

(**Q09 8MA0/01, Oct 2020**)

Q17.

Question	Scheme	Marks	AOs
(a)	Identifies an error for student A: They use $\frac{\cos\theta}{\sin\theta}=\tan\theta$ It should be $\frac{\sin\theta}{\cos\theta}=\tan\theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6°)\neq 2\sin(-26.6°)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
		(3 marks)	

Notes:

(a)

B1: Accept a response of the type 'They use $\frac{\cos\theta}{\sin\theta}=\tan\theta$. This is incorrect as $\frac{\sin\theta}{\cos\theta}=\tan\theta$'

It can be implied by a response such as 'They should get $\tan\theta=\frac{1}{2}$ not $\tan\theta=2$'

Accept also statements such as 'it should be $\cot\theta=2$'

(b)

B1: Accept a response where the candidate shows that $-26.6°$ is not a solution of $\cos\theta=2\sin\theta$. This can be shown by, for example, finding both $\cos(-26.6°)$ and $2\sin(-26.6°)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6°)=+ve$ and $2\sin(-26.6°)=-ve$ and stating that they therefore cannot be equal.

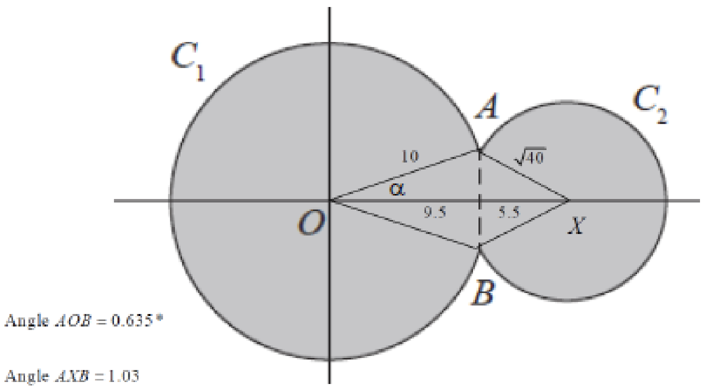
B1: Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x=5$ squared gives $x^2=25$ which has answers ±5

(**Q02 9MA0/02, Specimen papers**)

Q18.

Question	Scheme	Marks	AOs
(a)	Solves $x^2+y^2=100$ and $(x-15)^2+y^2=40$ simultaneously to find x or y E.g. $(x-15)^2+100-x^2=40 \Rightarrow x=...$	M1	3.1a
	Either $\Rightarrow -30x+325=40 \Rightarrow x=9.5$ Or $y=\frac{\sqrt{39}}{2}=$ awrt ± 3.12	A1	1.1b
	Attempts to find the angle AOB in circle C_1 Eg Attempts $\cos\alpha=\frac{\text{"9.5"}}{10}$ to find α then $\times 2$	M1	3.1a
	Angle $AOB = 2\times\arccos\left(\frac{9.5}{10}\right)=0.635$ rads (3sf) *	A1*	2.1
		(4)	
(b)	Attempts $10\times(2\pi-0.635)$ $= 56.48$	M1	1.1b
	Attempts to find angle AXB or AXO in circle C_2 (see diagram) E.g. $\cos\beta=\frac{15-\text{"9.5"}}{\sqrt{40}}\Rightarrow\beta=...$ (Note AXB =1.03 rads)	M1	3.1a
	Attempts $10\times(2\pi-0.635)+\sqrt{40}\times(2\pi-2\beta)$	dM1	2.1
	$= 89.7$	A1	1.1b
		(4)	
			(8 marks)

Notes:



(a)

M1: For the key step in an attempt to find either coordinate for where the two circles meet.
Look for an attempt to set up an equation in a single variable leading to a value for x or y.

A1: $x=9.5$ (or $y=\frac{\sqrt{39}}{2}=$ awrt ± 3.12)

M1: Uses the radius of the circle and correct trigonometry in an attempt to find angle AOB in circle C_1

E.g. Attempts $\cos\alpha = \frac{"9.5"}{10}$ to find α then $\times 2$

Alternatives include $\tan\alpha = \frac{\sqrt{100-"9.5"^2}}{"9.5"} = (0.3286...)$ to find α then $\times 2$

And $\cos AOB = \frac{10^2+10^2-\left(\sqrt{39}\right)^2}{2\times10\times10} = \frac{161}{200}$

A1*: Correct and careful work in proceeding to the given answer. Condone an answer with greater accuracy.
Condone a solution where the intermediate value has been truncated, provided the trig equation is correct.

E.g. $\sin\alpha = \frac{\sqrt{39}}{20} \Rightarrow \alpha = 0.317 \Rightarrow AOB = 2\alpha = 0.635$

Condone a solution written down from awrt $36.4°$ (without the need to shown any calculation.)
E

(b)

M1: Attempts to use the formula $s = r\theta$ with $r = 10$ and $\theta = 2\pi - 0.635$

The formula may be embedded. You may see $\underline{\underline{2\pi 10}}+2\pi\sqrt{40}\underline{\underline{-10\times0.635}}$... which is fine for this M1

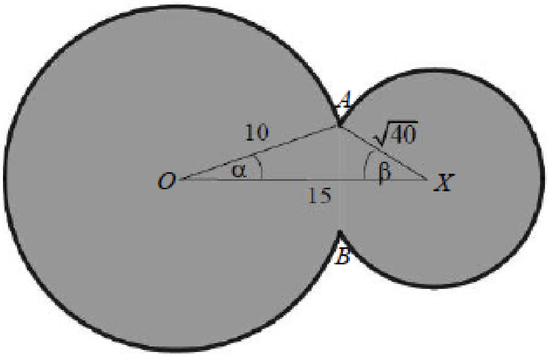
M1: Attempts to use a correct method in order to find angle AXB or AXO in circle C_2

Amongst many other methods are $\tan\beta = \frac{"3.12"}{15-9.5}$ and $\cos AXB = \frac{40+40-\left(\sqrt{39}\right)^2}{2\times\sqrt{40}\times\sqrt{40}} = \frac{41}{80}$

Note that many candidates believe this to be 0.635. This scores M0 dM0 A0

dM1: A full and complete attempt to find the perimeter of the region.
It is dependent upon having scored both M's.

A1: awrt 89.7



(a)

M1: For the key step in attempting to find all lengths in triangle OAX, condoning slips

A1: All three lengths correct

M1: Attempts cosine rule to find α then $\times 2$

A1*: Correct and careful work in proceeding to the given answer

---

(**Q11 9MA0/01, Oct 2020**)

Q19.

Question Number	Scheme	Notes	Marks
	(i) $\displaystyle\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y$, $y>0$, (ii) $\displaystyle\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x$, $x=4\sin^2\theta$		
(i) **Way 1**	$\dfrac{3y-4}{y(3y+2)} \equiv \dfrac{A}{y}+\dfrac{B}{(3y+2)} \Rightarrow 3y-4 = A(3y+2)+By$	See notes	M1
	$y=0 \Rightarrow -4=2A \Rightarrow A=-2$	At least one of their $A=-2$ **or** their $B=9$	A1
	$y=-\frac{2}{3} \Rightarrow -6=-\frac{2}{3}B \Rightarrow B=9$	Both their $A=-2$ **and** their $B=9$	A1
	$\displaystyle\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{-2}{y}+\frac{9}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of either $\dfrac{A}{y}\to\pm\lambda\ln y$ **or** $\dfrac{B}{(3y+2)}\to\pm\mu\ln(3y+2)$; $A\neq0$, $B\neq0$	M1
		At least one term correctly followed through from their A or from their B	A1 ft
	$= -2\ln y+3\ln(3y+2)\,\{+c\}$	$-2\ln y+3\ln(3y+2)$ **or** $-2\ln y+3\ln(y+\frac{2}{3})$ with correct bracketing, simplified or un-simplified. Can apply isw.	A1 **cao**
			[6]
(ii) (a) **Way 1**	$\left\{x=4\sin^2\theta \Rightarrow\right\} \dfrac{\mathrm{d}x}{\mathrm{d}\theta}=8\sin\theta\cos\theta$ or $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=4\sin2\theta$ or $\mathrm{d}x=8\sin\theta\cos\theta\,\mathrm{d}\theta$		B1
	$\displaystyle\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}.\,8\sin\theta\cos\theta\,\{\mathrm{d}\theta\}$ or $\displaystyle\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}.\,4\sin2\theta\,\{\mathrm{d}\theta\}$		M1
	$= \displaystyle\int \underline{\underline{\tan\theta}}.\,8\sin\theta\cos\theta\,\{\mathrm{d}\theta\}$ or $\displaystyle\int \underline{\underline{\tan\theta}}.\,4\sin2\theta\,\{\mathrm{d}\theta\}$	$\sqrt{\left(\frac{x}{4-x}\right)} \to \pm K\tan\theta$ or $\pm K\left(\frac{\sin\theta}{\cos\theta}\right)$	$\underline{\underline{\text{M1}}}$
	$= \displaystyle\int 8\sin^2\theta\,\mathrm{d}\theta$	$\displaystyle\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1
	$3=4\sin^2\theta$ **or** $\dfrac{3}{4}=\sin^2\theta$ **or** $\sin\theta=\dfrac{\sqrt{3}}{2} \Rightarrow \theta=\dfrac{\pi}{3}$ $\{x=0 \to \theta=0\}$	Writes down a correct equation involving $x=3$ leading to $\theta=\dfrac{\pi}{3}$ and no incorrect work seen regarding limits	B1
			[5]
(ii) (b)	$=\{8\}\displaystyle\int\left(\frac{1-\cos2\theta}{2}\right)\mathrm{d}\theta$ $\left\{= \displaystyle\int(4-4\cos2\theta)\,\mathrm{d}\theta\right\}$	Applies $\cos2\theta=1-2\sin^2\theta$ to their integral. (See notes)	M1
	$=\{8\}\left(\dfrac{1}{2}\theta-\dfrac{1}{4}\sin2\theta\right)$ $\{= 4\theta-2\sin2\theta\}$	For $\pm\alpha\theta\pm\beta\sin2\theta,\ \alpha,\beta\neq0$	M1
		$\sin^2\theta \to \left(\dfrac{1}{2}\theta-\dfrac{1}{4}\sin2\theta\right)$	A1
	$\left\{\displaystyle\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,\mathrm{d}\theta = 8\left[\frac{1}{2}\theta-\frac{1}{4}\sin2\theta\right]_0^{\frac{\pi}{3}}\right\} = 8\left(\left(\dfrac{\pi}{6}-\dfrac{1}{4}\left(\dfrac{\sqrt{3}}{2}\right)\right)-(0+0)\right)$		
	$=\dfrac{4}{3}\pi-\sqrt{3}$	"two term" exact answer of e.g. $\dfrac{4}{3}\pi-\sqrt{3}$ or $\dfrac{1}{3}(4\pi-3\sqrt{3})$	A1 o.e.
			[4]
			15

	Question Notes	
(i)	1st M1	Writing $\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)}$ and a complete method for finding the value of at least one of their A or their B.
	Note	M1A1 can be implied *for writing down* either $\frac{3y-4}{y(3y+2)} \equiv \frac{-2}{y} + \frac{\text{their } B}{(3y+2)}$ or $\frac{3y-4}{y(3y+2)} \equiv \frac{\text{their } A}{y} + \frac{9}{(3y+2)}$ with no working.
	Note	Correct bracketing is not necessary for the penultimate A1ft, but is required for the final A1 in (i)
	Note	Give 2nd M0 for $\frac{3y-4}{y(3y+2)}$ going directly to $\pm\alpha\ln(3y^2+2y)$
	Note	…but allow 2nd M1 for either $\frac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ or $\frac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$
(ii)(a)	1st M1	Substitutes $x = 4\sin^2\theta$ and their $\mathrm{d}x$ $\left(\text{from their correctly rearranged } \frac{\mathrm{d}x}{\mathrm{d}\theta}\right)$ into $\sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x$
	Note	$\mathrm{d}x \neq \lambda\mathrm{d}\theta$. For example $\mathrm{d}x \neq \mathrm{d}\theta$
	Note	Allow substituting $\mathrm{d}x = 4\sin 2\theta$ for the 1st M1 after a correct $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$ or $\mathrm{d}x = 4\sin 2\theta\,\mathrm{d}\theta$
	2nd M1	Applying $x = 4\sin^2\theta$ to $\sqrt{\left(\frac{x}{4-x}\right)}$ to give $\pm K\tan\theta$ or $\pm K\left(\frac{\sin\theta}{\cos\theta}\right)$
	Note	Integral sign is not needed for this mark.
	1st A1	Simplifies to give $\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$
	2nd B1	Writes down a correct equation involving $x = 3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits
	Note	Allow 2nd B1 for $x = 4\sin^2\left(\frac{\pi}{3}\right) = 3$ and $x = 4\sin^2 0 = 0$
	Note	Allow 2nd B1 for $\theta = \sin^{-1}\left(\sqrt{\frac{x}{4}}\right)$ followed by $x = 3,\ \theta = \frac{\pi}{3};\ x = 0,\ \theta = 0$

(ii)(b)	M1	Writes down a correct equation involving $\cos 2\theta$ and $\sin^2\theta$ **E.g.:** $\cos 2\theta = 1 - 2\sin^2\theta$ or $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ or $K\sin^2\theta = K\left(\frac{1-\cos 2\theta}{2}\right)$ and *applies* it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0,\ \beta \neq 0$ (can be simplified or un-simplified).
	1st A1	Integrating $\sin^2\theta$ to give $\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta$, un-simplified or simplified. Correct solution only. Can be implied by $k\sin^2\theta$ giving $\frac{k}{2}\theta - \frac{k}{4}\sin 2\theta$ or $\frac{k}{4}(2\theta - \sin 2\theta)$ un-simplified or simplified.
	2nd A1	A *correct solution in part (ii)* leading to a "two term" exact answer of e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{8}{6}\pi - \sqrt{3}$ or $\frac{4}{3}\pi - \frac{2\sqrt{3}}{2}$ or $\frac{1}{3}(4\pi - 3\sqrt{3})$
	Note	A decimal answer of 2.456739397… (without a correct **exact** answer) is A0.
	Note	Candidates can work in terms of λ (note that λ is not given in (ii)) and gain the 1st three marks (i.e. M1M1A1) in part (b).
	Note	If they incorrectly obtain $\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,\mathrm{d}\theta$ in part (i)(a) (or correctly guess that $\lambda = 8$) then the final A1 is available for a correct solution in part (ii)(b).

	Scheme	Notes	Marks
(i) Way 2	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{6y+2}{3y^2+2y}\,dy - \int \frac{3y+6}{y(3y+2)}\,dy$		
	$\frac{3y+6}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y+6 = A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow 6 = 2A \Rightarrow A = 3$	At least one of their $A = 3$ or their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$	Both their $A = 3$ and their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy$ $= \int \frac{6y+2}{3y^2+2y}\,dy - \int \frac{3}{y}\,dy + \int \frac{6}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ or $\frac{A}{y} \to \pm\lambda\ln y$ or $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$ $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 cao
			[6]
(i) Way 3	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{3y+1}{3y^2+2y}\,dy - \int \frac{5}{y(3y+2)}\,dy$		
	$\frac{5}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 5 = A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow 5 = 2A \Rightarrow A = \frac{5}{2}$	At least one of their $A = \frac{5}{2}$ or their $B = -\frac{15}{2}$	A1
	$y = -\frac{2}{3} \Rightarrow 5 = -\frac{2}{3}B \Rightarrow B = -\frac{15}{2}$	Both their $A = \frac{5}{2}$ and their $B = -\frac{15}{2}$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy$ $= \int \frac{3y+1}{3y^2+2y}\,dy - \int \frac{\frac{5}{2}}{y}\,dy + \int \frac{\frac{15}{2}}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ or $\frac{A}{y} \to \pm\lambda\ln y$ or $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$ $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)\ \{+c\}$	$\frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 cao
			[6]

	Scheme	Notes	
(i) Way 4	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{3y}{y(3y+2)}\,dy - \int \frac{4}{y(3y+2)}\,dy$		
	$= \int \frac{3}{(3y+2)}\,dy - \int \frac{4}{y(3y+2)}\,dy$		
	$\frac{4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 4 = A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow 4 = 2A \Rightarrow A = 2$	At least one of their $A = 2$ or their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$	Both their $A = 2$ and their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy$ $= \int \frac{3}{3y+2}\,dy - \int \frac{2}{y}\,dy + \int \frac{6}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{C}{(3y+2)} \to \pm\alpha\ln(3y+2)$ or $\frac{A}{y} \to \pm\lambda\ln y$ or $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$, $A \neq 0, B \neq 0, C \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \ln(3y+2) - 2\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y+2) - 2\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 cao
			[6]

	Alternative methods for B1M1M1A1 in (ii)(a)		
(ii)(a) Way 2	$\left\{x = 4\sin^2\theta \Rightarrow\right\} \dfrac{dx}{d\theta} = 8\sin\theta\cos\theta$	As in Way 1	B1
	$\displaystyle\int \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,8\sin\theta\cos\theta\,\{d\theta\}$	As before	M1
	$\displaystyle= \int \sqrt{\frac{\sin^2\theta}{(1-\sin^2\theta)}}\,.\,8\cos\theta\sin\theta\,\{d\theta\}$		
	$\displaystyle= \int \frac{\sin\theta}{\sqrt{(1-\sin^2\theta)}}\,.\,8\sqrt{(1-\sin^2\theta)}\sin\theta\,\{d\theta\}$		
	$\displaystyle= \int \sin\theta\,.\,8\sin\theta\,\{d\theta\}$	Correct method leading to $\sqrt{(1-\sin^2\theta)}$ being cancelled out	M1
	$\displaystyle= \int 8\sin^2\theta\,d\theta$	$\displaystyle\int 8\sin^2\theta\,d\theta$ including $d\theta$	A1 cso
(ii)(a) Way 3	$\left\{x = 4\sin^2\theta \Rightarrow\right\} \dfrac{dx}{d\theta} = 4\sin 2\theta$	As in Way 1	B1
	$x = 4\sin^2\theta = 2-2\cos 2\theta\,,\ \ 4-x = 2+2\cos 2\theta$		
	$\displaystyle\int \sqrt{\frac{2-2\cos 2\theta}{2+2\cos 2\theta}}\,.\,4\sin 2\theta\,\{d\theta\}$		M1
	$\displaystyle= \int \frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2+2\cos 2\theta}}\,.\,\frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2-2\cos 2\theta}}4\sin 2\theta\,\{d\theta\} = \int \frac{2-2\cos 2\theta}{\sqrt{4-4\cos^2 2\theta}}\,.\,4\sin 2\theta\,\{d\theta\}$		
	$\displaystyle= \int \frac{2-2\cos 2\theta}{2\sin 2\theta}\,.\,4\sin 2\theta\,\{d\theta\} = \int 2(2-2\cos 2\theta)\,.\,\{d\theta\}$	Correct method leading to $\sin 2\theta$ being cancelled out	M1
	$\displaystyle= \int 8\sin^2\theta\,d\theta$	$\displaystyle\int 8\sin^2\theta\,d\theta$ including $d\theta$	A1 cso

(**Q34 6666/01, June 2016**)

Q20.

