## Questions

**Q1.**

Show, using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, that

$$\sum_{r=1}^{n} 3(2r-1)^2 = n(2n+1)(2n-1)$$

for all positive integers *n*.

**(5)**

**(Total 5 marks)**

**Q2.**

(a) Show, using the formulae for $\sum r$ and $\sum r^2$, that

$$\sum_{r=1}^{n} (6r^2 + 4r - 1) = n(n+2)(2n+1)$$

**(5)**

(b) Hence, or otherwise, find the value of

$$\sum_{r=11}^{n=20} (6r^2 + 4r - 1)$$

**(2)**

**(Total 7 marks)**

**Q3.**

(a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to show that

$$\sum_{r=1}^{n} r(r^2 - 3) = \frac{1}{4}n(n + 1)(n + 3)(n - 2)$$

**(5)**

(b) Calculate the value of

$$\sum_{r=10}^{50} r(r^2 - 3)$$

**(3)**

**(Total 8 marks)**

**Q4.**

(a) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that

$$\sum_{r=1}^{n}(r+1)(r+4) = \frac{n}{3}(n+4)(n+5)$$

for all positive integers *n*.

**(5)**

(b) Hence show that

$$\sum_{r=n+1}^{2n}(r+1)(r+4) = \frac{n}{3}(n+1)(an+b)$$

where *a* and *b* are integers to be found.

**(3)**

**(Total for question = 8 marks)**

**Q5.**

(a) Use the results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(2r-1)^2 = \frac{1}{3}n(2n+1)(2n-1)$$

for all positive integers *n*.

**(6)**

(b) Hence show that

$$\sum_{r=n+1}^{3n}(2r-1)^2 = \frac{2}{3}n\left(an^2+b\right)$$

where *a* and *b* are integers to be found.

**(4)**

**(Total 10 marks)**

**Q6.**

(a) Use the standard results for summations to show that for all positive integers $n$

$$\sum_{r=1}^{n}(5r-2)^2 = \frac{1}{6}n(an^2+bn+c)$$

where $a$, $b$ and $c$ are integers to be determined.

**(5)**

(b) Hence determine the value of $k$ for which

$$\sum_{r=1}^{k}(5r-2)^2 = 94k^2$$

**(4)**

**(Total for question = 9 marks)**

**Q7.**

(a) Use the standard results for summations to show that, for all positive integers $n$,

$$\sum_{r=1}^{n} r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(an+b)$$

where $a$ and $b$ are integers to be determined.

**(4)**

(b) Hence show that, for all positive integers $k$,

$$\sum_{r=k+1}^{3k} r^2(r+1) = \frac{1}{3}k(3k+1)(Ak^2+Bk+C)$$

where $A$, $B$ and $C$ are integers to be determined.

**(3)**

(c) Hence, using algebra and making your method clear, determine the value of $k$ for which

$$25\sum_{r=k+1}^{3k} r^2(r+1) = 192k^3(3k+1)$$

**(3)**

**(Total for question = 10 marks)**