Mark Scheme

Q1.

Question Number	Scheme	Marks
a	$\dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}} \times \dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{3}-\sqrt{5}} = \dfrac{16\sqrt{3}-8\sqrt{5}-2\sqrt{45}+\sqrt{75}}{12-5}$	M1
	e.g. $\dfrac{21\sqrt{3}-14\sqrt{5}}{7}$	dM1
	$3\sqrt{3}-2\sqrt{5}$	A1
		(3)
b	$\left(x+5\sqrt{3}\right)\sqrt{5} = 40-2x\sqrt{3} \Rightarrow x\sqrt{5}+2x\sqrt{3} = 40-5\sqrt{15}$	M1
	$(x=)\ \dfrac{40-5\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$	A1
	$(x=)\ 15\sqrt{3}-10\sqrt{5}$	A1ft
		(3)
		(6 marks)

(a) Note that $3\sqrt{3}-2\sqrt{5}$ with no working scores M0dM0A0

M1: Attempts to rationalise the denominator by multiplying both numerator and denominator by $k\left(2\sqrt{3}-\sqrt{5}\right)$, where k is an integer usually 1, and proceeds to a fraction such as $\dfrac{...\sqrt{3}\pm...\sqrt{5}\pm...\sqrt{45}\pm...\sqrt{75}}{...}$ or $\dfrac{...}{12-5}$ or $\dfrac{...}{7}$

but not for e.g. $\dfrac{...}{12+2\sqrt{15}-2\sqrt{15}-5}$ or $\dfrac{...}{\left(2\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2}$ as these still have surds on the denominator.

Allow $\sqrt{45}$ and $\sqrt{75}$ to be written in terms of $\sqrt{3}$ and $\sqrt{5}$ as well.
Condone slips in multiplying out as well as miscopying errors.

Note $\dfrac{21\sqrt{3}-14\sqrt{5}}{7}$ with no intermediate working can still score M1 for the denominator.

Attempting to multiply by a multiple of $\dfrac{2\sqrt{3}+\sqrt{5}}{2\sqrt{3}+\sqrt{5}}$ is M0dM0A0

dM1: Attempts to simplify surds and may collect terms to achieve a fraction where

- the numerator is in terms of $\sqrt{3}$ and $\sqrt{5}$ only.
- the denominator is a multiple of 7 (which may be unsimplified)

It is dependent on the previous method mark.
If they have not fully multiplied out the numerator (or implied) then this mark cannot be scored. e.g.
$\dfrac{(8-\sqrt{15})(2\sqrt{3}-\sqrt{5})}{...} = \dfrac{16\sqrt{3}+8\sqrt{5}}{...}$ scores a maximum of M1dM0A0.

Note $\dfrac{21\sqrt{3}-14\sqrt{5}}{7}$ **with no previous working seen** scores M1dM0A0 because they have not shown any multiplying out of the brackets on the numerator.

Note $\dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}} = \dfrac{16\sqrt{3}-8\sqrt{5}-2\sqrt{45}+\sqrt{75}}{7} = 3\sqrt{3}-2\sqrt{5}$ is M1dM0A0 because they have not collected terms on the numerator (or changed them all to be in terms of $\sqrt{3}$ and $\sqrt{5}$ before simplifying to the final answer).

A1: $3\sqrt{3}-2\sqrt{5}$. The answer does not imply the method marks. Do not withhold this mark for slips in working such as invisible brackets provided they are recovered/implied by further work.

Note that a number of candidates are misreading $\frac{8-\sqrt{15}}{2\sqrt{3}+5}\times\frac{2\sqrt{3}-5}{2\sqrt{3}-5}=\frac{16\sqrt{3}-40-2\sqrt{45}+5\sqrt{15}}{12-25}$ this can score SC100 condoning one sign slip in their multiplying out of the brackets in the numerator.

(b) Note that $x=15\sqrt{3}-10\sqrt{5}$ with no working scores M0A0A0

M1: Multiplies out the brackets and isolates the x terms on one side. Condone for this mark if the surds are converted to rounded decimals.

e.g. $(\sqrt{5}+2\sqrt{3})x=5(8-\sqrt{15})$ scores M1

Alternatively divides both sides by $\sqrt{5}$ and isolates the x terms on one side.

e.g. $x+2x\frac{\sqrt{3}}{\sqrt{5}}=\frac{40}{\sqrt{5}}-5\sqrt{3}$

Condone slips in their working and invisible brackets, but there must be two terms on each side of the equation.

A1: $x=\frac{40-5\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$ or exact equivalent (which cannot be $15\sqrt{3}-10\sqrt{5}$).

In the alternative method it may be seen as $x=\frac{\frac{40}{\sqrt{5}}-5\sqrt{3}}{1+2\frac{\sqrt{3}}{\sqrt{5}}}$ which can score this mark.

Do not accept proceeding from $(\sqrt{5}+2\sqrt{3})x=5(8-\sqrt{15})\Rightarrow x=15\sqrt{3}-10\sqrt{5}$ without any intermediate expression for x. This scores A0 as the use of calculator technology is not acceptable.

An expression in decimals only is A0.

A1ft: $x=15\sqrt{3}-10\sqrt{5}$ only (or exact simplified equivalent) e.g. $x=5\left(3\sqrt{3}-2\sqrt{5}\right)$ This mark cannot be awarded without the previous A mark being scored.

Follow through on their part (a) answer of the form $a\sqrt{3}+b\sqrt{5}$ so award for $5a\sqrt{3}+5b\sqrt{5}$ oe including $5\left(a\sqrt{3}+b\sqrt{5}\right)$ where a and b may be fractions. Isw once they have achieved the correct answer.

Note: If a candidate has an unsimplified answer in part (a) e.g. $\frac{21\sqrt{3}-14\sqrt{5}}{7}$ then do not withhold this mark in part (b) if they proceed to e.g. $\frac{105\sqrt{3}-70\sqrt{5}}{7}$ as they have already been penalised once for not giving an answer in simplest form.

Alternative method – squaring both sides (send to review if unsure)

M1: Attempts to square both sides and proceeds to a three term quadratic in x with terms collected on one side of the equation (condone slips in multiplying out and collecting terms.

A1: As above in main scheme

A1ft: As above in main scheme

Q2.

Question	Scheme	Marks
	$a = 162$	B1
	$b = 5$	B1
	$c = 12$	B1
		(3 marks)

Notes

Make sure you mark in this order on epen.

B1 $a = 162$

B1 $b = 5$ condone $p = 5$

B1 $c = 12$ condone $q = 12$

Note: The values may be implied by their expression. If there is a contradiction between what appears to be their final expression and values for a, b or c which are incorrectly stated afterwards then treat this as isw.

Q3.

Question Number	Scheme	Marks
(a)	$3^{3x}=\left(3^x\right)^3=y^3$	B1
		(1)
(b)	$\dfrac{1}{3^{x-2}}=\dfrac{1}{3^x\times 3^{-2}}=\dfrac{9}{y}$	M1 A1
		(2)
(c)	$\dfrac{81}{9^{2-3x}}=\dfrac{9^2}{9^{2-3x}}=9^{2-(2-3x)}=9^{3x}=3^{6x}=y^6$	M1 A1
		(2)
		(5 marks)

Note: Correct answer in any part implies full marks for that part

(a)

B1 y^3 Condone $(y)^3$. Ignore once correct answer seen.

