

Mark Scheme

Q1.

Question Number	Scheme	Notes	Marks
	$x = 3\theta \sin \theta, y = \sec^3 \theta, 0 \leq \theta < \frac{\pi}{2}$		
(a)	<p>{When $y = 8$,} $8 = \sec^3 \theta \Rightarrow \cos^3 \theta = \frac{1}{8} \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$</p> <p>$k$ (or x) $= 3\left(\frac{\pi}{3}\right) \sin\left(\frac{\pi}{3}\right)$</p>	Sets $y = 8$ to find θ and attempts to substitute their θ into $x = 3\theta \sin \theta$	M1
	so k (or x) $= \frac{\sqrt{3}\pi}{2}$	$\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	A1
	Note: Obtaining two value for k without accepting the correct value is final A0		[2]
(b)	$\frac{dx}{d\theta} = 3 \sin \theta + 3\theta \cos \theta$	$3\theta \sin \theta \rightarrow 3 \sin \theta + 3\theta \cos \theta$ Can be implied by later working	B1
	$\left\{ \int y \frac{dx}{d\theta} \{d\theta\} \right\} = \int (\sec^3 \theta)(3 \sin \theta + 3\theta \cos \theta) \{d\theta\}$	Applies $(\pm K \sec^3 \theta) \left(\text{their } \frac{dx}{d\theta} \right)$ Ignore integral sign and $d\theta$; $K \neq 0$	M1
	$= 3 \int \theta \sec^2 \theta + \tan \theta \sec^2 \theta d\theta$	Achieves the correct result no errors in their working, e.g. bracketing or manipulation errors. Must have integral sign and $d\theta$ in their final answer.	A1 *
	$x = 0$ and $x = k \Rightarrow \underline{\alpha = 0}$ and $\underline{\beta = \frac{\pi}{3}}$	$\alpha = 0$ and $\beta = \frac{\pi}{3}$ or evidence of $0 \rightarrow 0$ and $k \rightarrow \frac{\pi}{3}$	B1
	Note: The work for the final B1 mark must be seen in part (b) only.		[4]
(c) Way 1	$\left\{ \int \theta \sec^2 \theta d\theta \right\} = \theta \tan \theta - \int \tan \theta \{d\theta\}$	$\theta \sec^2 \theta \rightarrow A\theta g(\theta) - B \int g(\theta), A > 0, B > 0$, where $g(\theta)$ is a trigonometric function in θ and $g(\theta) = \text{their } \int \sec^2 \theta d\theta$. [Note: $g(\theta) \neq \sec^2 \theta$]	M1
		dependent on the previous M mark Either $\lambda \theta \sec^2 \theta \rightarrow A\theta \tan \theta - B \int \tan \theta, A > 0, B > 0$ or $\theta \sec^2 \theta \rightarrow \theta \tan \theta - \int \tan \theta$	dM1
	$= \theta \tan \theta - \ln(\sec \theta)$ or $= \theta \tan \theta + \ln(\cos \theta)$	$\theta \sec^2 \theta \rightarrow \theta \tan \theta - \ln(\sec \theta)$ or $\theta \tan \theta + \ln(\cos \theta)$ or $\lambda \theta \sec^2 \theta \rightarrow \lambda \theta \tan \theta - \lambda \ln(\sec \theta)$ or $\lambda \theta \tan \theta + \lambda \ln(\cos \theta)$	A1
	Note: Condone $\theta \sec^2 \theta \rightarrow \theta \tan \theta - \ln(\sec x)$ or $\theta \tan \theta + \ln(\cos x)$ for A1		
	$\left\{ \int \tan \theta \sec^2 \theta d\theta \right\}$	$\tan \theta \sec^2 \theta$ or $\lambda \tan \theta \sec^2 \theta \rightarrow \pm C \tan^2 \theta$ or $\pm C \sec^2 \theta$ or $\pm C u^{-2}$, where $u = \cos \theta$	M1
	$= \frac{1}{2} \tan^2 \theta$ or $\frac{1}{2} \sec^2 \theta$ or $\frac{1}{2u^2}$ where $u = \cos \theta$ or $\frac{1}{2} u^2$ where $u = \tan \theta$	$\tan \theta \sec^2 \theta \rightarrow \frac{1}{2} \tan^2 \theta$ or $\frac{1}{2} \sec^2 \theta$ or $\frac{1}{2 \cos^2 \theta}$ or $\tan^2 \theta - \frac{1}{2} \sec^2 \theta$ or $0.5u^{-2}$, where $u = \cos \theta$ or $0.5u^2$, where $u = \tan \theta$ or $\lambda \tan \theta \sec^2 \theta \rightarrow \frac{\lambda}{2} \tan^2 \theta$ or $\frac{\lambda}{2} \sec^2 \theta$ or $\frac{\lambda}{2 \cos^2 \theta}$ or $0.5\lambda u^{-2}$, where $u = \cos \theta$ or $0.5\lambda u^2$, where $u = \tan \theta$	A1
	$\{\text{Area}(R)\} = \left[3\theta \tan \theta - 3 \ln(\sec \theta) + \frac{3}{2} \tan^2 \theta \right]_0^{\frac{\pi}{3}}$ or $\left[3\theta \tan \theta - 3 \ln(\sec \theta) + \frac{3}{2} \sec^2 \theta \right]_0^{\frac{\pi}{3}}$		
	$= \left(3\left(\frac{\pi}{3}\right) \sqrt{3} - 3 \ln 2 + \frac{3}{2}(3) \right) - (0)$ or $\left(3\left(\frac{\pi}{3}\right) \sqrt{3} - 3 \ln 2 + \frac{3}{2}(4) \right) - \left(\frac{3}{2}\right)$		
	$= \frac{9}{2} + \sqrt{3}\pi - 3 \ln 2$ or $\frac{9}{2} + \sqrt{3}\pi + 3 \ln\left(\frac{1}{2}\right)$ or $\frac{9}{2} + \sqrt{3}\pi - \ln 8$ or $\ln\left(\frac{1}{8} e^{3+\sqrt{3}\pi}\right)$		A1 o.e.
			[6]
			12