Question Number	Scheme		Marks
(a)	$A = \int_0^3 \sqrt{(3-x)(x+1)}\,\mathrm{d}x$, $x = 1 + 2\sin\theta$		
	$\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$	$\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$ or $2\cos\theta$ used correctly in their working. Can be implied.	B1
	$\left\{\int \sqrt{(3-x)(x+1)}\,\mathrm{d}x \text{ or } \int \sqrt{(3+2x-x^2)}\,\mathrm{d}x\right\}$		
	$= \int \sqrt{(3-(1+2\sin\theta))((1+2\sin\theta)+1)}\;2\cos\theta\,\{\mathrm{d}\theta\}$	Substitutes for both x and $\mathrm{d}x$, where $\mathrm{d}x \neq \lambda \mathrm{d}\theta$. Ignore $\mathrm{d}\theta$	M1
	$= \int \sqrt{(2-2\sin\theta)(2+2\sin\theta)}\;2\cos\theta\,\{\mathrm{d}\theta\}$ $= \int \sqrt{(4-4\sin^2\theta)}\;2\cos\theta\,\{\mathrm{d}\theta\}$		
	$= \int \sqrt{(4-4(1-\cos^2\theta)}\;2\cos\theta\,\{\mathrm{d}\theta\}$ or $\int \sqrt{4\cos^2\theta}\;2\cos\theta\,\{\mathrm{d}\theta\}$	Applies $\cos^2\theta = 1 - \sin^2\theta$ **see notes**	M1
	$= 4\int \cos^2\theta\,\mathrm{d}\theta,\ \{k = 4\}$	$4\int \cos^2\theta\,\mathrm{d}\theta$ or $\int 4\cos^2\theta\,\mathrm{d}\theta$ **Note:** $\mathrm{d}\theta$ is required here.	A1
	$0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\dfrac{1}{2} \Rightarrow \underline{\theta = -\dfrac{\pi}{6}}$ **and** $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1 \Rightarrow \underline{\theta = \dfrac{\pi}{2}}$	See notes	B1
			[5]
(b)	$\left\{k\int \cos^2\theta\,\{\mathrm{d}\theta\}\right\} = \{k\}\int \left(\dfrac{1+\cos 2\theta}{2}\right)\{\mathrm{d}\theta\}$	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral	M1
	$= \{k\}\left(\dfrac{1}{2}\theta + \dfrac{1}{4}\sin 2\theta\right)$	Integrates to give $\pm\alpha\theta \pm \beta\sin 2\theta,\ \alpha \neq 0,\ \beta \neq 0$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$	M1 **(A1 on ePEN)**
	$\left\{\text{So } 4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\,\mathrm{d}\theta = \left[2\theta + \sin 2\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}}\right\}$ $= \left(2\left(\dfrac{\pi}{2}\right) + \sin\left(\dfrac{2\pi}{2}\right)\right) - \left(2\left(-\dfrac{\pi}{6}\right) + \sin\left(-\dfrac{2\pi}{6}\right)\right)$		
	$\left\{= (\pi) - \left(-\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{2}\right)\right\} = \dfrac{4\pi}{3} + \dfrac{\sqrt{3}}{2}$	$\dfrac{4\pi}{3} + \dfrac{\sqrt{3}}{2}$ or $\dfrac{1}{6}\left(8\pi + 3\sqrt{3}\right)$	A1 cao cso
			[3] 8

		Question Notes
(a)	B1	$\frac{dx}{d\theta} = 2\cos\theta$. Also allow $dx = 2\cos\theta d\theta$. This mark can be implied by later working.
	Note	You can give B1 for $2\cos\theta$ used correctly in their working.
	M1	Substitutes $x = 1 + 2\sin\theta$ and their dx $\left(\text{from their rearranged } \frac{dx}{d\theta}\right)$ into $\sqrt{(3-x)(x+1)}\,dx$.
	Note	Condone bracketing errors here.
	Note	$dx \neq \lambda d\theta$. For example $dx \neq d\theta$.
	Note	Condone substituting $dx = \cos\theta$ for the 1st M1 after a correct $\frac{dx}{d\theta} = 2\cos\theta$ or $dx = 2\cos\theta d\theta$
	M1	Applies either • $1 - \sin^2\theta = \cos^2\theta$ • $\lambda - \lambda\sin^2\theta$ or $\lambda(1 - \sin^2\theta) = \lambda\cos^2\theta$ • $4 - 4\sin^2\theta = 4 + 2\cos 2\theta - 2 = 2 + 2\cos 2\theta = 4\cos^2\theta$ to their expression where λ is a numerical value.
	A1	Correctly proves that $\int \sqrt{(3-x)(x+1)}\,dx$ is equal to $4\int \cos^2\theta d\theta$ or $\int 4\cos^2\theta d\theta$
	Note	All three previous marks must have been awarded before A1 can be awarded.
	Note	Their final answer must include $d\theta$.
	Note	You can ignore limits for the final A1 mark.
	B1	Evidence of a correct equation in $\sin\theta$ or $\sin^{-1}\theta$ for both x-values leading to both θ values. Eg: • $0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2}$ which then leads to $\theta = -\frac{\pi}{6}$, **and** • $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1$ which then leads to $\theta = \frac{\pi}{2}$
	Note	Allow B1 for $x = 1 + 2\sin\left(-\frac{\pi}{6}\right) = 0$ **and** $x = 1 + 2\sin\left(\frac{\pi}{2}\right) = 3$
	Note	Allow B1 for $\sin\theta = \left(\frac{x-1}{2}\right)$ or $\theta = \sin^{-1}\left(\frac{x-1}{2}\right)$ followed by $x = 0,\ \theta = -\frac{\pi}{6}$; $x = 3,\ \theta = \frac{\pi}{2}$
(b)	NOTE	**Part (b) appears as M1A1A1 on ePEN, but is now marked as M1M1A1.**
	M1	Writes down a correct equation involving $\cos 2\theta$ and $\cos^2\theta$ **Eg:** $\cos 2\theta = 2\cos^2\theta - 1$ or $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ or $\lambda\cos^2\theta = \lambda\left(\frac{1+\cos 2\theta}{2}\right)$ and *applies* it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0$, $\beta \neq 0$ (can be simplified or un-simplified).
	A1	A *correct solution in part (b)* leading to a "two term" exact answer. Eg: $\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ **or** $\frac{8\pi}{6} + \frac{\sqrt{3}}{2}$ **or** $\frac{1}{6}(8\pi + 3\sqrt{3})$
	Note	5.054815… from no working is M0M0A0.
	Note	Candidates can work in terms of k (note that k is not given in (a)) for the M1M1 marks in part (b).
	Note	If they incorrectly obtain $4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta d\theta$ in part (a) (or guess $k = 4$) then the final A1 is available for a correct solution in part (b) only.

(**Q31 6666/01, June 2015**)

Q21.

Question Number	Scheme	Marks
	$\int x\sin 2x\,dx = -\frac{x\cos 2x}{2} + \int \frac{\cos 2x}{2}\,dx$	M1 A1 A1
	$= \quad \ldots \quad + \frac{\sin 2x}{4}$	M1
	$\left[\ \ldots\ \right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$	M1 A1
		[6]

(**Q28 6666/01, Jan 2011**)

Q22.

Question Number	Scheme	Marks
(a)	$\int x\sin 3x\,dx = -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x\,\{dx\}$	M1 A1
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\,\{+c\}$	A1
		[3]
(b)	$\int x^2\cos 3x\,dx = \frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x\,\{dx\}$	M1 A1
	$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right)\ \{+c\}$	A1 isw
	$\left\{= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x\ \{+c\}\right\}$ *Ignore subsequent working*	[3]
		6
(a)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x \to u' = 1$ and $v' = \sin 3x \to v = k\cos 3x$ (seen or implied), where k is a positive or negative constant. (Allow $k = 1$). This means that the candidate must achieve $x(k\cos 3x) - \int (k\cos 3x)$, where k is a consistent constant. If x^2 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $-\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x\,\{dx\}$. Can be un-simplified. Ignore the $\{dx\}$. **A1:** $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$ with/without $+c$. Can be un-simplified.	
(b)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x^2 \to u' = 2x$ or x and $v' = \cos 3x \to v = \lambda\sin 3x$ (seen or implied), where λ is a positive or negative constant. (Allow $\lambda = 1$). This means that the candidate must achieve $x^2(\lambda\sin 3x) - \int 2x(\lambda\sin 3x)$, where $u' = 2x$ or $x^2(\lambda\sin 3x) - \int x(\lambda\sin 3x)$, where $u' = x$. If x^3 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $\frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x\,\{dx\}$. Can be un-simplified. Ignore the $\{dx\}$. **A1:** $\frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right)$ with/without $+c$, can be un-simplified. You can ignore subsequent working here. **Special Case:** If the candidate scores the first two marks of M1A1 in part (b), then you can award the final A1 as a follow through for $\frac{1}{3}x^2\sin 3x - \frac{2}{3}$(their follow through part(a) answer).	

(**Q27 6666/01, Jan 2012**)

Q23.

Question Number	Scheme		Marks
(a)	0.73508		B1 cao [1]
(b)	Area $\approx \frac{1}{2}\times\frac{\pi}{8}$; $\times\underline{\left[0+2\left(\text{their } 0.73508+1.17157+1.02280\right)+0\right]}$		B1 M1
	$=\frac{\pi}{16}\times 5.8589... = 1.150392325... = 1.1504$ (4 dp)	awrt 1.1504	A1 [3]
(c)	$\{u=1+\cos x\} \Rightarrow \underline{\frac{du}{dx}=-\sin x}$		B1
	$\left\{\int \frac{2\sin 2x}{(1+\cos x)}\,dx = \right\} \int \frac{2(2\sin x\cos x)}{(1+\cos x)}\,dx$	$\sin 2x = 2\sin x\cos x$	B1
	$= \int \frac{4(u-1)}{u}.(-1)\,du \quad \left\{= 4\int \frac{(1-u)}{u}\,du\right\}$		M1
	$= 4\int\left(\frac{1}{u}-1\right)du = 4(\ln u - u)+c$		dM1
	$= 4\ln(1+\cos x)-4(1+\cos x)+c \quad = 4\ln(1+\cos x)-4\cos x+k$	AG	A1 cso [5]
(d)	$=\left[4\ln\left(1+\cos\frac{\pi}{2}\right)-4\cos\frac{\pi}{2}\right]-\left[4\ln(1+\cos 0)-4\cos 0\right]$	Applying limits $x=\frac{\pi}{2}$ and $x=0$ either way round.	M1
	$=[4\ln 1-0]-[4\ln 2-4]$		
	$=4-4\ln 2 \;\{=1.227411278...\}$	$\pm 4(1-\ln 2)$ or $\pm(4-4\ln 2)$ or awrt ± 1.2, ***however*** found.	A1
	Error $= \left\|(4-4\ln 2)-1.1504..\right\|$ $=0.0770112776... = 0.077$ (2sf)	awrt ± 0.077 or awrt $\pm 6.3(\%)$	A1 cso [3]
			12

(a)
B1: 0.73508 correct answer only. Look for this on the table or in the candidate's working.

(b)
B1: Outside brackets $\frac{1}{2}\times\frac{\pi}{8}$ or $\frac{\pi}{16}$ or awrt 0.196

M1: <u>For structure of trapezium rule</u> $\left[\;\underline{............}\;\right]$; (0 can be implied).

A1: anything that rounds to 1.1504

<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly**

Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8}+2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 6.0552).

Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8}\,(0+0)+2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 5.8589).

<u>Alternative method for part (b): Adding individual trapezia</u>

$$\text{Area} \approx \frac{\pi}{8}\times\left[\frac{0+0.73508}{2}+\frac{0.73508+1.17157}{2}+\frac{1.17157+1.02280}{2}+\frac{1.02280+0}{2}\right] = 1.150392325...$$

B1: $\frac{\pi}{8}$ and a divisor of 2 on all terms inside brackets.

M1: One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.

A1: anything that rounds to 1.1504

(c)
B1: $\frac{du}{dx}=-\sin x$ or $du=-\sin x\,dx$ or $\frac{dx}{du}=\frac{1}{-\sin x}$ oe.

B1: For seeing, applying or implying $\sin 2x = 2\sin x\cos x$.

M1: After applying substitution candidate achieves $\pm k\int\frac{(u-1)}{u}(du)$ or $\pm k\int\frac{(1-u)}{u}(du)$.

Allow M1 for "invisible" brackets here, eg: $\pm\int\frac{(\lambda u-1)}{u}(du)$ or $\pm\int\frac{(-\lambda+u)}{u}(du)$, where λ is a positive constant.

dM1: An attempt to divide through each term by u and $\pm k\int\left(\frac{1}{u}-1\right)du \to \pm k(\ln u - u)$ with/without $+c$. Note that this mark is dependent on the previous M1 mark being awarded.

<u>Alternative method:</u> Candidate can also gain this mark for applying integration by parts followed by a correct method for integrating $\ln u$. (See below).

A1: Correctly combines their $+c$ and "-4" together to give $\underline{4\ln(1+\cos x)-4\cos x+k}$

As a minimum candidate must write either $4\ln(1+\cos x)-4(1+\cos x)+c \to 4\ln(1+\cos x)-4\cos x+k$ or $4\ln(1+\cos x)-4(1+\cos x)+k \to 4\ln(1+\cos x)-4\cos x+k$

Note: that this mark is also for a correct solution only.

Note: those candidates who attempt to find the value of k will usually achieve A0.

(d)
M1: Substitutes limits of $x=\frac{\pi}{2}$ and $x=0$ into $\{4\ln(1+\cos x)-4\cos x\}$ or their answer from part (c) and subtracts the either way round. **Note that:** $\left[4\ln\left(1+\cos\frac{\pi}{2}\right)-4\cos\frac{\pi}{2}\right]-[0]$ is M0.

A1: $4(1-\ln 2)$ or $4-4\ln 2$ or awrt 1.2, however found.
This mark can be implied by the final answer of either awrt ± 0.077 or awrt ± 6.3

A1: For either awrt ± 0.077 or awrt ± 6.3 (for percentage error). Note this mark is for a **correct solution only**. Therefore if there if a candidate substitutes limits the incorrect way round and final achieves (usually fudges) the final correct answer then this mark can be withheld. Note that awrt 6.7 (for percentage error) is A0.

<u>Alternative method for dM1 in part (c)</u>

$$\int\frac{(1-u)}{u}\,du = \left((1-u)\ln u - \int -\ln u\,du\right) = \left((1-u)\ln u + u\ln u - \int\frac{u}{u}\,du\right) = \left((1-u)\ln u + u\ln u - u\right)$$

or $$\int\frac{(u-1)}{u}\,du = \left((u-1)\ln u - \int\ln u\,du\right) = \left((u-1)\ln u - \left(u\ln u - \int\frac{u}{u}\,du\right)\right) = \left((u-1)\ln u - u\ln u + u\right)$$

So **dM1** is for $\int\frac{(1-u)}{u}\,du$ going to $\left((1-u)\ln u + u\ln u - u\right)$ or $\left((u-1)\ln u - u\ln u + u\right)$ oe.

<u>Alternative method for part (d)</u>

M1A1 for $\left\{4\int_2^1\left(\frac{1}{u}-1\right)du = \right\} 4\left[\ln u - u\right]_2^1 = 4\left[(\ln 1 - 1)-(\ln 2 - 2)\right] = 4(1-\ln 2)$

<u>Alternative method for part (d): Using an extra constant λ from their integration.</u>

$$\left[4\ln\left(1+\cos\frac{\pi}{2}\right)-4\cos\frac{\pi}{2}+\lambda\right]-\left[4\ln(1+\cos 0)-4\cos 0+\lambda\right]$$

λ is usually -4, but can be a value of k that the candidate has found in part (d).

Note: The extra constant λ should cancel out and so the candidate can gain all three marks using this method, even the final A1 cso.

(**Q30 6666/01, Jan 2012**)

Q24.

Question Number	Scheme		Marks
(a)	$\left\{\begin{array}{ll} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x \Rightarrow & v = \frac{1}{2}\sin 2x \end{array}\right\}$		
	$\text{Int} = \int x\cos 2x \, dx = \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x.1\, dx$	***(see note below)*** Use of 'integration by parts' formula in the correct direction.	M1
		Correct expression.	A1
	$= \frac{1}{2}x\sin 2x - \frac{1}{2}\left(-\frac{1}{2}\cos 2x\right) + c$	$\sin 2x \to -\frac{1}{2}\cos 2x$ or $\sin kx \to -\frac{1}{k}\cos kx$ with $k \neq 1,\ k > 0$	dM1
	$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$	Correct expression with $+c$	A1 [4]
(b)	$\int x\cos^2 x\, dx = \int x\left(\frac{\cos 2x + 1}{2}\right) dx$	Substitutes **correctly** for $\cos^2 x$ in the given integral	M1
	$= \frac{1}{2}\int x\cos 2x\, dx + \frac{1}{2}\int x\, dx$		
	$= \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x\, dx$	$\frac{1}{2}$(their answer to (a)); or <u>underlined expression</u>	A1; $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$	Completely correct expression with/without $+c$	A1 [3]
			7 marks
Notes:			
(b)	$\text{Int} = \int x\cos 2x\, dx = \frac{1}{2}x\sin 2x \pm \int \frac{1}{2}\sin 2x.1\, dx$	This is acceptable for M1	M1
	$\left\{\begin{array}{ll} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x \Rightarrow & v = \lambda\sin 2x \end{array}\right\}$		
	$\text{Int} = \int x\cos 2x\, dx = \lambda x\sin 2x \pm \int \lambda\sin 2x.1\, dx$	This is also acceptable for M1	M1

Aliter (b) **Way 2**	$\int x\cos^2 x\,dx = \int x\left(\frac{\cos 2x+1}{2}\right)dx$	Substitutes **correctly** for $\cos^2 x$ in the given integral …	M1
	$\begin{cases} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \frac{1}{2}\cos 2x + \frac{1}{2} & \Rightarrow v = \frac{1}{4}\sin 2x + \frac{1}{2}x \end{cases}$	… or $u = x$ and $\frac{dv}{dx} = \frac{1}{2}\cos 2x + \frac{1}{2}$	
	$= \frac{1}{4}x\sin 2x + \frac{1}{2}x^2 - \int\left(\frac{1}{4}\sin 2x + \frac{1}{2}x\right)dx$		
	$= \underline{\frac{1}{4}x\sin 2x} + \frac{1}{2}x^2 + \underline{\frac{1}{8}\cos 2x} - \frac{1}{4}x^2 + c$	$\frac{1}{2}$(their answer to (a)); or underlined expression	A1 $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$	Completely correct expression with/without $+c$	A1 **[3]**
Aliter (b) **Way 3**	$\int x\cos 2x\,dx = \int x\left(2\cos^2 x - 1\right)dx$	Substitutes **correctly** for $\cos 2x$ in $\int x\cos 2x\,dx$	M1
	$\Rightarrow 2\int x\cos^2 x\,dx - \int x\,dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$		
	$\Rightarrow \int x\cos^2 x\,dx = \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x\,dx$	$\frac{1}{2}$(their answer to (a)); or underlined expression	A1; $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$	Completely correct expression with/without $+c$	A1 **[3]**
			7 marks

(**Q31 6666/01, June 2007**)

Q25.

