Mark Scheme

Q1.

Question Number	Scheme	Marks	
a	At rest when $v=0$: $\left(2t^2-9t+4\right)=0$	M1	
	$=(2t-1)(t-4)$,	DM1	Solve for t. Dependent on the previous M1
	$t=\frac{1}{2},\ 4$	A1	Incorrect answers with no method shown score M0A0
		[3]	
b	$a=\frac{dv}{dt}=4t-9$	M1	Differentiate v to obtain a (at least one power of t going down)
		A1	Correct derivative
	$t=5,\ \ a=11\,(\text{m s}^{-2})$	A1	
		[3]	
c	$s=\int v\,dt=\frac{2}{3}t^3-\frac{9}{2}t^2+4t\,(+C)$	M1	Integrate v to obtain s (at least one power of t going up)
		A1	
	Use of $t=0$, $t=\frac{1}{2}$, $t=4$, $t=5$ (and $t=0, s=15$) as limits in integrals	DM1	Correct strategy for their limits - requires subtraction of the negative distance. Dependent on the previous M1 and at least one positive solution for t in (0,5) from (a)
	$\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t\left(+15\right)\right]_0^{\frac{1}{2}}$ $-\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t\left(+15\right)\right]_{\frac{1}{2}}^{4}$ $+\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t\left(+15\right)\right]_4^{5}$	A1	NB: $\int_0^5 v\,dt$ scores M0A0A0

	$(0, \frac{23}{24}, -\frac{40}{3}, \frac{-55}{6})$ $= \frac{23}{24} + \frac{343}{24} + \frac{100}{24} = 19.4$ (m) $\left(15, 15\frac{23}{24}\left(\frac{383}{24}\right), \frac{5}{3}, 5.8\dot{3}\left(\frac{35}{6}\right)\right)$	A1 [5]	$19\frac{5}{12}$ $\left(\frac{233}{12}\right)$ or better
		(11)	

Q2.

(a)	$a = 4t^3 - 12t$ Convincing attempt to integrate $v = t^4 - 6t^2 (+c)$ Use initial condition to get $v = t^4 - 6t^2 + 8\left(\text{ms}^{-1}\right)$.	M1 A1 A1 **(3)**
(b)	Convincing attempt to integrate $s = \frac{t^5}{5} - 2t^3 + 8t(+0)$ Integral of their v	M1 A1ft **(2)**
(c)	Set their $v = 0$ Solve a quadratic in t^2 $(t^2 - 2)(t^2 - 4) = 0 \Rightarrow$ at rest when $t = \sqrt{2},\ t = 2$	M1 DM1 A1 **(3)** **[8]**

Q3.

Question Number	Scheme	Marks
(a)	$0 \le t \le 4$: $a = 8 - 3t$	M1
	$a = 0 \Rightarrow t = 8/3$ s	DM1
	$\rightarrow v = 8.\frac{8}{3} - \frac{3}{2}\left(\frac{8}{3}\right)^2 = \frac{32}{3}$ (m/s)	DM1 A1
	second M1 dependent on the first, and third dependent on the second.	(4)
(b)	$s = 4t^2 - t^3/2$	M1
	$t = 4$: $s = 64 - 64/2 = \underline{32\text{ m}}$	M1 A1 (3)
(c)	$t > 4$: $v = 0 \Rightarrow t = \underline{8\text{ s}}$	B1 (1)
(d)	*Either*	
	$t > 4$ $s = 16t - t^2$ $(+ C)$	M1
	$t = 4, s = 32 \rightarrow C = -16 \Rightarrow s = 16t - t^2 - 16$	M1 A1
	$t = 10 \rightarrow s = 44$ m	M1 A1
	But direction changed, so: $t = 8, s = 48$	M1
	Hence total dist travelled $= 48 + 4 = \underline{52\text{ m}}$	DM1 A1 (8)
	Or (probably accompanied by a sketch?)	
	t=4 v=8, t=8 v=0, so area under line $= \frac{1}{2} \times (8-4) \times 8$	M1A1A1
	t=8 v=0, t=10 v=-4, so area above line $= \frac{1}{2} \times (10-8) \times 4$	M1A1A1
	$\therefore$ total distance = 32(from b) + 16 + 4 = $\underline{52\text{ m.}}$	M1A1 (8)