Question Number	Scheme	Notes	Marks
(c)	Way 2 for the first 5 marks: Applying integration by parts on $\int (\theta + \tan \theta) \sec^2 \theta d\theta$		
Way 2	$\int (\theta \sec^2 \theta + \tan \theta \sec^2 \theta) d\theta = \int (\theta + \tan \theta) \sec^2 \theta d\theta, \quad \left\{ \begin{array}{l} u = \theta + \tan \theta \Rightarrow \frac{du}{d\theta} = 1 + \sec^2 \theta \\ \frac{dv}{d\theta} = \sec^2 \theta \Rightarrow v = \tan \theta = g(\theta) \end{array} \right\}$		
	$h(\theta)$ and $g(\theta)$ are trigonometric functions in θ and $g(\theta) = \tan \theta$. [Note: $g(\theta) \neq \sec^2 \theta$]		
		$A(\theta + \tan \theta)g(\theta) - B \int (1 + h(\theta))g(\theta), A > 0, B > 0$	M1
	$= (\theta + \tan \theta) \tan \theta - \int (1 + \sec^2 \theta) \tan \theta \{d\theta\}$	dependent on the previous M mark Either $\lambda[(\theta + \tan \theta) \sec^2 \theta] \rightarrow$ $A(\theta + \tan \theta) \tan \theta - B \int (1 + h(\theta)) \tan \theta, A \neq 0, B > 0$ or $(\theta + \tan \theta) \tan \theta - \int (1 + h(\theta)) \tan \theta$	dM1
	$= (\theta + \tan \theta) \tan \theta - \int (\tan \theta + \tan \theta \sec^2 \theta) \{d\theta\}$		
	$= (\theta + \tan \theta) \tan \theta - \ln(\sec \theta) - \int \tan \theta \sec^2 \theta \{d\theta\}$	$(\theta + \tan \theta) \tan \theta - \ln(\sec \theta)$ o.e. or $\lambda[(\theta + \tan \theta) \tan \theta - \ln(\sec \theta)]$ o.e.	A1
	$= (\theta + \tan \theta) \tan \theta - \ln(\sec \theta) - \frac{1}{2} \tan^2 \theta$	$\tan \theta \sec^2 \theta \rightarrow \pm C \tan^2 \theta$ or $\pm C \sec^2 \theta$	M1
	or $= (\theta + \tan \theta) \tan \theta - \ln(\sec \theta) - \frac{1}{2} \sec^2 \theta$ etc.	$(\theta + \tan \theta) \tan \theta - \frac{1}{2} \tan^2 \theta$ or $(\theta + \tan \theta) \tan \theta - \frac{1}{2} \sec^2 \theta$	A1
	Note	Allow the first two marks in part (c) for $\theta \tan \theta - \int \tan \theta$ embedded in their working	
	Note	Allow the first three marks in part (c) for $\theta \tan \theta - \ln(\sec \theta)$ embedded in their working	
Note	Allow 3 rd M1 2 nd A1 marks for either $\tan^2 \theta - \frac{1}{2} \tan^2 \theta$ or $\tan^2 \theta - \frac{1}{2} \sec^2 \theta$ embedded in their working		
Question Notes			
(a)	Note	Allow M1 for an answer of $k = \arctan 2.72$ without reference to $\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	
	Note	Allow M1 for an answer of $k = 3\left(\arccos\left(\frac{1}{2}\right)\right)\sin\left(\arccos\left(\frac{1}{2}\right)\right)$ without reference to $\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	
	Note	E.g. allow M1 for $\theta = 60^\circ$, leading to $k = 3(60)\sin(60)$ or $k = 90\sqrt{3}$	