Question Number	Scheme		Marks
(i)	$\int \ln\left(\frac{x}{2}\right) dx = \int 1.\ln\left(\frac{x}{2}\right) dx \Rightarrow \begin{cases} u = \ln\left(\frac{x}{2}\right) \Rightarrow \frac{du}{dx} = \frac{\frac{1}{2}}{\frac{x}{2}} = \frac{1}{x} \\ \frac{dv}{dx} = 1 \Rightarrow v = x \end{cases}$		
	$\int \ln\left(\frac{x}{2}\right) dx = x\ln\left(\frac{x}{2}\right) - \int x.\frac{1}{x}\, dx$	Use of 'integration by parts' formula in the correct direction. Correct expression.	M1 A1
	$= x\ln\left(\frac{x}{2}\right) - \int \underline{1}\, dx$	An attempt to multiply x by a candidate's $\frac{a}{x}$ or $\frac{1}{bx}$ or $\frac{1}{x}$.	dM1
	$= x\ln\left(\frac{x}{2}\right) - x + c$	Correct integration with $+c$	A1 aef [4]
(ii)	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\, dx$		
	$\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ gives } \sin^2 x = \frac{1-\cos 2x}{2}\right]$	Consideration of double angle formula for $\sin^2 x$	M1
	$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\, dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\underline{1 - \cos 2x})\, dx$		
	$= \frac{1}{2}\left[\underline{x - \frac{1}{2}\sin 2x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	Integrating to give $\underline{\pm ax \pm b\sin 2x}$; Correct result of anything equivalent to $\frac{1}{2}x - \frac{1}{4}\sin 2x$	dM1 A1
	$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin(\frac{\pi}{2})}{2}\right)\right]$ $= \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)\right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{8} + \frac{1}{4}$	$\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)$ or $\frac{\pi}{8} + \frac{1}{4}$ Candidate must collect their π term and constant term together for A1	A1 aef [5]
			9 marks

Question Number	Scheme		Marks
Aliter (i) **Way 2**	$\int \ln\left(\frac{x}{2}\right) dx = \int (\ln x - \ln 2)\, dx = \int \ln x\, dx - \int \ln 2\, dx$		
	$\int \ln x\, dx = \int 1.\ln x\, dx \Rightarrow \begin{cases} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = 1 \Rightarrow v = x \end{cases}$		
	$\int \ln x\, dx = x\ln x - \int x.\frac{1}{x}\, dx$	Use of 'integration by parts' formula in the correct direction.	M1
	$= x\ln x - x + c$	Correct integration of $\ln x$ with or without $+c$	A1
	$\int \ln 2\, dx = x\ln 2 + c$	Correct integration of $\ln 2$ with or without $+c$	M1
	Hence, $\int \ln\left(\frac{x}{2}\right) dx = x\ln x - x - x\ln 2 + c$	Correct integration with $+c$	A1 aef [4]

Question Number	Scheme	Marks
Aliter (ii) **Way 2**	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x . \sin x \, dx$ and $I = \int \sin^2 x \, dx$	
	$\begin{Bmatrix} u = \sin x & \Rightarrow & \frac{du}{dx} = \cos x \\ \frac{dv}{dx} = \sin x & \Rightarrow & v = -\cos x \end{Bmatrix}$	
	$\therefore I = \underline{\left\{ -\sin x \cos x + \int \cos^2 x \, dx \right\}}$ An attempt to use the correct by parts formula.	M1
	$\therefore I = \left\{ -\sin x \cos x + \int (1 - \sin^2 x) dx \right\}$	
	$\int \sin x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx - \int \sin^2 x \, dx \right\}$	
	$2 \int \sin^2 x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx \right\}$ For the LHS becoming $2I$	dM1
	$2 \int \sin^2 x \, dx = \{ -\sin x \cos x + x \}$	
	$\int \sin^2 x \, dx = \{ \underline{-\frac{1}{2} \sin x \cos x + \frac{x}{2}} \}$ <u>Correct integration</u>	A1
	$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \left[\left(-\frac{1}{2} \sin(\frac{\pi}{2}) \cos(\frac{\pi}{2}) + \frac{(\frac{\pi}{2})}{2} \right) - \left(-\frac{1}{2} \sin(\frac{\pi}{4}) \cos(\frac{\pi}{4}) + \frac{(\frac{\pi}{4})}{2} \right) \right]$ $= \left[(0 + \frac{\pi}{4}) - (-\frac{1}{4} + \frac{\pi}{8}) \right]$ Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \frac{\pi}{8} + \frac{1}{4}$ $\underline{\frac{1}{8}(\pi + 2)}$ or $\underline{\frac{\pi}{8} + \frac{1}{4}}$ Candidate must collect their pi term and constant term together for A1	A1 aef **[5]**

(**Q30 6666/01, Jan 2008**)

Q26.

Question Number	Scheme		Marks	
	(a) $\mathrm{f}(\theta)=4\cos^2\theta-3\sin^2\theta$			
	$=4\left(\frac{1}{2}+\frac{1}{2}\cos 2\theta\right)-3\left(\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)$		M1 M1	
	$=\frac{1}{2}+\frac{7}{2}\cos 2\theta$ *	cso	A1	(3)
	(b) $\int\theta\cos 2\theta\,\mathrm{d}\theta=\frac{1}{2}\theta\sin 2\theta-\frac{1}{2}\int\sin 2\theta\,\mathrm{d}\theta$		M1 A1	
	$=\frac{1}{2}\theta\sin 2\theta+\frac{1}{4}\cos 2\theta$		A1	
	$\int\theta\,\mathrm{f}(\theta)\,\mathrm{d}\theta=\frac{1}{4}\theta^2+\frac{7}{4}\theta\sin 2\theta+\frac{7}{8}\cos 2\theta$		M1 A1	
	$\left[\ \ldots\ \right]_0^{\frac{\pi}{2}}=\left[\frac{\pi^2}{16}+0-\frac{7}{8}\right]-\left[0+0+\frac{7}{8}\right]$		M1	
	$=\frac{\pi^2}{16}-\frac{7}{4}$		A1	(7)
				[10]

(**Q33 6666/01, June 2010**)

Q27.

Question	Scheme	Marks	AOs
(a)	$\cos 3A=\cos(2A+A)=\cos 2A\cos A-\sin 2A\sin A$	M1	3.1a
	$=\left(2\cos^2 A-1\right)\cos A-\left(2\sin A\cos A\right)\sin A$	dM1	1.1b
	$=\left(2\cos^2 A-1\right)\cos A-2\cos A\left(1-\cos^2 A\right)$	ddM1	2.1
	$=4\cos^3 A-3\cos A$*	A1*	1.1b
		(4)	
(b)	$1-\cos 3x=\sin^2 x\Rightarrow\cos^2 x+3\cos x-4\cos^3 x=0$	M1	1.1b
	$\Rightarrow\cos x\left(4\cos^2 x-\cos x-3\right)=0$ $\Rightarrow\cos x\left(4\cos x+3\right)\left(\cos x-1\right)=0$ $\Rightarrow\cos x=\ldots$	dM1	3.1a
	Two of $-90°, 0, 90°$, awrt $139°$	A1	1.1b
	All four of $-90°, 0, 90°$, awrt $139°$	A1	2.1
		(4)	
			(8 marks)

Notes:

(a)

Allow a proof in terms of x rather than A

M1: Attempts to use the compound angle formula for $\cos(2A + A)$ or $\cos(A + 2A)$
Condone a slip in sign

dM1: Uses correct double angle identities for $\cos 2A$ and $\sin 2A$
$\cos 2A = 2\cos^2 A - 1$ must be used. If either of the other two versions are used expect to see an attempt to replace $\sin^2 A$ by $1 - \cos^2 A$ at a later stage.
Depends on previous mark.

ddM1: Attempts to get all terms in terms of $\cos A$ using correct and appropriate identities.
Depends on both previous marks.

A1*: A completely correct and rigorous proof including correct notation, no mixed variables, missing brackets etc.

Alternative right to left is possible:

$4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right) = \cos A\left(2\cos^2 A - 1 + 2\left(1 - \sin^2 A\right) - 2\right) = \cos A\left(\cos 2A - 2\sin^2 A\right)$

$= \cos A\cos 2A - 2\sin A\cos A\sin A = \cos A\cos 2A - \sin 2A\sin A = \cos(2A + A) = \cos 3A$

Score M1: For $4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right)$

dM1: For $\cos A\left(2\cos^2 A - 1 + 2\left(1 - \sin^2 A\right) - 2\right)$ (Replaces $4\cos^2 A - 1$ by $2\cos^2 A - 1$ and $2\left(1 - \sin^2 A\right)$)

ddM1: Reaches $\cos A\cos 2A - \sin 2A\sin A$

A1: $\cos(2A + A) = \cos 3A$

(b)

M1: For an attempt to produce an equation just in $\cos x$ using both part (a) and the identity $\sin^2 x = 1 - \cos^2 x$
Allow one slip in sign or coefficient when copying the result from part (a)

dM1: Dependent upon the preceding mark. It is for taking the cubic equation in $\cos x$ and making a valid attempt to solve. This could include factorisation or division of a $\cos x$ term followed by an attempt to solve the 3 term quadratic equation in $\cos x$ to reach at least one non zero value for $\cos x$.
May also be scored for solving the cubic equation in $\cos x$ to reach at least one non zero value for $\cos x$.

A1: Two of $-90°, 0, 90°$, awrt $139°$ **Depends on the <u>first</u> method mark.**

A1: All four of $-90°, 0, 90°$, awrt $139°$ with no extra solutions offered within the range.

Note that this is an alternative approach for obtaining the cubic equation in (b):

$$1 - \cos 3x = \sin^2 x \Rightarrow 1 - \cos 3x = \frac{1}{2}(1 - \cos 2x)$$

$$\Rightarrow 2 - 2\cos 3x = 1 - \cos 2x$$

$$\Rightarrow 1 = 2\cos 3x - \cos 2x$$

$$\Rightarrow 1 = 2\left(4\cos^3 x - 3\cos x\right) - \left(2\cos^2 x - 1\right)$$

$$\Rightarrow 0 = 4\cos^3 x - 3\cos x - \cos^2 x$$

The M1 will be scored on the penultimate line when they use part (a) and use the correct identity for $\cos 2x$

(**Q10 9MA0/02, Oct 2020**)

Q28.

Question Number	Scheme		Marks
(i)	$\int xe^{4x}\,dx = \frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{dx\}$	$\pm\alpha xe^{4x} - \int \beta e^{4x}\{dx\},\ \alpha \neq 0,\ \beta > 0$	M1
		$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{dx\}$	A1
	$= \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}\,\{+c\}$	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$	A1
			[3]
(ii)	$\int \frac{8}{(2x-1)^3}\,dx = \frac{8(2x-1)^{-2}}{(2)(-2)}\,\{+c\}$	$\pm\lambda(2x-1)^{-2}$	M1
		$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or equivalent.	A1
	$\{= -2(2x-1)^{-2}\,\{+c\}\}$	*{Ignore subsequent working}.*	[2]
(iii)	$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y \qquad y = \frac{\pi}{6}$ at $x = 0$		
	Main Scheme		
	$\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}\,dy = \int e^x\,dx$ or $\int \sin 2y \sin y\,dy = \int e^x\,dx$		B1 oe
	$\int 2\sin y\cos y\sin y\,dy = \int e^x\,dx$	Applying $\frac{1}{\operatorname{cosec}2y}$ or $\sin 2y \to 2\sin y\cos y$	M1
		Integrates to give $\pm\mu\sin^3 y$	M1
	$\frac{2}{3}\sin^3 y = e^x\,\{+c\}$	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$	A1
		$e^x \to e^x$	B1
	$\frac{2}{3}\sin^3\left(\frac{\pi}{6}\right) = e^0 + c$ or $\frac{2}{3}\left(\frac{1}{8}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$	$\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$	A1
			[7]
	Alternative Method 1		
	$\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}\,dy = \int e^x\,dx$ or $\int \sin 2y \sin y\,dy = \int e^x\,dx$		B1 oe
	$\int -\frac{1}{2}(\cos 3y - \cos y)\,dy = \int e^x\,dx$	$\sin 2y\sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$	M1
		Integrates to give $\pm\alpha\sin 3y \pm \beta\sin y$	M1
	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right) = e^x\,\{+c\}$	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$	A1
		$e^x \to e^x$ *as part of solving their DE.*	B1
	$-\frac{1}{2}\left(\frac{1}{3}\sin\left(\frac{3\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right) = e^0 + c$ or $-\frac{1}{2}\left(\frac{1}{3} - \frac{1}{2}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	A1
			[7]
			12

	Question Notes	
(i)	M1	Integration by parts is applied in the form $\pm \alpha x e^{4x} - \int \beta e^{4x} \{dx\}$, where $\alpha \neq 0$, $\beta > 0$. (must be in this form).
	A1	$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x} \{dx\}$ or equivalent.
	A1	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$ with/without $+ c$. Can be un-simplified.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	**SPECIAL CASE:** A candidate who uses $u = x$, $\frac{dv}{dx} = e^{4x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1.
(ii)	M1	$\pm \lambda(2x-1)^{-2}$, $\lambda \neq 0$. **Note that** λ can be 1.
	A1	$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or $-2(2x-1)^{-2}$ or $\frac{-2}{(2x-1)^2}$ with/without $+ c$. Can be un-simplified.
	Note	You can ignore subsequent working which follows from a correct answer.
(iii)	B1	Separates variables as shown. dy and dx should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Allow B1 for** $\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y} = \int e^x$ **or** $\int \sin 2y \sin y = \int e^x$
	M1	$\frac{1}{\operatorname{cosec} 2y} \to 2\sin y \cos y$ **or** $\sin 2y \to 2\sin y \cos y$ **or** $\sin 2y \sin y \to \pm \lambda \cos 3y \pm \lambda \cos y$ seen anywhere in the candidate's working to (iii).
	M1	Integrates to give $\pm \mu \sin^3 y$, $\mu \neq 0$ **or** $\pm \alpha \sin 3y \pm \beta \sin y$, $\alpha \neq 0$, $\beta \neq 0$
	[illegible]	$2\sin^2 y \cos y \to \frac{2}{3}\sin^3 y$ (with no extra terms) or integrates to give $\frac{1}{2}\left(\frac{1}{3}\sin 3y\right.$ [illegible] $\left.\sin y\right)$
	B1	Evidence that e^x has been integrated to give e^x ***as part of solving their DE.***
	M1	Some evidence of using both $y = \frac{\pi}{6}$ and $x = 0$ in an integrated or changed equation containing c.
	Note	that is mark can be implied by the correct value of c.
	A1	$\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$ or $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$ **or any equivalent correct answer.**
	Note	You can ignore subsequent working which follows from a correct answer.

Alternative Method 2 (Using integration by parts twice)

$\int \sin 2y \sin y \, dy = \int e^x \, dx$		B1 oe
	Applies integration by parts twice to give $\pm \alpha \cos y \sin 2y \pm \beta \sin y \cos 2y$	M2
$\frac{1}{3}\cos y \sin 2y - \frac{2}{3}\sin y \cos 2y = e^x \{+c\}$	$\frac{1}{3}\cos y \sin 2y - \frac{2}{3}\sin y \cos 2y$ (simplified or un-simplified)	A1
	$e^x \to e^x$ ***as part of solving their DE.***	B1
	as in the main scheme	M1
$\frac{1}{3}\cos y \sin 2y - \frac{2}{3}\sin y \cos 2y = e^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	A1
		[7]

(**Q35 6666/01, June 2014**)

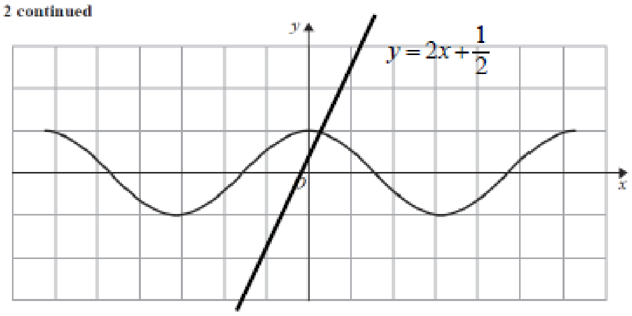
Q29.

Question	Scheme	Marks	AOs
(a)	$R=\sqrt{109}$	B1	1.1b
	$\tan\alpha=\frac{3}{10}$	M1	1.1b
	$\alpha=16.70°$ so $\sqrt{109}\cos(\theta+16.70°)$	A1	1.1b
		(3)	
(b)	(i) e.g $H=11-10\cos(80t)°+3\sin(80t)°$ or $H=11-\sqrt{109}\cos(80t+16.70)°$	B1	3.3
	(ii) $11+\sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t+"16.70"=540$	M1	3.4
	$t=\frac{540-"16.70"}{80}=(6.54)$	M1	1.1b
	t = 6 mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the '80' in the formula For example use $H=11-10\cos(90t)°+3\sin(90t)°$		3.3
		(1)	
			(9 marks)

Notes:
(a) **B1:** $R=\sqrt{109}$ Do not allow decimal equivalents **M1:** Allow for $\tan\alpha=\pm\frac{3}{10}$ **A1:** $\alpha=16.70°$
(b)(i) **B1:** see scheme **(b)(ii)** **B1ft:** their $11+$ their $\sqrt{109}$ Allow decimals here.
(c) **M1:** Sets $80t+"16.70"=540$. Follow through on their 16.70 **M1:** Solves their $80t+"16.70"=540$ correctly to find t **A1:** t = 6 mins 32 seconds
(d) B1: States that to increase the speed of the wheel the 80's in the equation would need to be increased.

