Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	$x^2 + y^2 = r^2$	B1	1.2
	$\{V\} = \pi \int_{-r}^{r} r^2 - x^2 \, dx$ or $\{V\} = 2\pi \int_{0}^{r} r^2 - x^2 \, dx$	B1	2.1
	Integrates to the form $\alpha x \pm \beta x^3$ $\left[\text{note: the correct integration gives } r^2 x - \frac{1}{3}x^3\right]$	M1	1.1b
	Substitutes limits of $-r$ and r and subtracts the correct way round $\left(r^2(r) - \frac{1}{3}(r)^3\right) - \left(r^2(-r) - \frac{1}{3}(-r)^3\right)$ or Substitutes limits of 0 and r and subtracts the correct way round with twice the volume. Note the limit of 0 can be implied if gives and answer of 0 $\left(r^2(r) - \frac{1}{3}(r)^3\right) - (0)$	dM1	1.1b
	$V = \frac{4}{3}\pi r^3$ * cso	A1*	1.1b
		(5)	
			(5 marks)

Notes:

B1: Correct equation of the circle, may be implied by correct integral

B1: Correct expression for the volume, including limits, dx may be implied and if using limits r and 0 the 2 could appear later with reasoning

M1: Integrates to the form $\alpha x \pm \beta x^3$. Do not award if $r^2 \to \lambda r^3$

dM1: Dependent on previous method mark. Correct use of limits r and r or limits of 0 and r with twice the volume.

A1*: $V = \frac{4}{3}\pi r^3$ * cso

Note: rotation about the y-axis all marks are available, however for the final accuracy mark must refer to symmetry

(**Q03 8FM0/01, Oct 2020**)

Q2.

Question Number	Scheme	Notes	Marks
Way 1	$y = e^x + 2e^{-x},\ x \geq 0$		
	$\{V =\}\ \pi\int_0^{\ln 4}\left(e^x + 2e^{-x}\right)^2 dx$	For $\pi\int\left(e^x + 2e^{-x}\right)^2$. Ignore limits and dx. Can be implied.	B1
	$= \{\pi\}\int_0^{\ln 4}\left(e^{2x} + 4e^{-2x} + 4\right)dx$	Expands $\left(e^x + 2e^{-x}\right)^2 \to \pm\alpha e^{2x} \pm \beta e^{-2x} \pm \delta$ where $\alpha, \beta, \delta \neq 0$. Ignore π, integral sign, limits and dx. This can be implied by later work.	M1
	$= \{\pi\}\left[\frac{1}{2}e^{2x} - 2e^{-2x} + 4x\right]_0^{\ln 4}$	Integrates at least one of either $\pm\alpha e^{2x}$ to give $\pm\frac{\alpha}{2}e^{2x}$ or $\pm\beta e^{-2x}$ to give $\pm\frac{\beta}{2}e^{-2x}$ $\alpha, \beta \neq 0$	M1
		dependent on the 2nd M mark $e^{2x} + 4e^{-2x} \to \frac{1}{2}e^{2x} - 2e^{-2x}$, which can be simplified or un-simplified	A1
		$4 \to 4x$ or $4e^0x$	B1 cao
	$= \{\pi\}\left(\left(\frac{1}{2}e^{2(\ln 4)} - 2e^{-2(\ln 4)} + 4(\ln 4)\right) - \left(\frac{1}{2}e^0 - 2e^0 + 4(0)\right)\right)$	**dependent on the previous method mark.** Some evidence of applying limits of $\ln 4$ o.e. and 0 to a changed function in x and subtracts the correct way round. **Note:** A proper consideration of the limit of 0 is required.	dM1
	$= \{\pi\}\left(\left(8 - \frac{1}{8} + 4\ln 4\right) - \left(\frac{1}{2} - 2\right)\right)$		
	$= \frac{75}{8}\pi + 4\pi\ln 4$ or $\frac{75}{8}\pi + 8\pi\ln 2$ or $\pi\left(\frac{75}{8} + 4\ln 4\right)$ or $\pi\left(\frac{75}{8} + 8\ln 2\right)$ or $\frac{75}{8}\pi + \ln 2^{8\pi}$ or $\frac{75}{8}\pi + \pi\ln 256$ or $\ln\left(2^{8\pi}e^{\frac{75}{8}\pi}\right)$ or $\frac{1}{8}\pi\left(75 + 32\ln 4\right)$, etc		A1 isw
			[7]
			7