Or M1, A1 for t > 4 $\frac{dv}{dt} = -2$, =constant

t=4, v=8; t=8, v=0; t=10, v=-4

M1, A1 $s = \frac{u+v}{2}t = \frac{32}{2}t$, =16 working for t = 4 to t = 8

M1, A1 $s = \frac{u+v}{2}t = \frac{-4}{2}t$, =-4 working for t = 8 to t = 10

M1, A1 total = 32+14+4, =52

M1 Differentiate to obtain acceleration
DM1 set acceleration. = 0 and solve for t
DM1 use their t to find the value of v
A1 32/3, 10.7oro better

OR using trial an improvement:
M1 Iterative method that goes beyond integer values
M1 Establish maximum occurs for t in an interval no bigger than 2.5<t<3.5
M1 Establish maximum occurs for t in an interval no bigger than 2.6<t<2.8
A1

Or M1 Find/state the coordinates of both points where the curve cuts the x axis.
DM1 Find the midpoint of these two values.
M1A1 as above.

Or M1 Convincing attempt to complete the square:

DM1 substantially correct $8t - \frac{3t^2}{2} = -\frac{3}{2}(t - \frac{8}{3})^2 + \frac{3}{2} \times \frac{64}{9}$

DM1 Max value = constant term
A1 CSO

M1 Integrate the correct expression
DM1 Substitute t = 4 to find distance (s=0 when t=0 - condone omission / ignoring of constant of integration)
A1 32(m) only

B1 t = 8 (s) only

M1 Integrate 16-2t
M1 Use t=4, s= their value from (b) to find the value of the constant of integration.
or 32 + integral with a lower limit of 4 (in which case you probably see these two marks
occurring with the next two. First A1 will be for 4 correctly substituted.)
A1 $s = 16t - t^2 - 16$ or equivalent
M1 substitute t = 10
A1 44
M1 Substitute t = 8 (their value from (c))
DM1 Calculate total distance (M mark dependent on the previous M mark.)
A1 52 (m)

OR the candidate who recognizes v = 16 – 2t as a straight line can divide the shape into two triangles:
M1 distance for t = 4 to t = candidates's 8 = ½ x change in time x change in speed.
A1 8-4
A1 8-0
M1 distance for t = their 8 to t = 10 =½ x change in time x change in speed.
A1 10-8
A1 0-(-4)
M1 Total distance = their (b) plus the two triangles (=32 + 16 + 4).
A1 52(m)

NB: This order on epen grid (the A's and M's will not match up.)

Q4.

Question	Scheme	Marks	AOs	Notes
(a)	$v = 12 + 4t - t^2 = 0$ and solving	M1	3.1a	Equating v to 0 and solving the quadratic If no evidence of solving, and at least one answer wrong, M0
	$t = 6$ (or -2)	A1	1.1b	6 but allow -2 as well at this stage
	Differentiate v wrt t	M1	1.1a	For differentiation (both powers decreasing by 1)
	$(a = \frac{dv}{dt} =)\ 4 - 2t$	A1	1.1b	Cao; only need RHS
	When $t = 6$, $a = -8$; Magnitude is 8 (m s^{-2})	A1	1.1b	Substitute in $t = 6$ and get 8 (m s^{-2}) as the answer . Must be **positive.** (A0 if two answers given)
		(5)		
(b)	Integrate v wrt t $(s =) 12t + 2t^2 - \frac{1}{3}t^3 (+C)$	M1 A1	3.1a 1.1b	For integration (at least two powers increasing by 1) Correct expression (ignore C) only need RHS Must be used in part (b)
	$t = 3$ => distance $= 45$ (m)	A1	1.1b	Correct distance. Ignore units
		(3)		
		(8 marks)		

Q5.

