Exam Questions – Chapter 8 Further Kinematics (A2)

Q1.

A curve has equation

$$y = \frac{2}{3}x^3 - \frac{7}{2}x^2 - 4x + 5$$

(a) Find $\frac{dy}{dx}$ writing your answer in simplest form.

(2)

(b) Hence find the range of values of x for which y is decreasing.

(4)

(Total for question = 6 marks)

Q2.

At time t seconds, where $t \geq 0$, a particle P is moving on a horizontal plane with acceleration $[(3t^2 - 4t)\mathbf{i} + (6t - 5)\mathbf{j}]$ m s^{-2}.

When $t = 3$ the velocity of P is $(11\mathbf{i} + 10\mathbf{j})$ m s^{-1}.

Find

(a) the velocity of P at time t seconds,

(5)

(b) the speed of P when it is moving parallel to the vector $\mathbf{i}$.

(4)

(Total 9 marks)

Q3.

[*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A particle P moves in such a way that its velocity $\mathbf{v}$ m s^{-1} at time t seconds is given by

$$\mathbf{v} = (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}$$

(5)

Given that, when $t = 0$, the position vector of P is $\mathbf{i}$ metres,

(b) find the position vector of P when $t = 3$

(5)

(Total 10 marks)

Q4.

[*In this question position vectors are given relative to a fixed origin O*]

At time t seconds, where $t \geq 0$, a particle, P, moves so that its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j})$ m .

(a) Find the acceleration of P when $t = 4$

(3)

(b) Find the position vector of P when $t = 4$

(3)

(Total for question = 6 marks)

Q5.

[*In this question,* **i** *is a horizontal unit vector and* **j** *is an upward vertical unit vector.*]

A particle P is projected from a fixed origin O with velocity $(3\mathbf{i} + 4\mathbf{j})$m s^{-1}. The particle moves freely under gravity and passes through the point A with position vector $\lambda(\mathbf{i} - \mathbf{j})$m, where λ is a positive constant.

(a) Find the value of λ.

(6)

(b) Find

(i) the speed of P at the instant when it passes through A,

(ii) the direction of motion of P at the instant when it passes through A.

(7)

(Total for question = 13 marks)

Q6.

Unless otherwise indicated, wherever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

A particle, P, moves along the x-axis. At time t seconds, $t \geq 0$, the displacement, x metres, of P from the origin O, is given by $x = \frac{1}{2}t^2(t^2 - 2t + 1)$

(a) Find the times when P is instantaneously at rest.

(5)

(b) Find the total distance travelled by P in the time interval $0 \leq t \leq 2$

(3)

(c) Show that P will never move along the negative x-axis.

(2)

(Total for question = 10 marks)

Q7.

[*In this question, position vectors are given relative to a fixed origin*.]

At time t seconds, where $t > 0$, a particle P has velocity $\mathbf{v}$ m s^{-1} where

$$\mathbf{v} = 3t^2\mathbf{i} - 6t^{\frac{1}{2}}\mathbf{j}$$

(a) Find the speed of P at time $t = 2$ seconds.

(2)

(b) Find an expression, in terms of t, $\mathbf{i}$ and $\mathbf{j}$, for the acceleration of P at time t seconds, where $t > 0$

(2)

At time $t = 4$ seconds, the position vector of P is $(\mathbf{i} - 4\mathbf{j})$ m.

(c) Find the position vector of P at time $t = 1$ second.

(4)

(Total for question = 8 marks)

Q8.

(i) At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = (1 - 4t)\mathbf{i} + (3 - t^2)\mathbf{j}$$

At the instant when $t = 0$, the velocity of P is $36\mathbf{i}$ m s^{-1}

(a) Find the velocity of P when $t = 4$

(3)

(b) Find the value of t at the instant when P is moving in a direction perpendicular to $\mathbf{i}$

(3)

(ii) At time t seconds, where $t \geq 0$, a particle Q moves so that its position vector $\mathbf{r}$ metres, relative to a fixed origin O, is given by

$$\mathbf{r} = (t^2 - t)\mathbf{i} + 3t\mathbf{j}$$

Find the value of t at the instant when the speed of Q is 5 m s^{-1}

(6)

(Total for question = 12 marks)