(b)

M1 For correct application of the addition/subtraction law so award for eg:

$\dfrac{1}{3^x\times 3^{-2}}$ or $\dfrac{1}{3^x\div 3^2}$ or $\dfrac{1}{\left(\dfrac{3^x}{3^2}\right)}$ or $\dfrac{1}{y\times 3^{-2}}$ or $\dfrac{3^2}{3^x}$ or sight of 9 or $\dfrac{1}{y}$ oe

A1 For $\dfrac{9}{y}$ or $9y^{-1}$ but NOT expressions that still contain $\div$ or fractions within fractions or 3^2y^{-1}. Ignore once correct answer seen.

(c)

M1 For simplifying the indices to expressions of the form $9^{\cdots}$ or $3^{\cdots}$ (not as a denominator) so award for 9^{3x}, $\left(9^x\right)^3$, 3^{6x}, $\left(3^x\right)^6$, or $k\times y^6$, $k\neq 0$ which must come from correct working. Also allow unsimplified equivalent expressions of the final answer eg $\dfrac{1}{y^{-6}}$ as long as it in terms of y

A1 y^6 only (not eg $\dfrac{1}{y^{-6}}$). Ignore once correct answer seen. Condon $(y)^6$

Note: In all parts they may work from $y=3^x$ and manipulate both sides to the given answer which is acceptable.

Eg part (b): $y=3^x\Rightarrow y\times 3^{-2}=3^{x-2}$ (M1) $\Rightarrow\dfrac{1}{y\times 3^{-2}}=\dfrac{1}{3^{x-2}}\Rightarrow\dfrac{9}{y}=\dfrac{1}{3^{x-2}}$ (A1)

Eg part (c): $y=3^x\Rightarrow y^{-6}=3^{-6x}\Rightarrow 81y^{-6}=3^{4-6x}$ (M1) $\Rightarrow 81y^{-6}=9^{2-3x}\Rightarrow\dfrac{81}{9^{2-3x}}=y^6$ (A1)

Q4.

Question	Scheme	Marks
	$\dfrac{21x^3-5x}{2\sqrt{x}} = \alpha x^{\frac{5}{2}} + \ldots$ or $\dfrac{21x^3-5x}{2\sqrt{x}} = \ldots + \beta x^{\frac{1}{2}}$	M1
	$\mathrm{f}(x) = \dfrac{27}{3}x^3 - \dfrac{21}{2}\times\dfrac{2}{7}x^{\frac{7}{2}} + \dfrac{5}{2}\times\dfrac{2}{3}x^{\frac{3}{2}}(+c) \quad \left(= 9x^3 - 3x^{\frac{7}{2}} + \dfrac{5}{3}x^{\frac{3}{2}}(+c)\right)$	M1A1A1
	$\mathrm{f}(9) = 10 \Rightarrow 9(9)^3 - 3(9)^{\frac{7}{2}} + \dfrac{5}{3}(9)^{\frac{3}{2}} + c = 10 \Rightarrow c = \ldots$	dM1
	$(\mathrm{f}(x) =)\ 9x^3 - 3x^{\frac{7}{2}} + \dfrac{5}{3}x^{\frac{3}{2}} - 35$	A1
		(6 marks)

Notes

On EPEN it is B1M1A1A1dM1A1. We are marking this as M1M1A1A1dM1A1

M1 Uses correct index laws to obtain at least one correct index from splitting the fraction. Award for $\alpha x^{\frac{5}{2}} + \ldots$ or $\ldots + \beta x^{\frac{1}{2}}$

M1 $x^n \to x^{n+1}$ correctly seen on one term (usually the $27x^2 \to \ldots x^3$). The indices do not need to be processed. This mark can also be awarded for integrating terms from incorrect attempts to split the fraction. Eg $\dfrac{21x^3-5x}{2x^{\frac{1}{2}}} = 42x^{\frac{7}{2}} - 10x^{\frac{3}{2}}$ $\quad 42x^{\frac{7}{2}} \to \ldots x^{\frac{9}{2}}$ or $-10x^{\frac{3}{2}} \to \ldots x^{\frac{5}{2}}$

This mark cannot be awarded for only seeing:

$$\frac{21x^3-5x}{2x^{\frac{1}{2}}} \to \frac{\ldots x^4 \pm \ldots x^2}{\ldots x^{\frac{3}{2}}}$$

A1 Two correct terms simplified or unsimplified, but the indices must have been processed.

A1 All correct simplified or unsimplified (+ c not required)

dM1 Uses f (9) = 10 and attempts to find c. Do not be too concerned by the mechanics of their arrangement. It is dependent on the previous method mark.

A1 $9x^3 - 3x^{\frac{7}{2}} + \dfrac{5}{3}x^{\frac{3}{2}} - 35$ All correct and simplified. Accept other simplified equivalent expressions for $\mathrm{f}(x)$ such as $\dfrac{1}{3}(27x^3 - 9x^{\frac{7}{2}} + 5x^{\frac{3}{2}} - 105)$

Q5.

Question Number	Scheme	Marks
(a)	$\left(2\sqrt{2}\right)^2 = p^2 + q^2 - 2pq\cos 60°$ oe	M1
	$p^2 + q^2 - pq = 8$ *	A1*
		(2)
(b)	$q = p + 2 \Rightarrow 8 = p^2 + ("p+2")^2 - p("p+2")$	M1
	$p^2 + 2p - 4 = 0$ or $q^2 - 2q - 4 = 0$	A1
	$p = \dfrac{-2 \pm \sqrt{2^2 - 4\times1\times(-4)}}{2}$ or $q = \dfrac{2 \pm \sqrt{(-2)^2 - 4\times1\times(-4)}}{2}$	M1
	$p = -1 + \sqrt{5}$ or $q = 1 + \sqrt{5}$	B1 (A1 on EPEN)
	$p = -1 + \sqrt{5}$ and $q = 1 + \sqrt{5}$ only	A1cso
		(5)
(c)	$\text{Area} = \dfrac{1}{2} \times ("-1+\sqrt{5}")("1+\sqrt{5}") \times \sin 60°$	M1
	$\text{Area} = \sqrt{3}$ (m^2)	A1
		(2)

Alt(a)	Forming a line BX which is perpendicular to AC where X is on the line AC. $AX = p\cos 60 = \dfrac{p}{2}$ $BX = \sqrt{p^2 - \left(\dfrac{p}{2}\right)^2} = \dfrac{\sqrt{3}}{2}p$ or $BX = p\sin 60$ $\left(\dfrac{\sqrt{3}}{2}p\right)^2 + \left(q - \dfrac{p}{2}\right)^2 = \left(2\sqrt{2}\right)^2$ or $\left(p\sin 60\right)^2 + \left(q - \dfrac{p}{2}\right)^2 = \left(2\sqrt{2}\right)^2$	M1
	$\dfrac{3p^2}{4} + q^2 - pq + \dfrac{p^2}{4} = 8$	
	$p^2 + q^2 - pq = 8$ *	A1*
		(9 marks)

(a)

M1 $\left(2\sqrt{2}\right)^2 = p^2+q^2-2pq\cos 60°$ or $\left(2\sqrt{2}\right)^2 = p^2+q^2-2pq\times\frac{1}{2}$ or $8 = p^2+q^2-2pq\cos 60°$

They may carry this out in two stages by forming two right angled triangles with BX being perpendicular to AC (see Alt(a)). To score this mark they must proceed as far as

$\left(\frac{\sqrt{3}}{2}p\right)^2+\left(q-\frac{p}{2}\right)^2=\left(2\sqrt{2}\right)^2$ or $\left(p\sin 60\right)^2+\left(q-\frac{p}{2}\right)^2=\left(2\sqrt{2}\right)^2$

Condone missing brackets for M1.