	Question Notes	
	Note	π is only required for the 1st B1 mark and the final A1 mark.
	Note	Give 1st B0 for writing $\pi\int y^2\,dx$ followed by $2\pi\int\left(e^x + 2e^{-x}\right)^2 dx$
	Note	Give 1st M1 for $\left(e^x + 2e^{-x}\right)^2 \to e^{2x} + 4e^{-2x} + 2e^0 + 2e^0$ because $\delta = 2e^0 + 2e^0$
	Note	A decimal answer of 46.8731... or $\pi(14.9201...)$ (without a correct **exact** answer) is A0
	Note	$\pi\left[\frac{1}{2}e^{2x} - 2e^{-2x} + 4x\right]_0^{\ln 4}$ followed by awrt 46.9 (without a correct **exact** answer) is final dM1A0
	Note	Allow exact equivalents which should be in the form $a\pi + b\pi\ln c$ or $\pi(a + b\ln c)$, where $a = \frac{75}{8}$ or $9\frac{3}{8}$ or 9.375. Do not allow $a = \frac{150}{16}$ or $9\frac{6}{16}$
	Note	Give B1M0M1A1B0M1A0 for the common response $\pi\int_0^{\ln 4}\left(e^x + 2e^{-x}\right)^2 dx \to \pi\int_0^{\ln 4}\left(e^{2x} + 4e^{-2x}\right)dx = \pi\left[\frac{1}{2}e^{2x} - 2e^{-2x}\right]_0^{\ln 4} = \frac{75}{8}\pi$

Question Number	Scheme	Notes	Marks
	$y = e^x + 2e^{-x},\ x \geq 0$		
Way 2	$\{V =\}\ \pi\int_0^{\ln 4}\left(e^x + 2e^{-x}\right)^2 dx$	For $\pi\int\left(e^x + 2e^{-x}\right)^2$. Ignore limits and dx. Can be implied.	B1
	$u = e^x \Rightarrow \frac{du}{dx} = e^x = u$ and $x = \ln 4 \Rightarrow u = 4, x = 0 \Rightarrow u = e^0 = 1$		
	$V = \{\pi\}\int_1^4\left(u + \frac{2}{u}\right)^2\frac{1}{u}du = \{\pi\}\int_1^4\left(u^2 + \frac{4}{u^2} + 4\right)\frac{1}{u}du$		
	$= \{\pi\}\int_1^4\left(u + \frac{4}{u^3} + \frac{4}{u}\right)du$	$\left(e^x + 2e^{-x}\right)^2 \to \pm\alpha u \pm \beta u^{-3} \pm \delta u^{-1}$ where $u = e^x, \alpha, \beta, \delta \neq 0$. Ignore π, integral sign, limits and du. This can be implied by later work.	M1
	$= \{\pi\}\left[\frac{1}{2}u^2 - \frac{2}{u^2} + 4\ln u\right]_1^4$	Integrates at least one of either $\pm\alpha u$ to give $\pm\frac{\alpha}{2}u^2$ or $\pm\beta u^{-3}$ to give $\pm\frac{\beta}{2}u^{-2}$ $\alpha, \beta \neq 0$, where $u = e^x$	M1
		dependent on the 2nd M mark $u + 4u^{-3} \to \frac{1}{2}u^2 - 2u^{-2}$, simplified or un-simplified, where $u = e^x$	A1
		$4u^{-1} \to 4\ln u$, where $u = e^x$	B1 cao
	$= \{\pi\}\left(\left(\frac{1}{2}(4)^2 - \frac{2}{(4)^2} + 4\ln 4\right) - \left(\frac{1}{2}(1)^2 - \frac{2}{(1)^2} + 4\ln 1\right)\right)$	**dependent on the previous method mark.** Some evidence of applying limits of 4 and 1 to a changed function in u [or $\ln 4$ o.e. and 0 to an integrated function in x] and subtracts the correct way round.	dM1
	$= \{\pi\}\left(\left(8 - \frac{1}{8} + 4\ln 4\right) - \left(\frac{1}{2} - 2\right)\right)$		
	$= \frac{75}{8}\pi + 4\pi\ln 4$ or $\frac{75}{8}\pi + 8\pi\ln 2$ or $\pi\left(\frac{75}{8} + 4\ln 4\right)$ or $\pi\left(\frac{75}{8} + 8\ln 2\right)$ or $\frac{75}{8}\pi + \ln 2^{8\pi}$ or $\frac{75}{8}\pi + \pi\ln 256$ or $\ln\left(2^{8\pi}e^{\frac{75}{8}\pi}\right)$ or $\frac{1}{8}\pi(75 + 32\ln 4)$, etc		A1 isw
			[7]

(**Q01 6666/01, June 2017**)

Q3.