Question Notes Continued		
(b)	Note	To gain A1, $d\theta$ does not need to appear until they obtain $3 \int (\theta \sec^2 \theta + \tan \theta \sec^2 \theta) d\theta$
	Note	For M1, their $\frac{dx}{d\theta}$, where their $\frac{dx}{d\theta} \neq 3\theta \sin \theta$, needs to be a trigonometric function in θ
	Note	Writing $\int (\sec^3 \theta)(3 \sin \theta + 3\theta \cos \theta) = 3 \int (\theta \sec^2 \theta + \tan \theta \sec^2 \theta) d\theta$ is sufficient for B1M1A1
	Note	Writing $\frac{dx}{d\theta} = 3 \sin \theta + 3\theta \cos \theta$ followed by writing $\int y \frac{dx}{d\theta} d\theta = 3 \int (\theta \sec^2 \theta + \tan \theta \sec^2 \theta) d\theta$ is sufficient for B1M1A1
	Note	The final A mark would be lost for $\int \frac{1}{\cos^3 \theta} 3 \sin \theta + 3\theta \cos \theta = 3 \int (\theta \sec^2 \theta + \tan \theta \sec^2 \theta) d\theta$ [lack of brackets in this particular case].
	Note	Give 2 nd B0 for $\alpha = 0$ and $\beta = 60^\circ$, without reference to $\beta = \frac{\pi}{3}$
(c)	Note	A decimal answer of 7.861956551... (without a correct exact answer) is A0.
	Note	First three marks are for integrating $\theta \sec^2 \theta$ with respect to θ
	Note	Fourth and fifth marks are for integrating $\tan \theta \sec^2 \theta$ with respect to θ
	Note	Candidates are not penalised for writing $\ln \sec \theta $ as either $\ln(\sec \theta)$ or $\ln \sec \theta$
	Note	$\theta \sec^2 \theta \rightarrow \theta \tan \theta + \ln(\sec \theta)$ WITH NO INTERMEDIATE WORKING is M0M0A0
	Note	$\theta \sec^2 \theta \rightarrow \theta \tan \theta - \ln(\cos \theta)$ WITH NO INTERMEDIATE WORKING is M0M0A0
	Note	$\theta \sec^2 \theta \rightarrow \theta \tan \theta - \ln(\sec \theta)$ WITH NO INTERMEDIATE WORKING is M1M1A1
	Note	$\theta \sec^2 \theta \rightarrow \theta \tan \theta + \ln(\cos \theta)$ WITH NO INTERMEDIATE WORKING is M1M1A1
	Note	Writing a correct $uv - \int v \frac{du}{dx}$ with $u = \theta$, $\frac{dv}{d\theta} = \tan \theta$, $\frac{du}{d\theta} = 1$ and $v = \text{their } g(\theta)$ and making one error in the direct application of this formula is 1 st M1 only.