A1* Achieves $p^2+q^2-pq=8$ with no errors including omission of brackets. One of the lines above must have been seen for M1A1. If they state = 8 without showing any working then A1 cannot be scored.

(b)

M1 Substitutes $q=p+2$ (oe) into the given equation

A1 $p^2+2p-4=0$ or the equivalent equation in q ($q^2-2q-4=0$)

M1 Attempts to solve to find p using the formula or completing the square using their values. Alternatively, they achieve a quadratic equation in q and attempt to solve to find q using their values. Usual rules for solving quadratics apply. If they state the roots or factorise then M0. If they use the quadratic formula then the values must be embedded.

B1 $p=-1+\sqrt{5}$ **or** $q=1+\sqrt{5}$ (ignore any other solutions) Must be exact. This is independent of the previous method mark so if the roots are just stated this mark can be scored. (Note this is A1 on EPEN)

A1 $p=-1+\sqrt{5}$ **and** $q=1+\sqrt{5}$ only cso (all other marks must have been scored to award A1)

(c)

M1 Attempts to find the area of the triangle using $\frac{1}{2}\times(\text{"}-1+\sqrt{5}\text{"})(\text{"}1+\sqrt{5}\text{"})\times\sin 60°$. Must see at least one stage of working using their p and their q

A1 $\sqrt{3}$ (m^2) condone lack of units. Do not accept rounded answers.

Q6.

Question Number	Scheme	Marks
	$x-6x^{\frac{1}{2}}+4=0$	
	$x^{\frac{1}{2}}=3\pm\sqrt{5}$ oe	M1 A1
	$x=\left(3\pm\sqrt{5}\right)^2 \Rightarrow x=14\pm6\sqrt{5}$	M1 A1 A1
		(5 marks)

M1 For attempting to solve an equation of the form $y^2-6y+4=0$ by completing the square or quadratic formula to reach at least one solution. There must be some working shown for this mark to be awarded, accept as a minimum identifying $y=x^{\frac{1}{2}}$ and writing the quadratic in y before solutions.

A1 $\left(x^{\frac{1}{2}}\right)=3\pm\sqrt{5}$ Both required (though one may be later rejected) but need not be simplified, so accept $\dfrac{6\pm2\sqrt{5}}{2}$

M1 For attempting to square a solution of the form $p\pm q\sqrt{r}$ with 2 (out of 4) correct terms (may be implied by correct answers for their terms, but must have seen at least one solution for $x^{\frac{1}{2}}$)

A1 $x=14+6\sqrt{5}$ **or** $x=14-6\sqrt{5}$ as an answer Accept equivalents for this mark.

A1 $x=14+6\sqrt{5}$ **and** $x=14-6\sqrt{5}$ as answers, must be simplified.

Special Case: For candidates who show no initial working and write $x^{\frac{1}{2}}=3\pm\sqrt{5}$ as their first step, M0A0M1A1A1 is possible if they go on to achieve correct answers

Question Number	Scheme	Marks
Alt	$x+4=6x^{\frac{1}{2}}$	
	$(x+4)^2=36x$	M1 A1
	$x^2-28x+16=0 \Rightarrow (x-14)^2=180 \Rightarrow x=14\pm\sqrt{180} \Rightarrow x=14\pm6\sqrt{5}$	M1 A1 A1
		(5)

M1 Isolates the square root term and squares both sides.

A1 Correct squared expression, $(x+4)^2$ need not be expanded (as in scheme).

M1 Expands and solves the quadratic in x Note that candidates who square term by term will score no marks.

A1A1 As main scheme. Note for the final A both solutions must be fully simplified.

Q7.

Question Number	Scheme	Marks
	Attempts both sides as powers of 3 $\frac{3^x}{3^{4y}} = 3^3 \times 3^{0.5} \Rightarrow 3^{x-4y} = 3^{"3.5"}$	M1
	Sets powers equal and attempts to makes y the subject : $x - 4y = "3.5" \Rightarrow y = \ldots$	dM1
	$y = \frac{1}{4}x - \frac{7}{8}$	A1
		(3) **(3 marks)**
Alt1	Multiplies by 3^{4y} first:	
	Attempts both sides as powers of 3 $3^x = 27\sqrt{3} \times 3^{4y} \Rightarrow 3^x = 3^{"3.5"+4y}$ (Addition law on RHS)	M1
	Sets powers equal and makes y the subject $x = "3.5" + 4y \Rightarrow y = \ldots$	dM1
	$y = \frac{1}{4}x - \frac{7}{8}$	A1

Alt2	Divides by $27\sqrt{3}$ first: Attempts both sides as powers of 3 $\frac{3^x}{3^{4y}\times 27\sqrt{3}}=1\Rightarrow 3^{x-4y-"3.5"}=3^0$ (Subtraction law on LHS) Sets powers equal and makes y the subject $x-"3.5"-4y=0\Rightarrow y=...$ $y=\frac{1}{4}x-\frac{7}{8}$	M1 dM1 A1
Alt3	Takes logs of both sides Eg Base 3: $\log_3\left(\frac{3^x}{3^{4y}}\right)=\log_3\left(27\sqrt{3}\right)$ $\log_3 3^x-\log_3 3^{4y}=\log_3 27\sqrt{3}$ $x-4y=3.5\Rightarrow y=...$ $y=\frac{1}{4}x-\frac{7}{8}$	 M1 dM1 A1

M1 Attempts to use the subtraction law on the LHS and the addition law on the RHS to achieve a form of $3^{\cdots}=3^{\cdots}$

Condone errors writing $27\sqrt{3}$ as a single power of 3 but it must be clear what the two indices are before adding if they make an error ($27=3^a$ and $\sqrt{3}=3^b$ so $27\sqrt{3}=3^{a+b}$)
A common mistake is to write $27\sqrt{3}\Rightarrow\sqrt{9}\times\sqrt{3}\times\sqrt{3}=3^2$ which can be condoned and they can still get M1M1A0.

They may rearrange the equation first so look for attempts at the appropriate index laws being applied (see alternatives). In Alt2 allow the RHS=1.

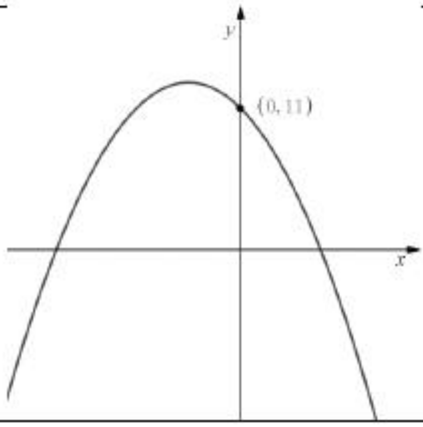
They may use logs on both sides (the most likely would be base 3 - see Alt3) To score M1 they would need to take logs and then apply the laws of logs to either add or subtract.

dM1 Dependent upon the previous M mark, it is for an attempt to make y the subject. For this mark follow through their power for $27\sqrt{3}$ but they must have 3 terms in their equation relating to the powers and they cannot "lose" one in the rearrangement. (i.e $ax+by+c=0$ oe where $a,b,c\neq 0$) Do not award this mark if they rearrange to make x the subject. Condone sign slips only.