Question	Scheme	Marks	AOs
	A correct overall strategy, an attempt at integrating y^2 with respect to x combine in some way with the volume of revolution formula (use of $\pi\int y^2\mathrm{d}x$ or $\alpha\int y^2\mathrm{d}x$ for any variable α is fine) followed by attempt to find an angle/form an equation in θ	M1	3.1a
	$y^2 = kx^{\frac{2}{3}} + \ldots + \frac{m}{x^{\frac{4}{3}}}$ or $y^2 = kx^{\frac{2}{3}} + \ldots + mx^{-\frac{4}{3}}$ where … is one or two more terms.	M1	1.1b
	$y^2 = 4x^{\frac{2}{3}} + 4x^{-\frac{1}{3}} + x^{-\frac{4}{3}}$ or $y^2 = 4x^{\frac{2}{3}} + 2x^{-\frac{1}{3}} + x^{-\frac{4}{3}} + 2x^{-\frac{1}{3}}$ (oe)	A1	1.1b
	$\int y^2\mathrm{d}x = \int 4x^{\frac{2}{3}} + \frac{4}{x^{\frac{1}{3}}} + \frac{1}{x^{\frac{4}{3}}}\mathrm{d}x = \alpha x^{\frac{5}{3}} + \beta x^{\frac{2}{3}} + \gamma x^{-\frac{1}{3}}$	M1	1.1b
	$= \frac{12x^{\frac{5}{3}}}{5} + 6x^{\frac{2}{3}} - \frac{3}{x^{\frac{1}{3}}}$ (oe)	A1ft A1	1.1b 1.1b
	$\frac{\theta}{2}\left[\frac{12x^{\frac{5}{3}}}{5} + 6x^{\frac{2}{3}} - \frac{3}{x^{\frac{1}{3}}}\right]_{\frac{1}{8}}^{8} = \frac{461}{2}$ $\Rightarrow \frac{\theta}{2}\left[\left(\frac{12\times 8^{\frac{5}{3}}}{5} + 6\times 8^{\frac{2}{3}} - \frac{3}{8^{\frac{1}{3}}}\right) - \left(\frac{12\times\left(\frac{1}{8}\right)^{\frac{5}{3}}}{5} + 6\times\left(\frac{1}{8}\right)^{\frac{2}{3}} - \frac{3}{\left(\frac{1}{8}\right)^{\frac{1}{3}}}\right)\right] = \frac{461}{2} \Rightarrow \theta = \ldots$ OR $\pi\left[\frac{12x^{\frac{5}{3}}}{5} + 6x^{\frac{2}{3}} - \frac{3}{x^{\frac{1}{3}}}\right]_{\frac{1}{8}}^{8} = \pi\left[\left(\frac{12\times 8^{\frac{5}{3}}}{5} + 6\times 8^{\frac{2}{3}} - \frac{3}{8^{\frac{1}{3}}}\right) - \left(\frac{12\times\left(\frac{1}{8}\right)^{\frac{5}{3}}}{5} + 6\times\left(\frac{1}{8}\right)^{\frac{2}{3}} - \frac{3}{\left(\frac{1}{8}\right)^{\frac{1}{3}}}\right)\right] = \ldots$ followed by $\frac{\theta}{2\pi}\times\ldots = \frac{461}{2} \Rightarrow \theta = \ldots$	M1	3.1a
	$\theta = \frac{40}{9}$ (radians)	A1	1.1b
		(8)	
	(8 marks)		

Notes		
	M1	A correct overall strategy, either finding full volume rotated by 2π first, then performing some kind of scaling, or using $\alpha\int y^2\mathrm{d}x$ for a variable α (ideally $\frac{\theta}{2}$, but for the strategy accept with any variable multiple), to form an equation in just the angle.
	M1	Attempting to square y to a three or four term expression. Look for correct powers on first and last term with some term(s) in the middle.
	A1	Correct expansion in three or four terms – award when first seen.
	M1	Integrates y^2 w.r.t. x. Must have at least two terms in their y^2 with fractional indices. Power to be increased by 1 in at least two terms.
	A1ft	Two terms of integral correct. Follow through on their expansion. Need not be simplified.
	A1	Fully correct integral. Need not be simplified. May still be four terms
	M1	Either : Substitutes limits and subtracts correct way round (must be seen or implied by the answer), and equates to $\frac{461}{2}$ if using $\frac{1}{2}\theta\int y^2\mathrm{d}x$ and proceeds to find θ. Or : Substitutes limits and subtracts correct way round (seen or implied) and multiplies by π to get the full volume AND then multiplies the result by $\frac{\theta}{2\pi}$ before equating to $\frac{461}{2}$. **The method must be correct for this mark – so they must be using $\frac{\theta}{2}\int y^2\mathrm{d}x$ directly or $\pi\int y^2\mathrm{d}x$ and scale by $\frac{\theta}{2\pi}$ when setting equal to $\frac{461}{2}$**
	A1	Correct angle found. Accept $\frac{40}{9}$, awrt 4.44 or awrt $255°$ (as long as the degrees units are made clear – do not accept just 255) isw once a correct value of θ is found.

<u>**Special case**</u> The question specified that algebraic integration must be used, so use of a calculator to find the integral cannot score the marks for integration but may be allowed the strategy and answer marks. A maximum of M1M0A0M0A0A0M1A1 is available in such cases.
Expanding y^2 first but showing no integration can score the second M and first A (if earned) as well.

Note that $\int_{1/8}^{8}\left(2x^{\frac{1}{3}}+x^{-\frac{2}{3}}\right)^2\,\mathrm{d}x=\frac{4149}{40}=103.725$ but just this alone is worth **no marks**. There must be an attempt to incorporate this within a strategy to gain access to marks.

(**Q09 8FM0/01, June 2019**)

Q4.

