Mark Scheme

Q1.

Question Number	Scheme	Marks
	$\sum_{r=1}^{n} 3(4r^2 - 4r + 1) = 12\sum_{r=1}^{n} r^2 - 12\sum_{r=1}^{n} r + \sum_{r=1}^{n} 3$	M1
	$= \frac{12}{6}n(n+1)(2n+1) - \frac{12}{2}n(n+1), \quad +3n$	A1, B1
	$= n\left[2(n+1)(2n+1) - 6(n+1) + 3\right]$	M1
	$= n\left[4n^2 - 1\right] = n(2n+1)(2n-1)$	A1 cso
		[5]
Notes:	Induction is not acceptable here First M for expanding given expression to give a 3 term quadratic and attempt to substitute. First A for first two terms correct or equivalent. B for $+3n$ appearing Second M for factorising by n Final A for completely correct solution	

Q2.

Question Number	Scheme	Marks
(a)	$6\sum r^2 + 4\sum r - \sum 1 = 6\frac{n}{6}(n+1)(2n+1) + 4\frac{n}{2}(n+1), -n$	M1 A1, B1
	$= \frac{n}{6}(12n^2 + 18n + 6 + 12n + 12 - 6)$ or $n(n+1)(2n+1) + (2n+1)n$	M1
	$= \frac{n}{6}(12n^2 + 30n + 12) = n(2n^2 + 5n + 2) = n(n+2)(2n+1)$ *	A1 (5)
(b)	$\sum_{r=1}^{20}(6r^2 + 4r - 1) - \sum_{r=1}^{10}(6r^2 + 4r - 1) = 20 \times 22 \times 41 - 10 \times 12 \times 21$	M1
	$= 15520$	A1 (2) [7]

Q3.

Question Number	Scheme		Marks
(a)	$r(r^2-3)=r^3-3r$	r^3-3r	B1
	$\sum_{r=1}^{n} r(r^2-3)=\sum_{r=1}^{n} r^3-3\sum_{r=1}^{n} r$		
	$=\frac{1}{4}n^2(n+1)^2-\frac{3}{2}n(n+1)$	M1: An attempt to use at least one of the standard formulae correctly. A1: Correct expression	M1A1
	$=\frac{1}{4}n(n+1)(n(n+1)-6)$	Attempt factor of $\frac{1}{4}n(n+1)$ before given answer	M1
	$=\frac{1}{4}n(n+1)(n^2+n-6)$		
	$=\frac{1}{4}n(n+1)(n+3)(n-2)$	cso	A1
			(5)
(b)	$\sum_{r=10}^{50} r(r^2-3)=\mathrm{f}(50)-\mathrm{f}(9\,or\,10)$	Require some use of the result in part (a) for method.	M1
	$=\frac{1}{4}(50)(51)(53)(48)-\frac{1}{4}(9)(10)(12)(7)$	Correct expression	A1
	= 1621800 - 1890		
	= 1619910	cao	A1
			(3)
			Total 8

Q4.

Question Number	Scheme	Marks
(a)	$\sum_{r=1}^{n}(r+1)(r+4)$	
	$=\sum_{r=1}^{n}r^2+5r+4$	B1
	$=\frac{n}{6}(n+1)(2n+1)+5\frac{n}{2}(n+1)+4n$	M1 A1
	$=\frac{n}{6}\{(n+1)(2n+1)+15(n+1)+24\}$	dM1
	$=\frac{n}{6}\{(2n^2+3n+1)+15n+15+24\}$	
	$=\frac{n}{6}\left(2n^2+18n+40\right)$ or $=\frac{n}{3}\left(n^2+9n+20\right)$	
	$=\frac{n}{3}(n+4)(n+5)$ ** **given answer** **	A1*
		(5)
(b)	$\sum_{r=n+1}^{2n}(r+1)(r+4) \quad =\frac{2n}{3}(2n+4)(2n+5)-\frac{n}{3}(n+4)(n+5)$	M1
	$=\frac{n}{3}\{8n^2+36n+40-n^2-9n-20\}$	dM1
	$=\frac{n}{3}\{7n^2+27n+20\}=\frac{n}{3}(n+1)(7n+20)$ or $a=7$, $b=20$	A1
		(3)
		(8 marks)

Notes

(a)

B1: Expands bracket correctly to r^2+5r+4

M1: Uses $\frac{n}{6}(n+1)(2n+1)$ or $\frac{n}{2}(n+1)$ correctly.

A1: Completely correct expression.

dM1: Attempts to remove factor $\frac{n}{6}$ or $\frac{n}{3}$ to obtain a quadratic factor. Need not be 3 term.