A1 $y=\frac{1}{4}x-\frac{7}{8}$ or exact simplified equivalent eg $y=\frac{2x-7}{8}$, $y=0.25x-0.875$

DO NOT ACCEPT $y=\frac{x-\frac{7}{2}}{4}$ or $y=\frac{x-3.5}{4}$

Q8.

Question Number	Scheme	Marks
(a)	$\mathrm{f}(x) = 11 - 4x - 2x^2$ $\Rightarrow \ldots \underline{-2}(2x + x^2)\ldots$ or $\Rightarrow \ldots \underline{-2}(2x + x^2 \ldots)$	B1
	$\ldots(2x + x^2) \Rightarrow \ldots\left((x+1)^2 \pm \ldots\right)$	M1
	$(\mathrm{f}(x) =)\ 13 - 2(x+1)^2$	A1
		(3)
(b)		M1 A1
		(2)
(c)	$x = -1$	B1ft
		(1)
		Total 6
Alt(a)	$a + b(x^2 + 2cx + c^2) = 11 - 4x - 2x^2$ $b = -2$	B1
	$2bc = -4 \Rightarrow c = \ldots\ (= 1)$	M1
	$a + bc^2 = -4 \Rightarrow a = \ldots\ (= 13)$ $(\mathrm{f}(x) =)\ 13 - 2(x+1)^2$	A1

(a)

B1: $b = -2$

M1: Attempts to complete the square on $x^2 \pm 2x$ so score for $(x \pm 1)^2 \pm \ldots$ or alternatively attempts to compare coefficients to find a value for *c*.
Condone $11 - 4x - 2x^2 \Rightarrow \ldots - 2(2x - x^2)\ldots \Rightarrow -2(x \pm 1)^2 \pm \ldots$

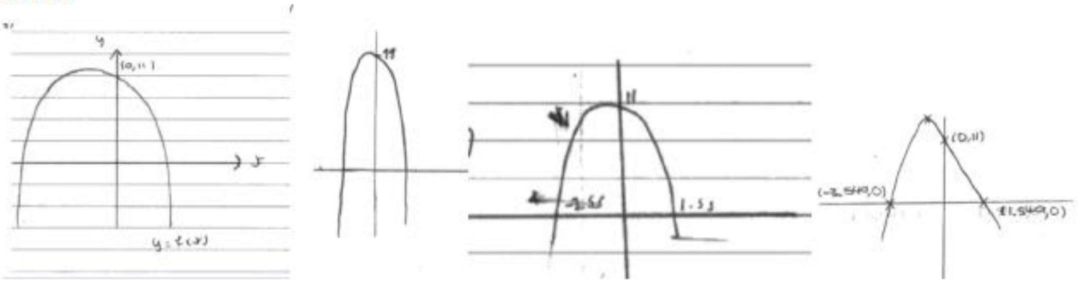
A1: $13 - 2(x+1)^2$. If *a*, *b* and *c* are stated and there is a contradiction then mark their final expression. Condone $13 + -2(x+1)^2$ or just the values of *a*, *b* and *c* being stated.

(b)

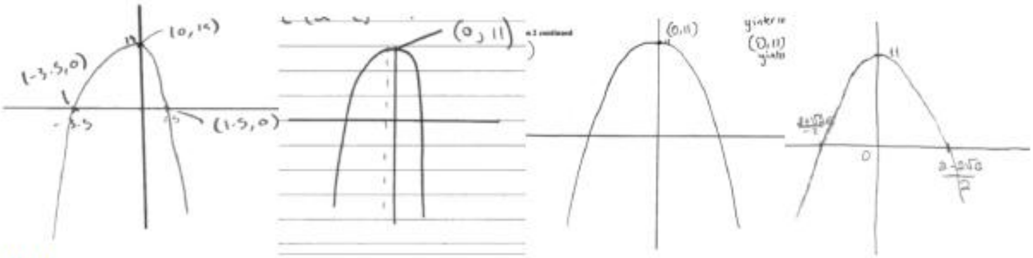
M1: ∩ shape anywhere on a set of axes Cannot be a part of other functions (eg a cubic). See examples below

A1: Correct shape that cuts x-axis once either side of the origin, maximum in quadrant 2 and y intercept (0, 11) or 11 marked on y-axis. Condone $(11, 0)$ if the intercept is in the correct place. Do not be concerned with any labelled x-intercepts. Condone aspects of the graph which may appear linear (but not a ^ shape). Its line of symmetry must appear to the left of the origin and its curve should broadly appear symmetrical about this line. Ignore x-intercepts or the maximum stated.
The y-intercept may be stated underneath their graph instead, but if there is a contradiction between the graph and what is marked on the graph then the graph takes precedence.
Do not accept graphs which appear to be symmetrical about the y-axis.
Examples

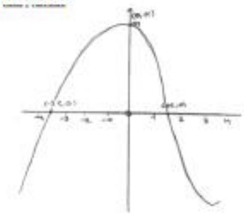
M1A1



M1A0



M0A0



(c)

B1ft: Correct equation seen in part (c) (follow through their numeric c so allow $x = -$ "c").

Q9.

Question	Scheme	Marks
(a)	$x^2+kx-9=-3x^2-5x+k \Rightarrow 4x^2+kx+5x-9-k(=0)$	M1
	$b^2-4ac=0 \Rightarrow (k+5)^2-4\times4(-9-k)=0$	M1
	$k^2+26k+169=0$*	A1*
		(3)
(b)	$k^2+26k+169=0 \Rightarrow k=-13$	B1
	$k=-13 \Rightarrow 4x^2-8x+4=0 \Rightarrow x=...$	M1
	$(1, -21)$	A1
		(3)
		Total 6

Notes:

Mark (a) and (b) together

(a)

M1: Equates both curves and collects terms to one side. Allow slips but there must be some attempt to collect terms to one side. The " = 0" may be implied by their attempt at the discriminant.

M1: Attempts to find the discriminant "b^2-4ac" of a 3TQ. May be seen as an attempt at e.g. $b^2-4ac=0$ or equivalent e.g. $b^2=4ac$ or may be seen embedded in an attempt at the quadratic formula.
It requires b and c both of the form $pk+q$, $p,q\neq0$ and requires a as a constant.
If clearly a wrong formula/expression is used e.g. "b^2+4ac" this scores M0

A1*: Achieves the printed answer with no errors and sufficient working shown.
The " = 0" must appear at least once before the printed answer unless they start with $b^2=4ac$.

(b)

B1: Correct value of k. Condone $x=-13$ if it is subsequently used as a value for k.

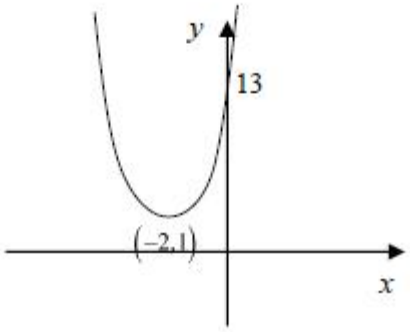
M1: Either:

- substitutes their value of k into their equation obtained by equating the curves and collecting terms from part (a) and solves a 3TQ for x by any correct method including a calculator or
- substitutes their k into the 2 given equations, equates, collects terms to one side and solves a 3TQ for x by any correct method including a calculator

A1: Correct coordinates. Allow as e.g. $x=1$, $y=-21$

Correct answer only in (b) scores 3/3

Q10.