Question	Scheme	Marks	AOs
(a)	$(5, 15) \Rightarrow 15 = \dfrac{\sqrt{225\times 5^2 - 2025}}{a} \Rightarrow a = \ldots$	M1	3.3
	$a = 4$	A1	1.1b
		(2)	
(b)	Evidence of the use of $\pi\int x^2\,\mathrm{d}y$ for the curve BC or the curve CD	M1	3.1b
	For BC $V_1 = \dfrac{\pi}{225}\int(16y^2 + 2025)\,\mathrm{d}y$ or $\pi\int\left(\dfrac{16}{225}y^2 + 9\right)\mathrm{d}y$	A1ft	1.1b
	For CD $V_2 = 25\pi\int(16 - y)\,\mathrm{d}y$ or $\pi\int(400 - 25y)\,\mathrm{d}y$	A1	1.1b
	$V_1 = \dfrac{\pi}{225}\int_0^{15}(16y^2 + 2025)\,\mathrm{d}y$ or $\pi\int_0^{15}\left(\dfrac{16}{225}y^2 + 9\right)\mathrm{d}y$	M1	3.3
	$V_2 = 25\pi\int_{15}^{16}(16 - y)\,\mathrm{d}y$ or $\pi\int_{15}^{16}(400 - 25y)\,\mathrm{d}y$	M1	3.3
	$V_1 = \dfrac{\{\pi\}}{225}\left[\dfrac{16y^3}{3} + 2025y\right]_0^{15}$ or $\{\pi\}\left[\dfrac{16y^3}{675} + 9y\right]_0^{15}$	A1ft	1.1b
	$V_2 = 25\{\pi\}\left[16y - \dfrac{y^2}{2}\right]_{15}^{16}$ or $\{\pi\}\left[400y - \dfrac{25y^2}{2}\right]_{15}^{16}$	A1ft	1.1b
	$V = V_1 + V_2 = \dfrac{\pi}{225}(18000 + 30375) + 25\pi\left(128 - \dfrac{255}{2}\right)$ $V = V_1 + V_2 = 215\pi + 12.5\pi$	M1	3.4
	$V = \dfrac{455\pi}{2}\,\mathrm{cm}^3$ or $227.5\pi\,\mathrm{cm}^3$	A1	2.2b
		(9)	

(c)	E.g. • The equation of the curve may not be a suitable model • The sides of the candle will not be perfectly curved/smooth • There will be a whole in the middle for the wick	B1	3.5b
		(1)	
(d)	Makes an appropriate comment that is consistent with their value for the volume and 700 cm^3. E.g. a good estimate as 700 cm^3 is only 15 cm^3 less than 715 cm^3	B1ft	3.5a
		(1)	
			(13 marks)

Notes

(a)

M1: Substitutes (5, 15) into the equation modelling the curve in an attempt to find the value of a

A1: Infers from the data in the model, the value of a

(b)

M1: Uses either model to obtain x^2 in terms of y and applies $\pi\int x^2\,dy$

A1ft: Correct expression for the volume generated by the curve BC (follow through their a value)

A1: Correct expression for the volume generated by the curve CD

M1: Chooses limits appropriate to their model for the curve BC

M1: Chooses limits appropriate to their model for the curve CD

A1ft: Correct integration (follow through their a value)

A1ft: Correct integration follow through on their volume as long it is of the form $Ay - By^2$

M1: Uses the model to find the sum of volumes

A1: $\dfrac{455\pi}{2}$

Note: Use of calculator for integration maximum score M1 A1ft A1 M1 M1 A0ft A0ft M1 A1

(c)

B1: States an acceptable limitation of the model

(d)

B1ft: Compares the actual volume to their answer to part (b) and makes an assessment of the model with a reason.

(**Q09 8FM0/01, Oct 2021**)

Q5.

Question	Scheme	Marks	AOs
(a)	$\{V=\}\pi\int_0^2\left[(2-y)^{\frac{1}{2}}\right]^2 \mathrm{d}y$ or $\{V=\}\pi\int_0^2(2-y)\mathrm{d}y$	B1	3.3
	Integrates to the form $\alpha y \pm \beta y^2$	M1	1.1b
	Correct integration $2y-\frac{1}{2}y^2$	A1	1.1b
	Uses their y limits correctly in a changed expression $\pi\left[2y-\frac{1}{2}y^2\right]_0^2=\pi\left(2(2)-\frac{1}{2}(2^2)\right)-0=...\{2\pi \text{ or } 6.28...\}$	M1	3.4
	mass = 'their volume' $\times 900$	M1	3.1b
	Mass = 5700 (kg) 2 s.f. cao	A1	2.2b
		(6)	
(b)	eg The surface will not be smooth The pile will not follow the shape of the curve The pile will not be solid Equation of the curves may not be a suitable model Concrete is likely to be uneven/may have bumps The pile is unlikely to be symmetrical	B1	3.5b
		(1)	
(c)	Makes a comparison about the difference between their mass and 5500 and draws a conclusion e.g. 200 difference which is a lot of concrete therefore not a good model e.g. the mass of 5700 is very close to 5500 kg and draws a conclusion about the model – e.g. therefore a good model e.g. Finds the percentage error and draw a conclusion about the model e.g. The masses are very close/significantly different and draws an appropriate conclusion Not sufficient to say 5700 > 5500 B0	B1ft	3.5a
		(1)	
		(8 marks)	

Notes:

(a)

B1: Sets up the model to find a correct expression for the volume, including limits, d*y* may be implied. The limits may be seen later.

