Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	$\alpha+(\alpha+3\beta)+(\alpha+6\beta)=21$ and $\alpha(\alpha+3\beta)(\alpha+6\beta)=91$	M1	1.1b
	$3\alpha+9\beta=21 \Rightarrow \alpha+3\beta=7 \Rightarrow \alpha=7-3\beta$ $(7-3\beta)(7-3\beta+3\beta)(7-3\beta+6\beta)=91$ $(7-3\beta)^3+9(7-3\beta)^2\beta+18(7-3\beta)\beta^2=91$ Or $3\beta=7-\alpha$ and $\alpha^3+9\alpha^2\beta+18\alpha\beta^2=91$ leading to $\alpha^3+3\alpha^2(7-\alpha)+2\alpha(7-\alpha)^2=91$	M1	3.1a
	$343-63\beta^2=91$ or $49-9\beta^2=13$ Or $7\alpha^2-98\alpha+91=0$	A1	1.1b
	$49-9\beta^2=13 \Rightarrow \beta=...\{\pm 2\}$ or $7\alpha^2-98\alpha+91=0 \Rightarrow \alpha=...\{1, 13\}$	dM1	1.1b
	$\alpha=7-3(\pm 2)=...\{1, 13\}$ Or $\beta=\dfrac{7-'\alpha'}{3}$	ddM1	1.1b
	Roots = 1, 7, 13 only	dddM1 A1	1.1b 1.1b
		(7)	

	Alternative $\alpha+(\alpha+3\beta)+(\alpha+6\beta)=21$	**M1**	1.1b
	$\alpha+3\beta=7$ therefore one root must be 7	**M1** **A1**	3.1a 1.1b
	$(x-7)(x^2-14x+13)$	**dM1**	1.1b
	$(x-7)(x-1)(x-13)$	**ddM1**	1.1b
	Roots = 1, 13	**dddM1** **A1**	1.1b 1.1b
			(7 marks)

Notes:

M1: Sets the sum of the roots = 21 and the product of the roots = 91
M1: Forms an equation in one variable using their sum of roots and product of roots equations
A1: Correct simplified quadratic equation
dM1: Dependent on the previous method mark, solves their equation to find **real roots**, may be a cubic if slips earlier
ddM1: Dependent on previous method mark. Finds the value of the other variable
dddM1: Dependent on previous method mark. Uses their values of α and β to find the value of the roots
A1: Correct roots
Alternative
M1: Sets the sum of the roots = 21
M1: Simplifies and deduces one root
A1: One root = 7
dM1: Factorises into linear and quadratic, achieves correct first and last term
ddM1: Factorises their quadratic, which must be factorisable
dddM1: Uses their factorised quadratic to find the remining 2 roots
A1: Correct roots 1 and 13

Q2.

Question	Scheme	Marks	AOs
	$w = 2z+1 \Rightarrow z = \frac{w-1}{2}$	B1	3.1a
	$\left(\frac{w-1}{2}\right)^3 - 3\left(\frac{w-1}{2}\right)^2 + \left(\frac{w-1}{2}\right) + 5 = 0$	M1	3.1a
	$\frac{1}{8}(w^3 - 3w^2 + 3w - 1) - \frac{3}{4}(w^2 - 2w + 1) + \frac{w-1}{2} + 5 = 0$		
	$w^3 - 9w^2 + 19w + 29 = 0$	M1 A1 A1	1.1b 1.1b 1.1b
		(5)	
ALT 1	$\alpha + \beta + \gamma = 3, \alpha\beta + \beta\gamma + \alpha\gamma = 1, \alpha\beta\gamma = -5$	B1	3.1a
	New sum = $2(\alpha + \beta + \gamma) + 3 = 9$ New pair sum = $4(\alpha\beta + \beta\gamma + \gamma\alpha) + 4(\alpha + \beta + \gamma) + 3 = 19$ New product = $8\alpha\beta\gamma + 4(\alpha\beta + \beta\gamma + \gamma\alpha) + 2(\alpha + \beta + \gamma) + 1 = -29$	M1	3.1a
	$w^3 - 9w^2 + 19w + 29 = 0$	M1 A1 A1	1.1b 1.1b 1.1b
		(5)	
			(5 marks)

Notes
B1: Selects the method of making a connection between z and w by writing $z=\frac{w-1}{2}$ M1: Applies the process of substituting their $z=\frac{w-1}{2}$ into $z^3-3z^2+z+5=0$ (Allow $z = 2w + 1$) M1: Manipulates their equation into the form $w^3+pw^2+qw+r(=0)$ having substituted their z in terms of w. Note that the "= 0" can be missing for this mark. A1: At least two of p, q, r correct. Note that the "= 0" can be missing for this mark. A1: Fully correct equation including "= 0" The first 4 marks are available if another letter is used instead of w but the final answer must be in terms of w. **ALT1** B1: Selects the method of giving three correct equations containing α, β and γ M1: Applies the process of finding the new sum, new pair sum, new product M1: Applies $w^3-(\text{new sum})w^2+(\text{new pair sum})w-(\text{new product})(=0)$ or identifies p as –(new sum) q as (new pair sum) and r as –(new product) A1: At least two of p, q, r correct. A1: Fully correct equation including "= 0" The first 4 marks are available if another letter is used instead of w but the final answer must be in terms of w.

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Q3.