(**Q13 9MA0/02, Specimen papers**)

Q30.

Question	Scheme	Marks	AOs
(a)	2 continued  Diagram 1	B1	3.1a
	For an allowable linear graph and explaining that there is only one intersection	B1	2.4
		(2)	
(b)	$\cos x - 2x - \frac{1}{2} = 0 \Rightarrow 1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$	M1	1.1b
	Solves their $x^2 + 4x - 1 = 0$	dM1	1.1b
	Allow awrt 0.236 but accept $-2+\sqrt{5}$	A1	1.1b
		(3)	
			(5 marks)

(a)

B1: Draws $y = 2x + \frac{1}{2}$ on Figure 1 or Diagram 1 with an attempt at the correct gradient and the correct intercept. Look for a straight line with an intercept at $\approx \frac{1}{2}$ and a further point at $\approx \left(\frac{1}{2}, 1\frac{1}{2}\right)$ Allow a tolerance of 0.25 of a square in either direction on these two points. It must appear in quadrants 1, 2 and 3.

B1: There must be an allowable linear graph on Figure 1 or Diagram1 for this to be awarded
Explains that as there is only one intersection so there is just one root.
This requires a reason and a minimal conclusion.
The question asks candidates to explain but as a bare minimum allow one "intersection"

Note: An allowable linear graph is one with intercept of $\pm\frac{1}{2}$ with one intersection with $\cos x$ **OR** gradient of ± 2 with one intersection with $\cos x$

(b)

M1: Attempts to use the small angle approximation $\cos x = 1 - \frac{x^2}{2}$ in the given equation.
The equation must be in a single variable but may be recovered later in the question.

dM1: Proceeds to a 3TQ in a single variable and attempts to solve. See General Principles
The previous M must have been scored. Allow completion of square or formula or calculator. Do not allow attempts via factorisation unless their equation does factorise. You may have to use your calculator to check if a calculator is used.

A1: Allow $-2+\sqrt{5}$ or awrt 0.236.

Do not allow this where there is another root given and it is not obvious that 0.236 has been chosen.

(**Q02 9MA0/01, June 2019**)

Q31.

Question Number	Scheme	Marks
	$x = 4\sin\left(t + \frac{\pi}{6}\right), \quad y = 3\cos 2t, \quad 0 \leqslant t < 2\pi$	
(a)	$\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right), \quad \frac{dy}{dt} = -6\sin 2t$	B1 B1
	So, $\frac{dy}{dx} = \frac{-6\sin 2t}{4\cos\left(t + \frac{\pi}{6}\right)}$	B1$\sqrt{}$ oe
		[3]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\} -6\sin 2t = 0$	M1 oe
	@ $t = 0$, $x = 4\sin\left(\frac{\pi}{6}\right) = 2$, $y = 3\cos 0 = 3 \quad \to (2, 3)$	M1
	@ $t = \frac{\pi}{2}$, $x = 4\sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2}$, $y = 3\cos\pi = -3 \to (2\sqrt{3}, -3)$	
	@ $t = \pi$, $x = 4\sin\left(\frac{7\pi}{6}\right) = -2$, $y = 3\cos 2\pi = 3 \to (-2, 3)$	
	@ $t = \frac{3\pi}{2}$, $x = 4\sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2}$, $y = 3\cos 3\pi = -3 \to (-2\sqrt{3}, -3)$	A1A1A1
		[5] 8

(a)

B1: Either one of $\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right)$ or $\frac{dy}{dt} = -6\sin 2t$. They do not have to be simplified.

B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. They do not have to be simplified.

Any or both of the first two marks can be implied.
Don't worry too much about their notation for the first two B1 marks.

B1: Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. **Note:** This is a follow through mark.

Alternative differentiation in part (a)

$$x = 2\sqrt{3}\sin t + 2\cos t \Rightarrow \frac{dx}{dt} = 2\sqrt{3}\cos t - 2\sin t$$

$$y = 3(2\cos^2 t - 1) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$$

or $y = 3\cos^2 t - 3\sin^2 t \Rightarrow \frac{dy}{dt} = -6\cos t\sin t - 6\sin t\cos t$

or $y = 3(1 - 2\sin^2 t) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$

(b)

M1: Candidate sets their numerator from part (a) or their $\frac{dy}{dt}$ equal to 0.

Note that their numerator must be a trig function. Ignore $\frac{dx}{dt}$ equal to 0 at this stage.

M1: Candidate substitutes a found value of t, to attempt to find either one of x or y.
The first two method marks can be implied by ONE correct set of coordinates for (x, y) or (y, x) interchanged.
A correct point coming from NO WORKING can be awarded M1M1.
A1: At least TWO sets of coordinates.
A1: At least THREE sets of coordinates.
A1: ONLY FOUR correct sets of coordinates. If there are more than 4 sets of coordinates then award A0.
Note: Candidate can use the diagram's symmetry to write down some of their coordinates.

Note: When $x = 4\sin\left(\frac{\pi}{6}\right) = 2$, $y = 3\cos 0 = 3$ is acceptable for a pair of coordinates.

Also it is fine for candidates to display their coordinates on a table of values.
Note: The coordinates must be exact for the accuracy marks. Ie (3.46..., -3) or (-3.46..., -3) is A0.

Note: $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0$ ONLY is fine for the first M1, and potentially the following M1A1A0A0.

Note: $\frac{dy}{dx} = 0 \Rightarrow \cos t = 0$ ONLY is fine for the first M1 and potentially the following M1A1A0A0.

Note: $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0$ & $\cos t = 0$ has the potential to achieve all five marks.

Note: It is possible for a candidate to gain full marks in part (b) if they make sign errors in part (a).

(b) An alternative method for finding the coordinates of the two maximum points.

Some candidates may use $y = 3\cos 2t$ to write down that the y-coordinate of a maximum point is 3.
They will then deduce that $t = 0$ or π and proceed to find the x-coordinate of their maximum point. These candidates will receive no credit until they attempt to find one of the x-coordinates for the maximum point.

M1M1: Candidate states $y = 3$ and attempts to substitute $t = 0$ or π into $x = 4\sin\left(t + \frac{\pi}{6}\right)$.

M1M1 can be implied by candidate stating either $(2, 3)$ or $(2, -3)$.

Note: these marks can only be awarded together for a candidate using this method.

A1: For both $(2, 3)$ or $(-2, 3)$.

A0A0: Candidate cannot achieve the final two marks by using this method. They can, however, achieve these marks by subsequently solving their numerator equal to 0.

(**Q29 6666/01, Jan 2012**)

Q32.

Question Number	Scheme	Marks
(*a*)	At $P(4, 2\sqrt{3})$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leq t \leq \frac{\pi}{2}$	A1
(*b*)	$x = 8\cos t$, $\quad y = 4\sin 2t$	
	$\frac{dx}{dt} = -8\sin t$, $\frac{dy}{dt} = 8\cos 2t$	M1 A1
	At P, $\frac{dy}{dx} = \underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	M1
	$\left\{ = \frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58 \right\}$	
	Hence m(N) = $-\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	M1
	N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$	M1
	N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ (*)	A1 cso (6)
(*c*)	$A = \int_0^4 y\,dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.(-8\sin t)\,dt$	M1 A1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\,dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t\cos t).\sin t\,dt$	M1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\,dt$	
	$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\,dt$ (*)	A1 (4)
(*d*)	$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	M1 A1
	$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	M1
	$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$	A1 (4)
		(16 marks)

(**Q33 6666/01, June 2008**)

Q33.

Question	Scheme		Marks	AOs
	$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\ 0 \le t < 2\pi;\ C_2: x^2 + y^2 = 66$			
Way 1	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$		M1	3.1a
	$100(1-\sin^2 t) + 32\sin^2 t = 66$	$100\cos^2 t + 32(1-\cos^2 t) = 66$	M1	2.1
			A1	1.1b
	$100 - 68\sin^2 t = 66 \Rightarrow \sin^2 t = \frac{1}{2}$ $\Rightarrow \sin t = ...$	$68\cos^2 t + 32 = 66 \Rightarrow \cos^2 t = \frac{1}{2}$ $\Rightarrow \cos t = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	

Way 2	$\{\cos^2 t + \sin^2 t = 1 \Rightarrow\}\ \left(\frac{x}{10}\right)^2 + \left(\frac{y}{4\sqrt{2}}\right)^2 = 1\ \{\Rightarrow 32x^2 + 100y^2 = 3200\}$		M1	3.1a
	$\frac{x^2}{100} + \frac{66-x^2}{32} = 1$	$\frac{66-y^2}{100} + \frac{y^2}{32} = 1$	M1	2.1
			A1	1.1b
	$32x^2 + 6600 - 100x^2 = 3200$ $x^2 = 50 \Rightarrow x = ...$	$2112 - 32y^2 + 100y^2 = 3200$ $y^2 = 16 \Rightarrow y = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding x-coordinate or y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
Way 3	$\{C_2: x^2 + y^2 = 66 \Rightarrow\}\ x = \sqrt{66}\cos\alpha,\ y = \sqrt{66}\sin\alpha$ $\{C_1 = C_2 \Rightarrow\}\ 10\cos t = \sqrt{66}\cos\alpha,\ 4\sqrt{2}\sin t = \sqrt{66}\sin\alpha$ $\{\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow\}\ \left(\frac{10\cos t}{\sqrt{66}}\right)^2 + \left(\frac{4\sqrt{2}\sin t}{\sqrt{66}}\right)^2 = 1$		M1	3.1a
	then continue with applying the mark scheme for Way 1			
Way 4	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$		M1	3.1a
	$100\left(\frac{1+\cos 2t}{2}\right) + 32\left(\frac{1-\cos 2t}{2}\right) = 66$		M1	2.1
			A1	1.1b
	$50 + 50\cos 2t + 16 - 16\cos 2t = 66 \Rightarrow 34\cos 2t + 66 = 66$ $\Rightarrow \cos 2t = ...$		dM1	1.1b
	Substitutes their solution back into the original equation(s) to get the value of the x-coordinate and value of the y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
	Note: Give final A0 for writing $x = 5\sqrt{2},\ y = -4$ followed by $S = (-4, 5\sqrt{2})$			
				(6 marks)

Notes for Question	
	Way 1
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 1: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only or $\cos^2 t$ only
A1:	A correct equation in $\sin^2 t$ only or $\cos^2 t$ only
dM1: **Note:**	**dependent on both the previous M marks** Rearranges to make $\sin t = ...$ where $-1 \le \sin t \le 1$ or $\cos t = ...$ where $-1 \le \cos t \le 1$ Condone 3rd M1 for $\sin^2 t = \frac{1}{2} \Rightarrow \sin t = \frac{1}{4}$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$

	Way 2
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 2: A complete process of using $\cos^2 t + \sin^2 t \equiv 1$ to convert the parametric equation for C_1 into a Cartesian equation for C_1
M1:	Complete valid attempt to write an equation in terms of x only or y only not involving trigonometry
A1:	A correct equation in x only or y only not involving trigonometry
dM1: **Note:**	**dependent on both the previous M marks** Rearranges to make $x = ...$ or $y = ...$ their x^2 or their y^2 must be >0 for this mark
M1: **Note:**	See scheme their x^2 and their y^2 must be >0 for this mark
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$
	Way 3
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 3: A complete process of writing C_2 in parametric form, combining the parametric equations of C_1 and C_2 and applying $\cos^2\alpha + \sin^2\alpha \equiv 1$ to give an equation in one variable (i.e. t) only.
	then continue with applying the mark scheme for Way 1
	Way 4
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 4: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1: **Note:**	Uses the identities $\cos 2t \equiv 2\cos^2 t - 1$ and $\cos 2t \equiv 1 - 2\sin^2 t$ to achieve an equation in $\cos 2t$ only At least one of $\cos 2t \equiv 2\cos^2 t - 1$ or $\cos 2t \equiv 1 - 2\sin^2 t$ must be correct for this mark.
A1:	A correct equation in $\cos 2t$ only
dM1:	**dependent on both the previous M marks** Rearranges to make $\cos 2t = ...$ where $-1 \le \cos 2t \le 1$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

	$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\ \ 0 \le t < 2\pi;\ \ C_2: x^2 + y^2 = 66$		
Way 5	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66(\sin^2 t + \cos^2 t)$	M1	2.1
		A1	1.1b
	$100\cos^2 t + 32\sin^2 t = 66\sin^2 t + 66\cos^2 t \Rightarrow 34\cos^2 t = 34\sin^2 t$ $\Rightarrow \tan t = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
		(6)	

	Way 5
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 5: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only and $\cos^2 t$ only **with no constant term**
A1:	A correct equation in $\sin^2 t$ and $\cos^2 t$ containing no constant term
dM1:	**dependent on both the previous M marks** Rearranges to make $\tan t = ...$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

(**Q04 9MA0/02, June 2019**)

Q34.

Question	Scheme	Marks
(a)	$2\cot 2x + \tan x \equiv \dfrac{2}{\tan 2x} + \tan x$	B1
	$\equiv \dfrac{(1-\tan^2 x)}{\tan x} + \dfrac{\tan^2 x}{\tan x}$	M1
	$\equiv \dfrac{1}{\tan x}$	M1
	$\equiv \cot x$	A1*
		(4)
(b)	$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2 \Rightarrow 3\cot x = \operatorname{cosec}^2 x - 2$	
	$\Rightarrow 3\cot x = 1 + \cot^2 x - 2$	M1
	$\Rightarrow 0 = \cot^2 x - 3\cot x - 1$	A1
	$\Rightarrow \cot x = \dfrac{3 \pm \sqrt{13}}{2}$	M1
	$\Rightarrow \tan x = \dfrac{2}{3 \pm \sqrt{13}} \Rightarrow x = ..$	M1
	$\Rightarrow x = 0.294, -2.848, -1.277, 1.865$	A2,1,0
		(6) **(10 marks)**
(a)alt 1	$2\cot 2x + \tan x \equiv \dfrac{2\cos 2x}{\sin 2x} + \tan x$	B1
	$\equiv 2\dfrac{\cos^2 x - \sin^2 x}{2\sin x \cos x} + \dfrac{\sin x}{\cos x}$	M1
	$\equiv \dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x} + \dfrac{\sin^2 x}{\sin x \cos x} \equiv \dfrac{\cos^2 x}{\sin x \cos x}$	M1
	$\equiv \dfrac{\cos x}{\sin x}$	
	$\equiv \cot x$	A1*
(a)alt 2	$2\cot 2x + \tan x \equiv 2\dfrac{(1-\tan^2 x)}{2\tan x} + \tan x$	B1M1
	$\equiv \dfrac{2}{2\tan x} - \dfrac{2\tan^2 x}{2\tan x} + \tan x$ or $\dfrac{(1-\tan^2 x) + \tan^2 x}{\tan x}$	
	$\equiv \dfrac{2}{2\tan x} = \cot x$	M1A1*

Alt (b)	$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2 \Rightarrow \dfrac{3\cos x}{\sin x} = \dfrac{1}{\sin^2 x} - 2$	
	$(\times \sin^2 x) \Rightarrow 3\sin x \cos x = 1 - 2\sin^2 x$	M1
	$\Rightarrow \dfrac{3}{2}\sin 2x = \cos 2x$	M1A1
	$\Rightarrow \tan 2x = \dfrac{2}{3} \Rightarrow x = ..$	M1
	$\Rightarrow x = 0.294, -2.848, -1.277, 1.865$	A2,1,0 **(6)**

(a)

B1 States or uses the identity $2\cot 2x = \dfrac{2}{\tan 2x}$ or alternatively $2\cot 2x = \dfrac{2\cos 2x}{\sin 2x}$

This may be implied by $2\cot 2x = \dfrac{1-\tan^2 x}{\tan x}$. Note $2\cot 2x = \dfrac{1}{2\tan 2x}$ is B0

M1 Uses the correct double angle identity $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

Alternatively uses $\sin 2x = 2\sin x\cos x$, $\cos 2x = \cos^2 x - \sin^2 x$ oe and $\tan x = \dfrac{\sin x}{\cos x}$

M1 Writes their two terms with a single common denominator and simplifies to a form $\dfrac{ab}{cd}$.

For this to be scored the expression must be in either $\sin x$ and $\cos x$ or just $\tan x$.

In alternative 2 it is for splitting the complex fraction into parts and simplifying to a form $\dfrac{ab}{cd}$.

You are awarding this for a correct method to proceed to terms like $\dfrac{\cos^2 x}{\sin x\cos x}$, $\dfrac{2\cos^3 x}{2\sin x\cos^2 x}$, $\dfrac{2}{2\tan x}$

A1* cso. For proceeding to the correct answer. This is a given answer and all aspects must be correct including the consistent use of variables. If the candidate approaches from both sides there must be a conclusion for this mark to be awarded. Occasionally you may see a candidate attempting to prove $\cot x - \tan x \equiv 2\cot 2x$. This is fine but again there needs to be a conclusion for the A1*
If you are unsure of how some items should be marked then please use review

(b)

M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)

WITH an attempt at using $\text{cosec}^2 x = \pm 1 \pm \cot^2 x$ to produce a quadratic equation in just $\cot x$ / $\tan x$

A1 $\cot^2 x - 3\cot x - 1 = 0$ The = 0 may be implied by subsequent working

Alternatively accept $\tan^2 x + 3\tan x - 1 = 0$

M1 Solves a 3TQ=0 in $\cot x$ (or tan) using the formula or any suitable method for their quadratic to find at least one solution. Accept answers written down from a calculator. You may have to check these from an incorrect quadratic. FYI answers are $\cot x =$ awrt 3.30, -0.30

Be aware that $\cot x = \dfrac{3 \pm \sqrt{13}}{2} \Rightarrow \tan x = \dfrac{-3 \pm \sqrt{13}}{2}$

M1 For $\tan x = \dfrac{1}{\cot x}$ and using arctan producing at least one answer for x in degrees or radians.

You may have to check these with your calculator.

A1 Two of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3dp) in radians or degrees.