M1: Integrates to the form $\alpha y \pm \beta y^2$

A1: Correct integration

M1: Substitutes their *y* limits the correct way round and subtracts, must be a changed expression

M1: Multiplies their volume by 900 to find the mass

A1: 5700 cao

Note incorrect upper limit of $\sqrt{2}$ leads to 5200kg Scores B0 M1 A1 M1 M1 A0

Note: Finding the volume around the *x*-axis can score B0 M0 A0 M0 M1 A0 only

Note If they use their calculator to find the value of the definite integration and achieve the correct answer the maximum they can score is B1M0A0M1M1A1

(b)

B1: See scheme, must be referring to the model and not the value of the density etc

(c)

B1ft: See scheme, follow through on their answer to (a). If using a calculation, it must be correct. Ignore any contradictory comments e.g. 3.6% out so it's fairly close so it's a good model but it's an overestimate which isn't good. You may need to use your own judgement but any sensible comment comparing their value to 5500 is acceptable.

(**Q05 8FM0/01, June 2023**)

Q6.

Question	Scheme		Marks	AOs
(a)	Depth = 0.16 (m)		B1	2.2b
			(1)	
(b)	$y = 1 + kx^2 \Rightarrow 1.16 = 1 + k(0.2)^2 \Rightarrow k = ...$		M1	3.3
	$\Rightarrow k = 4$ **cao** $\left\{\text{So } y = 1 + 4x^2\right\}$		A1	1.1b
			(2)	
(c)	$\frac{\pi}{4}\int (y-1)\mathrm{d}y$	$\frac{\pi}{4}\int y\,\mathrm{d}y$	B1ft	1.1a
	$= \left\{\frac{\pi}{4}\right\}\int_1^{1.16} (y-1)\mathrm{d}y$	$= \left\{\frac{\pi}{4}\right\}\int_0^{0.16} y\mathrm{d}y$	M1	3.3
	$= \left\{\frac{\pi}{4}\right\}\left[\frac{y^2}{2} - y\right]_1^{1.16}$	$= \left\{\frac{\pi}{4}\right\}\left[\frac{y^2}{2}\right]_0^{0.16}$	M1	1.1b
			A1	1.1b
	$= \frac{\pi}{4}\left(\left(\frac{1.16^2}{2} - 1.16\right) - \left(\frac{1}{2} - 1\right)\right) \{= 0.0032\pi\}$	$= \frac{\pi}{4}\left(\left(\frac{0.16^2}{2}\right) - (0)\right) \{= 0.0032\pi\}$		
	$V_{\text{cylinder}} = \pi(0.2)^2(1.16)\ \{= 0.0464\pi\}$		B1	1.1b
	Volume $= 0.0464\pi - 0.0032\pi\ \{= 0.0432\pi\}$		M1	3.4
	$= 0.1357168026... = 0.136(\text{m}^3)$ (3sf)		A1	1.1b
			(7)	
(d)	Any one of e.g. The measurements may not be accurate. The inside surface of the bowl may not be smooth. There may be wastage of concrete when making the bird bath.		B1	3.5b
			(1)	
(e)	Some comment consistent with their values. We do need a reason. e.g. $\left[\left(\frac{0.136 - 0.127}{0.127}\right) \times 100 = 7.0866...\right]$ so not a good estimate because the volume of concrete needed to make the bird bath is approximately 7% lower than that predicted by the model. **or** We might expect the actual amount of concrete to exceed that which the model predicts due to wastage, so the model does not look suitable since it predicts more concrete than was used.		B1ft	3.5a
			(1)	
			(12 marks)	

	Question Notes	
(a)	B1	Infers that the maximum depth of the bird bath could be 0.16 (m).
(b)	M1	Substitutes $y=1.16$ and $x=0.2$ or $x=-0.2$ into $y=1+kx^2$ and rearranges to give $k=....$
	A1	$k=4$ **cao**
(c)	B1ft	Uses the model to obtain either $\frac{\pi}{(\text{their } k)}\int (y-1)\,\mathrm{d}y$ or $\frac{\pi}{(\text{their } k)}\int y\,\mathrm{d}y$
	M1	Chooses limits that are appropriate to their model.
	M1	Integrates y (with respect to y) to give $\pm\lambda y^2$, where $\lambda \neq 0$ is a constant.
	A1	Uses their model correctly to give either $y-1 \to \frac{y^2}{2}-y$ or $y \to \frac{y^2}{2}$
	B1	$V_{\text{cylinder}} = \pi(0.2)^2(1.16)$ or 0.0464π or $\frac{29}{625}\pi$, o.e.
	M1	Depends on **both** previous M marks. Uses the model to find $V_{\text{their cylinder}}$ − their integrated volume.
	A1	0.136 **cao**
(d)	B1	States an acceptable limitation of the model.
(e)	B1ft	Compares the actual volume with their answer to (c). Makes an assessment of the model. E.g. evaluates the percentage error and uses this to make a sensible comment about the model with a reason.