Question	Scheme		Marks
(a)	$a=3$		B1
	$b=\pm 2$		M1
	$3x^2+12x+13=3(x+2)^2+1$ or $a=3,\ b=2,\ c=1$		A1
			(3)
(b)		Correct U shape with minimum in second quadrant	B1
		Intercept 13 on y-axis.	B1
		Vertex at $(-2,1)$	B1ft
			(3)
			(6 marks)

Notes:

(a)

B1: Correct value for a stated or shown by working.

M1: Obtains $b=\pm 2$. Allow unsimplified e.g. $b=\pm\frac{4}{2},\ \pm\frac{12}{6}$. May be implied by e.g. $3(x\pm 2)^2+\ldots$

A1: Fully correct expression or correct values.

Note that there are various methods e.g. $ax^2+bx+c=a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}\Rightarrow 3x^2+12x+13=\ldots$

or $a(x+b)^2+c=ax^2+2abx+ab^2+c\Rightarrow a=3,\ 2ab=12, ab^2+c=13\Rightarrow b,c=\ldots$

(b)

There must be a sketch to score marks in (b)

Labelling on the sketch takes precedence

Treat part (b) as Hence, or otherwise, i.e. part (a) may be incorrect but full marks are available in part (b) for a correct sketch of $y=3x^2+12x+13$

B1: Correct U shape with minimum point in the second quadrant but **not** on the x-axis. Do not allow a clear V shape.

B1: Correct y intercept labelled or stated. Allow as just 13 or (0, 13) or (13, 0) as long as it is in the correct position. If stated away from the sketch it must be as (0, 13) and correspond to the sketch. Their curve must at least touch at (0, 13). Any other intercepts can be ignored.

B1ft: Correct vertex labelled in some way, or follow through their b and c i.e. $x=-b$ and $y=c$. Must be a turning point but could be a maximum or a minimum. Allow the labelling as shown or as $x=..,\ y=\ldots$ or as the ordinates shown on the axes. It must correspond to the sketch. If stated away from the sketch it must be correct or correct follow through and correspond to the sketch.

Q11.

Question Number	Scheme	Marks
(a)(i)	$-a+6a+8+a^2=32 \Rightarrow a^2+5a-24=0$	M1
	$(a+8)(a-3)=0$	dM1
	$a=3$ or $a=-8$ and chooses $a=3$ with reason *	A1* cso
		(3)
(ii)	$3x^3+26x^2-9x=0 \Rightarrow x(3x^2+26x-9)=0$	M1
	$x(3x-1)(x+9)$	
	$(x=)\ 0, \frac{1}{3}, -9$	A1
		(2)
(b)(i)	$(y=)\ 0$	B1
	$y^{\frac{1}{3}}="\frac{1}{3}"$ or $y^{\frac{1}{3}}="-9" \Rightarrow y=\ldots$ (or $(-9)^3=\ldots$ or $\left(\frac{1}{3}\right)^3=\ldots$)	M1
	$(y=)\ \frac{1}{27}, -729$	A1
		(3)
(b)(ii)	$9^z="\frac{1}{3}" \rightarrow z=\ldots$	M1
	$(z=)\ -\frac{1}{2}$ only	A1
		(2)
		(10 marks)

(a)(i)

M1 Substitutes in $x = \pm 1, y = 32$ and proceeds to a 3TQ in terms of a with all terms on one side of the equation. Condone the lack of = 0 and condone slips in their rearrangement.

dM1 Attempts to solve their quadratic equation by either factorising, completing the square or the quadratic formula. They cannot just state the roots. It is dependent on the first method mark. In all cases they must show their working to score this mark so:

- Solving by factorising requires the factorised form of their $a^2 + 5a - 24 = 0$ to be stated before proceeding to the roots i.e $(a+"8")(a-"3")$ $(=0)$
- Solving by using the quadratic formula requires the values for a, b and c to be stated in the formula before proceeding to the roots. They cannot just state the values of a, b and c.
- Solving by completing the square requires eg $\left(a \pm "\frac{5}{2}"\right)^2 \pm \ldots$ before rearranging to find the roots.

A1* $a = 3$ or $a = -8$ and chooses $a = 3$ with a minimal reason. Eg "as a is a positive constant" or "Since $a > 0$", "a cannot be negative".

The final mark cannot be scored without the previous method marks being scored and there cannot be any errors even if missing brackets are recovered. Just crossing out -8 without a reason is A0.

(a)(ii)

M1 Takes out a factor of x from the given cubic with $a = 3$ (or divides through by x) and attempts to solve the resulting quadratic equation. You must see at least one intermediate line of working before proceeding to the roots eg the factorised form, values in the quadratic formula or completed square form.

A1 $(x =)$ $0, \frac{1}{3}, -9$ provided M1 has been scored.

Solutions with no working in this part scores 0 marks.

(b)(i)

B1 $(y =)$ 0

M1 Sets $y^{\frac{1}{3}}$ equal to any of their non-zero solutions from part (a) and attempts to cube their value to find a value for y. You must see at least one stage of working to score this mark. $(-9)^3 = \ldots$ on its own scores M1.

A1 $(y =)$ $\frac{1}{27}, -729$ and no others other than 0.

Solutions without any working will score a maximum of B1M0A0 in this part.

(ii)

M1 Sets 9^z equal to any of their positive solutions found in part (a) and proceeds to find a value for z. They may write 9^z as 3^{2z} before proceeding to find a value for z.
Alternatively, you may see attempts to link b(i) and b(ii) together:
Eg $y = 9^{3z} \Rightarrow \frac{1}{27} = 729^z \Rightarrow z = \ldots$ which can score M1.
You must see at least one stage of working to score this mark.
The method may include log statements which is acceptable.

A1 $(z =)$ $-\frac{1}{2}$ only (provided M1 has been scored).

Answer stated without working scores 0 marks. Evidence of calculator use will also be A0.

Q12.

Question	Scheme		Marks
	$y = -4x - 1$ $\Rightarrow (-4x-1)^2 + 5x^2 + 2x = 0$	Attempts to makes y the subject of the linear equation and substitutes into the other equation.	M1
	$21x^2 + 10x + 1 = 0$	Correct 3 term quadratic	A1
	$(7x+1)(3x+1) = 0 \Rightarrow (x=) -\frac{1}{7}, -\frac{1}{3}$	dM1: Solves a **3 term** quadratic by the usual rules A1: $(x=)\ -\frac{1}{7}, -\frac{1}{3}$	dM1A1
	$y = -\frac{3}{7}, \frac{1}{3}$	M1: Substitutes to find at least one y value A1: $y = -\frac{3}{7}, \frac{1}{3}$	M1 A1
			(6)
	Alternative		
	$x = -\frac{1}{4}y - \frac{1}{4}$ $\Rightarrow y^2 + 5\left(-\frac{1}{4}y - \frac{1}{4}\right)^2 + 2\left(-\frac{1}{4}y - \frac{1}{4}\right) = 0$	Attempts to makes x the subject of the linear equation and substitutes into the other equation.	M1
	$\frac{21}{16}y^2 + \frac{1}{8}y - \frac{3}{16} = 0$ $(21y^2 + 2y - 3 = 0)$	Correct 3 term quadratic	A1
	$(7y+3)(3y-1) = 0 \Rightarrow (y=) -\frac{3}{7}, \frac{1}{3}$	Solves a **3 term** quadratic	dM1
		$(y=) -\frac{3}{7}, \frac{1}{3}$	A1
	$x = -\frac{1}{7}, -\frac{1}{3}$	Substitutes to find at least one x value.	M1
		$x = -\frac{1}{7}, -\frac{1}{3}$	A1
			(6)
			(6 marks)

Q13.

