Exam Questions – Chapter 6 + 7 Trig (A2)

Q1.

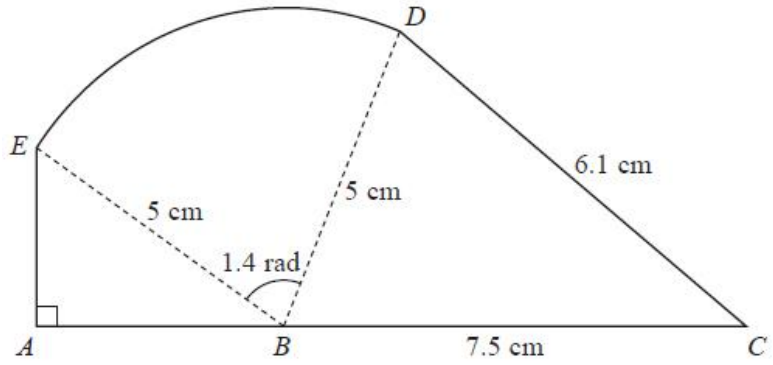

Figure 2

The shape *ABCDEA*, as shown in Figure 2, consists of a right-angled triangle *EAB* and a triangle *DBC* joined to a sector *BDE* of a circle with radius 5 cm and centre *B*.

The points *A*, *B* and *C* lie on a straight line with *BC* = 7.5 cm.

Angle *EAB* = $\pi/2$ radians, angle *EBD* = 1.4 radians and *CD* = 6.1 cm.

(a) Find, in cm^2, the area of the sector *BDE*.

(2)

(b) Find the size of the angle *DBC*, giving your answer in radians to 3 decimal places.

(2)

(c) Find, in cm^2, the area of the shape *ABCDEA*, giving your answer to 3 significant figures.

(5)

(Total 9 marks)

Q2.

The depth of water, *D* metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)° \qquad 0 \leqslant t < 24$$

where *t* is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total for question = 5 marks)

Q3.

On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where a and b are positive constants.

(a) Find a complete equation for the model.

(3)

(b) Use the model to determine the height of the carriage above the ground when $t = 40$

(1)

In an alternative model, the vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is given by

$$H = 29\cos(9t + \alpha)^\circ + \beta \qquad 0 \leq \alpha < 360^\circ$$

where α and β are constants.

(c) Find a complete equation for the alternative model.

(2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

(1)

(Total for question = 7 marks)

Q4.

(a) Prove that

$$2\cot 2x + \tan x \equiv \cot x \qquad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $-\pi \leq x < \pi$,

$$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2$$

Give your answers to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(Total for question = 10 marks)

Q5.

$$g(\theta) = 4\cos 2\theta + 2\sin 2\theta$$

Given that $g(\theta) = R\cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(a) find the exact value of R and the value of α to 2 decimal places.

(3)

(b) Hence solve, for $-90° < \theta < 90°$,

$$4\cos 2\theta + 2\sin 2\theta = 1$$

giving your answers to one decimal place.

(5)

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

(c) state the range of possible values of k.

(2)

(Total for question = 10 marks)

Q6.

(a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + a)$, where R and a are constants, $R > 0$ and $0 < a$ 90° Give the exact value of R and give the value of a to 2 decimal places.

(3)

(b) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15$$

Give your answers to one decimal place.

(5)

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15$$

Give your answer to one decimal place.

(2)

(Total for question = 10 marks)

Q7.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos\theta} + \tan\theta \equiv \frac{\cos\theta}{1-\sin\theta} \qquad \theta \neq (2n+1)90° \quad n \in \mathbb{Z}$$

(3)

Given that $\cos 2x \neq 0$

(b) solve for $0 < x < 90°$

$$\frac{1}{\cos 2x} + \tan 2x = 3\cos 2x$$

giving your answers to one decimal place.

(5)

(Total for question = 8 marks)

Q8.

(a) Show that the equation

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

can be written in the form

$$(3\sin x - 1)^2 = 2$$

(3)

(b) Hence solve, for $0 \leq x < 360°$,

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

giving your answers to 2 decimal places.

(5)

(Total for question = 8 marks)

Q9.

(a) Prove that

$\sin 2x - \tan x \equiv \tan x \cos 2x, \quad x \neq (2n + 1)90°, \quad n \in \mathbb{Z}$

(4)

(b) Given that $x \neq 90°$ and $x \neq 270°$, solve, for $0 \leq x < 360°$,

$$\sin 2x - \tan x = 3\tan x \sin x$$

Give your answers in degrees to one decimal place where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(Total for question = 9 marks)

Q10.

(i) Solve, for $0 \leq \theta < 360°$, the equation

$$9\sin(\theta + 60°) = 4$$

giving your answers to 1 decimal place.
You must show each step of your working.

(4)

(ii) Solve, for $-\pi \leq x < \pi$, the equation

$$2\tan x - 3\sin x = 0$$

giving your answers to 2 decimal places where appropriate.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

(Total 9 marks)

Q11.

In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$4\tan x = 5\cos x$$

can be written as

$$5\sin^2 x + 4\sin x - 5 = 0$$

(3)

(b) Hence solve, for $0 < x \leq 360°$

$$4\tan x = 5\cos x$$

giving your answers to one decimal place.

(4)

(c) Hence find the **number of solutions** of the equation

$$4\tan 3x = 5\cos 3x$$

in the interval $0 < x \leq 1800°$, explaining briefly the reason for your answer.

(2)

(Total for question = 9 marks)

Q12.

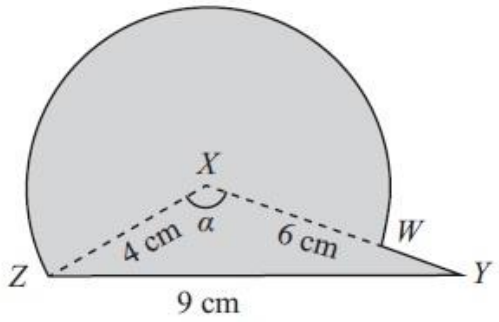

Figure 1

The triangle *XYZ* in Figure 1 has *XY* = 6 cm, *YZ* = 9 cm, *ZX* = 4 cm and angle *ZXY* = *a*. The point *W* lies on the line *XY*.

The circular arc *ZW*, in Figure 1 is a major arc of the circle with centre *X* and radius 4 cm.

(a) Show that, to 3 significant figures, *a* = 2.22 radians.

(2)

(b) Find the area, in cm^2, of the major sector *XZWX*.

(3)

The region enclosed by the major arc *ZW* of the circle and the lines *WY* and *YZ* is shown shaded in Figure 1.

Calculate

(c) the area of this shaded region,

(3)

(d) the perimeter *ZWYX* of this shaded region.

(4)

(Total 12 marks)