(c)	Alternative method for finding $\int \tan \theta \sec^2 \theta d\theta$		
	$\left\{ \begin{array}{l} u = \tan \theta \quad \Rightarrow \quad \frac{du}{d\theta} = \sec^2 \theta \\ \frac{dv}{d\theta} = \sec^2 \theta \quad \Rightarrow \quad v = \tan \theta \end{array} \right\}$		
	$\int \tan \theta \sec^2 \theta d\theta = \tan^2 \theta - \int \tan \theta \sec^2 \theta d\theta$ $\Rightarrow 2 \int \tan \theta \sec^2 \theta d\theta = \tan^2 \theta$		
	$\int \tan \theta \sec^2 \theta d\theta = \frac{1}{2} \tan^2 \theta$	$\tan \theta \sec^2 \theta \text{ or } \rightarrow \pm C \tan^2 \theta$	M1
		$\tan \theta \sec^2 \theta \rightarrow \frac{1}{2} \tan^2 \theta$	A1
	or $\left\{ \begin{array}{l} u = \sec \theta \quad \Rightarrow \quad \frac{du}{d\theta} = \sec \theta \tan \theta \\ \frac{dv}{d\theta} = \sec \theta \tan \theta \quad \Rightarrow \quad v = \sec \theta \end{array} \right\}$		
	$\Rightarrow \int \tan \theta \sec^2 \theta d\theta = \sec^2 \theta - \int \sec^2 \theta \tan \theta d\theta$ $\Rightarrow 2 \int \tan \theta \sec^2 \theta d\theta = \sec^2 \theta$		
	$\int \tan \theta \sec^2 \theta d\theta = \frac{1}{2} \sec^2 \theta$	$\tan \theta \sec^2 \theta \text{ or } \rightarrow \pm C \sec^2 \theta$	M1
		$\tan \theta \sec^2 \theta \rightarrow \frac{1}{2} \sec^2 \theta$	A1

Q2.

Question Number	Scheme	Marks
(a)	$\left[x = \ln(t+2), y = \frac{1}{t+1} \right], \Rightarrow \frac{dx}{dt} = \frac{1}{t+2}$ <p>Must state $\frac{dx}{dt} = \frac{1}{t+2}$</p> $\text{Area}(R) = \int_{\ln 2}^{\ln 4} \frac{1}{t+1} dx = \int_0^2 \left(\frac{1}{t+1} \right) \left(\frac{1}{t+2} \right) dt$ <p>Area = $\int \frac{1}{t+1} dx$. Ignore limits.</p> <p>$\int \left(\frac{1}{t+1} \right) \times \left(\frac{1}{t+2} \right) dt$. Ignore limits.</p> <p>Changing limits, when: $x = \ln 2 \Rightarrow \ln 2 = \ln(t+2) \Rightarrow 2 = t+2 \Rightarrow t = 0$ $x = \ln 4 \Rightarrow \ln 4 = \ln(t+2) \Rightarrow 4 = t+2 \Rightarrow t = 2$</p> <p>Hence, $\text{Area}(R) = \int_0^2 \frac{1}{(t+1)(t+2)} dt$</p> <p>changes limits $x \rightarrow t$ so that $\ln 2 \rightarrow 0$ and $\ln 4 \rightarrow 2$</p>	<p>B1</p> <p>M1;</p> <p>A1 AG</p> <p>B1</p> <p>[4]</p>
(b)	$\left(\frac{1}{(t+1)(t+2)} \right) = \frac{A}{(t+1)} + \frac{B}{(t+2)}$ $1 = A(t+2) + B(t+1)$ <p>Let $t = -1, 1 = A(1) \Rightarrow \underline{A = 1}$</p> <p>Let $t = -2, 1 = B(-1) \Rightarrow \underline{B = -1}$</p> $\int_0^2 \frac{1}{(t+1)(t+2)} dt = \int_0^2 \frac{1}{(t+1)} - \frac{1}{(t+2)} dt$ $= [\ln(t+1) - \ln(t+2)]_0^2$ $= (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$ $= \ln 3 - \ln 4 + \ln 2 = \ln 3 - \ln 2 = \ln\left(\frac{3}{2}\right)$ <p>Either $\pm a \ln(t+1)$ or $\pm b \ln(t+2)$ Both \ln terms correctly ft.</p> <p>Substitutes <i>both</i> limits of 2 and 0 and subtracts the correct way round.</p> <p>$\frac{\ln 3 - \ln 4 + \ln 2}{\text{or } \ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{2}\right)}$ $\text{or } \underline{\ln 3 - \ln 2}$ or $\underline{\ln\left(\frac{3}{2}\right)}$ (must deal with $\ln 1$)</p>	<p>M1</p> <p>A1</p> <p>dM1</p> <p>A1 $\sqrt{\quad}$</p> <p>ddM1</p> <p>A1 aef isw</p> <p>[6]</p>

