

Questions

Q1.

Show, using the formulae for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$, that

$$\sum_{r=1}^n 3(2r - 1)^2 = n(2n + 1)(2n - 1)$$

for all positive integers n .

(5)

(Total 5 marks)

Q2.

(a) Show, using the formulae for $\sum r$ and $\sum r^2$, that

$$\sum_{r=1}^n (6r^2 + 4r - 1) = n(n + 2)(2n + 1)$$

(5)

(b) Hence, or otherwise, find the value of

$$\sum_{r=11}^{n=20} (6r^2 + 4r - 1)$$

(2)

(Total 7 marks)

Q3.

$$\sum_{r=1}^n \sum_{r=1}^n$$

(a) Use the standard results for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^3$ to show that

$$\sum_{r=1}^n r(r^2 - 3) = \frac{1}{4}n(n + 1)(n + 3)(n - 2)$$

(5)

(b) Calculate the value of

$$\sum_{r=10}^{50} r(r^2 - 3)$$

(3)

(Total 8 marks)

Q4.

(a) Using the formulae for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$, show that

$$\sum_{r=1}^n (r+1)(r+4) = \frac{n}{3} (n+4)(n+5)$$

for all positive integers n .

(5)

(b) Hence show that

$$\sum_{r=n+1}^{2n} (r+1)(r+4) = \frac{n}{3} (n+1)(an+b)$$

where a and b are integers to be found.

(3)

(Total for question = 8 marks)

Q5.

(a) Use the results for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$ to show that

$$\sum_{r=1}^n (2r-1)^2 = \frac{1}{3} n(2n+1)(2n-1)$$

for all positive integers n .

(6)

(b) Hence show that

$$\sum_{r=n+1}^{3n} (2r-1)^2 = \frac{2}{3} n(an^2 + b)$$

where a and b are integers to be found.

(4)

(Total 10 marks)

Q6.

(a) Use the standard results for summations to show that for all positive integers n

$$\sum_{r=1}^n (5r-2)^2 = \frac{1}{6}n(an^2 + bn + c)$$

where a , b and c are integers to be determined.

(5)

(b) Hence determine the value of k for which

$$\sum_{r=1}^k (5r-2)^2 = 94k^2$$

(4)

(Total for question = 9 marks)

Q7.

(a) Use the standard results for summations to show that, for all positive integers n ,

$$\sum_{r=1}^n r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(an+b)$$

where a and b are integers to be determined.

(4)

(b) Hence show that, for all positive integers k ,

$$\sum_{r=k+1}^{3k} r^2(r+1) = \frac{1}{3}k(3k+1)(Ak^2+Bk+C)$$

where A , B and C are integers to be determined.

(3)

(c) Hence, using algebra and making your method clear, determine the value of k for which

$$25 \sum_{r=k+1}^{3k} r^2(r+1) = 192k^3(3k+1)$$

(3)

(Total for question = 10 marks)