(**Q07 8FM0/01, Specimen papers**)

Q7.

Question	Scheme	Marks	AOs
(a)	$x=0$ and $y=12$ $0=(12+12)^2(A-12^2) \Rightarrow A=144$	B1	3.3
		(1)	
(b)	$V=\frac{\pi}{350}\int_{-12}^{12}(12+y)^2(144-y^2)\ \{\mathrm{d}y\}$	B1	3.4
	$(12+y)^2(144-y^2)=...\{20736+3456y-24y^3-y^4\}$ $V=\left\{\frac{\pi}{350}\right\}\int(20736+3456y-24y^3-y^4)\ \mathrm{d}y$ $=\left\{\frac{\pi}{350}\right\}\left(20736y+1728y^2-6y^4-\frac{1}{5}y^5\right)$ $=\{\pi\}\left(\frac{10368}{175}y+\frac{864}{175}y^2-\frac{3}{175}y^4-\frac{1}{1750}y^5\right)$ **Alternative by parts** $(12+y)^2(144-y^2)=(12+y)^3(12-y)$ $V=\int(12+y)^3(12-y)\ \mathrm{d}y$ $=\frac{1}{4}(12+y)^4(12-y)+\int\frac{1}{4}(12+y)^4\ \mathrm{d}y$ $=\frac{1}{4}(12+y)^4(12-y)+\frac{1}{20}(12+y)^5$	M1 A1	1.1b 1.1b
	$\left[20736(12)+1728(12)^2-6(12)^4-\frac{1}{5}(12)^5\right]-$ $\left[20736(-12)+1728(-12)^2-6(-12)^4-\frac{1}{5}(-12)^5\right]$ $=(323481.6)-(-74659.6)$ **Alternative by parts** $\left[\frac{1}{4}(12+12)^4(12-12)+\frac{1}{20}(12+12)^5\right]-\left[\frac{1}{4}(12-12)^4(12+12)+\frac{1}{20}(12-12)^5\right]$	M1	3.4
	$=3600\ \mathrm{m}^3$ cao	A1	2.2b
		(5)	

(c)	$350x^2 = (13+y)^2(169-y^2)$ Or $411x^2 = (13+y)^2(169-y^2)$	**B1**	3.5c
		(1)	
(d)	e.g. the balloon may not be exactly the same shape as the curve The balloon's material will have some thickness the balloon is not the same shape as the curve as the balloon does not taper to nothing at the bottom. Balloon may stretch B0 for balloon may not be smooth, comments on the basket	**B1**	3.5b
		(1)	
			(8 marks)

Notes:

(a)

B1: Uses $x=0$ and $y=12$ in the **full** equation to show that $A=144$

Note Uses $x=0$ and $y=-12$ is B0

(b)

B1: Uses the model to set up the volume for the balloon, with limits, the d*y* may be implied

M1: Multiplies out the brackets and integrates $\int x^n \, dx \to x^{n+1}$. Alternatively uses integration by parts the correct way. Condone a slip when multiplying out

A1: Correct integration

M1: Uses the limits of – 12 and 12, subtracts the correct way round.

If the integration is correct this can be evidenced by for example $(323481.6)-(-74659.6)$ or

2903.56 – (– 670.05) if including $\dfrac{\pi}{350}$

If the integration is incorrect we must see the substitution of 12 and –12 into their integrated function

Candidates may use limits of – 12 to 0 and then 0 to 12 and add which is fine.

A1: Correct volume 3600 m^3, unit required and 2 s.f.

Note: No Evidence of integration maximum B1 M0A0 M0A0 if a correct answer stated

(c)

B1: See scheme for equation

(d)

B1: Correct limitation, see scheme, must be about the balloon part not the basket. Balloon might not be smooth is B0

Q8.

