Exam Questions – Complex Numbers (Chapter 1 and 2)

Q1.

$$z = \frac{4}{1+i}$$

Find, in the form $a + ib$ where $a, b \in \mathbb{R}$

(a) z

(2)

(b) z^2

(2)

Given that z is a complex root of the quadratic equation $x^2 + px + q = 0$, where p and q are real integers,

(c) find the value of p and the value of q.

(3)

(Total for question = 7 marks)

Q2.

$$f(x) = 9x^3 - 33x^2 - 55x - 25$$

Given that $x = 5$ is a solution of the equation $f(x) = 0$, use an algebraic method to solve $f(x) = 0$ completely.

(5)

(Total for question = 5 marks)

Q3.

Given that 4 and $2i - 3$ are roots of the equation

$$x^3 + ax^2 + bx - 52 = 0$$

where a and b are real constants,

(a) write down the third root of the equation,

(1)

(b) find the value of a and the value of b.

(5)

(Total for question = 6 marks)

Q4.

The complex numbers *z* and *w* are given by

$$z = 8 + 3i, \quad w = -2i$$

Express in the form *a* + *b*i, where *a* and *b* are real constants,

(a) *z* − *w*,

(1)

(b) *zw*.

(2)

(Total 3 marks)

Q5.

The complex number *w* is given by

$$w = 10 - 5i$$

(a) Find |*w*|.

(1)

(b) Find arg *w*, giving your answer in radians to 2 decimal places.

(2)

The complex numbers *z* and *w* satisfy the equation

$$(2 + i)(z + 3i) = w$$

(c) Use algebra to find *z*, giving your answer in the form *a* + *b*i, where *a* and *b* are real numbers.

(4)

Given that

$$\arg(\lambda + 9i + w) = \frac{\pi}{4}$$

where *λ* is a real constant,

(d) find the value of *λ*.

(2)

(Total 9 marks)

Q6.

Given that $x = \dfrac{1}{2}$ is a root of the equation

$$2x^3 - 9x^2 + kx - 13 = 0, \quad k \in \mathbb{R}$$

find

(a) the value of k,

(3)

(a) the other 2 roots of the equation.

(4)

(Total 7 marks)

Q7.

$$z = \dfrac{50}{3+4i}$$

Find, in the form $a + i\,b$ where $a,b \in$ R,

(a) z,

(2)

(b) z^2 .

(2)

Find

(c) $|z|$,

(2)

(d) $\arg z^2$, giving your answer in degrees to 1 decimal place.

(2)

(Total 8 marks)

Q8.

The roots of the equation

$$z^3 - 8z^2 + 22z - 20 = 0$$

are z_1, z_2 and z_3.

(a) Given that $z_1 = 3 + i$, find z_2 and z_3.

(4)

(b) Show, on a single Argand diagram, the points representing z_1, z_2 and z_3.

(2)

(Total 6 marks)

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Q9.

(a) Shade on an Argand diagram the set of points

$$\left\{ z \in \mathbb{C} : \left| z - 4\mathrm{i} \right| \leqslant 3 \right\} \cap \left\{ z \in \mathbb{C} : -\frac{\pi}{2} < \arg(z + 3 - 4\mathrm{i}) \leqslant \frac{\pi}{4} \right\}$$

(6)

The complex number w satisfies

$$\left| w - 4\mathrm{i} \right| = 3$$

(b) Find the maximum value of arg w in the interval ($-\pi$, π].

Give your answer in radians correct to 2 decimal places.

(2)

(Total for question = 8 marks)

Q10.

A complex number z is represented by the point P in the complex plane.

Given that z satisfies

$$\left| z - 1 \right| = 1$$

(a) sketch on an Argand diagram the locus of P as z varies.

(2)

Given that z also satisfies

$$\arg (z + 1) = \theta$$

(b) determine the possible values of θ such that the locus arg $(z + 1) = \theta$ is a tangent to the locus $\left| z - 1 \right| = 1$

(3)

(c) Hence determine the exact possible complex numbers z.

(3)

(Total for question = 8 marks)

(i) The point P is one vertex of a regular pentagon in an Argand diagram. The centre of the pentagon is at the origin.

Given that P represents the complex number $6 + 6i$, determine the complex numbers that represent the other vertices of the pentagon, giving your answers in the form $re^{i\theta}$

(5)

(ii) (a) On a single Argand diagram, shade the region, R, that satisfies both

$$|z - 2i| \leqslant 2 \qquad \text{and} \qquad \frac{1}{4}\pi \leqslant \arg z \leqslant \frac{1}{3}\pi$$

(2)

(b) Determine the exact area of R, giving your answer in simplest form.

(4)

(Total for question = 11 marks)

Q12.

Given that on an Argand diagram the locus of points defined by $|z + 5 - 12i| = 10$ is a circle,

(a) write down,

(i) the coordinates of the centre of this circle,
(ii) the radius of this circle.

(2)

(b) Show, by shading on an Argand diagram, the set of points defined by

$$|z + 5 - 12i| \leqslant 10$$

(1)

(c) For the set of points defined in part (b), determine the maximum value of $|z|$

(3)

The set of points A is defined by

$$A = \{z : 0 \leqslant \arg(z + 5 - 20i) \leqslant \pi\} \cap \{z : |z + 5 - 12i| \leqslant 10\}$$

(d) Determine the area of the region defined by A, giving your answer to 3 significant figures.

(4)

(Total for question = 14 marks)

The locus *C* is given by

$$|z - 4| = 4$$

The locus *D* is given by

$$\arg z = \frac{\pi}{3}$$

(a) Sketch, on the same Argand diagram, the locus *C* and the locus *D*

(4)

The set of points *A* is defined by

$$A = \{z \in \mathbb{C} : |z - 4| \leqslant 4\} \cap \left\{z \in \mathbb{C} : 0 \leqslant \arg z \leqslant \frac{\pi}{3}\right\}$$

(b) Show, by shading on your Argand diagram, the set of points *A*

(1)

(c) Find the area of the region defined by *A*, giving your answer in the form *pπ* + *q*√3 where *p* and *q* are constants to be determined.

(4)

(Total for question = 9 marks)