Takes out brackets.

Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} + \frac{1}{(t+2)}$ means first M1A0 in (b).

Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} - \frac{1}{(t+2)}$ means first M1A1 in (b).



Question Number	Scheme	Marks
(c)	$x = \ln(t+2), \quad y = \frac{1}{t+1}$	
	$e^x = t+2 \Rightarrow t = e^x - 2$	Attempt to make $t = \dots$ the subject giving $t = e^x - 2$ M1 A1
	$y = \frac{1}{e^x - 2 + 1} \Rightarrow y = \frac{1}{e^x - 1}$	Eliminates t by substituting in y giving $y = \frac{1}{e^x - 1}$ dM1 A1
	[4]	
<i>Aliter</i> (c) Way 2	$t+1 = \frac{1}{y} \Rightarrow t = \frac{1}{y} - 1 \text{ or } t = \frac{1-y}{y}$	Attempt to make $t = \dots$ the subject M1
	$y(t+1) = 1 \Rightarrow yt + y = 1 \Rightarrow yt = 1 - y \Rightarrow t = \frac{1-y}{y}$	Giving either $t = \frac{1}{y} - 1$ or $t = \frac{1-y}{y}$ A1
	$x = \ln\left(\frac{1}{y} - 1 + 2\right) \text{ or } x = \ln\left(\frac{1-y}{y} + 2\right)$	Eliminates t by substituting in x dM1
	$x = \ln\left(\frac{1}{y} + 1\right)$	
(d)	$e^x = \frac{1}{y} + 1$	
	$e^x - 1 = \frac{1}{y}$	
	$y = \frac{1}{e^x - 1}$	giving $y = \frac{1}{e^x - 1}$ A1
	[4]	
(d)	Domain : $x > 0$	$x > 0$ or just > 0 B1 [1]
		15 marks



Question Number	Scheme	Marks
<i>Aliter</i> (c) Way 3	$e^x = t + 2 \Rightarrow t + 1 = e^x - 1$	Attempt to make $t + 1 = \dots$ the subject M1 giving $t + 1 = e^x - 1$ A1
	$y = \frac{1}{t+1} \Rightarrow y = \frac{1}{e^x - 1}$	Eliminates t by substituting in y dM1 giving $y = \frac{1}{e^x - 1}$ A1 [4]
<i>Aliter</i> (c) Way 4	$t + 1 = \frac{1}{y} \Rightarrow t + 2 = \frac{1}{y} + 1 \text{ or } t + 2 = \frac{1+y}{y}$	Attempt to make $t + 2 = \dots$ the subject M1 Either $t + 2 = \frac{1}{y} + 1$ or $t + 2 = \frac{1+y}{y}$ A1
	$x = \ln\left(\frac{1}{y} + 1\right) \text{ or } x = \ln\left(\frac{1+y}{y}\right)$ $x = \ln\left(\frac{1}{y} + 1\right)$ $e^x = \frac{1}{y} + 1 \Rightarrow e^x - 1 = \frac{1}{y}$ $y = \frac{1}{e^x - 1}$	Eliminates t by substituting in x dM1 giving $y = \frac{1}{e^x - 1}$ A1 [4]

Q3.