Q.	Scheme	Marks	Notes
a	$v=0 \Rightarrow 3t^2-16t+21=0$	M1	Set $v=0$ and attempt to solve
	$((3t-7)(t-3)=0)$ $t_1=\frac{7}{3}$, $t_2=3$	A1	
		(2)	
b	$a=\frac{d}{dt}(3t^2-16t+21)$	M1	Differentiate v to obtain a
	$=6t-16$	A1	
	$t=t_1$, $a=6\times\frac{7}{3}-16=-2$ (m s^{-2}) Magnitude 2 (m s^{-2})	A1	No errors seen. Must be positive - the Q asks for magnitude.
		(3)	
c	$s=\int(3t^2-16t+21)dt$	M1	Integrate v to find s
	$=t^3-8t^2+21t(+C)$	A1	
	$\pm\left((3^3-8\times9+21\times3)-\left(\left(\frac{7}{3}\right)^3-8\times\frac{49}{9}+21\times\frac{7}{3}\right)\right)$	M1	Correct use of their limits
	$s=0.148$ (m) $\left(\frac{4}{27}\right)$	A1	Final answer must be positive. 0.15 or better
		(4)	
d	Return to $O \Rightarrow s=0=t(t^2-8t+21)$	B1	seen or implied
	Discriminant of quadratic $=64-4\times21(=-20)<0$	M1	Or equivalent. ***given answer so must show some evidence of method***
	No real roots $\Rightarrow$ does not return to O	A1	Sufficient correct working to justify ***given answer***
		(3)	
dalt	Travels away until $t_1=\frac{7}{3}$, turns back at $t_2=3$ then turns away again	M1	Complete story
	$s_3=18$	B1	Seen or implied
	Complete argument	A1	
		(3)	
dalt	Distance time graph	B1	
	Locate min turning point	M1	
	Complete argument	A1	
		(3)	
		[12]	

Q6.

Question	Scheme	Marks	AOs
(a)	Multiply out and differentiate *wrt* to time (or use of product rule i.e. must have two terms with correct structure)	M1	1.1a
	$v = 2t^3 - 3t^2 + t$	A1	1.1b
	$2t^3 - 3t^2 + t = 0$ and solve: $t(2t-1)(t-1) = 0$	**DM1**	1.1b
	$t = 0$ or $t = \frac{1}{2}$ or $t = 1$; any two	A1	1.1b
	All three	A1	1.1b
		(5)	
(b)	Find x when $t = 0, \frac{1}{2}$, 1 and 2 : $(0, \frac{1}{32}, 0, 2)$	M1	2.1
	Distance = $\frac{1}{32} + \frac{1}{32} + 2$	M1	2.1
	$2\frac{1}{16}$ (m) oe or 2.06 or better	A1	1.1b
		(3)	
(c)	$x = \frac{1}{2}t^2(t-1)^2$	M1	3.1a
	$\frac{1}{2}$ perfect square so $x \geq 0$ i.e. never negative	A1 cso	2.4
		(2)	
			(10 marks)

Notes:
(a) **M1:** Must have 3 terms and at least two powers going down by 1 **A1:** A correct expression **DM1:** Dependent on first M, for equating to zero and attempting to solve a <u>cubic</u> **A1:** Any two of the three values (Two correct answers can imply a correct method) **A1:** The third value
(b) **M1:** For attempting to find the values of x (at least two) at their t values found in (a) or at $t=2$ or equivalent e.g. they may integrate their v and sub in at least two of their t values **M1:** Using a correct strategy to combine their distances (must have at least 3 distances) **A1:** $2\frac{1}{16}$ (m) oe or 2.06 or better
(c) **M1:** Identify strategy to solve the problem such as: (i) writing x as $\frac{1}{2}\times$ perfect square (ii) or using x values identified in (b). (iii) or using calculus i.e. identifying min points on x–t graph. (iv) or using x-t graph. **A1 cso :** Fully correct explanation to show that $x \geq 0$ i.e. never negative