Question	Scheme	Marks	AOs
(a)	$(0.4,\ 4) \Rightarrow 0.4 = k \times 4^2 + \sqrt{4} \Rightarrow k = ...$	M1	3.3
	$k = -0.1$	A1	1.1b
		(2)	
(b)	Cylinder volume $= \pi \times 0.4^2 \times 0.5 = 0.08\ \pi = \dfrac{2}{25}\pi$	B1	3.4
	Volume generated by curve $= \pi\int x^2\,\mathrm{d}y$ $\pi\int\left(\sqrt{y}+ky^2\right)^2\{\mathrm{d}y\} = \pi\int\left(\sqrt{y}-0.1y^2\right)^2\{\mathrm{d}y\}$	M1	3.1b
	$=\{\pi\}\displaystyle\int\left(y+2ky^{\frac{5}{2}}+k^2y^4\right)\{\mathrm{d}y\}$ $=\{\pi\}\displaystyle\int\left(y-0.2y^{\frac{5}{2}}+0.01y^4\right)\{\mathrm{d}y\}$	A1ft	1.1b
	$=\{\pi\}\displaystyle\int_0\left(y-0.2y^{\frac{5}{2}}+0.01y^4\right)\{\mathrm{d}y\}$ $\Rightarrow\{\pi\}\left[Ay^2+By^{\frac{7}{2}}+Cy^5\right]$ at least one of their terms with the correct power	M1	3.4
	$=\{\pi\}\left[\dfrac{y^2}{2}+\dfrac{4k}{7}y^{\frac{7}{2}}+\dfrac{k^2}{5}y^5\right]$ $=\{\pi\}\left[\dfrac{y^2}{2}-\dfrac{2}{35}y^{\frac{7}{2}}+\dfrac{1}{500}y^5\right]$	A1ft	1.1b
	$V=\pi\left(8-\dfrac{256}{35}+\dfrac{256}{125}\right)-(0)+\dfrac{2}{25}\pi$ $V=\dfrac{2392}{875}\pi+\dfrac{2}{25}\pi$	M1	3.4
	$V=\dfrac{2462\pi}{875}\text{cm}^3$	A1	2.2b
		(7)	
(c)	E.g. • The equation of the curve may not be a suitable model • The sides of the ornament will not be perfectly smooth • There may be flaws/bubbles within the glass • The corner (ABC) may not be a perfect right angle	B1	3.5b
		(1)	

(d)	Makes an appropriate comment that is consistent with their value for the volume and 9 cm^3. Some evidence of making a comparison and draws a conclusion E.g. a good estimate as 8.84 cm^3 is only 0.16 cm^3 less than 9 cm^3 • A volume between 8.5 and 9.5 is a good model • A volume between 8 and 10 can be either a good or bad model • A volume less than 8 or more than 10 is a bad model, over estimate or underestimate • model volume is less, not enough glass would be ordered so it is a bad model, following a correct answer to (b)	B1ft	3.5a
		(1)	
			(11 marks)

Notes
(a) M1: Substitutes (0.4, 4) into the equation modelling the curve in an attempt to find the value of k A1: Infers from the data in the model, the value of k (b) B1: Uses the information given in the model to establish the correct volume of the cylinder M1: Uses the model and applies $\pi\int x^2\{\mathrm{d}y\}$, dy not required and π may appear later in their solution. If they find an expression for x^2 first and then substitutes into the formula score M1 even if an incorrect expansion. A1ft: Correct expression for the volume generated by the curve with the bracket expanded (follow through their k value), dy not required and π may appear later in their solution. Indices need to be processed for this mark, may be seen later in the solution. M1: Attempts to integrate with at least one power raised by 1 A1ft: Correct integration (follow through on their expression for x^2 as long as there are 3 terms). Need not be simplified. M1: Uses the correct limits and finds the sum of the 2 volumes. Must come from an attempt at $\pi\int_0^4 x^2\{\mathrm{d}y\}$ and an attempt at the volume of the cylinder, condone incorrect formula used as long as it is 3 dimensional not an area. A1: $\frac{2462\pi}{875}$ **Use of calculator scores a maximum of B1M1A0M0A0M1A0 volume = π2.7337…** (c) B1: States an acceptable limitation of the model, which is the curve but accept flaws/bubbles in the glass. Measurements may not be accurate, or anything related to thickness is B0 (d) B1ft: Compares the actual volume to their answer to part (b) and makes an assessment of the model with a reason. If using a percentage error then they must use 9 as the true volume.

(**Q08 8FM0/01, June 2024**)

Q9.

Question	Scheme	Marks	AOs
(a)	$a = 4$	B1	3.3
		(1)	
(b)	Model A: (i) Widest point will be 4 (cm) from the base	B1	3.4
	(ii) Width at widest point is 12 (cm) $(2\times('a'+2)\text{ ft})$	B1ft	3.4
	Model B: (i) $y = 4+\frac{x^3-64x}{100} \Rightarrow \frac{dy}{dx}=\frac{3x^2-64}{100}$	M1	3.1b
	$\frac{dy}{dx}=0 \Rightarrow x=\pm\sqrt{\frac{64}{3}}=\pm\frac{8\sqrt{3}}{3}=\pm\text{awrt}4.62$	A1	1.1b
	So max width is a distance $8-\frac{8}{\sqrt{3}}=8-\frac{8\sqrt{3}}{3}\approx 3.38$ (cm) from base.	A1	3.4
	(ii) $y\big\|_{-4.61..}=4+\frac{(-4.62...)^3-64(-4.62...)}{100}=...$	dM1	3.4
	$=5.97...$ so diameter is approximately 11.9 (cm) $[2a+3.94...\text{ ft}]$	A1ft	3.2a
		(7)	
(c)	Model A and model B both have diameters closed to 12 Model B distance from base is closer to 3 than Model A so is more appropriate.	B1ft	3.5b
		(1)	