Question Number	Scheme	Marks
5.	Working parametrically: $x = 1 - \frac{1}{2}t$, $y = 2^t - 1$ or $y = e^{t \ln 2} - 1$	
(a)	$\{x = 0 \Rightarrow\} 0 = 1 - \frac{1}{2}t \Rightarrow t = 2$ Applies $x = 0$ to obtain a value for t . When $t = 2$, $y = 2^2 - 1 = 3$ Correct value for y .	M1 A1 [2]
(b)	$\{y = 0 \Rightarrow\} 0 = 2^t - 1 \Rightarrow t = 0$ Applies $y = 0$ to obtain a value for t . (Must be seen in part (b)). When $t = 0$, $x = 1 - \frac{1}{2}(0) = 1$ $x = 1$	M1 A1 [2]
(c)	$\frac{dx}{dt} = -\frac{1}{2}$ and either $\frac{dy}{dt} = 2^t \ln 2$ or $\frac{dy}{dt} = e^{t \ln 2} \ln 2$ $\frac{dy}{dx} = \frac{2^t \ln 2}{-\frac{1}{2}}$ Attempts their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$. At A , $t = "2"$, so $m(T) = -8 \ln 2 \Rightarrow m(N) = \frac{1}{8 \ln 2}$ Applies $t = "2"$ and $m(N) = \frac{-1}{m(T)}$ $y - 3 = \frac{1}{8 \ln 2} (x - 0)$ or $y = 3 + \frac{1}{8 \ln 2} x$ or equivalent. See notes.	B1 M1 M1 M1 A1 oe cso [5]
(d)	$\text{Area}(R) = \int (2^t - 1) \cdot \left(-\frac{1}{2}\right) dt$ Complete substitution for both y and dx $x = -1 \rightarrow t = 4$ and $x = 1 \rightarrow t = 0$ $= \left\{ -\frac{1}{2} \right\} \left(\frac{2^t}{\ln 2} - t \right)$ Either $2^t \rightarrow \frac{2^t}{\ln 2}$ or $(2^t - 1) \rightarrow \frac{(2^t)}{\pm \alpha (\ln 2)} - t$ or $(2^t - 1) \rightarrow \pm \alpha (\ln 2)(2^t) - t$ $(2^t - 1) \rightarrow \frac{2^t}{\ln 2} - t$ $\left\{ -\frac{1}{2} \left[\frac{2^t}{\ln 2} - t \right]_4^0 \right\} = -\frac{1}{2} \left(\left(\frac{1}{\ln 2} \right) - \left(\frac{16}{\ln 2} - 4 \right) \right)$ Depends on the previous method mark. Substitutes their changed limits in t and subtracts either way round. $= \frac{15}{2 \ln 2} - 2$ $\frac{15}{2 \ln 2} - 2$ or equivalent.	M1 B1 M1* A1 dM1* A1 [6]