Question Number	Scheme	Marks
(a)	Attempts perimeter of garden = $2\times5x+2\times(6x-2)$	M1
	Sets $2\times5x+2\times(6x-2)>29\Rightarrow22x>33$	dM1
	$\Rightarrow x>\frac{33}{22}\Rightarrow x>1.5$ *	A1*
		(3)
(b)	Attempts area of garden = $2x(2x-1)+3x(6x-2)$	M1
	Sets $A<72\Rightarrow\ 22x^2-8x-72<0$	A1
	Finds critical values $11x^2-4x-36\Rightarrow x=-\frac{18}{11},2$	M1
	Chooses inside region	ddM1
	$-\frac{18}{11}<x<2$	A1
		(5)
(c)	$1.5<x<2$	B1
		(1)
		(9 marks)

(a)

M1 An attempt at finding the perimeter of the garden.
Scored for sight of $5x+2x-1+2x+6x-2+$ additional term(s) involving x
Individual lengths may not be seen so imply for sight of a total of $ax+b$, where $a>15$

dM1 Sets their $P>29$ and attempts to solve by proceeding to $ax>c$
You may condone an attempt in which $P=29\Rightarrow ax=c$

A1* cso with at least one correct intermediate (simplified) line $22x>33$ or $x>\frac{33}{22}$ before $x>1.5$ seen.
Condone an attempt in which you see $P=29\Rightarrow x=1.5$ before $x>1.5$ seen
Note that it is possible to start with $x>1.5$ and prove $P>29$ but for the A1* to be scored there must be a final statement of the type "hence $x>1.5$". There is no requirement for any units

(b) **Mark part (b) and (c) together**

M1 For an attempt at finding the area of the garden. For this to be scored look for
The sum of two areas $2x(2x-1)+...x(6x-2)$ condoning slips
The sum of two areas $5x(2x-1)+...x(..x\pm...)$ condoning slips
The difference between two areas $5x(6x-2)-2x(..x\pm...)$ condoning slips.

A1 A "correct and simplified" equality or inequality, condoning $<\leftrightarrow\leqslant\leftrightarrow=$ Eg. $22x^2-8x-72<0$ oe

M1 A valid attempt to find the critical values of their 3TQ. Allow factorisation, formula, completion of square or use of calculator. If a calculator is used then the answer(s) must be correct for their 3TQ.
Condone candidates who fail to state the negative root of their quadratic.

ddM1 Dependent upon both M's. For choosing the inside region for their critical values. Condone $<\leftrightarrow\leqslant$
Condone for this mark replacing a negative root with 0, 0.5 or 1.5. So accept for example one of either $1.5<x<\text{"2"}$, $0<x<\text{"2"}$ or $0.5<x<\text{"2"}$

A1 $-\frac{18}{11}<x<2$ Allow $0<x<2$ or $0.5<x<2$ due to context. Allow alternative notation. See below

(c)

B1 $1.5<x<2$. Accept versions such as $(1.5,2)$, $x>1.5$ **and** $x<2$, $x>1.5\cap\ x<2$
Do not allow $x>1.5$ or $x<2$ $\quad x>1.5,\ x<2$

Q14.

Question Number	Scheme	Marks
(a)	$2\times 4^{x}-2^{x+3}=17\times 2^{x-1}-4$	
	Uses an index law and states or implies any of	
	$4^{x}=p^{2},\quad 2^{x+3}=8p\quad\text{or}\quad 2^{x-1}=\frac{p}{2}$	B1
	Writes the given equation in terms of p	
	$2\times 4^{x}-2^{x+3}=17\times 2^{x-1}-4\Rightarrow 2p^{2}-2^{3}\times p=\frac{17p}{2^{1}}-4$	M1
	Proceeds to $4p^{2}-33p+8=0$ via $2p^{2}-8p=\frac{17p}{2}-4$ * CSO	A1*
		(3)
(b)	$4p^{2}-33p+8=0\Rightarrow(4p-1)(p-8)=0\Rightarrow p=...,...$	M1
	Sets $2^{x}=\frac{1}{4},8\Rightarrow x=...$	M1
	$x=-2,3$	A1
		(3)
		(6 marks)

(a) Watch out here as this is a given answer. All stages of working are required to score the 3 marks

B1: Uses an index law and states or implies any of $4^{x}=p^{2},\quad 2^{x+3}=8p$ or $2^{x-1}=\frac{p}{2}$

Allow equivalents such as $4^{x}=p\times p,\ 2^{x+3}=8\times p, p\times 8$ or $2^{x-1}=\frac{1}{2}p, p\div 2, 0.5p$

M1: Attempts to write the given equation in x to a quadratic equation in terms of p

$2\times 4^{x}-2^{x+3}=17\times 2^{x-1}-4\Rightarrow 2p^{2}-2^{3}\times p=\frac{17p}{2^{1}}-4$ or $2p^{2}-2^{3}\times p=17p\times 2^{-1}-4$

All three index laws must be seen but condone slips on signs or on the 2^{3} if there was an incorrect attempt to process.

$2\times 4^{x}-2^{x+3}=17\times 2^{x-1}-4\Rightarrow 2p^{2}-6\times p=\frac{17p}{2^{1}}-4$ would be fine for the M1

Allow a recovery for this M1, e.g $2^{x+3}=2^{x}+2^{3}=8p$ but not the A1*

Watch for candidates who manipulate the given equation first. This is acceptable.

$2\times 4^{x}-2^{x+3}=17\times 2^{x-1}-4\Rightarrow 4^{x}-2^{x+2}=17\times 2^{x-2}-2$

So $\Rightarrow p^{2}-2^{2}\times p=\frac{17p}{2^{2}}-2$

A1*: CSO Proceeds to the given answer of $4p^{2}-33p+8=0$ with no errors or omissions.

Condone working such as $4^{x}=2^{x^{2}}$ leading to p^{2}. It is often hard to decipher the relative heights of the indices

An intermediate line of $2p^{2}-8p=\frac{17p}{2}-4$ o.e. must be seen.

(b) **This is a non calculator part so the use of a calculator is penalised**

M1: Valid **non calculator** attempt at solving $4p^2 - 33p + 8 = 0$. Allow slips/miscopies on $4p^2 - 33p + 8 = 0$ for example, $4p^2 - 33p - 8 = 0$. The roots cannot just appear.

Examples such as $4p^2 - 33p + 8 = (p-8)\left(p - \frac{1}{4}\right) = 0 \Rightarrow p = 8, \frac{1}{4}$ is obvious calculator work and scores M0

Award for an attempt to

- factorise (usual rules) leading to values for (p)
- use the quadratic formula condoning this calculation $\frac{33 \pm \sqrt{(-33)^2 - 4(4)(8)}}{2(4)}$ followed by $\frac{1}{4}, 8$
- complete the square leading to values for (p)

M1: Valid non calculator attempt at solving an equation of the form $2^x = k, \quad k > 0$

This can be implied for a correct solution for either root.