A1: Completely correct work including a step with a collected **3 term** quadratic prior in the bracket with correct printed answer.

Accept approach which starts with LHS and then RHS which meet at $\frac{n^3}{3}+3n^2+\frac{20n}{3}$. Award marks as above.

NB If induction attempted then typically this may only score the first B1.
However, consider the solution carefully and award as above if seen in the body of the induction attempt.

(b) M1: Uses f($2n$) – f(n) or f($2n$) – f(n+1) correctly. Require all 3 terms in $2n$ (and n+1 if used).

dM1: Attempts to remove factor $\frac{n}{6}$ or $\frac{n}{3}$ to obtain a quadratic factor. Need not be 3 term.

A1: Either in expression or as above.

Q5.

Question Number	Scheme	Notes	Marks
	$\{S_n =\} \sum_{r=1}^{n} (2r-1)^2$		
(a)	$= \sum_{r=1}^{n} 4r^2 - 4r + 1$	Multiplying out brackets and an attempt to use at least one of the two standard formulae correctly.	M1
	$= 4.\frac{1}{6}n(n+1)(2n+1) - 4.\frac{1}{2}n(n+1) + n$	First two terms correct.	A1
		$+n$	B1
	$= \frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$		
	$= \frac{1}{3}n\{2(n+1)(2n+1) - 6(n+1) + 3\}$	Attempt to factorise out $\frac{1}{3}n$	M1
		Correct expression with $\frac{1}{3}n$ factorised out with no errors seen.	A1
	$= \frac{1}{3}n\{2(2n^2+3n+1) - 6(n+1) + 3\}$ $= \frac{1}{3}n\{4n^2+6n+2 - 6n - 6 + 3\}$ $= \frac{1}{3}n(4n^2-1)$		
	$= \frac{1}{3}n(2n+1)(2n-1)$	Correct proof. No errors seen.	A1 *
			(6)
	Note that there are no marks for proof by induction.		
(b)	$\sum_{r=n+1}^{3n} (2r-1)^2 = S_{3n} - S_n$		
	$= \frac{1}{3}.3n(6n+1)(6n-1) - \frac{1}{3}n(2n+1)(2n-1)$	Use of $S_{3n} - S_n$ or $S_{3n} - S_{n+1}$ with the result from (a) used at least once.	M1
		Correct unsimplified expression. E.g. Allow $2(3n)$ for $6n$.	A1
	Note that (b) says **hence** so they have to be using the result from (a)		
	$= n(36n^2-1) - \frac{1}{3}n(4n^2-1)$		
	$= \frac{1}{3}n(108n^2 - 3 - 4n^2 + 1)$	Factorising out $\frac{1}{3}n$ (or $\frac{2}{3}n$)	dM1
	$= \frac{1}{3}n(104n^2 - 2)$		
	$= \frac{2}{3}n(52n^2 - 1)$	$\frac{2}{3}n(52n^2-1)$	A1
	$\{a = 52,\ b = -1\}$		(4)
			10

Q6.

Question	Scheme	Marks	AOs
(a)	$(5r-2)^2 = 25r^2 - 20r + 4$	B1	1.1b
	$\sum_{r=1}^{n} 25r^2 - 20r + 4 = \frac{25}{6}n(n+1)(2n+1) - \frac{20}{2}n(n+1) + ...$	M1	2.1
	$= \frac{25}{6}n(n+1)(2n+1) - \frac{20}{2}n(n+1) + 4n$	A1	1.1b
	$= \frac{1}{6}n\left[25\left(2n^2+3n+1\right) - 60(n+1) + 24\right]$	dM1	1.1b
	$= \frac{1}{6}n\left[50n^2 + 15n - 11\right]$	A1	1.1b
		(5)	
(b)	$\frac{1}{6}k\left[50k^2 + 15k - 11\right] = 94k^2$	M1	1.1b
	$50k^3 - 549k^2 - 11k = 0$ or $50k^2 - 549k - 11 = 0$	A1	1.1b
	$(k-11)(50k+1) = 0 \Rightarrow k = ...$	M1	1.1b
	$k = 11$(only)	A1	2.3
		(4)	
		(9 marks)	

Notes

(a)
B1: Correct expansion
M1: Substitutes at least one of the standard formulae into their expanded expression
A1: Fully correct expression
dM1: Attempts to factorise $\frac{1}{6}n$ having used at least one standard formula correctly. Dependent on the first M mark.
A1: Obtains the correct expression or the correct values of a, b and c
(b)
M1: Uses their result from part (a) and sets equal to $94k^2$ and attempt to expand and collect terms.
A1: Correct cubic or quadratic
M1: Attempts to solve their 3TQ or cubic equation
A1: Identifies the correct value of k with no other values offered

Q7.