<p>5. (a)</p>	<p>M1: Applies $x = 0$ and obtains a value of t.</p> <p>A1: For $y = 2^2 - 1 = 3$ or $y = 4 - 1 = 3$</p> <p>Alternative Solution 1:</p> <p>M1: For substituting $t = 2$ into either x or y.</p> <p>A1: $x = 1 - \frac{1}{2}(2) = 0$ and $y = 2^2 - 1 = 3$</p> <p>Alternative Solution 2:</p> <p>M1: Applies $y = 3$ and obtains a value of t.</p> <p>A1: For $x = 1 - \frac{1}{2}(2) = 0$ or $x = 1 - 1 = 0$.</p> <p>Alternative Solution 3:</p> <p>M1: Applies $y = 3$ or $x = 0$ and obtains a value of t.</p> <p>A1: Shows that $t = 2$ for both $y = 3$ and $x = 0$.</p> <p>(b) M1: Applies $y = 0$ and obtains a value of t. Working must be seen in part (b).</p> <p>A1: For finding $x = 1$.</p> <p>Note: Award M1A1 for $x = 1$.</p> <p>(c) B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. This mark can be implied by later working.</p> <p>M1: Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. Note: their $\frac{dy}{dt}$ must be a function of t.</p> <p>M1: Uses their value of t found in part (a) and applies $m(N) = \frac{-1}{m(T)}$.</p> <p>M1: $y - 3 = (\text{their normal gradient})x$ or $y = (\text{their normal gradient})x + 3$ or equivalent.</p> <p>A1: $y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or $y - 3 = \frac{1}{\ln 256}(x - 0)$ or $(8\ln 2)y - 24\ln 2 = x$</p> <p>or $\frac{y - 3}{(x - 0)} = \frac{1}{8\ln 2}$. You can apply isw here.</p> <p>Working in decimals is ok for the three method marks. B1, A1 require exact values.</p> <p>(d) M1: Complete substitution for both y and dx. So candidate should write down $\int (2^t - 1) \cdot \left(\text{their } \frac{dx}{dt}\right)$</p> <p>B1: Changes limits from $x \rightarrow t$. $x = -1 \rightarrow t = 4$ and $x = 1 \rightarrow t = 0$. Note $t = 4$ and $t = 0$ seen is B1.</p> <p>M1*: Integrates 2^t correctly to give $\frac{2^t}{\ln 2}$</p> <p>... or integrates $(2^t - 1)$ to give either $\frac{(2^t)}{\pm \alpha(\ln 2)} - t$ or $\pm \alpha(\ln 2)(2^t) - t$.</p> <p>A1: Correct integration of $(2^t - 1)$ with respect to t to give $\frac{2^t}{\ln 2} - t$.</p> <p>dM1*: Depends upon the previous method mark.</p> <p>Substitutes their limits in t and subtracts either way round.</p> <p>A1: Exact answer of $\frac{15}{2\ln 2} - 2$ or $\frac{15}{\ln 4} - 2$ or $\frac{15 - 4\ln 2}{2\ln 2}$ or $\frac{7.5}{\ln 2} - 2$ or $\frac{15}{2}\log_2 e - 2$ or equivalent.</p>
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Question Number	Scheme	Marks
5.	<u>Alternative: Converting to a Cartesian equation:</u> $t = 2 - 2x \Rightarrow y = 2^{2-2x} - 1$	
(a)	$\{x = 0 \Rightarrow\} y = 2^2 - 1$ $y = 3$	<p>Applies $x = 0$ in their Cartesian equation... ... to arrive at a correct answer of 3.</p> <p>M1 A1</p>
(b)	$\{y = 0 \Rightarrow\} 0 = 2^{2-2x} - 1 \Rightarrow 0 = 2 - 2x \Rightarrow x = ...$ $x = 1$	<p>Applies $y = 0$ to obtain a value for x. (Must be seen in part (b)). $x = 1$</p> <p>M1 A1</p>
(c)	$\frac{dy}{dx} = -2(2^{2-2x}) \ln 2$	<p>$\pm \lambda 2^{2-2x}, \lambda \neq 1$ $-2(2^{2-2x}) \ln 2$ or equivalent</p> <p>M1 A1</p>
	<p>At A, $x = 0$, so $m(T) = -8 \ln 2 \Rightarrow m(N) = \frac{1}{8 \ln 2}$ $y - 3 = \frac{1}{8 \ln 2} (x - 0)$ or $y = 3 + \frac{1}{8 \ln 2} x$ or equivalent.</p>	<p>Applies $x = 0$ and $m(N) = \frac{-1}{m(T)}$ As in the original scheme.</p> <p>M1 M1 A1 oe</p>
(d)	$\text{Area}(R) = \int (2^{2-2x} - 1) dx$ $= \int_{-1}^1 (2^{2-2x} - 1) dx$ $= \left(\frac{2^{2-2x}}{-2 \ln 2} - x \right)$ $\left\{ \left[\frac{2^{2-2x}}{-2 \ln 2} - x \right]_{-1}^1 \right\} = \left(\left(\frac{1}{-2 \ln 2} - 1 \right) - \left(\frac{16}{-2 \ln 2} + 1 \right) \right)$ $= \frac{15}{2 \ln 2} - 2$	<p>Form the integral of their Cartesian equation of C. For $2^{2-2x} - 1$ with limits of $x = -1$ and $x = 1$. I.e. $\int_{-1}^1 (2^{2-2x} - 1)$</p> <p>M1 B1</p> <div style="border: 1px solid black; padding: 5px; margin: 5px;"> <p>Either $2^{2-2x} \rightarrow \frac{2^{2-2x}}{-2 \ln 2}$ or $(2^{2-2x} - 1) \rightarrow \frac{2^{2-2x}}{\pm \alpha (\ln 2)} - x$ or $(2^{2-2x} - 1) \rightarrow \pm \alpha (\ln 2) (2^{2-2x}) - x$</p> </div> <p>M1*</p> <p>$(2^{2-2x} - 1) \rightarrow \frac{2^{2-2x}}{-2 \ln 2} - x$</p> <p>A1</p> <div style="border: 1px solid black; padding: 5px; margin: 5px;"> <p>Depends on the previous method mark. Substitutes limits of -1 and their x_B and subtracts either way round.</p> </div> <p>dM1*</p> <p>$\frac{15}{2 \ln 2} - 2$ or equivalent.</p> <p>A1</p>
(d)	<u>Alternative method: In Cartesian and applying $u = 2 - 2x$</u>	
	$\text{Area}(R) = \int (2^u - 1) \left\{ \frac{du}{-2} \right\}$, where $u = 2 - 2x$ $= \int_4^0 (2^u - 1) \left(-\frac{1}{2} \right) \{ du \}$	<p>M0: Unless a candidate <i>writes</i> $\int (2^{2-2x} - 1) \{ dx \}$ Then apply the “working parametrically” mark scheme.</p>