In degrees the answers you would accept are (awrt 2dp) $x = 16.8°, 106.8°, -73.2°, -163.2°$

A1 All four of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3 dp) with no extra solutions in the range $-\pi \leqslant x \leqslant \pi$

See main scheme for Alt to (b) using Double Angle formulae still entered M A M M A A in epen

1st M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)

then using $\cot x = \dfrac{\cos x}{\sin x}$, $\text{cosec}^2 x = \dfrac{1}{\sin^2 x}$ and $\times \sin^2 x$ to form an equation sin and cos

1st A1 For $\dfrac{3}{2}\sin 2x = \cos 2x$ or equivalent. **Attached to the next M**

2nd M1 For using both correct double angle formula
3rd M1 For moving from $\tan 2x = C$ to x=.using the correct order of operations.

(**Q27 6665/01, June 2016**)

Q35.

Question Number	Scheme	Marks
(a)	$\sec 2A + \tan 2A = \dfrac{1}{\cos 2A} + \dfrac{\sin 2A}{\cos 2A}$	B1
	$= \dfrac{1+\sin 2A}{\cos 2A}$	M1
	$= \dfrac{1+2\sin A\cos A}{\cos^2 A - \sin^2 A}$	M1
	$= \dfrac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$	
	$= \dfrac{(\cos A+\sin A)(\cos A+\sin A)}{(\cos A+\sin A)(\cos A-\sin A)}$	M1
	$= \dfrac{\cos A+\sin A}{\cos A-\sin A}$	A1*
		(5)
(b)	$\sec 2\theta + \tan 2\theta = \dfrac{1}{2} \Rightarrow \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \dfrac{1}{2}$	
	$\Rightarrow 2\cos\theta + 2\sin\theta = \cos\theta - \sin\theta$	
	$\Rightarrow \tan\theta = -\dfrac{1}{3}$	M1 A1
	$\Rightarrow \theta = awrt\ 2.820, 5.961$	dM1A1 (4)
		(9 marks)

(a)

B1 A correct identity for $\sec 2A = \dfrac{1}{\cos 2A}$ **OR** $\tan 2A = \dfrac{\sin 2A}{\cos 2A}$.

It need not be in the proof and it could be implied by the sight of $\sec 2A = \dfrac{1}{\cos^2 A - \sin^2 A}$

M1 For setting their expression as a single fraction. The denominator must be correct for their fractions and at least two terms on the numerator.

This is usually scored for $\dfrac{1+\cos 2A\tan 2A}{\cos 2A}$ or $\dfrac{1+\sin 2A}{\cos 2A}$

M1 For getting an expression in just $\sin A$ and $\cos A$ by using the double angle identities

$\sin 2A = 2\sin A\cos A$ and $\cos 2A = \cos^2 A - \sin^2 A$, $2\cos^2 A - 1$ or $1 - 2\sin^2 A$.

Alternatively for getting an expression in just $\sin A$ and $\cos A$ by using the double angle identities $\sin 2A = 2\sin A\cos A$ and $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ with $\tan A = \dfrac{\sin A}{\cos A}$.

For example $= \dfrac{1}{\cos^2 A - \sin^2 A} + \dfrac{2\sin A/\cos A}{1 - \sin^2 A/\cos^2 A}$ is B1M0M1 so far

M1 In the main scheme it is for replacing 1 by $\cos^2 A + \sin^2 A$ and factorising both numerator and denominator

A1* Cancelling to produce given answer with no errors.
Allow a consistent use of another variable such as θ, but mixing up variables will lose the A1*.

(b)

M1 For using part (a), cross multiplying, dividing by $\cos\theta$ to reach $\tan\theta = k$
Condone $\tan 2\theta = k$ for this mark only

A1 $\tan\theta = -\frac{1}{3}$

dM1 Scored for $\tan\theta = k$ leading to at least one value (with 1 dp accuracy) for θ between 0 and 2π. You may have to use a calculator to check. Allow answers in degrees for this mark.

A1 $\theta =$ awrt $2.820, 5.961$ with no extra solutions within the range. Condone 2.82 for 2.820.

You may condone different/ mixed variables in part (b)

....

There are some long winded methods. Eg. M1, dM1 applied as in main scheme

$$\Rightarrow (2\cos\theta + 2\sin\theta)^2 = (\cos\theta - \sin\theta)^2 \Rightarrow 4 + 4\sin 2\theta = 1 - \sin 2\theta$$

$\Rightarrow \sin 2\theta = -\frac{3}{5}$ is M1 (for $\sin 2\theta = k$) A1

$\Rightarrow \theta = 2.820, 5.961$ for dM1 (for $\theta = \frac{\arcsin k}{2}$) A1

....

$\cos\theta + 3\sin\theta = 0 \Rightarrow (\sqrt{10})\cos(\theta - 1.25) = 0$ M1 for..$\cos(\theta - \alpha) = 0, \alpha = \arctan\left(\pm\frac{3}{1} \text{ or } \pm\frac{1}{3}\right)$ A1

$\Rightarrow \theta = 2.820, 5.961$ dM1 A1

$\cos\theta + 3\sin\theta = 0 \Rightarrow (\sqrt{10})\sin(\theta + 0.32) = 0$ M1 A1

$\Rightarrow \theta = 2.820, 5.961$ dM1 A1

....

$\cos\theta = -3\sin\theta \Rightarrow \cos^2\theta = 9\sin^2\theta \Rightarrow \sin^2\theta = \frac{1}{10} \Rightarrow \sin\theta = (\pm)\sqrt{\frac{1}{10}}$ M1 A1

$\Rightarrow \theta = 2.820, 5.961$ dM1 A1

$\cos\theta = -3\sin\theta \Rightarrow \cos^2\theta = 9\sin^2\theta \Rightarrow \cos^2\theta = \frac{9}{10} \Rightarrow \cos\theta = (\pm)\sqrt{\frac{9}{10}}$ M1 A1

$\Rightarrow \theta = 2.820, 5.961$ dM1 A1

....

Question Number	Scheme	Marks
Alt I **From RHS**	$\frac{\cos A+\sin A}{\cos A-\sin A}=\frac{\cos A+\sin A}{\cos A-\sin A}\times\frac{\cos A+\sin A}{\cos A+\sin A}$ $=\frac{\cos^2 A+\sin^2 A+2\sin A\cos A}{\cos^2 A-\sin^2 A}$ $=\frac{1+\sin 2A}{\cos 2A}$ $=\frac{1}{\cos 2A}+\frac{\sin 2A}{\cos 2A}$ $=\sec 2A+\tan 2A$	(Pythagoras) **M1** (Double Angle) **M1** (Single Fraction) **M1** **B1** (Identity), **A1***
Alt II **Both sides**	Assume true $\sec 2A+\tan 2A=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\frac{1}{\cos 2A}+\frac{\sin 2A}{\cos 2A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\frac{1+\sin 2A}{\cos 2A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\frac{1+2\sin A\cos A}{\cos^2 A-\sin^2 A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\times(\cos A-\sin A)\Rightarrow\frac{1+2\sin A\cos A}{\cos A+\sin A}=\cos A+\sin A$ $1+2\sin A\cos A=\cos^2 A+2\sin A\cos A+\sin^2 A=1+2\sin A\cos A$ True	B1 (identity) M1 (single fraction) M1 (double angles) M1 (Pythagoras) A1*
Alt 111 **Very difficult**	$\sec 2A+\tan 2A=\frac{1}{\cos 2A}+\tan 2A$ $=\frac{1}{\cos 2A}+\frac{2\tan A}{1-\tan^2 A}$ $=\frac{1-\tan^2 A+2\tan A\cos 2A}{\cos 2A(1-\tan^2 A)}$ $=\frac{1-\tan^2 A+2\tan A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(1-\tan^2 A)}$ $=\frac{1-\frac{\sin^2 A}{\cos^2 A}+2\frac{\sin A}{\cos A}(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)\left(1-\frac{\sin^2 A}{\cos^2 A}\right)}$ $\times\cos^2 A=\frac{\cos^2 A-\sin^2 A+2\sin A\cos A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(\cos^2 A-\sin^2 A)}$ $=\frac{(\cos^2 A-\sin^2 A)(1+2\sin A\cos A)}{(\cos^2 A-\sin^2 A)(\cos^2 A-\sin^2 A)}$ Final two marks as in main scheme	(Identity) **B1** (Single fraction) **M1** (Double Angle and in just sin and cos) **M1** **M1A1***

(**Q25 6665/01, June 2015**)

Q36.

Question	Scheme	Marks	AOs
	$1-\cos 2\theta \equiv \tan\theta\sin 2\theta,\ \theta \neq \frac{(2n+1)\pi}{2},\ n\in\mathbb{Z}$		
(a) Way 1	$\tan\theta\sin 2\theta = \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x-5)(1-\cos 2x) = 3\tan^2 x\sin 2x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$		
(b) Way 1	$(\sec^2 x-5)\tan x\sin 2x = 3\tan^2 x\sin 2x$ **or** $(\sec^2 x-5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$		
	Deduces $x=0$	B1	2.2a
	Uses $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ e.g. $(1+\tan^2 x-3\tan x-5)\tan x = 0$ **or** $(1+\tan^2 x-3\tan x-5)(1-\cos 2x) = 0$ **or** $1+\tan^2 x-5 = 3\tan x$	M1	2.1
	$\tan^2 x-3\tan x-4=0$	A1	1.1b
	$(\tan x-4)(\tan x+1)=0 \Rightarrow \tan x = ...$	M1	1.1b
	$x = -\frac{\pi}{4},\ 1.326$	A1	1.1b
		A1	1.1b
		(6)	
			(9 marks)

Notes for Question

(a)	**Way 1**
M1:	Applies $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to $\tan\theta\sin 2\theta$
M1:	Cancels as scheme (may be implied) and attempts to use $\cos 2\theta = 1-2\sin^2\theta$
A1*:	For a correct proof showing all steps of the argument
(a) Way 2	
M1:	For using $\cos 2\theta = 1-2\sin^2\theta$
Note:	If the form $\cos 2\theta = \cos^2\theta - \sin^2\theta$ or $\cos 2\theta = 2\cos^2\theta - 1$ is used, the mark cannot be awarded until $\cos^2\theta$ has been replaced by $1-\sin^2\theta$
M1:	Attempts to write their $2\sin^2\theta$ in terms of $\tan\theta$ and $\sin 2\theta$ using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and
	$\sin 2\theta = 2\sin\theta\cos\theta$ within the given expression
A1*:	For a correct proof showing all steps of the argument
Note:	If a proof meets in the middle; e.g. they show LHS $= 2\sin^2\theta$ and RHS $= 2\sin^2\theta$; then some indication must be given that the proof is complete. E.g. $1-\cos 2\theta \equiv \tan\theta\sin 2\theta$, QED, box

Notes for Question Continued	
(b)	
B1:	Deduces that the given equation yields a solution $x = 0$
M1:	For using the key step of $\sec^2 x = 1 + \tan^2 x$ and cancels/factorises out $\tan x$ or $(1 - \cos 2x)$ or $\sin 2x$ to produce a quadratic factor or quadratic equation in just $\tan x$
Note:	Allow the use of $\pm\sec^2 x = \pm 1 \pm \tan^2 x$ for M1
A1:	Correct 3TQ in $\tan x$. E.g. $\tan^2 x - 3\tan x - 4 = 0$
Note:	E.g. $\tan^2 x - 4 = 3\tan x$ or $\tan^2 x - 3\tan x = 4$ are acceptable for A1
M1:	For a correct method of solving their 3TQ in $\tan x$
A1:	Any one of $-\frac{\pi}{4}$, awrt -0.785, awrt 1.326, $-45°$, awrt $75.964°$
A1:	Only $x = -\frac{\pi}{4}, 1.326$ **cao** stated in the range $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Note: **Alternative Method (Alt 1)**

$(\sec^2 x - 5)\tan x \sin 2x = 3\tan^2 x \sin 2x$ **or** $(\sec^2 x - 5)(1 - \cos 2x) = 3\tan x(1 - \cos 2x)$			
Deduces $x = 0$		B1	2.2a
$\sec^2 x - 5 = 3\tan x \Rightarrow \frac{1}{\cos^2 x} - 5 = 3\left(\frac{\sin x}{\cos x}\right)$ $1 - 5\cos^2 x = 3\sin x\cos x$ $1 - 5\left(\frac{1 + \cos 2x}{2}\right) = \frac{3}{2}\sin 2x$	Complete process (as shown) of using the identities for $\sin 2x$ and $\cos 2x$ to proceed as far as $\pm A \pm B\cos 2x = \pm C\sin 2x$	M1	2.1
$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ $\{3\sin 2x + 5\cos 2x = -3\}$	$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ o.e.	A1	1.1b
$\sqrt{34}\sin(2x + 1.03) = -3$	Expresses their answer in the form $R\sin(2x + \alpha) = k;\ k \neq 0$ with values for R and α	M1	1.1b
$\sin(2x + 1.03) = -\frac{3}{\sqrt{34}}$			
$x = -\frac{\pi}{4}, 1.326$		A1 A1	1.1b 1.1b

(**Q12 9MA0/02, June 2018**)

Q37.

Question	Scheme	Marks
(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=\frac{1}{2}\Rightarrow\alpha=26.57°$	M1A1
		(3)
(b)	$\frac{2}{2\cos\theta-\sin\theta-1}=15\Rightarrow\frac{2}{\sqrt{5}\cos(\theta+26.6°)-1}=15$	
	$\Rightarrow\cos(\theta+26.6°)=\frac{17}{15\sqrt{5}}=(awrt\ 0.507)$	M1A1
	$\theta+26.57°=59.54°$	
	$\Rightarrow\theta=awrt\ 33.0°$ or $awrt\ 273.9°$	A1
	$\theta+26.6°=360°-$ their '59.5°'	dM1
	$\Rightarrow\theta=awrt\ 273.9°$ and $awrt\ 33.0°$	A1
		(5)
(c)	$\theta-$ their $26.57°=$ their $59.54°\Rightarrow\theta=...$	M1
	$\theta=$ awrt $86.1°$	A1
		(2)
		(10 marks)

(a)

B1 $R=\sqrt{5}$. Condone $R=\pm\sqrt{5}$ Ignore decimals

M1 $\tan\alpha=\pm\frac{1}{2}$, $\tan\alpha=\pm\frac{2}{1}\Rightarrow\alpha=...$

If their value of R is used to find the value of α only accept $\cos\alpha=\pm\frac{2}{R}$ OR $\sin\alpha=\pm\frac{1}{R}\Rightarrow\alpha=...$

A1 $\alpha=$ awrt $26.57°$

(b)

M1 Attempts to use part (a) $\Rightarrow\cos(\theta\pm$ their $26.6°)=K$, $|K|\leqslant 1$

A1 $\cos(\theta\pm$ their $26.6°)=\frac{17}{15\sqrt{5}}=($awrt $0.507)$. Can be implied by $(\theta\pm$ their $26.6°)=$ awrt $59.5°/59.6°$

A1 One solution correct, $\theta=awrt\ 33.0°$ or $\theta=awrt\ 273.9°$ Do not accept 33 for 33.0.

dM1 Obtains a second solution in the range. It is dependent upon having scored the previous M.
Usually for $\theta\pm$ their $26.6°=360°-$ their $59.5°\Rightarrow\theta=...$

A1 Both solutions $\theta=awrt\ 33.0°$ and $awrt\ 273.9°$. Do not accept 33 for 33.0.
Extra solutions inside the range withhold this A1. Ignore solutions outside the range $0\leqslant\theta<360°$

(c)

M1 $\theta-$ their $26.57°=$ their $59.54°\Rightarrow\theta=...$
Alternatively $-\theta+$ their $26.6°=-$their $59.5°\Rightarrow\theta=...$
If the candidate has an incorrect sign for α, for example they used $\cos(\theta-26.57°)$ in part (b) it would be scored for $\theta+$ their $26.57°=$ their $59.54°\Rightarrow\theta=...$

A1 awrt $86.1°$ ONLY. Allow both marks following a correct (a) and (b)
They can restart the question to achieve this result. Do not award if 86.1 was their smallest answer in (b). This occurs when they have $\cos(\theta-26.57°)$ instead of $\cos(\theta+26.57°)$ in (b)

Answers in radians: Withhold only one A mark, the first time a solution in radians appears
FYI (a) $\alpha=0.46$ (b) $\theta_1=$ awrt 0.58 and $\theta_2=$ awrt 4.78 (c) $\theta_3=$ awrt 1.50. Require 2 dp accuracy

(**Q22 6665/01, June 2016**)

Q38.

Question Number	Scheme	Marks
(a)	$R=\sqrt{20}$	B1
	$\tan\alpha=\frac{4}{2}\Rightarrow\alpha=\text{awrt }1.107$	M1A1
		(3)
(b)(i)	$'4+5R^2'=104$	B1ft
(ii)	$3\theta-'1.107'=\frac{\pi}{2}\Rightarrow\theta=\text{awrt }0.89$	M1A1
		(3)
(c)(i)	4	B1
(ii)	$3\theta-'1.107'=2\pi\Rightarrow\theta=\text{awrt }2.46$	M1A1
		(3)
		(9 marks)

(a)

B1 Accept $R=\sqrt{20}$ or $2\sqrt{5}$ or awrt 4.47

Do not accept $R=\pm\sqrt{20}$

This could be scored in parts (b) or (c) as long as you are certain it is R

M1 for sight of $\tan\alpha=\pm\frac{4}{2}$, $\tan\alpha=\pm\frac{2}{4}$. Condone $\sin\alpha=4, \cos\alpha=2\Rightarrow\tan\alpha=\frac{4}{2}$

If R is found first only accept $\sin\alpha=\pm\frac{4}{R}$, $\cos\alpha=\pm\frac{2}{R}$

A1 $\alpha=$ awrt 1.107. The degrees equivalent $63.4°$ is A0.

If a candidate does all the question in degrees they will lose just this mark.

(b)(i)

B1ft Either 104 or if R was incorrect allow for the numerical value of their '$4+5R^2$'.

Allow a tolerance of 1 dp on decimal R's.

(b)(ii)

M1 Using $3\theta\pm\text{their '}1.107\text{'}=\frac{\pi}{2}\Rightarrow\theta=..$

Accept $3\theta\pm\text{their '}1.107\text{'}=(2n+1)\frac{\pi}{2}\Rightarrow\theta=..$ where n is an integer

Allow slips on the lhs with an extra bracket such as

$3(\theta\pm\text{their '}1.107\text{'})=\frac{\pi}{2}\Rightarrow\theta=..$

The degree equivalent is acceptable $3\theta-\text{their '}63.4°\text{'}=90°\Rightarrow\theta=$

Do not allow mixed units in this question

A1 awrt 0.89 radians or $51.1°$. Do not allow multiple solutions for this mark.