If the value of k is not a power of 2 then score for $2^x = k \Rightarrow x = \log_2 k$

A1: Both solutions $x = -2, 3$ following the correct quadratic equation in p. There is no need to state $x =$ ..

If they then go on to reject $x = -2$ say then A0

Note that 011 is possible in part (b) for candidates who don't show a non calculator solution of $4p^2 - 33p + 8 = 0$

Q15.

Question Number	Scheme	Marks
(i)	$\sqrt{8}=2\sqrt{2}$ seen anywhere in the solution (see notes)	B1
	$\left(x+\sqrt{2}\right)^2+\left(3x-5\sqrt{8}\right)^2$ $=x^2+2x\sqrt{2}+2+9x^2-30x\sqrt{8}+25\times 8$	M1
	$=10x^2-58x\sqrt{2}+202$	A1
		(3)
(ii)	$\sqrt{3}\left(4y-3\sqrt{3}\right)=5y+\sqrt{3}$ $\Rightarrow 4\sqrt{3}y-9=5y+\sqrt{3}$ $\Rightarrow 4\sqrt{3}y-5y=9+\sqrt{3}$	M1
	$\Rightarrow y=\ldots$ or $\Rightarrow ky=\ldots$ eg $y=\text{"}\dfrac{9+\sqrt{3}}{4\sqrt{3}-5}\text{"}$ or $\text{"}23\text{"}y=\text{"}9+\sqrt{3}\text{"}$	dM1
	$y=\dfrac{9+\sqrt{3}}{4\sqrt{3}-5}\times\dfrac{4\sqrt{3}+5}{4\sqrt{3}+5}\Rightarrow y=\dfrac{\ldots}{\text{"}23\text{"}}$	ddM1
	$y=\dfrac{57}{23}+\dfrac{41}{23}\sqrt{3}$ (or $y=2\dfrac{11}{23}+1\dfrac{18}{23}\sqrt{3}$)	A1
		(4)
	(ii) Alternative 1:	
	$\sqrt{3}\left(4p+4q\sqrt{3}-3\sqrt{3}\right)=5\left(p+q\sqrt{3}\right)+\sqrt{3}$ $\Rightarrow 4p\sqrt{3}+12q-9=5p+5q\sqrt{3}+\sqrt{3}$ $\Rightarrow 4p=5q+1,\ 12q-9=5p$	M1 dM1
	$\Rightarrow 4p=5q+1,\ 12q-9=5p$ $\Rightarrow p=\ldots,\ q=\ldots$	ddM1
	$y=\dfrac{57}{23}+\dfrac{41}{23}\sqrt{3}$ (or $y=2\dfrac{11}{23}+1\dfrac{18}{23}\sqrt{3}$)	A1
		Total 7

Note solutions relying on calculator technology are not acceptable.
(i)

B1: May be stated in the margin or seen/implied anywhere in the solution.
It does not need to be explicitly stated.
Eg. sight of $30\sqrt{8} \to 60\sqrt{2}$ is fine as is $\left(3x-5\sqrt{8}\right)^2 \to ...x^2 - 60\sqrt{2} + ...$

M1: Expands both brackets so look for:
$x^2 + ax\sqrt{2} + 2 + bx^2 - cx\sqrt{8} + d$ or $x^2 + ax\sqrt{2} + 2 + bx^2 - cx\sqrt{2} + d$
where a, b, c and d are non-zero. **The x terms do not need to be collected to score this mark.**

A1: $10x^2 - 58x\sqrt{2} + 202$ or equivalent isw after a correct answer. $58\sqrt{2}x$ is acceptable

Eg $x^2 + 2x\sqrt{2} + 2 + 9x^2 - 30x\sqrt{8} + 25 \times 8 \Rightarrow 10x^2 - 58x\sqrt{2} + 202$ scores B1M1A1 as $\sqrt{8} = 2\sqrt{2}$ is correctly seen by the correct collection of the terms.
Eg $x^2 + 2x\sqrt{2} + 2 + 9x^2 - 30x\sqrt{8} + 25 \times 8 \Rightarrow 10x^2 - 60x\sqrt{2} + 202$ scores B0M1A0 as $\sqrt{8} = 2\sqrt{2}$ is not correctly seen or implied as the coefficient of x is incorrect

(ii) On EPEN this is M1A1M1A1 we are marking this M1M1M1A1

M1: Attempts to multiply out and isolates the two y terms on one side of the equation.
Condone sign slips only in their rearrangement.

dM1: Attempts to make y or ky the subject (where k is an integer or fraction which would simplify to an integer). Score for $y = \mathrm{f}(x)$ or $ky = \mathrm{g}(x)$ where the function includes $\sqrt{3}$ but it cannot be scored for directly proceeding to $y = \frac{57}{23} + \frac{41}{23}\sqrt{3}$ or $y = \frac{57 + 41\sqrt{3}}{23}$.
It is dependent on the previous method mark.
Note $y\left(4\sqrt{3} - 5\right) = 9 + \sqrt{3} \Rightarrow y = \frac{57}{23} + \frac{41}{23}\sqrt{3}$ is M1dM0ddM0A0

ddM1: Proceeds to $y = ...$ with a rational denominator. The denominator does not need to be simplified. It is dependent on the previous method mark. They must have shown the step of either rationalising the denominator or showing how they achieve $ky = ...$ and proceeds to $y = ...$

A1: Correct answer in the correct form with all stages of working seen
- Collects y terms on one side of the equation
- Makes y the subject
- Rationalises and proceeds to the correct answer. Somewhere in their solution they will have had to have multiplied two brackets involving surds together and they must have shown this being multiplied out (calculators are not allowed)

  eg $\frac{9+\sqrt{3}}{4\sqrt{3}-5} \times \frac{4\sqrt{3}+5}{4\sqrt{3}+5} = \frac{36\sqrt{3} + 45 + 12 + 5\sqrt{3}}{23}$

Condone solutions with invisible brackets to score full marks, provided the general method is sound and the answer has not just come from a calculator or incorrect working.

Do not accept $y = \frac{57 + 41\sqrt{3}}{23}$

Alt(ii)1 (main scheme alternative)

M1: Substitutes $y = p + q\sqrt{3}$ expands, collects terms. Condone sign slips.

dM1: Compares rational/irrational parts to form two equations. It is dependent on the previous method mark.

ddM1: Solves 2 linear equations in p and q using an acceptable method. They cannot just state the values. Condone slips in their working. It is dependent on the previous method mark.

A1: $y = \frac{57}{23} + \frac{41}{23}\sqrt{3}$ (or $y = 2\frac{11}{23} + 1\frac{18}{23}\sqrt{3}$) with full working shown.

Alt(ii)2 Squaring and completing the square

M1: Squares both sides and multiplies out brackets (condone slips)
eg. $(48y^2 - 72\sqrt{3}y + 81 = 25y^2 + 3 + 10\sqrt{3}y)$

dM1: Rearranges to form a 3TQ = 0 $(23y^2 - 82\sqrt{3}y + 78 = 0)$. It is dependent on the previous method mark.

ddM1: Attempts to complete the square and proceeds to make y the subject. It is dependent on the previous method mark.

