

## Questions

### Q1.

Show, using the formulae for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^2$ , that

$$\sum_{r=1}^n 3(2r - 1)^2 = n(2n + 1)(2n - 1)$$

for all positive integers  $n$ .

(5)

(Total 5 marks)

### Q2.

(a) Show, using the formulae for  $\sum r$  and  $\sum r^2$ , that

$$\sum_{r=1}^n (6r^2 + 4r - 1) = n(n + 2)(2n + 1)$$

(5)

(b) Hence, or otherwise, find the value of

$$\sum_{r=11}^{n=20} (6r^2 + 4r - 1)$$

(2)

(Total 7 marks)

### Q3.

(a) Use the standard results for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^3$  to show that

$$\sum_{r=1}^n r(r^2 - 3) = \frac{1}{4}n(n + 1)(n + 3)(n - 2)$$

(5)

(b) Calculate the value of

$$\sum_{r=10}^{50} r(r^2 - 3)$$

(3)

(Total 8 marks)

**Q4.**

(a) Using the formulae for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^2$ , show that

$$\sum_{r=1}^n (r+1)(r+4) = \frac{n}{3} (n+4)(n+5)$$

for all positive integers  $n$ .

**(5)**

(b) Hence show that

$$\sum_{r=n+1}^{2n} (r+1)(r+4) = \frac{n}{3} (n+1)(an+b)$$

where  $a$  and  $b$  are integers to be found.

**(3)**

**(Total for question = 8 marks)**

**Q5.**

(a) Use the results for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^2$  to show that

$$\sum_{r=1}^n (2r-1)^2 = \frac{1}{3} n(2n+1)(2n-1)$$

for all positive integers  $n$ .

**(6)**

(b) Hence show that

$$\sum_{r=n+1}^{3n} (2r-1)^2 = \frac{2}{3} n(an^2+b)$$

where  $a$  and  $b$  are integers to be found.

**(4)**

**(Total 10 marks)**

**Q6.**

(a) Use the standard results for summations to show that for all positive integers  $n$

$$\sum_{r=1}^n (5r - 2)^2 = \frac{1}{6}n(an^2 + bn + c)$$

where  $a$ ,  $b$  and  $c$  are integers to be determined.

**(5)**

(b) Hence determine the value of  $k$  for which

$$\sum_{r=1}^k (5r - 2)^2 = 94k^2$$

**(4)**

**(Total for question = 9 marks)**

**Q7.**

(a) Use the standard results for summations to show that, for all positive integers  $n$ ,

$$\sum_{r=1}^n r^2(r + 1) = \frac{1}{12}n(n + 1)(n + 2)(an + b)$$

where  $a$  and  $b$  are integers to be determined.

**(4)**

(b) Hence show that, for all positive integers  $k$ ,

$$\sum_{r=k+1}^{3k} r^2(r + 1) = \frac{1}{3}k(3k + 1)(Ak^2 + Bk + C)$$

where  $A$ ,  $B$  and  $C$  are integers to be determined.

**(3)**

(c) Hence, using algebra and making your method clear, determine the value of  $k$  for which

$$25 \sum_{r=k+1}^{3k} r^2(r + 1) = 192k^3(3k + 1)$$

**(3)**

**(Total for question = 10 marks)**