Question	Scheme	Marks	AOs
	Complex roots are e.g. $\alpha \pm \beta i$ or $(z^3+z^2+pz+q)\div(z-3)=z^2+4z+p+12$ or $f(3)=0 \Rightarrow 3^3+3^2+3p+q=0$ or One of: $3+z_2+z_3=-1,\ \ 3z_2z_3=-q,\ \ 3z_2+3z_3+z_2z_3=p$	B1	3.1a
	Sum of roots $\alpha+\beta i+\alpha-\beta i+3=-1 \Rightarrow \alpha=...$ or $\alpha+\beta i+\alpha-\beta i=-4 \Rightarrow \alpha=...$	M1	1.1b
	$\alpha=-2$	A1	1.1b
	So $\frac{1}{2}\times 2\beta\times 5=35 \Rightarrow \beta=7$	M1	1.1b
	$q=-3("-2+7i")("-2-7i")=...$ or $p=3("-2+7i")+3("-2-7i")+("-2+7i")("-2-7i")$ or $(z-3)(z-("-2+7i"))(z-("-2-7i"))=...$	M1	3.1a
	$q=-159$ or $p=41$	A1	1.1b
	$3p+q=-36 \Rightarrow p=\frac{-36-q}{3}=41$ and $q=-159$	A1	1.1b
		(7)	

	Alternative		
	$(z^3+z^2+pz+q)\div(z-3)=z^2+4z+p+12$	B1	3.1a
	$z^2+4z+p+12=0 \Rightarrow z=\frac{-4\pm\sqrt{4^2-4(p+12)}}{2}\left(=-2\pm i\sqrt{p+8}\right)$	M1	1.1b
	$\alpha=-2$	A1	1.1b
	$\beta=\sqrt{p+8}$	M1	1.1b
	$\frac{1}{2}\times(3+2)\times 2\sqrt{p+8}=35 \Rightarrow p=...$	M1	3.1a
	$p=41$	A1	1.1b
	$3p+q=-36 \Rightarrow q=-159$	A1	1.1b
		(7)	
		(7 marks)	

Notes
B1: Recognises that the other roots must form a conjugate pair **or** obtains $z^2+4z+p+12$ (or $z^2+4z-\frac{q}{3}$) as the quadratic factor **or** writes down a correct equation for p and q **or** writes down a correct equation involving "z_2" and "z_3" M1: Uses the sum of the roots of the cubic or the sum of the roots of their quadratic to find a value for "α" A1: Correct value for "α" M1: Uses their value for "α" and the given area to find a value for "β". Must be using the area and triangle dimensions correctly e.g. $\frac{1}{2}\times\beta\times5=35\Rightarrow\beta=14$ scores M0 M1: Uses an appropriate method to find p or q A1: A correct value for p or q A1: Correct values for p and q **Alternative** B1: Obtains $z^2+4z+p+12$ (or $z^2+4z-\frac{q}{3}$) as the quadratic factor M1: Solves their quadratic factor by completing the square or using the quadratic formula A1: Correct value for "α" M1: Uses their imaginary part to find "β" in terms of p M1: Draws together the fact that the imaginary parts of their complex conjugate pair and the real root form the sides of the required triangle and forms an equation in terms of p, sets equal to 35 and solves for p A1: A correct value for p or q A1: Correct values for p and q

Q4.

Question	Scheme	Marks	AOs
	$\{w = x - 1 \Rightarrow\}\ x = w + 1$	B1	3.1a
	$(w+1)^3 + 3(w+1)^2 - 8(w+1) + 6 = 0$	M1	3.1a
	$w^3 + 3w^2 + 3w + 1 + 3(w^2 + 2w + 1) - 8w - 8 + 6 = 0$		
	$w^3 + 6w^2 + w + 2 = 0$	M1	1.1b
		A1	1.1b
		A1	1.1b
		(5)	
ALT 1	$\alpha + \beta + \gamma = -3,\ \alpha\beta + \beta\gamma + \alpha\gamma = -8,\ \alpha\beta\gamma = -6$	B1	3.1a
	sum roots = $\alpha - 1 + \beta - 1 + \gamma - 1$	M1	3.1a
	$= \alpha + \beta + \gamma - 3 = -3 - 3 = -6$		
	pair sum = $(\alpha-1)(\beta-1) + (\alpha-1)(\gamma-1) + (\beta-1)(\gamma-1)$		
	$= \alpha\beta + \alpha\gamma + \beta\gamma - 2(\alpha+\beta+\gamma) + 3$		
	$= -8 - 2(-3) + 3 = 1$		
	product = $(\alpha - 1)(\beta - 1)(\gamma - 1)$		
	$= \alpha\beta\gamma - (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha+\beta+\gamma) - 1$		
	$= -6 - (-8) - 3 - 1 = -2$		
	$w^3 + 6w^2 + w + 2 = 0$	M1	1.1b
		A1	1.1b
		A1	1.1b
		(5)	
		(5 marks)	

	Question Notes	
	B1	Selects the method of making a connection between x and w by writing $x = w + 1$
	M1	Applies the process of substituting their $x = w + 1$ into $x^3 + 3x^2 - 8x + 6 = 0$
	M1	Depends on previous M mark. Manipulating their equation into the form $w^3 + pw^2 + qw + r = 0$
	A1	At least two of p, q, r are correct.
	A1	Correct final equation.
ALT 1	B1	Selects the method of giving three correct equations each containing α, β and γ.
	M1	Applies the process of finding sum roots, pair sum and product.
	M1	Depends on previous M mark. Applies $w^3 - (\text{their sum roots})w^2 + (\text{their pair sum})w - \text{their } \alpha\beta\gamma = 0$
	A1	At least two of p, q, r are correct.
	A1	Correct final equation.