(c)(i)

B1 4

(c)(ii)

M1 Using $3\theta\pm\text{their '}1.107\text{'}=2\pi\Rightarrow\theta=...$

Accept $3\theta\pm\text{their '}1.107\text{'}=n\pi\Rightarrow\theta=..$ where n is an integer, including 0

Allow slips on the lhs with an extra bracket such as

$3(\theta\pm\text{their '}1.107\text{'})=2\pi\Rightarrow\theta=..$

The degree equivalent is acceptable $3\theta-\text{their '}63.4°\text{'}=360°\Rightarrow\theta=$ but

Do not allow mixed units in this question

A1 $\theta=$ awrt 2.46 radians or $141.1°$ Do not allow multiple solutions for this mark.

(**Q29 6665/01, June 2014**)

Q39.

Question Number	Scheme	Marks
(a)	$R=\sqrt{29}$	B1
	$\tan\alpha=\frac{2}{5}\Rightarrow\alpha=$ awrt 0.381	M1A1
		(3)
(b)	$5\cot 2x-3\operatorname{cosec}2x=2\Rightarrow 5\frac{\cos 2x}{\sin 2x}-\frac{3}{\sin 2x}=2$	M1
	$\Rightarrow 5\cos 2x-2\sin 2x=3$	A1
		(2)
(c)	$5\cos 2x-2\sin 2x=3\Rightarrow\cos(2x+0.381)=\frac{3}{\sqrt{29}}$	M1
	$2x+0.381=\arccos\left(\frac{3}{\sqrt{29}}\right)\Rightarrow x=...$	dM1
	$x=$ awrt 0.30, 2.46	A1A1
		(4)
		(9 marks)
Alt I (c)	$5\cos 2x-2\sin 2x=3\Rightarrow 10\cos^2 x-5-4\sin x\cos x=3$	
	$\Rightarrow 4\tan^2 x+2\tan x-1=0$	M1
	$\Rightarrow\tan x=\frac{-1\pm\sqrt{5}}{4}\Rightarrow x=..$	dM1
	$x=$ awrt 0.30, 2.46	A1A1
		(4)
Alt II (c)	$5\cos 2x-2\sin 2x=3\Rightarrow(5\cos 2x)^2=(3+2\sin 2x)^2$ & $\cos^2 2x=1-\sin^2 2x$	
	$\Rightarrow 29\sin^2 2x+12\sin 2x-16=0$	M1
	$\Rightarrow\sin 2x=\frac{-12\pm\sqrt{2000}}{58}\Rightarrow 2x=..\Rightarrow x=..$	dM1
	$x=$ awrt 0.30, 2.46	A1A1
		(4)

(a)

B1 $R=\sqrt{29}$

Condone $R=\pm\sqrt{29}$ (Do not allow decimals for this mark Eg 5.39 but remember to isw after $\sqrt{29}$)

M1 $\tan\alpha=\pm\frac{2}{5}$, $\tan\alpha=\pm\frac{5}{2}\Rightarrow\alpha=...$

If R is used to find α accept $\sin\alpha=\pm\frac{2}{R}$ or $\cos\alpha=\pm\frac{5}{R}\Rightarrow\alpha=...$

A1 $\alpha =$ awrt 0.381 Note that the degree equivalent $\alpha =$ awrt $21.8°$ is A0

(b)

M1 Replaces $\cot 2x$ by $\dfrac{\cos 2x}{\sin 2x}$ **and** $\operatorname{cosec} 2x$ by $\dfrac{1}{\sin 2x}$ in the lhs

Do not be concerned by the coefficients 5 and -3.

Replacing $\cot 2x$ by $\dfrac{1}{\tan 2x}$ does not score marks until the $\tan 2x$ has been replaced by $\dfrac{\sin 2x}{\cos 2x}$

They may state $\times \sin 2x \Rightarrow 5\cos 2x - 3 = 2\sin 2x$ which implies this mark

A1 cso $5\cos 2x - 2\sin 2x = 3$ There is no need to state the value of 'c'

The notation must be correct. They cannot mix variables within their equation

Do not accept for the final A1 $\tan 2x = \dfrac{\sin}{\cos} 2x$ within their equations

(c)

M1 Attempts to use part (a) and (b). They must be using their R and α from part (a) and their c from part (b)

Accept $\cos(2x \pm '\alpha') = \dfrac{'c'}{'R'}$ Condone $\cos(\theta \pm '\alpha') = \dfrac{'c'}{'R'}$ or even $\cos(x \pm '\alpha') = \dfrac{'c'}{'R'}$ for the first M

dM1 Score for dealing with the cos, the α and the 2 **correctly** and in that order to reach $x = ..$

Don't be concerned if they change the variable in the question and solve for $\theta =$ (as long as all operations have been undone).You may not see any working. It is implied by one correct answer.

You may need to check with a calculator.

Eg for an incorrect α $\cos(2x + 1.19) = \dfrac{3}{\sqrt{29}} \Rightarrow x = -0.105$ would score M1 dM1 A0 A0

A1 One solution correct, usually $x = 0.3 / 0.30$ or $x = 2.46$ or in degrees $17.2°$ **or** $141.(0)°$

A1 Both solutions correct awrt $x =$ awrt $0.30, 2.46$ and no extra values in the range.

Condone candidates who write 0.3 and 2.46 without any (more accurate) answers

In degrees accept awrt 1 dp $17.2°, 141.(0)°$ and no extra values in the range.

....

Special case: For candidates who are misreading the question and using their part (a) with 2 on the rhs.

They will be allowed to score a maximum of SC M1 dM1 A0 A0

M1 Attempts to use part (a) with 2. They must be using their R and α from part (a)

Accept $\cos(2x \pm '\alpha') = \dfrac{2}{'R'}$ Condone $\cos(\theta \pm '\alpha') = \dfrac{2}{'R'}$ or even $\cos(x \pm '\alpha') = \dfrac{2}{'R'}$ for the first M

dM1 Score for dealing with the cos, the α and the 2 **correctly** and in that order to reach $x = ..$

You may not see any working. It is implied by one correct answer. You may need to check with a calculator.

Eg for an correct α and R $\cos(2x + 0.381) = \dfrac{2}{\sqrt{29}} \Rightarrow x = 0.405$

....

Alt to part (c)

M1 Attempts both double angle formulae condoning sign slips on cos2x , divides by $\cos^2 x$ and forms a quadratic in tan by using the identity $\pm 1 \pm \tan^2 x = \sec^2 x$

dM1 Attempts to solve their quadratic in tan$x$ leading to a solution for x.

A1 A1 As above

....

(**Q24 6665/01, June 2017**)

Q40.

Question	Scheme		Marks	AOs
(a)	$\frac{1}{\cos\theta}+\tan\theta=\frac{1+\sin\theta}{\cos\theta}$ or $\frac{(1+\sin\theta)\cos\theta}{\cos^2\theta}$		M1	1.1b
	$=\frac{1+\sin\theta}{\cos\theta}\times\frac{1-\sin\theta}{1-\sin\theta}=\frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}=\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$ or $\frac{(1+\sin\theta)\cos\theta}{\cos^2\theta}=\frac{(1+\sin\theta)\cos\theta}{1-\sin^2\theta}=\frac{(1+\sin\theta)\cos\theta}{(1+\sin\theta)(1-\sin\theta)}$		dM1	2.1
	$=\frac{\cos\theta}{1-\sin\theta}$ *		A1*	1.1b
			(3)	
(b)	$\frac{1}{\cos 2x}+\tan 2x=3\cos 2x$ $\Rightarrow 1+\sin 2x=3\cos^2 2x=3\left(1-\sin^2 2x\right)$	$\frac{\cos 2x}{1-\sin 2x}=3\cos 2x$ $\Rightarrow \cos 2x=3\cos 2x(1-\sin 2x)$	M1	2.1
	$\Rightarrow 3\sin^2 2x+\sin 2x-2=0$	$\Rightarrow \cos 2x(2-3\sin 2x)=0$	A1	1.1b
	$\sin 2x=\frac{2}{3},\ (-1)\Rightarrow 2x=\ldots\Rightarrow x=\ldots$		M1	1.1b
	$x=20.9°,\ 69.1°$		A1 A1	1.1b 1.1b
			(5)	
			(8 marks)	

Notes
(a) If starting with the LHS: Condone if another variable for θ is used except for the final mark

M1: Combines terms with a common denominator. The numerator must be correct for their common denominator.

dM1: Either:

- $\frac{1+\sin\theta}{\cos\theta}$: Multiplies numerator and denominator by $1-\sin\theta$, uses the difference of two squares and applies $\cos^2\theta=1-\sin^2\theta$
- $\frac{(1+\sin\theta)\cos\theta}{\cos^2\theta}$: Uses $\cos^2\theta=1-\sin^2\theta$ on the denominator, applies the difference of two squares

It is dependent on the previous method mark.

A1*: Fully correct proof with correct notation and no errors in the main body of their work. Withhold this mark for writing eg sin instead of $\sin\theta$ anywhere in the solution and for eg $\sin\theta^2$ instead of $\sin^2\theta$

Alt(a) If starting with the RHS: Condone if another variable is used for θ except for the final mark

M1: Multiplies by $\frac{1+\sin\theta}{1+\sin\theta}$ leading to $\frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$ or

Multiplies by $\frac{\cos\theta}{\cos\theta}$ leading to $\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$

dM1: Applies $\cos^2\theta = 1-\sin^2\theta$ and cancels the $\cos\theta$ factor from the numerator and denominator leading to $\frac{1+\sin\theta}{\cos\theta}$ or

Applies $\cos^2\theta = 1-\sin^2\theta$ and uses the difference of two squares leading to $\frac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$

It is dependent on the previous method mark.

A1*: Fully correct proof with correct notation and no errors in the main body of their work. If they work from both the LHS and the RHS and meet in the middle with both sides the same then they need to conclude at the end by stating the original equation.

(b) ***Be aware that this can be done entirely on their calculator which is not acceptable***

M1: Either multiplies through by cos $2x$ and applies $\cos^2 2x = 1-\sin^2 2x$ to obtain an equation in sin $2x$ only or alternatively sets $\frac{\cos 2x}{1-\sin 2x} = 3\cos 2x$ and multiplies by $1-\sin 2x$

A1: Correct equation or equivalent. The = 0 may be implied by their later work (Condone notational slips in their working)

M1: Solves for sin $2x$, uses arcsin to obtain at least one value for $2x$ and divides by 2 to obtain at least one value for x. The roots of the quadratic can be found using a calculator. They cannot just write down values for x from their quadratic in sin$2x$

A1: For 1 of the required angles. Accept awrt 21 or awrt 69. Also accept awrt 0.36 rad or awrt 1.21 rad

A1: For both angles (awrt 20.9 and awrt 69.1) and no others inside the range. If they find $x = 45$ it must be rejected. (Condone notational slips in their working)

(**Q13 8MA0/01, June 2022**)

Q41.

Question	Scheme	Marks	AOs
(a)(i)	$y\times\frac{dx}{dt} = 5\sin 2t\times 6\cos t$ or $5\times 2\sin t\cos t\times 6\cos t$	M1	1.2

	(Area =) $\int 5\sin 2t \times 6\cos t\, dt = \int 5\times 2\sin t\cos t \times 6\cos t\, dt$ or $\int 5\sin 2t \times 6\cos t\, dt = \int 60\sin t\cos^2 t\, dt$	dM1	1.1b
	(Area =) $\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t\, dt$ *	A1*	2.1*
		(3)	
(a)(ii)	$\int 60\sin t\cos^2 t\, dt = -20\cos^3 t$	M1 A1	1.1b 1.1b
	Area $= \left[-20\cos^3 t\right]_0^{\frac{\pi}{2}} = 0-(-20) = 20$ *	A1*	2.1
		(3)	
(b)	$5\sin 2t = 4.2 \Rightarrow \sin 2t = \frac{4.2}{5}$	M1	3.4
	$t = 0.4986...,1.072...$	A1	1.1b
	Attempts to finds the x values at both t values	dM1	3.4
	$t = 0.4986... \Rightarrow x = 2.869...$ $t = 1.072 \Rightarrow x = 5.269...$	A1	1.1b
	Width of path = 2.40 metres	A1	3.2a
		(5)	
			(11 marks)

Notes:

(a)(i)

M1: Attempts to multiply y by $\frac{dx}{dt}$ to obtain $A\sin 2t\cos t$ but may apply $\sin 2t = 2\sin t\cos t$ here

dM1: Attempts to use $\sin 2t = 2\sin t\cos t$ within an integral which may be implied by

e.g. $A\int \sin 2t \times \cos t\, dt = \int k\sin t\cos^2 t\, dt$

A1*: Fully correct work leading to the given answer.

This must include $\sin 2t = 2\sin t\cos t$ or e.g. $5\sin 2t = 10\sin t\cos t$ seen explicitly in their proof and a correct intermediate line that includes an integral sign and the "dt"

Allow the limits to just "appear" in the final answer e.g. working need not be shown for the limits.

(a)(ii)

M1: Obtains $\int 60\sin t\cos^2 t\, dt = k\cos^3 t$. This may be attempted via a substitution of $u = \cos t$ to obtain

$\int 60\sin t\cos^2 t\, dt = ku^3$

A1: Correct integration $-20\cos^3 t$ or equivalent e.g. $-20u^3$

A1*: Rigorous proof with all aspects correct including the correct limits and the $0-(-20)$ and

not just: $-20\cos^3\frac{\pi}{2} - \left(-20\cos^3 0\right) = 20$

(b)

M1: Uses the given model and attempts to find value(s) of t when $\sin 2t = \frac{4.2}{5}$. Look for $2t = \sin^{-1}\frac{4.2}{5} \Rightarrow t = ...$

A1: At least one correct value for t, correct to 2 dp. FYI $t = 0.4986...,1.072...$ or in degrees $t = 28.57..., 61.42...$

dM1: Attempts to find **TWO** distinct values of x when $\sin 2t = \dfrac{4.2}{5}$. Condone poor trig work and allow this mark if 2 values of x are attempted from 2 values of t.
A1: Both values correct to 2 dp. NB $x = 2.869\ldots,\ 5.269\ldots$

Or may take Cartesian approach

$$5\sin 2t = 4.2 \Rightarrow 10\sin t\cos t = 4.2 \Rightarrow 10\frac{x}{6}\sqrt{1-\frac{x^2}{36}} = 4.2 \Rightarrow x^4 - 36x^2 + 228.6144 = 0 \Rightarrow x = 2.869\ldots, 5.269\ldots$$

M1: For converting to Cartesian form A1: Correct quartic M1: Solves quartic A1: Correct values

A1: 2.40 metres or 240 cm
Allow awrt 2.40 m or allow 2.4m (not awrt 2.4 m) and allow awrt 240 cm. **Units are required.**

(**Q12 9MA0/02, Oct 2020**)

Q42.

Question Number	Scheme	Marks
(a)	$\operatorname{cosec} 2x + \cot 2x = \dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x}$	M1
	$= \dfrac{1+\cos 2x}{\sin 2x}$	M1
	$= \dfrac{1+2\cos^2 x - 1}{2\sin x\cos x}$	
	$= \dfrac{2\cos^2 x}{2\sin x\cos x}$	M1 A1
	$= \dfrac{\cos x}{\sin x} = \cot x$	A1*
		(5)
(b)	$\operatorname{cosec}(4\theta+10°) + \cot(4\theta+10°) = \sqrt{3}$	
	$\cot(2\theta \pm ..°) = \sqrt{3}$	M1
	$2\theta \pm \ldots = 30° \Rightarrow \theta = 12.5°$	dM1, A1
	$2\theta \pm \ldots = 180 + PV° \Rightarrow \theta = ..°$	dM1
	$\theta = 102.5°$	A1
		(5)
		(10 marks)

(a)

M1 Writing $\operatorname{cosec}2x = \frac{1}{\sin 2x}$ **and** $\cot 2x = \frac{\cos 2x}{\sin 2x}$ *or* $\frac{1}{\tan 2x}$

M1 Writing the lhs as a single fraction $\frac{a+b}{c}$. The denominator must be correct for their terms.

M1 Uses the appropriate double angle formulae/trig identities to produce a fraction in a form containing no addition or subtraction signs. A form $\frac{p \times q}{s \times t}$ or similar

A1 A correct intermediate line. Accept $\frac{2\cos^2 x}{2\sin x \cos x}$ or $\frac{2\sin x \cos x}{2\sin x \cos x \tan x}$ or similar

This cannot be scored if errors have been made

A1* Completes the proof by cancelling and using either $\frac{\cos x}{\sin x} = \cot x$ or $\frac{1}{\tan x} = \cot x$

The cancelling could be implied by seeing $\frac{2}{2}\frac{\cos x}{\sin x}\frac{\cos x}{\cos x} = \cot x$

The proof cannot rely on expressions like $\cot = \frac{\cos}{\sin}$ (with missing x's) for the final A1

(b)

M1 Attempt to use the solution to part (a) with $2x = 4\theta + 10 \Rightarrow$ to write or imply $\cot(2\theta \pm ...°) = \sqrt{3}$

Watch for attempts which start $\cot \alpha = \sqrt{3}$. The method mark here is not scored until the α has been replaced by $2\theta \pm ...°$

Accept a solution from $\cot(2x \pm ...°) = \sqrt{3}$ where θ has been replaced by another variable.

dM1 Proceeds from the previous method and uses $\tan.. = \frac{1}{\cot..}$ and $\arctan\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$ to solve $2\theta \pm ...^\circ = 30^\circ \Rightarrow \theta = ..$

A1 $\theta = 12.5^\circ$ or exact equivalent. Condone answers such as $x = 12.5^\circ$

dM1 This mark is for the correct method to find a second solution to θ. It is dependent upon the first M only.
Accept $2\theta \pm ... = 180 + PV^\circ \Rightarrow \theta = ..^\circ$

A1 $\theta = 102.5^\circ$ or exact equivalent. Condone answers such as $x = 102.5^\circ$
Ignore any solutions outside the range. This mark is withheld for any extra solutions within the range.
If radians appear they could just lose the answer marks. So for example

$2\theta \pm ... = \frac{\pi}{6}(0.524) \Rightarrow \theta = ..$ is M1dM1A0 followed by

$2\theta \pm ... = \pi + '\frac{\pi}{6}' \Rightarrow \theta = ..$ dM1A0

Special case 1: For candidates in (b) who solve $\cot(4\theta \pm ...^\circ) = \sqrt{3}$ the mark scheme is severe, so we are awarding a special case solution, scoring 00011.