(d)	$V_B = \pi\int_{-8}^{8} y^2\,dx = \pi\int_{-8}^{8}\left(4+\frac{x^3-64x}{100}\right)^2 dx = \ldots$	B1	1.1b
	$=\frac{\{\pi\}}{10000}\int_{(-8)}^{(8)} 400^2+x^6+64^2x^2+2\left(400x^3-400\times 64x-64x^4\right)dx$ $=\frac{\{\pi\}}{10000}\int_{(-8)}^{(8)} 160000+x^6+4096x^2+800x^3-51200x-128x^4\ dx$ $=\{\pi\}\int_{(-8)}^{(8)} 16+\frac{x^6}{10000}+\frac{4096}{10000}x^2+\frac{8}{100}x^3-\frac{512}{100}x-\frac{128}{10000}x^4\ dx$ $=\{\pi\}\int_{(-8)}^{(8)} 16+\frac{x^6}{1000}+\frac{256}{625}x^2+\frac{2}{25}x^3-\frac{128}{25}x-\frac{8}{625}x^4\ dx$ $=\{\pi\}\int_{(-8)}^{(8)} 16+\frac{8x(x-8)(x+8)}{100}+\left(\frac{x(x-8)(x+8)}{100}\right)^2 dx$	M1	1.1b
	$=\frac{\{\pi\}}{10000}\left[160000x+\frac{x^7}{7}+4096\frac{x^3}{3}+800\frac{x^4}{4}-51200\frac{x^2}{2}-128\frac{x^5}{5}\right]_{(-8)}^{(8)}$	dM1	1.1b
	$=\{\pi\}\left[16x+\frac{x^7}{70000}+\frac{256}{1875}x^3+\frac{1}{50}x^4-\frac{64}{25}x^2-\frac{8}{3125}x^5\right]_{(-8)}^{(8)}$		
	$=\frac{\{\pi\}}{10000}(620583.00....--2258983.01...)\approx\frac{2879566\pi}{10000}$	M1	3.4
	$=\text{awrt}\,905\left(\text{cm}^3\right)$ cso	A1	1.1b
		(5)	
(e)	Compares their volume to 900 or compares their volume + 100 to 1 litre or 1000 and comments appropriately.	B1ft	3.5a
		(1)	
			(15 marks)

Notes:

Units not required in this question

(a)

B1: For $a = 4$, ignore any reference to units.

(b)

B1: Correct distance from base for Model A is 4

B1ft: Correct width at widest point. Follow through their 'a', so $2\times('a'+2)$.

M1: Attempts the derivative for Model B's equation, reduce any power by 1

A1: Sets $\frac{dy}{dx}=0$ and finds correct x coordinate of the stationary point (accept ±)

A1: For $8-\frac{8}{\sqrt{3}}$ or awrt 3.38 cso

dM1: Dependent on previous M mark. Uses their value of x to find the value of y. If no working shown the value of y must come from their x value.

Note using $x = 4.62$ give y = 2.029…

A1: Correct diameter, awrt 11.9 follow through their 'a', so $[2a+3.94\ldots \text{ft}]$

Note: Correct answers with no working send to review

Trial and error approach

Candidates could score B1 B1 for model A however if working in integers it is unlikely that they will find the correct value for x (they are using $x = -5$) not a valid method M0A0A0dM0A0

(c)

B1ft: They must have answers for all parts in (b). Accept any well-reasoned comment that follows their answers to (b) If the answers are correct, they must conclude that model B is more appropriate.

- If answers for one model are correct ish but other incorrect, or one value is clearly closer For example

	Distance (3)	**Diameter (12)**	**Distance (3)**	**Diameter (12)**
A	9.4	9.05	4	6
B	3.38	12.06	4.62	4.06
Conclusion	Selects B as distance/diameter closet		Select A as diameter closest	

- If distances and diameters are similar selects the model which has the most appropriate value for distance or diameter

  For example

	Distance (3)	**Diameter (12)**	**Distance (3)**	**Diameter (12)**

A	0.76	6.8	4	20
B	1.28	10.5	3.38	19.94
Conclusion	selects B as the diameter is closet		Selects B as distance is closet	

- If all values of the distances and diameters are varied any sensible reason stated for selecting a model.

(d)

B1: Applies $\pi\int_{-8}^{8} y^2\,dx$ to the model. Must have π and correct limits, with y substituted in.

Alternatively attempts to square y first and then substitute in.

M1: Attempts to expand y^2 this can be a poor attempt but must include at least a constant and x^6 terms as long a clear attempt at y^2 (Limits not required for this mark.)

dM1: Attempts the integration, must first be rearranged to an integrable form then look for power increasing by at least 1 in at least two terms. (Limits not required for this mark.)

M1: Applies correct limits to their integral following an attempt at y^2 with at least a constant and x^6 terms.

If there is no working shown, allow this method mark if the correct answer appears from a calculator as it implies correct limits have been applied the correct way round. (So M0dM0M1 is possible.)

A1: awrt 905 cso note it must come from a fully correct solution

Note: For answers that appear from calculator B1M0dM0M1A0 is possible, the question specifies algebraic integration to be used so the integration needs to be seen to score the other marks.

(e)

B1ft: Compares their volume to 900 or compares their volume + 100 to 1 litre or 1000 and comments appropriately. Correct answer in (d) needs to conclude that it is suitable.

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