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Q5.

Question	Scheme	Marks	AOs
(a)	$\alpha+\beta+\left(\alpha+\frac{12}{\alpha}-\beta\right)=8$ so $2\alpha+\frac{12}{\alpha}=8$	M1	1.1b
		A1	1.1b
	$\Rightarrow 2\alpha^2-8\alpha+12=0$ or $\alpha^2-4\alpha+6=0$ $\Rightarrow \alpha=\frac{4\pm\sqrt{(-4)^2-4(1)(6)}}{2(1)}$ or $(\alpha-2)^2-4+6=0 \Rightarrow \alpha=...$	M1	1.1b
	$\Rightarrow \alpha=2\pm i\sqrt{2}$ are the two complex roots	A1	1.1b
	A correct full method to find the third root. Common methods are: Sum of roots $=8 \Rightarrow$ third root $=8-\left(2+i\sqrt{2}\right)-\left(2-i\sqrt{2}\right)=...$ third root $=2+i\sqrt{2}+\frac{12}{2+i\sqrt{2}}-\left(2-i\sqrt{2}\right)=...$ Product of roots $=24 \Rightarrow$ third root $=\frac{24}{\left(2+i\sqrt{2}\right)\left(2-i\sqrt{2}\right)}=...$ $(z-\alpha)(z-\beta)=z^2-4z+6 \Rightarrow f(z)=\left(z^2-4z+6\right)(z-\gamma)\Rightarrow \gamma=...$ (or long division to find third factor).	M1	3.1a
	Hence the roots of $f(z)=0$ are $2\pm i\sqrt{2}$ and 4	A1	1.1b
		(6)	
(b)	E.g. $f(4)=0 \Rightarrow 4^3-8\times4^2+4p-24=0\Rightarrow p=...$ Or $p=\left(2+i\sqrt{2}\right)\left(2-i\sqrt{2}\right)+4\left(2+i\sqrt{2}\right)+4\left(2-i\sqrt{2}\right)\Rightarrow p=...$ Or $f(z)=(z-4)(z^2-4z+6)\Rightarrow p=...$	M1	3.1a
	$\Rightarrow p=22$ cso	A1	1.1b
		(2)	
		(8 marks)	

Notes		
(a)	M1	Equates sum of roots to 8 and obtains an equation in just α.
	A1	Obtains a correct equation in α.
	M1	Forms a three term quadratic equation in α and attempts to solve this equation by either completing the square or using the quadratic formula to give $\alpha=....$
	A1	$\alpha=2\pm i\sqrt{2}$
	M1	Any correct method for finding the remaining root. There are various routes possible. See scheme for common ones. Allow this mark if -24 is used as the product. See note below for a less common approach.
	A1	Third root found with all three roots correct. Note α and β need not be identified.
(b)	M1	Any correct method of finding p. For example, applies the factor theorem, process of finding the pair sum of roots, or uses the roots to form f(z).
	A1	$p=22$ by correct solution only. Note: this can be found using only their complex roots from (a) (e.g. by factor theorem)

Note for (a) final M – it is possible to find the second and third roots using only one initial root (e.g. if second root forgotten or error leads to only one initial root being found).

Product of roots $=\alpha\beta\left(\alpha+\frac{12}{\alpha}-\beta\right)=24\Rightarrow\alpha\beta^2-\left(\alpha^2+12\right)\beta+24=0$, substitutes in α and attempts to solve the quadratic in β to achieve remaining roots. The final M can be gained once three roots in total have been obtained. (This is unlikely to be seen as part of a correct answer.) Allow if -24 has been used for the product.

Q6.

Question	Scheme	Marks	AOs
(i)	$p+q+r=2, \quad pq+pr+qr=4, \quad pqr=5$	B1	3.1a
	$\frac{2}{p}+\frac{2}{q}+\frac{2}{r}=\frac{2(pq+pr+qr)}{pqr}$	M1	1.1b
	$=\frac{8}{5}$	A1ft	1.1b
		(3)	
	Alternative for part (i)		
	$x=\frac{2}{y} \Rightarrow \frac{8}{y^3}-\frac{8}{y^2}+\frac{8}{y}-5=0 \Rightarrow 5y^3-8y^2+8y-8=0$	B1	3.1a
	$\frac{2}{p}+\frac{2}{q}+\frac{2}{r}=-\frac{-8}{5}$	M1	1.1b
	$=\frac{8}{5}$	A1ft	1.1b
		(3)	
(ii)	$(p-4)(q-4)(r-4)=(pq-4p-4q+16)(r-4)$ $=pqr-4pq-4pr-4qr+16p+16q+16r-64$	M1 A1	1.1b 1.1b
	$(=pqr-4(pq+pr+qr)+16(p+q+r)-64)$		
	$=5-4(4)+16(2)-64=-43$	A1	1.1b
		(3)	
	Alternative for part (ii)		
	$(x+4)^3-2(x+4)^2+4(x+4)-5=0$	M1	1.1b
	$=\ldots 64+\ldots-32+\ldots 16+\ldots-5=43$	A1	1.1b
	$\therefore (p-4)(q-4)(r-4)=-43$	A1	1.1b
		(3)	