$\cot(4\theta + \beta^\circ) = \sqrt{3} \Rightarrow 4\theta + \beta = 30^\circ \Rightarrow \theta = ..$ is M0M0A0 where $\beta = 5^\circ$ or 10°

$\Rightarrow 4\theta + \beta = 210^\circ \Rightarrow \theta = ..$ can score M1A1 Special case.

If $\beta = 5^\circ, \theta = 51.25$ If $\beta = 10^\circ, \theta = 50$

Special case 2: Just answers in (b) **with no working** scores 1 1 0 0 0 for 12.5 and 102.5
BUT $\cot(2\theta \pm 5^\circ) = \sqrt{3} \Rightarrow \theta = 12.5^\circ, 102.5^\circ$ scores all available marks.

Question Number	Scheme	Marks
(a)Alt 1	$\text{cosec}2x + \cot 2x = \frac{1}{\sin 2x} + \frac{1}{\tan 2x}$	1^{ST} M1
	$= \frac{1}{2\sin x\cos x} + \frac{1-\tan^2 x}{2\tan x}$	
	$= \frac{\tan x + (1-\tan^2 x)\sin x\cos x}{2\sin x\cos x\tan x}$ or $= \frac{2\tan x + 2(1-\tan^2 x)\sin x\cos x}{4\sin x\cos x\tan x}$	2^{nd} M1
	$= \frac{\tan x + \sin x\cos x - \tan^2 x\sin x\cos x}{2\sin x\cos x\tan x}$	
	$= \frac{\tan x + \sin x\cos x - \tan x\sin^2 x}{2\sin x\cos x\tan x}$	
	$= \frac{\tan x(1-\sin^2 x) + \sin x\cos x}{2\sin x\cos x\tan x}$	
	$= \frac{\tan x\cos^2 x + \sin x\cos x}{2\sin x\cos x\tan x}$	
	$= \frac{\sin x\cos x + \sin x\cos x}{2\sin x\cos x\tan x}$	
	$= \frac{2\sin x\cos x}{2\sin x\cos x\tan x}$ oe	3^{rd} M1A1
	$= \frac{1}{\tan x} = \cot x$	A1* (5)
(a)Alt 2	**Example of how main scheme could work in a roundabout route**	
	$\text{cosec}2x + \cot 2x = \cot x \Leftrightarrow \frac{1}{\sin 2x} + \frac{1}{\tan 2x} = \frac{1}{\tan x}$	1^{st} M1
	$\Leftrightarrow \tan 2x\tan x + \sin 2x\tan x = \sin 2x\tan 2x$	2^{nd} M1
	$\Leftrightarrow \frac{2\tan x}{1-\tan^2 x}\times\tan x + 2\sin x\cos x\times\frac{\sin x}{\cos x} = 2\sin x\cos x\times\frac{2\tan x}{1-\tan^2 x}$	
	$\Leftrightarrow \frac{2\tan^2 x}{1-\tan^2 x} + 2\sin^2 x = \frac{4\sin^2 x}{1-\tan^2 x}$	
	$\times(1-\tan^2 x) \Leftrightarrow 2\tan^2 x + 2\sin^2 x(1-\tan^2 x) = 4\sin^2 x$	
	$\Leftrightarrow 2\tan^2 x - 2\sin^2 x\tan^2 x = 2\sin^2 x$	
	$\Leftrightarrow 2\tan^2 x(1-\sin^2 x) = 2\sin^2 x$	3^{rd} M1
	$\div 2\tan^2 x \Leftrightarrow 1-\sin^2 x = \cos^2 x$	A1
	As this is true, initial statement is true	A1*
		(5)

(**Q27 6665/01, June 2014**)

Q43.

Question Number	Scheme		Marks
	(i) $9\sin(\theta+60^\circ)=4;\ \ 0\le\theta<360^\circ$ (ii) $2\tan x-3\sin x=0;\ -\pi\le x<\pi$		
(i)	$\sin(\theta+60^\circ)=\frac{4}{9}$, so $(\theta+60^\circ)=26.3877...$ $(\alpha=26.3877...)$	Sight of $\sin^{-1}\left(\frac{4}{9}\right)$ or awrt $26.4°$ or 0.461^c Can also be implied for $\theta=$ awrt -33.6 (i.e. $26.4-60$)	M1
	So, $\theta+60^\circ=\{153.6122...,\ 386.3877...\}$	$\theta+60^\circ$ = either "$180-$ their α" or "$360^\circ+$ their α" and not for $\theta=$ either "$180-$ their α" or "$360^\circ+$ their α". This can be implied by later working. The candidate's α could also be in radians but do not allow mixing of degrees and radians.	M1
	and $\theta=\{93.6122...,\ 326.3877...\}$	A1: At least one of awrt 93.6° or awrt 326.4° A1: Both awrt 93.6° and awrt 326.4°	A1 A1
	Both answers are cso and must come from correct work		
	Ignore extra solutions outside the range. In an otherwise fully correct solution deduct the final A1for any extra solutions in range		
			[4]
(ii)	$2\left(\frac{\sin x}{\cos x}\right)-3\sin x=0$	Applies $\tan x=\frac{\sin x}{\cos x}$	M1
	Note: Applies $\tan x=\frac{\sin x}{\cos x}$ can be implied by $2\tan x-3\sin x=0\Rightarrow\tan x(2-3\cos x)$ $2\sin x-3\sin x\cos x=0$		
	$\sin x(2-3\cos x)=0$		
	$\cos x=\frac{2}{3}$	$\cos x=\frac{2}{3}$	A1
	$x=$ awrt $\{0.84,\ -0.84\}$	A1: One of either awrt 0.84 or awrt -0.84 A1ft: You can apply ft for $x=\pm\alpha$, where $\alpha=\cos^{-1}k$ and $-1\le k\le 1$	A1A1ft
	In this part of the solution, if there are any extra answers in range in an otherwise correct solution withhold the A1ft.		
	$\{\sin x=0\Rightarrow\}\ x=0$ **and** $-\pi$	**Both** $x=0$ **and** $-\pi$ or awrt -3.14 **from** $\sin x=0$ **In this part of the solution, ignore extra solutions in range.**	B1
	Note solutions are: $x=\{-3.1415...,\ -0.8410...,\ 0,\ 0.8410...\}$ Ignore extra solutions outside the range		
	For all answers in degrees in (ii) M1A1A0A1ftB0 is possible		
	Allow the use of θ in place of x in (ii)		
			[5]
			Total 9

(**Q17 6664/01, June 2014**)

Q44.

Question	Scheme	Marks	AOs
(a)	$\dfrac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\dfrac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{3+2\cos\theta}$	M1	1.1b
	$\equiv\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$	A1	1.1b
	$\equiv\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$	M1	1.1b
	$\equiv 4-5\cos\theta$ *	A1*	2.1
		(4)	
(b)	$4+3\sin x=4-5\cos x\Rightarrow\tan x=-\dfrac{5}{3}$	M1	2.1
	$x=\text{awrt }121°, 301°$	A1 A1	1.1b 1.1b
		(3)	
			(7 marks)

Notes

(a)

M1: Uses the identity $\sin^2\theta=1-\cos^2\theta$ within the fraction

A1: Correct (simplified) expression in just $\cos\theta$ $\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$ or exact equivalent such as $\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$ Allow for $\dfrac{12-7u-10u^2}{3+2u}$ where they introduce $u=\cos\theta$

We would condone mixed variables here.

M1: A correct attempt to factorise the numerator, usual rules. Allow candidates to use $u=\cos\theta$ oe

A1*: A fully correct proof with correct notation and no errors.

Only withhold the last mark for (1) Mixed variable e.g. θ and x's (2) Poor notation $\cos\theta^2\leftrightarrow\cos^2\theta$ or $\sin^2=1-\cos^2$ within the solution.

Don't penalise incomplete lines if it is obvious that it is just part of their working

E.g. $\dfrac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\dfrac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{}=\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$

(b)

M1: Attempts to use part (a) and proceeds to an equation of the form $\tan x=k,\quad k\neq 0$

Condone $\theta\leftrightarrow x$ Do not condone $a\tan x=0\Rightarrow\tan x=b\Rightarrow x=...$

Alternatively squares $3\sin x=-5\cos x$ and uses $\sin^2 x=1-\cos^2 x$ oe to reach $\sin x=A, -1<A<1$ or $\cos x=B, -1<B<1$

A1: Either $x=\text{awrt }121°$ **or** $301°$. Condone awrt 2.11 or 5.25 which are the radian solutions

A1: Both $x=\text{awrt }121°$ **and** $301°$ and no other solutions.

Answers without working, or with no incorrect working in (b).

Question states hence or otherwise so allow

For 3 marks both $x=\text{awrt }121°$ **and** $301°$ and no other solutions.

For 1 marks scored SC 100 for either $x=\text{awrt }121°$ **or** $301°$

Alternative proof in part (a):

M1: Multiplies across and form 3TQ in $\cos\theta$ on rhs

$10\sin^2\theta - 7\cos\theta + 2 = (4 - 5\cos\theta)(3 + 2\cos\theta) \Rightarrow 10\sin^2\theta - 7\cos\theta + 2 = A\cos^2\theta + B\cos\theta + C$

A1: Correct identity formed $10\sin^2\theta - 7\cos\theta + 2 = -10\cos^2\theta - 7\cos\theta + 12$

dM1: Uses $\cos^2\theta = 1 - \sin^2\theta$ on the rhs or $\sin^2\theta = 1 - \cos^2\theta$ on the lhs

Alternatively proceeds to $10\sin^2\theta + 10\cos^2\theta = 10$ and makes a statement about $\sin^2\theta + \cos^2\theta = 1$ oe

A1*: Shows that $(4 - 5\cos\theta)(3 + 2\cos\theta) \equiv 10\sin^2\theta - 7\cos\theta + 2$ oe AND makes a minimal statement "hence true"

(**Q12 8MA0/01, June 2019**)

Q45.

Question	Scheme	Marks	AOs
(a)	e.g. $2\dfrac{\sin\theta}{\cos\theta}(8\cos\theta+23(\underline{1-\cos^2\theta}))=8\times\underline{2\sin\theta\cos\theta\sec^2\theta}$	B1	1.2
	$2\tan\theta(8\cos\theta+23\sin^2\theta)=8\sin2\theta\sec^2\theta$ $\Rightarrow 2\sin\theta\cos\theta(8\cos\theta+23(1-\cos^2\theta))=8\sin2\theta$ $\sin2\theta(8\cos\theta+23(1-\cos^2\theta))=8\sin2\theta$ $\sin2\theta(23\cos^2\theta-8\cos\theta-15)=0$	M1A1	2.1 2.2a
		(3)	
(b)	$\sin2x(23\cos^2x-8\cos x-15)=0$		
	$\sin2x=0\Rightarrow x=360°$ or $540°$	B1	2.2a
	$23\cos^2x-8\cos x-15\Rightarrow\cos x=-\dfrac{15}{23}$	M1	1.1b
	$\cos x=-\dfrac{15}{23}\Rightarrow x=...$	dM1	1.1b
	$x=360°, 540°$ and awrt $491°$ only	A1	2.3
		(4)	
			(7 marks)

Notes

(a) Allow use of e.g. x but the final mark requires the equation to be in terms of θ

B1(M1 on EPEN): For recalling and using at least one correct trigonometric identity in the given equation.

e.g. one of: $\sin^2\theta+\cos^2\theta=1$, $1+\tan^2\theta=\sec^2\theta$, $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$, $\sin2\theta=2\sin\theta\cos\theta$

This may be seen explicitly or may be implied by their working by e.g. $\tan\theta\cos\theta=\sin\theta$ or they might multiply both sides by $\cos^2\theta$ leaving $8\sin2\theta$ on the rhs implying $1+\tan^2\theta=\sec^2\theta$

M1: For manipulating the equation using trigonometric identities (condoning sign slips only in the identities and arithmetic slips) to obtain an expression of the form:

$A\sin2\theta\cos^2\theta+B\sin2\theta\cos\theta+C\sin2\theta\ (=0)$ or $\sin2\theta\left(A\cos^2\theta+B\cos\theta+C\right)\ (=0)$ with $A, B, C\neq0$

A1: $\sin2\theta(23\cos^2\theta-8\cos\theta-15)=0$ oe e.g. $\sin2\theta(-23\cos^2\theta+8\cos\theta+15)=0$ cao

Note that this is not a given answer so condone notational slips e.g. $\cos\theta^2$ for $\cos^2\theta$ provided the intention is clear but the final equation must have no notational errors.

Note that the "$=0$" is not required for the M1 but is required for the A1

Note: some candidates arrive at the correct final answer fortuitously following errors in their work.

(b) Allow all marks in (b) to score if the correct equation is obtained fortuitously in part (a)
Also allow use of θ instead of x throughout in part (b). Correct answers, no working scores max 1000

B1: For one of $x=360(°)$ or $x=540(°)$ Condone $x=2\pi$ or $x=3\pi$ for this mark.

The degrees symbol is not required. This may come from $\cos x=1$

M1: Attempts to solve their 3TQ from part (a) or a "made up" 3TQ (which may only be seen in (b)) leading to a value for $\cos x$. The general guidance for solving a 3 term quadratic equation can be applied. Allow solution(s) from a calculator which may be implied by at least one correct value for their 3TQ. Must be a value for $\cos x$ and not e.g. x.

dM1: Attempts to find one of their angles in the range $360<x<540$ (but not 450) for their $\cos x=k$ where $|k|<1$ May be implied by their value(s) but must be in degrees.

Requires them to state a value for cos$x$. Must be checked (you can check cos(their x) = their k (1sf))

A1: $x=360°, 540°$ and awrt $491°$ only with no other values in range (including 450).

The degrees symbol is not required. awrt 491 must come from $\cos x=-\dfrac{15}{23}$

(**Q14 9MA0/02, June 2023**)

Q46.

Question	Scheme	Marks	AOs
(a)	$R=\sqrt{2^2+8^2}=\sqrt{68}=2\sqrt{17}$	B1	1.1b
	$2\cos\theta+8\sin\theta=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$ $2=R\cos\alpha \quad 8=R\sin\alpha$ $\tan\alpha=\frac{8}{2}\Rightarrow\alpha=...$	M1	1.1b
	$\alpha=$ awrt 1.326	A1	2.2a
		(3)	
(b)(i)	$4.5\times"2\sqrt{17}"$	M1	1.1b
	$9\sqrt{17}$	A1	2.2a
(ii)	awrt 1.33	B1ft	2.2a
		(3)	
		(6 marks)	

Notes

(a)

B1: $R=2\sqrt{17}$ or $\sqrt{68}$.

$\pm2\sqrt{17}$ or $\pm\sqrt{68}$ score B0

(Condone if this comes from e.g., $8=R\cos\alpha \quad 2=R\sin\alpha$)

Decimal answers score B0 unless the exact value is seen then apply isw.

M1: Proceeds to a value for α from $\tan\alpha=\pm\frac{8}{2}$, $\cos\alpha=\pm\frac{2}{"\sqrt{68}"}$, $\sin\alpha=\pm\frac{8}{"\sqrt{68}"}$

May be implied by awrt 1.33 radians or 76 degrees

A1: awrt 1.326 for α. Apply isw if this is then subsequently rounded to e.g. 1.33

(b)(i)

M1: For a value of $\pm4.5\times$ their R or allow $\pm4.5R$ (with the letter R)

But not embedded in an expression e.g. $9\sqrt{17}\cos(\theta-\alpha)$ unless extracted later.

Note that the sum may be found as $9\cos x+36\sin x$ with the maximum then found using calculus

e.g. $S=9\cos x+36\sin x\Rightarrow\frac{dS}{dx}=-9\sin x+36\cos x=0\Rightarrow\tan x=4\Rightarrow\sin x=\frac{4}{\sqrt{17}},\ \cos x=\frac{1}{\sqrt{17}}$

$\Rightarrow 9\cos x+36\sin x=9\sqrt{17}$. This will score M1 once they reach $\pm4.5\times$ their R

May be implied by $9\sqrt{17}$ or awrt 37.1 (which may come from a graphical method)

May also see e.g. $\text{Max}(9\cos x+36\sin x)=\sqrt{9^2+36^2}=...$

A1: $9\sqrt{17}$ or exact equivalent e.g. $\sqrt{1377}$, $4.5\sqrt{68}$, $4.5(2\sqrt{17})$ and apply isw once a correct answer is seen

(ii)

B1ft: awrt 1.33 (or follow through on their α even if in degrees (76), no matter how accurate)

(**Q08 9MA0/02, June 2023**)

Q47.

Question	Scheme	Marks	AOs
(a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha = 2 \Rightarrow \alpha = ...$	M1	1.1b
	$\alpha = 1.107$	A1	1.1b
		(3)	
	$\theta = 5+\sqrt{5}\sin\left(\frac{\pi t}{12}+1.107-3\right)$		
(b)	$\left(5+\sqrt{5}\right)$ °C or awrt 7.24 °C	B1ft	2.2a
		(1)	
(c)	$\frac{\pi t}{12}+1.107-3=\frac{\pi}{2}\Rightarrow t=$	M1	3.1b
	$t=$ awrt 13.2	A1	1.1b
	Either 13:14 or 1:14 pm or 13 hours 14 minutes after midnight.	A1	3.2a
		(3)	
			(7 marks)
Notes:			

(a)

B1: $R=\sqrt{5}$ only.

M1: Proceeds to a value of α from $\tan\alpha=\pm2, \tan\alpha=\pm\frac{1}{2}, \sin\alpha=\pm\frac{2}{"R"}$ OR $\cos\alpha=\pm\frac{1}{"R"}$

It is implied by either awrt 1.11 (radians) or 63.4 (degrees)

A1: $\alpha=$ awrt 1.107

(b)

B1ft: Deduces that the maximum temperature is $(5+\sqrt{5})$ °C or awrt 7.24 °C Remember to isw

Condone a lack of units. Follow through on their value of R so allow $(5+"R")$ °C

(c)

M1: An complete strategy to find t from $\frac{\pi t}{12}\pm1.107-3=\frac{\pi}{2}$.

Follow through on their 1.107 but the angle must be in radians.

It is possible via degrees but only using $15t\pm63.4-171.9=90$

A1: awrt $t=13.2$

A1: The question asks for the time of day so accept either 13:14, 1:14 pm, 13 hours 14 minutes after midnight, 13h 14, or 1 hour 14 minutes after midday. If in doubt use review

....

It is possible to attempt parts (b) and (c) via differentiation but it is unlikely to yield correct results.