A1: $y = \frac{57}{23} + \frac{41}{23}\sqrt{3}$ (or $y = 2\frac{11}{23} + 1\frac{18}{23}\sqrt{3}$)

Alt(ii)3 Dividing by $\sqrt{3}$, then rationalising the denominator and collecting terms:

(ii) $4y - 3\sqrt{3} = \frac{5y+\sqrt{3}}{\sqrt{3}} \frac{(\times\sqrt{3})}{(\sqrt{3})}$

$4y - 3\sqrt{3} = \frac{\sqrt{3}(5y+\sqrt{3})}{3}$

$4y - 3\sqrt{3} = \frac{5\sqrt{3}y+3}{3}$

$4y - 3\sqrt{3} = \frac{5\sqrt{3}}{3}y + 1$

$4y - \frac{5\sqrt{3}}{3}y = 1 + 3\sqrt{3}$ M1 for isolating y terms on one side of the equation

$y\left(4 - \frac{5\sqrt{3}}{3}\right) = 1 + 3\sqrt{3}$

$y = \frac{1+3\sqrt{3}}{4-\frac{5}{3}\sqrt{3}} \frac{(\times 4+\frac{5}{3}\sqrt{3})}{(\times 4+\frac{5}{3}\sqrt{3})}$ dM1 for making the y the subject

ddM1 for proceeds to $y = \ldots$ with a rational denominator

$y = \frac{4 + 12\sqrt{3} + \frac{5}{3}\sqrt{3} + 15}{16 - \frac{25}{3}}$

~~$y = 4 + 12\sqrt{3} + \frac{5}{3}\sqrt{3} + 15$~~

$y = \frac{19 + \frac{41}{3}\sqrt{3}}{\frac{23}{3}}$

$y = \frac{57}{23} + \frac{41}{23}\sqrt{3}$ A1 Correct answer with full working shown

Q16.

Question	Scheme	Marks
(a)	$\left(r-\frac{1}{r}\right)^2=r^2-r\times\frac{1}{r}-r\times\frac{1}{r}+\frac{1}{r^2}$	M1
	$=r^2+\frac{1}{r^2}-2$	A1
		(2)
(b)	$\frac{1}{3+2\sqrt{2}}=\frac{1}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}$	M1
	$=\frac{3-2\sqrt{2}}{3^2-\left(2\sqrt{2}\right)^2}=3-2\sqrt{2}$	A1
		(2)
(b) ALT	$\frac{1}{3+2\sqrt{2}}=p+q\sqrt{2}\Rightarrow 1=\left(p+q\sqrt{2}\right)\left(3+2\sqrt{2}\right)=3p+4q+2p\sqrt{2}+3q\sqrt{2}$ $\Rightarrow \begin{matrix}3p+4q=1\\2p+3q=0\end{matrix}$	M1
	$\Rightarrow\begin{matrix}3p+4q=1\\2p+3q=0\end{matrix}\Rightarrow\begin{matrix}6p+8q=2\\6p+9q=0\end{matrix}\Rightarrow q=-2\Rightarrow 3p-8=1\Rightarrow p=3$ $\frac{1}{3+2\sqrt{2}}=3-2\sqrt{2}$	A1
		(2)
(c)	$\left(\sqrt{3+2\sqrt{2}}-\frac{1}{\sqrt{3+2\sqrt{2}}}\right)^2=3+2\sqrt{2}+\frac{1}{3+2\sqrt{2}}-2$	M1
	$=3+2\sqrt{2}+3-2\sqrt{2}-2=...(=4)$	dM1
	so $\sqrt{3+2\sqrt{2}}-\frac{1}{\sqrt{3+2\sqrt{2}}}=2$	A1
		(3)
(c) Alt	$\sqrt{3+2\sqrt{2}}-\frac{1}{\sqrt{3+2\sqrt{2}}}=2\Rightarrow 3+2\sqrt{2}-1=2\sqrt{3+2\sqrt{2}}$	M1
	$\Rightarrow\left(2+2\sqrt{2}\right)^2=4\left(3+2\sqrt{2}\right)$	dM1
	$\Rightarrow 4+8\sqrt{2}+8=12+8\sqrt{2}$ ✓ Hence true	A1
		(3)
		(7 marks)

Notes:

(a)

M1: Expands the bracket to obtain 3 or 4 terms with at least 2 correct which may be unsimplified. Allow use of a different variable e.g. x for this mark.

A1: Correct simplified expansion in terms of r. Accept terms in any order.

Accept e.g. $r^2 + r^{-2} - 2$ and accept $r^2 + \left(\frac{1}{r}\right)^2 - 2$. Do **not** isw and mark their final answer.

Note that if a correct simplified expression is seen and they then re-write the expression correctly in a different way then this mark should be awarded provided their expression is correct.

(b)

M1: Correct process to rationalise the denominator, so look for multiplying numerator and denominator by $3-2\sqrt{2}$ or any multiple of this expression. No processing is required so just look for the statement as shown above.

A1: Shows an intermediate line before obtaining $3-2\sqrt{2}$

Examples of an acceptable intermediate line are:

$$\frac{3-2\sqrt{2}}{3^2-\left(2\sqrt{2}\right)^2},\ \frac{3-2\sqrt{2}}{9-8},\ \frac{3-2\sqrt{2}}{9-6\sqrt{2}+6\sqrt{2}-\left(2\sqrt{2}\right)^2},\ \frac{3-2\sqrt{2}}{9-6\sqrt{2}+6\sqrt{2}-8},\ \frac{3-2\sqrt{2}}{1}$$

Do **not** allow $\frac{3-2\sqrt{2}}{1}$ as the final answer.

(b) ALT

M1: Sets $\frac{1}{3+2\sqrt{2}} = p + q\sqrt{2}$, multiplies up, compares rational and irrational parts to produce 2 equations in p and q.

A1: Solves simultaneously **showing working** and obtains $3-2\sqrt{2}$.

(c)

M1: Applies their result from (a) or the correct result to $\left(\sqrt{3+2\sqrt{2}} - \frac{1}{\sqrt{3+2\sqrt{2}}}\right)^2$

dM1: Depends on the first mark. Applies their result from (b) and cancels $\sqrt{2}$ terms to achieve a constant.

A1: Reaches the correct answer from correct work. **There is no requirement to justify the positive square root.**

(c) ALT

M1: Multiplies equation through by $\sqrt{3+2\sqrt{2}}$ and applies $\sqrt{3+2\sqrt{2}} \times \sqrt{3+2\sqrt{2}} = 3+2\sqrt{2}$

dM1: Depends on the first mark. Squares both sides. May or may not have cancelled the 2 before squaring.

A1: Achieves similar expression for both sides and gives a (minimal) conclusion.

Note that there will be other methods for 6(c) e.g.:

$$\sqrt{3+2\sqrt{2}}-\frac{1}{\sqrt{3+2\sqrt{2}}}=\frac{3+2\sqrt{2}-1}{\sqrt{3+2\sqrt{2}}}$$

M1: Attempts common denominator and applies $\sqrt{3+2\sqrt{2}}\times\sqrt{3+2\sqrt{2}}=3+2\sqrt{2}$

$$\left(\frac{2+2\sqrt{2}}{\sqrt{3+2\sqrt{2}}}\right)^2=\frac{12+8\sqrt{2}}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}=\frac{36-32}{9-8}=4$$

dM1: Squares and attempts to rationalise the denominator

$$\text{So } \sqrt{3+2\sqrt{2}}-\frac{1}{\sqrt{3+2\sqrt{2}}}=2\,*$$

A1*: Fully correct work