(iii)	E.g. $p^3+q^3+r^3=$ $=(p+q+r)^3-3(p+q+r)(pq+pr+qr)+3pqr$ or $=(p+q+r)\left((p+q+r)^2-2(pq+pr+qr)-pq-pr-qr\right)+3pqr$ or $=2\left((p+q+r)^2-2(pq+pr+qr)\right)-4(p+q+r)+3pqr$ $\Rightarrow p^3+q^3+r^3=\ldots$	M1	3.1a
	$=2^3-3(2)(4)+3(5)=-1$ $=2\left(2^2-3(4)\right)+3(5)=-1$ $=2\left(2^2-2(4)\right)-4(2)+3(5)=-1$	A1	1.1b
		(2)	

	Alternative for part (iii)		
	$p^3-2p^2+4p-5=0,\ q^3-2q^2+4q-5=0,\ r^3-2r^2+4r-5=0$ $p^3+q^3+r^3-2(p^2+q^2+r^2)+4(p+q+r)-15=0$ $p^3+q^3+r^3=2\left((p+q+r)^2-2(pq+pr+qr)\right)-4(p+q+r)+15$ $\Rightarrow p^3+q^3+r^3=\ldots$	M1	3.1a
	$=2\left(2^2-2(4)\right)-4(2)+15=-1$	A1	1.1b
		(2)	
			(8 marks)

Notes
(i) B1: Identifies the correct values for all 3 expressions (can score anywhere). Allow notation such as $\sum p$, $\sum pq$ for the sum and pair sum. M1: Uses a correct identity for the sum A1ft: Correct value (follow through their 2, 4 and 5) **Alternative:** B1: Obtains the correct cubic in "y" M1: Uses a correct method A1ft: Correct value (follow through their 2, 4 and 5) (ii) M1: Attempt to expand – must have an expression that involves the sum, pair sum and product A1: Correct expansion A1: Correct value **Alternative:** M1: Substitutes $x+4$ for x in the given cubic A1: Calculates the correct constant term A1: Correct value (iii) M1: Establishes a correct identity that is in terms of the sum, pair sum and product and substitutes to reach a numerical expression for $p^3+q^3+r^3$ A1: Correct value

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Q7.

Question	Scheme	Marks	AOs
(i)	$\alpha+\beta+\gamma=8,\quad \alpha\beta+\beta\gamma+\gamma\alpha=28,\quad \alpha\beta\gamma=32$	B1	3.1a
	$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$	M1	1.1b
	$=\frac{7}{8}$	A1ft	1.1b
		(3)	
(ii)	$(\alpha+2)(\beta+2)(\gamma+2)=(\alpha\beta+2\alpha+2\beta+4)(\gamma+2)$	M1	1.1b
	$=\alpha\beta\gamma+2(\alpha\beta+\alpha\gamma+\beta\gamma)+4(\alpha+\beta+\gamma)+8$	A1	1.1b
	$=32+2(28)+4(8)+8=128$	A1	1.1b
		(3)	
	Alternative for part (ii)		
	$(x-2)^3-8(x-2)^2+28(x-2)-32=0$	M1	1.1b
	$=...-8+...-32+...-56-32=-128$	A1	1.1b
	$\therefore(\alpha+2)(\beta+2)(\gamma+2)=128$	A1	1.1b
		(3)	
(iii)	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$	M1	3.1a
	$=8^2-2(28)=8$	A1ft	1.1b
		(2)	
			(8 marks)

Notes

(i)
B1: Identifies the correct values for all 3 expressions (can score anywhere)
M1: Uses a correct identity
A1ft: Correct value (follow through their 8, 28 and 32)
(ii)
M1: Attempts to expand
A1: Correct expansion
A1: Correct value
Alternative:
M1: Substitutes $x-2$ for x in the given cubic
A1: Calculates the correct constant term
A1: Changes sign and so obtains the correct value
(iii)
M1: Establishes the correct identity
A1ft: Correct value (follow through their 8, 28 and 32)

Q8.

Question	Scheme	Marks	AOs
(a)	$\alpha\beta\gamma=-\frac{1}{3}$ and $\alpha\beta+\alpha\gamma+\beta\gamma=-\frac{4}{3}$	B1	3.1a
	$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}=\frac{-4/3}{-1/3}$	M1	1.1b
	$=4$	A1	1.1b
		(3)	
(b)	$\left\{\alpha+\beta+\gamma=-\frac{1}{3}\right\}$ New product $=\frac{1}{\alpha}\times\frac{1}{\beta}\times\frac{1}{\gamma}=\frac{1}{\alpha\beta\gamma}=\frac{1}{-1/3}=...(-3)$ New pair sum $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\alpha\gamma}=\frac{\gamma+\alpha+\beta}{\alpha\beta\gamma}=\frac{-1/3}{-1/3}=...(1)$	M1	3.1a
	$x^3-(\text{part (a)})x^2+(\text{new pair sum})x-(\text{new product})(=0)$	M1	1.1b
	$x^3-4x^2+x+3=0$	A1	1.1b
		(3)	
	Alternative e.g. $z=\frac{1}{x}\Rightarrow\frac{3}{x^3}+\frac{1}{x^2}-\frac{4}{x}+1=0$	M1	3.1a
	$x^3-4x^2+x+3=0$	M1 A1	1.1b 1.1b
		(3)	
			(6 marks)

Notes:
(a) **B1:** Correct values for the product and pair sum of the roots **M1:** A complete method to find the sum of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$. Must substitute in their values of the product and pair sum **A1:** correct value 4 Note: If candidate does not divide by 3 so that $\alpha\beta\gamma=-1$ and $\alpha\beta+\alpha\gamma+\beta\gamma=-4$ the maximum they can score is B0 M1 A0
(b) **M1:** A correct method to find the value of the new pair sum and the value of the new product **M1:** Applies $x^3-(\text{part (a)})x^2+(\text{their new pair sum})x-(\text{their new product})(=0)$ **A1:** Fully correct equation, in any variable, including = 0
(b) Alternative **M1:** Realises the connection between the roots and substitutes into the cubic equation **M1:** Manipulates their equation into the form $x^3+ax^2+bx+c=0$ **A1:** Fully correct equation in any variable, including = 0

Q9.