$\frac{d\theta}{dt}=\frac{\pi}{12}\cos\left(\frac{\pi t}{12}-3\right)-\frac{2\pi}{12}\sin\left(\frac{\pi t}{12}-3\right)=0\Rightarrow\tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2}\Rightarrow t=13.23=13:14$ scores M1 A1 A1

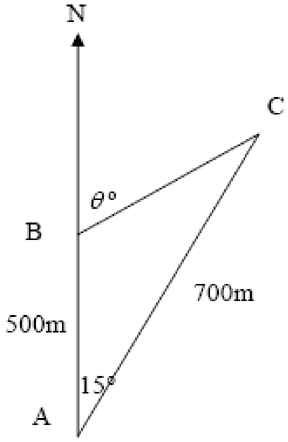
$\frac{d\theta}{dt}=\cos\left(\frac{\pi t}{12}-3\right)-2\sin\left(\frac{\pi t}{12}-3\right)=0\Rightarrow\tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2}\Rightarrow t=13.23=13:14$ they can score M1 A0 A1 (SC)

A value of $t=1.23$ implies the minimum value has been found and therefore incorrect method M0.

....

(**Q06 9MA0/01, Oct 2020**)

Q48.

Question Number	Scheme	Marks
		
(a)	$BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15°$	M1 A1
	$(= \ 63851.92\ldots)$	
	$BC = 253$ awrt	A1 (3)
(b)	$\dfrac{\sin B}{700} = \dfrac{\sin 15}{\text{candidate's } BC}$	M1
	$\sin B = \sin 15 \times 700 / 253_c = 0.716..$ and giving an **obtuse** B (134.2°) dep on 1[st] M	M1
	$\theta = 180° -$ candidate's angle B (Dep. on first M only, B can be acute)	M1
	$\theta = 180 - 134.2 = (0)45.8$ (allow 46 or awrt 45.7, 45.8, 45.9)	A1 (4) [7]

Notes:		
	(a) If use cos 15° = ..., then A1 not scored until written as $BC^2 = \ldots$ correctly *Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC* Finding value for *BX* and *CX* and using Pythagoras M1 $BC^2 = (500 \sin 15°)^2 + (700 - 500 \cos 15°)^2$ A1 $BC = 253$ awrt A1 (b) Several alternative methods: (Showing the M marks, 3[rd] M dep. on first M)) (i) $\cos B = \dfrac{500^2 + \text{candidate's}BC^2 - 700^2}{2\text{x}500\text{xcandidate's}BC}$ or $700^2 = 500^2 + BC_c{}^2 - 2x500xBC_c$ M1 Finding angle B M1 dep., then M1 as above (ii) 2 triangle approach, as defined in notes for (a) $\tan CBX = \dfrac{700 - valueforAX}{valueforBX}$ M1 Finding value for $\angle CBX$ ($\approx 59°$) dep M1 $\theta = [180° - (75° + candidate's\ \angle CBX)]$ M1 (iii) Using sine rule (or cos rule) to find C first: Correct use of sine or cos rule for C M1, Finding value for C M1 Either $B = 180° - (15° +$ candidate's $C)$ or $\theta = (15° +$ candidate's $C)$ M1 (iv) $700 \cos 15° = 500 + BC \cos\theta$ M2 {first two Ms earned in this case} Solving for θ; $\theta = 45.8$ (allow 46 or5.7, 45.8, 45.9) M1;A1 Note: S.C. In main scheme, if θ used in place of B, third M gained immediately; Other two marks likely to be earned, too, for correct value of θ stated.	

(**Q13 6664/01, Jan 2008**)

Q49.

Question Number	Scheme	Marks
(i)	$\text{cosec}2x = \dfrac{1}{\sin 2x}$	M1
	$= \dfrac{1}{2\sin x \cos x}$	M1
	$= \dfrac{1}{2}\text{cosec}x \sec x \Rightarrow \quad \lambda = \dfrac{1}{2}$	A1
		(3)
(ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\sec^2\theta + 3\sec\theta = 2(\sec^2\theta - 1)$	M1
	$\sec^2\theta + 3\sec\theta + 2 = 0$	
	$(\sec\theta + 2)(\sec\theta + 1) = 0$	M1
	$\sec\theta = -2, -1$	A1
	$\cos\theta = -0.5, -1$	M1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	A1A1
		(6)
		(9 marks)
ALT (ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\times\dfrac{1}{\cos^2\theta} + 3\times\dfrac{1}{\cos\theta} = 2\times\dfrac{\sin^2\theta}{\cos^2\theta}$	
	$3 + 3\cos\theta = 2\sin^2\theta$	
	$3 + 3\cos\theta = 2(1 - \cos^2\theta)$	M1
	$2\cos^2\theta + 3\cos\theta + 1 = 0$	
	$(2\cos\theta + 1)(\cos\theta + 1) = 0 \Rightarrow \cos\theta = -0.5, -1$	M1A1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	M1,A1,A1
		(6)
		(9 marks)

Notes for Question

(i)

M1 Uses the identity $\text{cosec}2x = \frac{1}{\sin 2x}$

M1 Uses the correct identity for $\sin 2x = 2\sin x\cos x$ in their expression.
Accept $\sin 2x = \sin x\cos x + \cos x\sin x$

A1 $\lambda = \frac{1}{2}$ following correct working

(ii)

M1 Replaces $\tan^2\theta$ by $\pm\sec^2\theta \pm 1$ to produce an equation in just $\sec\theta$

M1 Award for a forming a 3TQ=0 in $\sec\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\sec\theta$
If they replace $\sec\theta = \frac{1}{\cos\theta}$ it is for forming a 3TQ in $\cos\theta$ and applying a correct method for finding two answers to $\cos\theta$

A1 Correct answers to $\sec\theta = -2, -1$ or $\cos\theta = -\frac{1}{2}, -1$

M1 Award for using the identity $\sec\theta = \frac{1}{\cos\theta}$ and proceeding to find at least one value for θ.
If the 3TQ was in cosine then it is for finding at least one value of θ.

A1 Two correct values of θ. All method marks must have been scored.
Accept two of $120°, 180°, 240°$ or two of $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19

A1 All three answers correct. They must be given in terms of π as stated in the question.
Accept $0.\dot{6}\pi, 1.\dot{3}\pi, \pi$
Withhold this mark if further values in the range are given. All method marks must have been scored.
Ignore any answers outside the range.

Alt (ii)

M1 Award for replacing $\sec^2\theta$ with $\frac{1}{\cos^2\theta}$, $\sec\theta$ with $\frac{1}{\cos\theta}$, $\tan^2\theta$ with $\frac{\sin^2\theta}{\cos^2\theta}$ multiplying through by $\cos^2\theta$ (seen in at least 2 terms) and replacing $\sin^2\theta$ with $\pm 1 \pm \cos^2\theta$ to produce an equation in just $\cos\theta$

M1 Award for a forming a 3TQ=0 in $\cos\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\cos\theta$

A1 $\cos\theta = -\frac{1}{2}, -1$

M1 Proceeding to finding at least one value of θ from an equation in $\cos\theta$.

A1 Two correct values of θ. All method marks must have been scored
Accept two of $120°, 180°, 240°$ or two of $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19

A1 All three answers correct. They must be given in terms of π as stated in the question.

Notes for Question Continued
Accept $0.\dot{6}\pi, 1.\dot{3}\pi, \pi$ All method marks must have been scored. Withhold this mark if further values in the range are given. Ignore any answers outside the range

(**Q24 6665/01/R, June 2013**)

Q50.

Question Number	Scheme		Marks
	Note: A similar scheme would apply for T&I for candidates using their a and their r. So,... 1st M1: For attempting to find one of the correct S_n's either side (but next to) 1000. 2nd M1: For one of these S_n's correct for their a and their r. (You may need to get your calculators out!) 3rd M1: For attempting to find both of the correct S_n's either side (but next to) 1000. A1: Cannot be gained for wrong a and/or r. **Trial & Improvement Cumulative Approach:** A similar scheme to T&I will be applied here: 1st M1: For getting as far as the cumulative sum of 13 terms. 2nd M1: (1) S_{13} = awrt 999.7 or truncated 999. 3rd M1: For getting as far as the cumulative sum to 14 terms. Also at this stage $S_{13} < 1000$ and $S_{14} > 1000$. A1: BOTH (1) S_{13} = awrt 999.7 or truncated 999 AND (2) S_{14} = awrt 1005.8 or truncated 1005 AND $n = 14$. **Trial & Improvement Method:** for $(0.75)^n < \frac{6}{256} = 0.0234375$ 3rd M1: For evidence of examining both $n = 13$ and $n = 14$. Eg: $(0.75)^{13}\{= 0.023757...\}$ and $(0.75)^{14}\{= 0.0178179...\}$ A1: $n = 14$ **Any misreads,** S_n > 10000 etc, please escalate up to your Team Leader.		
	(a) $3\sin(x + 45^\circ) = 2$; $0 \le x < 360^\circ$ (b) $2\sin^2 x + 2 = 7\cos x$; $0 \le x < 2\pi$		
(a)	$\sin(x + 45^\circ) = \frac{2}{3}$, so $(x + 45^\circ) = 41.8103...$ $(\alpha = 41.8103...)$	$\sin^{-1}\left(\frac{2}{3}\right)$ or awrt 41.8 or awrt 0.73^c	M1
	So, $x + 45^\circ = \{138.1897..., 401.8103...\}$	$x + 45^\circ$ = either "180 – their α" or "360° + their α" (α could be in radians).	M1
	and $x = \{93.1897..., 356.8103...\}$	Either awrt 93.2° or awrt 356.8°	A1
		Both awrt 93.2° and awrt 356.8°	A1
			[4]

(b)	$2(1-\cos^2 x)+2 = 7\cos x$ — Applies $\sin^2 x = 1-\cos^2 x$	M1
	$2\cos^2 x + 7\cos x - 4 = 0$ — Correct 3 term, $2\cos^2 x + 7\cos x - 4 \{= 0\}$	A1 oe
	$(2\cos x - 1)(\cos x + 4)\{=0\}$, $\cos x = ...$ — Valid attempt at solving and $\cos x = ...$	M1
	$\cos x = \frac{1}{2}$, $\{\cos x = -4\}$ — $\cos x = \frac{1}{2}$ (See notes.)	A1 cso
	$\left(\beta = \frac{\pi}{3}\right)$	
	$x = \frac{\pi}{3}$ or $1.04719...^c$ — Either $\frac{\pi}{3}$ or awrt 1.05^c	B1
	$x = \frac{5\pi}{3}$ or $5.23598...^c$ — Either $\frac{5\pi}{3}$ or awrt 5.24^c or 2π – their β (See notes.)	B1 ft
		[6]

Question Number	Scheme	Marks
(a)	1st M1: can also be implied for $x =$ awrt -3.2 2nd M1: for $x + 45^\circ =$ either "180 – their α" or "360° + their α". This can be implied by later working. The candidate's α could also be in radians. **Note that this mark is not** for $x =$ either "180 – their α" or "360° + their α". **Note:** Imply the first two method marks or award M1M1A1 for either awrt 93.2° or awrt 356.8°. **Note:** Candidates who apply the following incorrect working of $3\sin(x+45^\circ) = 2$ $\Rightarrow 3(\sin x + \sin 45) = 2$, etc will usually score M0M0A0A0. If there are any EXTRA solutions inside the range $0 \le x < 360$ and the candidate would otherwise score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \le x < 360$. **Working in Radians:** Note the answers in radians are $x =$ awrt 1.6, awrt 6.2 If a candidate works in radians then mark part (a) as above awarding the A marks in the same way. If the candidate would then score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question.) **No working:** Award M1M1A1A0 for one of awrt 93.2° or awrt 356.8° seen without any working. Award M1M1A1A1 for both awrt 93.2° and awrt 356.8° seen without any working. Allow benefit of the doubt (FULL MARKS) for final answer of $\sin x$ {and not x} = {awrt 93.2, awrt 356.8}	

Question Number	Scheme	Marks
(b)	1st M1: for a correct method to use $\sin^2 x = 1 - \cos^2 x$ on the given equation. Give bod if the candidate omits the bracket when substituting for $\sin^2 x$, but $2 - \cos^2 x + 2 = 7\cos x$, without supporting working, (eg. seeing "$\sin^2 x = 1 - \cos^2 x$") would score 1st M0. Note that applying $\sin^2 x = \cos^2 x - 1$, scores M0. 1st A1: for obtaining either $2\cos^2 x + 7\cos x - 4$ or $-2\cos^2 x - 7\cos x + 4$. 1st A1: can also awarded for a correct three term equation eg. $2\cos^2 x + 7\cos x = 4$ or $2\cos^2 x = 4 - 7\cos x$ etc. 2nd M1: for a valid attempt at factorisation of a quadratic (either 2TQ or 3TQ) in cos, can use any variable here, c, y, x or $\cos x$, and an attempt to find at least one of the solutions. See introduction to the Mark Scheme. *Alternatively*, using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution. 2nd A1: for $\cos x = \frac{1}{2}$, BY A CORRECT SOLUTION ONLY UP TO THIS POINT. Ignore extra answer of $\cos x = -4$, but penalise if candidate states an incorrect result e.g. $\cos x = 4$. If they have used a substitution, a correct value of their c or their y or their x. **Note:** 2nd A1 for $\cos x = \frac{1}{2}$ can be implied by later working. 1st B1: for either $\frac{\pi}{3}$ or awrt 1.05^c 2nd B1: for either $\frac{5\pi}{3}$ or awrt 5.24^c or can be ft from $2\pi -$ their β or $360^\circ -$ their β where $\beta = \cos^{-1}(k)$, such that $0 < k < 1$ or $-1 < k < 0$, but $k \neq 0$, $k \neq 1$ or $k \neq -1$. If there are any EXTRA solutions inside the range $0 \le x < 2\pi$ and the candidate would otherwise score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$. **Working in Degrees:** Note the answers in degrees are $x = 60,\ 300$ If a candidate works in degrees then mark part (b) as above awarding the B marks in the same way. If the candidate would then score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question.)	
	$x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$ scores M0A0M0A0B1B1, $x = 60$ and $x = 300$ scores M0A0M0A0B1B0, $x = \frac{\pi}{3}$ ONLY or $x = 60$ ONLY scores M0A0M0A0B1B0, $x = \frac{5\pi}{3}$ ONLY or $x = 120$ ONLY scores M0A0M0A0B0B1. **No working:** You cannot apply the ft in the B1ft if the answers are given with NO working. Eg: $x = \frac{\pi}{5}$ and $x = \frac{9\pi}{3}$ FROM NO WORKING scores M0A0M0A0B0B0. **For candidates using trial & improvement, please forward these to your Team Leader.**	

(**Q15 6664/01, June 2011**)

Q51.

Question Number	Scheme	Marks
(a)	$\sin^2\theta + \cos^2\theta = 1$	
	$\div\sin^2\theta \qquad \frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$	M1
	$1 + \cot^2\theta = \mathrm{cosec}^2\theta$ ∗	A1 cso (2)
(b)	$2(\mathrm{cosec}^2\theta - 1) - 9\,\mathrm{cosec}\,\theta = 3$	M1
	$2\,\mathrm{cosec}^2\theta - 9\,\mathrm{cosec}\,\theta - 5 = 0$ *or* $5\sin^2\theta + 9\sin\theta - 2 = 0$	M1
	$(2\,\mathrm{cosec}\,\theta + 1)(\mathrm{cosec}\,\theta - 5) = 0$ *or* $(5\sin\theta - 1)(\sin\theta + 2) = 0$	M1
	$\mathrm{cosec}\,\theta = 5$ *or* $\sin\theta = \frac{1}{5}$	A1
	$\theta = 11.5°, 168.5°$	A1 A1 (6)
		(8 marks)

(**Q24 6665/01, June 2008**)

Q52.

Question Number	Scheme		Marks
(a)	$R^2 = 3^2 + 4^2$		M1
	$R = 5$		A1
	$\tan\alpha = \frac{4}{3}$		M1
	$\alpha = 53\ ...\ °$	awrt 53°	A1 (4)
(b)	Maximum value is 5	ft their R	B1 ft
	At the maximum, $\cos(\theta - \alpha) = 1$ *or* $\theta - \alpha = 0$		M1
	$\theta = \alpha = 53\ ...\ °$	ft their α	A1 ft (3)
(c)	$f(t) = 10 + 5\cos(15t - \alpha)°$		
	Minimum occurs when $\cos(15t - \alpha)° = -1$		M1
	The minimum temperature is $(10 - 5)° = 5°$		A1 ft (2)
(d)	$15t - \alpha = 180$		M1
	$t = 15.5$	awrt 15.5	M1 A1 (3)
			[12]

(**Q26 6665/01, Jan 2009**)

Q53.

Question Number	Scheme		Marks
Q (a)	$A = B \Rightarrow \cos(A+A) = \cos 2A = \underline{\cos A\cos A - \sin A\sin A}$	Applies $A = B$ to $\cos(A+B)$ to give the underlined equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$	M1
	$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives		
	$\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required)	Complete proof, with a link between LHS and RHS. No errors seen.	A1 AG
			(2)
(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$	Eliminating y correctly.	M1
	$3\sin 2x = 4\left(\frac{1-\cos 2x}{2}\right) - 2\cos 2x$	Using result in part (a) to substitute for $\sin^2 x$ as $\frac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\frac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles.	M1
	$3\sin 2x = 2(1 - \cos 2x) - 2\cos 2x$		
	$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$		
	$3\sin 2x + 4\cos 2x = 2$	Rearranges to give correct result	A1 AG
			(3)
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$		
	$3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$		
	Equate $\sin 2x$: $3 = R\sin\alpha$ Equate $\cos 2x$: $4 = R\cos\alpha$		
	$R = \sqrt{3^2 + 4^2}\,; = \sqrt{25} = 5$	$R = 5$	B1
	$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...^\circ$	$\tan\alpha = \pm\frac{3}{4}$ or $\tan\alpha = \pm\frac{4}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{4}{\text{their } R}$	M1
		awrt 36.87	A1
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$		
			(3)

Question Number	Scheme		Marks
(d)	$3\sin 2x + 4\cos 2x = 2$		
	$5\cos(2x - 36.87) = 2$		
	$\cos(2x - 36.87) = \frac{2}{5}$	$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$	M1
	$(2x - 36.87) = 66.42182...^{\circ}$	awrt 66	A1
	$(2x - 36.87) = 360 - 66.42182...^{\circ}$		
	Hence, $x = 51.64591...^{\circ}$, $165.22409...^{\circ}$	One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3	A1
		Both awrt 51.6 AND awrt 165.2	A1
			(4)
		If there are any EXTRA solutions inside the range $0 \le x < 180^{\circ}$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \le x < 180^{\circ}$.	
			[12]

(**Q24 6665/01, June 2009**)