Question	Scheme	Marks	AOs
(a)	$\sum_{r=1}^{n} r^2(r+1) = \sum_{r=1}^{n} r^3 + r^2 = \frac{1}{4}n^2(n+1)^2 + \frac{1}{6}n(n+1)(2n+1)$	M1 A1	1.1b 1.1b
	$= \frac{1}{12}n(n+1)\left[3n(n+1)+2(2n+1)\right]$	dM1	1.1b
	$= \frac{1}{12}n(n+1)\left[3n^2+7n+2\right] = \frac{1}{12}n(n+1)(n+2)(3n+1)$ cso	A1	2.1
		(4)	
(b)	$\sum_{r=k+1}^{3k} r^2(r+1) = \frac{1}{12}(3k)(3k+1)(3k+2)(9k+1) - \frac{1}{12}(k)(k+1)(k+2)(3k+1)$	M1	3.1a
	$= \frac{1}{12}k(3k+1)\left[3(3k+2)(9k+1)-(k+1)(k+2)\right]$ or $= \frac{1}{3}k(3k+1)\left[\frac{3}{4}(3k+2)(9k+1) - \frac{1}{4}(k+1)(k+2)\right]$	M1	1.1b
	$= \frac{1}{12}k(3k+1)\left[80k^2+60k+4\right]$ $= \frac{1}{3}k(3k+1)\left(20k^2+15k+1\right)$ cso	A1	1.1b
		(3)	
(c)	$\frac{25}{3}k(3k+1)\left(20k^2+15k+1\right) = 192k^3(3k+1)$ either $\Rightarrow 25\left(20k^2+15k+1\right) = 576k^2 \Rightarrow 76k^2 - 375k - 25 = 0$ Or $\Rightarrow 25k\left(20k^2+15k+1\right) = 576k^3 \Rightarrow 76k^3 - 375k^2 - 25k = 0$ Or $\Rightarrow 25(3k+1)\left(20k^2+15k+1\right) = 576k^2(3k+1) \Rightarrow 228k^3 - 1049k^2 - 450k - 25 = 0$ Or $-76k^4 + \frac{1049}{3}k^3 + 150k^2 + \frac{25}{3}k = 0$	M1	1.1b
	$76k^2 - 375k - 25 = 0 \Rightarrow k = \ldots$ $76k^3 - 375k^2 - 25k = 0 \Rightarrow k = \ldots$ $-76k^4 + \frac{1049}{3}k^3 + 150k^2 + \frac{25}{3}k = 0 \Rightarrow k = \ldots$	M1	1.1b
	$k = 5$ (only)	A1	2.3
		(3)	
		(10 marks)	

Notes
(a) M1: Substitutes at least one of the standard formulae into their expanded expression

A1: Fully correct expression

dM1: Attempts to factorise $\frac{1}{12}n(n+1)$ or $\frac{1}{3}n(n+1)$ having used at least one standard formula correctly at any stage. Dependent on the first M mark.

If they show no method for factorising (use a calculator) they can go from
$3n^3+10n^2+9n+2=(n+1)(n+2)(3n+1)$

A1: Obtains the correct expression or the correct values of a and b, with no errors seen

(b)

M1: Uses the result from part (a) and adopts a correct strategy by attempting
$\sum_{r=1}^{3k} r^2(r+1)-\sum_{r=1}^{k} r^2(r+1)$

M1: Factorises out at least $k(3k+1)$ at any stage, could be done by inspection

A1: Obtains the correct expression with no errors seen.

Note If a candidate does not use part (a) but restarts they can still score marks for the same reasons. If unsure please send to review

(c)

M1: Uses the given equation, substitutes their answer from part (b) and simplifies to either reach

- $Ak^4+Bk^3+Ck^2+Dk\{=0\}$
- $k(Ak^3+Bk^2+Ck+D)\{=0\}$ or $(3k+1)(Ak^3+Bk^2+Ck)\{=0\}$
- $Ak^3+Bk^2+Ck\{=0\}$
- a 3TQ or $k(3k+1)(Ak^2+Bk+C)\{=0\}$

this can be implied by a correct value for k if left unsimplified

M1: Solves their equation as long as solving their $(b)=192k^3(3k+1)$ to find a non zero value for k, including by calculator. You may need to check this.

A1: Selects the appropriate correct answer of $k=5$. Any other solutions must be clearly rejected.

Note: Correct answer with no working is M0M0A0