Question	Scheme	Marks	AOs
	$w = 3x - 2 \Rightarrow x = \frac{w+2}{3}$	B1	3.1a
	$9\left(\frac{w+2}{3}\right)^3 - 5\left(\frac{w+2}{3}\right)^2 + 4\left(\frac{w+2}{3}\right) + 7 = 0$	M1	3.1a
	$\frac{1}{3}\left(w^3 + 6w^2 + 12w + 8\right) - \frac{5}{9}\left(w^2 + 4w + 4\right) + \frac{4}{3}(w+2) + 7 = 0$		
	$3w^3 + 13w^2 + 28w + 91 = 0$	dM1 A1 A1	1.1b 1.1b 1.1b
		(5)	
	Alternative:		
	$\alpha + \beta + \gamma = \frac{5}{9}, \alpha\beta + \beta\gamma + \alpha\gamma = \frac{4}{9}, \alpha\beta\gamma = -\frac{7}{9}$	B1	3.1a
	New sum $= 3(\alpha+\beta+\gamma) - 6 = -\frac{13}{3}$		
	New pair sum $= 9(\alpha\beta + \beta\gamma + \gamma\alpha) - 12(\alpha+\beta+\gamma) + 12 = \frac{28}{3}$	M1	3.1a
	New product $= 27\alpha\beta\gamma - 18(\alpha\beta + \beta\gamma + \gamma\alpha) + 12(\alpha+\beta+\gamma) - 8 = -\frac{91}{3}$		
	$w^3 - \left(-\frac{13}{3}\right)w^2 + \frac{28}{3}w - \left(-\frac{91}{3}\right) = 0$	dM1	1.1b
	$3w^3 + 13w^2 + 28w + 91 = 0$	A1 A1	1.1b 1.1b
		(5)	
			(5 marks)

Notes
B1: Selects the method of making a connection between x and w by writing $x = \frac{w+2}{3}$ Condone the use of a different letter than w M1: Applies the process of substituting $x = \frac{w+2}{3}$ into $9x^3 - 5x^2 + 4x + 7 = 0$ dM1: Depends on the previous M mark. Manipulates their equation into the form $aw^3 + bw^2 + cw + d (= 0)$. Condone the use of a different letter then w consistent with B1 mark. A1: At least two of a, b, c, d correct A1: Fully correct equation, must be in terms of w **Alternative:** B1: Selects the method of giving three correct equations containing α, β and γ M1: Applies the process of finding the new sum, new pair sum, new product dM1: Depends on the previous M mark. Applies $w^3 - (\text{new sum})w^2 + (\text{new pair sum})w - (\text{new product})(= 0)$ condone the use of any letter here. A1: At least two of a, b, c, d correct A1: Fully correct equation in term of w

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Q10.

Question	Scheme	Marks	AOs
(i)	$\sum \alpha_i = -\frac{5}{3}$ and $\sum \alpha_i \alpha_j = 0$ This mark can be awarded if seen in part (ii) or part (iii)	B1	3.1a
	So $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = (\alpha + \beta + \gamma + \delta)^2 - 2\left(\sum \alpha_i \alpha_j\right) = \ldots$	M1	1.1b
	$= \frac{25}{9} - 2 \times 0 = \frac{25}{9}$	A1	1.1b
		(3)	
(ii)	$\sum \alpha_i \alpha_j \alpha_k = \frac{7}{3}$ and $\prod \alpha_i = 2$ or for $x = \frac{2}{w}$ used in equation This mark can be awarded if seen in part (i) or part (iii)	B1	2.2a
	So $2\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}\right) = 2 \times \frac{\sum \alpha_i \alpha_j \alpha_k}{\alpha\beta\gamma\delta} = 2 \times \frac{\left(\frac{7}{3}\right)}{\left(\frac{6}{3}\right)}$ or for $3\left(\frac{16}{w^4}\right) + 5\left(\frac{8}{w^3}\right) - 7\left(\frac{2}{w}\right) + 6 = 0 \Rightarrow 6w^4 - 14w^3 + \ldots = 0$ leading to $\frac{14}{6}$	M1	1.1b
	$\left(= 2 \times \frac{7/3}{2}\right)\left(= \frac{14}{6}\right) = \frac{7}{3}$	A1	1.1b
		(3)	

(iii)	$(3-\alpha)(3-\beta)(3-\gamma)(3-\delta)=...$ expands all four brackets Or equation with these roots is $3(3-x)^4+5(3-x)^3-7(3-x)+6=0$	M1	3.1a
	$=81-27\left(\sum\alpha_i\right)+9\left(\sum\alpha_i\alpha_j\right)-3\left(\sum\alpha_i\alpha_j\alpha_k\right)+\prod\alpha_i$ $=81-27\left(-\frac{5}{3}\right)+9(0)-3\left(\frac{7}{3}\right)+2$ Or expands to fourth power and constant terms and attempts product of roots $3x^4+...+3\times3^4+5\times3^3-7\times3+6\rightarrow\prod\alpha_i=\frac{"363"}{3}$	dM1	1.1b
	$=121$	A1	1.1b
		(3)	
			(9 marks)

Notes:

(i)

B1: Correct sum and pair sum of roots seen or implied. Must realise the pair sum is zero.

Note: These values can be seen anywhere in the candidate's solution

M1: Uses correct expression for the sum of squares.

A1: $\frac{25}{9}$. Allow this mark from incorrect sign on sum of squares (but they will score B0 if the sign is incorrect).

(ii)

B1: Correct triple sum and product of roots seen or implied. May be stated in (i). Alternatively, this may be scored for sight of $x=\frac{2}{w}$ used as a transformation in the equation.

Note: These values can be seen anywhere in the candidate's solution

M1: Substitutes their values into $2\times\frac{\sum\alpha_i\alpha_j\alpha_k}{\alpha\beta\gamma\delta}=...$ In the alternative it is for rearranging the equation to a quartic in w and uses to find the sum of the roots.

A1: $\frac{7}{3}$ Allow this mark from incorrect sign of both triple sum and product (but they will score B0 if the sign is incorrect).

(iii)

M1: A correct method to find the value used – may recognise structure as scheme, may expand the expression in stages, or may attempt to use a linear transformation $(3-x)$ or e.g. $(3-w)$ in original equation. Condone slips as long as the intention is clear.

dM1: Dependent on previous method mark. Uses at least 2 values of their sum of roots etc. in their expression. If using a linear shift this is for expanding to find the coefficient of x^4 and constant term and attempts product of roots by dividing the constant term by the coefficient of x^4.

A1: 121.

Q11.

Question	Scheme	Marks	AOs
(a)	$4x^3+px^2-14x+q=0 \Rightarrow x^3+\frac{p}{4}x^2-\frac{14}{4}x+\frac{q}{4}=0$ $\alpha+\beta+\gamma=-\frac{p}{4}$ $\alpha\beta+\alpha\gamma+\beta\gamma=-\frac{14}{4}$ or $-\frac{7}{2}$	B1	3.1a
	$(\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $\left(-\frac{p}{4}\right)^2=16+2\left(-\frac{7}{2}\right)\Rightarrow p=\ldots$ or $(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)=\alpha^2+\beta^2+\gamma^2$ $\left(-\frac{p}{4}\right)^2-2\left(-\frac{7}{2}\right)=16\Rightarrow p=\ldots$	M1	3.1a
	$p=12$ * cso	A1*	1.1b
		(3)	
(b)	$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$	M1	1.1b
	$\frac{\left(-\frac{7}{2}\right)}{\left(\frac{-q}{4}\right)}=\frac{14}{3}\Rightarrow q=\ldots$	M1	1.1b
	$q=3$	A1	1.1b
		(3)	
	Alternative $4(\frac{1}{w})^3+12(\frac{1}{w})^2-14(\frac{1}{w})+q\{=0\}$	M1	1.1b
	$qw^3-14w^2+12w+4=0\Rightarrow\frac{14}{3}=-\frac{-14}{q}\Rightarrow q=\ldots$	M1	1.1b
	$q=3$	A1	1.1b
		(3)	

(c)	$(\alpha-1)(\beta-1)(\gamma-1)=\ldots$ $=\alpha\beta\gamma-(\alpha\beta+\alpha\gamma+\beta\gamma)+(\alpha+\beta+\gamma)-1$	M1 A1	1.1a 1.1b
	$=\left(-\frac{\text{their }3}{4}\right)-\left(-\frac{7}{2}\right)+\left(-\frac{12}{4}\right)-1=\ldots$	dM1	1.1b
	$=-\frac{5}{4}$	A1	1.1b
		(4)	
Alt	$4(x+1)^3+12(x+1)^2-14(x+1)+'3'\{=0\}$ or substitutes in 1	M1	1.1a
	$=\ldots4+\ldots12+\ldots-14+'3'=5$ or $4x^3+24x^2+22x+2+$ 'their q'	A1ft	1.1b

	$= -\frac{\text{'their constant'}}{4}$	dM1	1.1b
	$= -\frac{5}{4}$	A1	1.1b
			(10 marks)

Notes:

(a)

B1: Identifies the correct values for the sum and pair sum. This may be implied by substituting into an equation, it must be clear

M1: Uses the correct identity and values of their sum **and** their pair sum to find a value of p

A1*: $p = 12$ cso there is no need to see a reason

(b)

M1: Establishes a correct identity

M1: Uses their identity and their pair sum and their product of roots to find a value of q. Condone a slip but the intention must be clear.

A1: $q = 3$ Allow this mark from incorrect sign of both pair sum and product

Alternative

M1: Uses $x = \frac{1}{w}$ the substitution

M1: Simplifies to an quartic equation of the form $aw^3 + bw^2 + cw + d = 0$ and uses $\frac{14}{3} = -\frac{b}{a}$ to find a value for q

A1: $q = 3$

(c)

M1: Attempts to multiply out the three brackets.

A1: Correct expansion.

dM1: Dependent on previous method. Substitutes in the value of their sum, pair sum and the value of their product as appropriate. Condone a slip but the intention must be clear

A1: Correct value

Alternative

M1: Substitutes $(x + 1)$ or $x = 1$ into the cubic with their value of q. Allow the use of different letters e.g. $(w + 1)$

A1ft: Correct constant terms, follow through on their value of q

dM1: Applies $-\frac{\text{'their constant'}}{4}$

A1: Correct value

Q12.

Question	Scheme	Marks	AOs
(i)	$\alpha+\beta+\gamma=\frac{3}{2},\ \alpha\beta+\alpha\gamma+\beta\gamma=\frac{5}{2}$	B1	3.1a
	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)=\left(\frac{3}{2}\right)^2-2\left(\frac{5}{2}\right)=\ldots$	M1	1.1b
	$=-\frac{11}{4}=-2.75$ cso	A1	1.1b
		(3)	
(ii)	$\alpha\beta\gamma=-\frac{7}{2}$ or $x=\frac{3}{w}$ used in the equation	B1	2.2a
	$\frac{3}{\alpha}+\frac{3}{\beta}+\frac{3}{\gamma}=\frac{3(\alpha\beta+\alpha\gamma+\beta\gamma)}{\alpha\beta\gamma}=\frac{3\left(\frac{5}{2}\right)}{\left(-\frac{7}{2}\right)}$ or $2\left(\frac{3}{w}\right)^3-3\left(\frac{3}{w}\right)^2+5\left(\frac{3}{w}\right)+7=0\Rightarrow 7w^3+15w^2-27w+54\{=0\}$ $\Rightarrow -\frac{'15'}{'7'}$	M1	1.1b
	$=-\frac{15}{7}$ cso	A1	1.1b
		(3)	
(iii)	$(5-\alpha)(5-\beta)(5-\gamma)=A\pm B(\alpha+\beta+\gamma)\pm C(\alpha\beta+\alpha\gamma+\beta\gamma)\pm(\alpha\beta\gamma)$ $=\{5^3-5^2(\alpha+\beta+\gamma)+5(\alpha\beta+\alpha\gamma+\beta\gamma)-\alpha\beta\gamma\}$ or $2(5-w)^3-3(5-w)^2+5(5-w)+7\{=0\}$ or $f(x)=A(x-\alpha)(x-\beta)(x-\gamma)\Rightarrow A=2$	M1	3.1a
	$(5-\alpha)(5-\beta)(5-\gamma)=125-25\left(\frac{3}{2}\right)+5\left(\frac{5}{2}\right)+\frac{7}{2}$ or $(5-\alpha)(5-\beta)(5-\gamma)=-\left(\frac{2\times125-3\times25+25+7}{-2}\right)$ Or $-2w^3+27w^2-125w+207\{=0\}\Rightarrow-\frac{'207'}{'-2'}$ Or $f(5)=2(5-\alpha)(5-\beta)(5-\gamma)$ $\Rightarrow(5-\alpha)(5-\beta)(5-\gamma)=\frac{f(5)}{2}$	M1	1.1b

	$= \frac{207}{2} = 103.5$ cso	A1	1.1b
		(3)	
			(9 marks)

Notes

(i)

B1: Correct sum and pair sum, they may be seen anywhere in the candidates working.

M1: Uses a correct identity and substitutes in their sum and pair sum to find a value.

A1: Correct value following B1, if uses $\alpha + \beta + \gamma = -\frac{3}{2}$ this can score B0 M1 A0 cso

(ii)

B1: Correct value for the product (may be seen anywhere in the candidates working) or for using $x = \frac{3}{w}$ in the given equation.

M1: Uses a correct identity and substitutes in their pair sum and product to obtain a value or multiplies through by w^3 to identify at least the required terms and finds their new sum.

A1: Correct value from correct pair sum and product cso

(iii)

M1: Correct strategy for obtaining the required value by expanding, must reach an expression for the form $A \pm B(\alpha + \beta + \gamma) \pm C(\alpha\beta + \alpha\gamma + \beta\gamma) \pm (\alpha\beta\gamma)$ may not be factorised for example.

$A \pm B\alpha \pm B\beta \pm B\gamma \pm C\alpha\beta \pm C\alpha\gamma \pm C\beta\gamma \pm (\alpha\beta\gamma)$

or

Attempts the correct linear transformation of the given equation and expands.

or

Uses $f(x) = A(x-\alpha)(x-\beta)(x-\gamma)$ to find a value for A

M1: Uses their sum, pair sum and product to obtain a value. Allow recovery from a sign slip as long as substituting into an expression of the form $A \pm B(\alpha + \beta + \gamma) \pm C(\alpha\beta + \alpha\gamma + \beta\gamma) \pm (\alpha\beta\gamma)$.

This would be A0 even if the correct answer is achieved.

or

Simplifies to obtain at least the required terms to find a value for the new product. Ignore the other terms whether correct or not.

Or

Uses $\frac{f(5)}{2}$

A1: Correct value with no errors seen cso

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Q13.

Question	Scheme	Marks	AOs
(a)	$2x^4 + Ax^3 - Ax^2 - 5x + 6 = 0 Þ x^4 + \frac{A}{2}x^3 - \frac{A}{2}x^2 - \frac{5}{2}x + \frac{6}{2} = 0$ $abgd = \frac{6}{2}$ $abg + abd + agd + bgd = \frac{5}{2}$	B1	3.1a
	$\frac{3}{\alpha}+\frac{3}{\beta}+\frac{3}{\gamma}+\frac{3}{\delta}=\frac{3(\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta)}{\alpha\beta\gamma\delta}=\frac{3\times\left(\frac{5}{2}\right)}{\left(\frac{6}{2}\right)}=\ldots$	M1	1.1b
	$= \frac{5}{2}$	A1	1.1b
		(3)	
(b)	$a + b + g + d = -\frac{A}{2}$ $ab + ag + ad + bg + bd + gd = -\frac{A}{2}$	B1	1.1b
	$(a + b + g + d)^2 =$ $a^2 + b^2 + g^2 + d^2 + 2(ab + ag + ad + bg + bd + gd)$	M1 A1	3.1a 1.1b
	$\frac{3}{\alpha}+\frac{3}{\beta}+\frac{3}{\gamma}+\frac{3}{\delta}=\frac{3(\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta)}{\alpha\beta\gamma\delta}=\frac{3\times\left(\frac{5}{2}\right)}{\left(\frac{6}{2}\right)}=\ldots$	dM1	1.1b
	$A = -1, -3$	A1	1.1b
		(5)	
		(8 marks)	

Notes:
(a) **B1:** Identifies the correct values for the product and triple pair sum. If seen these must be correct, but they may be implied if not explicitly extracted, e.g. $\sum \frac{3}{\alpha_i} = 3\frac{\sum \alpha_i \alpha_j \alpha_k}{\alpha\beta\gamma\delta} = 3 \times \frac{5}{6} = \frac{5}{2}$ does not technically contain an incorrect step if one realises the 2's will cancel. A good indication will be what happens in part (b) – if there they realise the 2's are involved then allow b.o.d. for a calculation such as here given in (a). Note $\sum \alpha\beta\gamma\delta$ is a correct notation for the product of roots. **M1:** Uses the correct identity and their values of the product and triple sum. (The values need not be correct for this mark.) **A1:** Correct value following correct identity with no indication of incorrect values for triple sum and product (see comment on the B mark).
(b) **B1:** Identifies the correct values for the sum and pair sum. Allow when first seen – some will list all these in part (a), which is fine for this mark. **M1:** Attempts to find the identity for $(a+b+g+d)^2$ in terms of the sum of squares and pair sum (seen or implied). Allow attempts where the "2" is incorrect with no other method shown. **A1:** Correct identity (seen or implied). **dM1:** Dependent on previous method mark. Substitutes the sum and pair sum of the roots into their identity and forms and solves a 3TQ for A (usual rules). **A1:** Correct values of A

(a) Alt	$x = \frac{3}{w} \Rightarrow 2\left(\frac{3}{w}\right)^4 + ... - 5\left(\frac{3}{w}\right) + 6 = 0$ or $x = \frac{1}{w} \Rightarrow 2\left(\frac{1}{w}\right)^4 + ... - 5\left(\frac{1}{w}\right) + 6 = 0$	B1	3.1a
	$\Rightarrow ... + ...w + ... - 15w^3 + 6w^4 = 0$ $\Rightarrow \sum \frac{3}{\alpha_i} = -\left(\frac{-15}{6}\right)$ or $\Rightarrow 2 + ... - 5w^3 + 6w^4 = 0$ $\Rightarrow 3\sum \frac{1}{\alpha_i} = 3 \times -\left(\frac{-5}{6}\right)$	M1	1.1b
	$= \frac{5}{2}$	A1	1.1b
		(3)	

Alt (a): Some may use a transformation. These can be scored as

B1: Makes a correct substitution into the equation to solve the problem. This will probably be $x = \frac{3}{w}$ but note that $x = \frac{1}{w}$ can also be used.

M1: Multiplies through by w^4 and extracts the correct sum of roots from the new equation. If using $x = \frac{1}{w}$ they must also multiply through by the 3 to gain this mark.

A1: Correct answer from correct work on the relevant coefficients (the others need not be seen).

Q14.

Question	Scheme	Marks	AOs
(a)	$\alpha\left(\dfrac{5}{\alpha}\right)\left(\alpha+\dfrac{5}{\alpha}-1\right)=15$	M1	1.1b
		A1	1.1b
	$\Rightarrow 5\alpha+\dfrac{25}{\alpha}-5=15 \Rightarrow \alpha^2-4\alpha+5=0$ $\Rightarrow \alpha=\dfrac{--4\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)}$ or $(\alpha-2)^2-4+5=0 \Rightarrow \alpha=...$	M1	3.1a
	$\Rightarrow \alpha=2\pm\text{i}$	A1	1.1b
	Hence the roots of $\text{f}(z)=0$ are $2+\text{i}, 2-\text{i}$ and 3	A1	2.2a
		(5)	
(b)	$p=-\left(\text{"}(2+\text{i})\text{"}+\text{"}(2-\text{i})\text{"}+\text{"}3\text{"}\right) \Rightarrow p=...$	M1	3.1a
	$\Rightarrow p=-7$ **cso**	A1	1.1b
		(2)	
(b) **ALT 1**	$\text{f}(z)=(z-3)(z^2-4z+5) \Rightarrow p=...$	M1	3.1a
	$\Rightarrow p=-7$ **cso**	A1	1.1b
		(2)	
	(7 marks)		

Question Notes

(a)	M1	Multiplies the three given roots together and sets the result equal to 15 or -15
	A1	Obtains a correct equation in α.
	M1	Forms a quadratic equation in α and attempts to solve this equation by either completing the square or using the quadratic formula to give $\alpha=....$
	A1	$\alpha=2\pm\text{i}$
	A1	Deduces the roots are $2+\text{i}, 2-\text{i}$ and 3
(b)	M1	Applies the process of finding $-\sum\left(\text{of their three roots found in part } (a)\right)$ to give $p=...$
	A1	$p=-7$ by correct solution only.
(b) **ALT 1**	M1	Applies the process expanding $(z-\text{"}3\text{"})(z-(\text{their sum})z+\text{their product})$ in order to find $p=...$
	A1	$p=-7$ by correct solution only.