Q4.

Question Number	Scheme	Marks
(a)	Note: You can mark parts (a) and (b) together.	
	$x = 4t + 3, y = 4t + 8 + \frac{5}{2t}$	
	$\frac{dx}{dt} = 4, \frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$ Both $\frac{dx}{dt} = 4$ or $\frac{dt}{dx} = \frac{1}{4}$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$	B1
	So, $\frac{dy}{dx} = \frac{4 - \frac{5}{2}t^{-2}}{4} \left\{ = 1 - \frac{5}{8}t^{-2} = 1 - \frac{5}{8t^2} \right\}$ Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$	M1 o.e.
	{When $t = 2,$ } $\frac{dy}{dx} = \frac{27}{32}$ $\frac{27}{32}$ or 0.84375 cao	A1
		[3]
	Way 2: Cartesian Method	
	$\frac{dy}{dx} = 1 - \frac{10}{(x-3)^2}$ $\frac{dy}{dx} = 1 - \frac{10}{(x-3)^2}$, simplified or un-simplified.	B1
		$\frac{dy}{dx} = \pm \lambda \pm \frac{\mu}{(x-3)^2}, \lambda \neq 0, \mu \neq 0$ M1
	{When $t = 2, x = 11$ } $\frac{dy}{dx} = \frac{27}{32}$ $\frac{27}{32}$ or 0.84375 cao	A1
		[3]
(b)	Way 3: Cartesian Method	
	$\frac{dy}{dx} = \frac{(2x+2)(x-3) - (x^2 + 2x - 5)}{(x-3)^2}$ Correct expression for $\frac{dy}{dx}$, simplified or un-simplified.	B1
	$\left\{ = \frac{x^2 - 6x - 1}{(x-3)^2} \right\}$ $\frac{dy}{dx} = \frac{f'(x)(x-3) - 1f(x)}{(x-3)^2}$, where $f(x) = \text{their } "x^2 + ax + b"$, $g(x) = x - 3$	M1
	{When $t = 2, x = 11$ } $\frac{dy}{dx} = \frac{27}{32}$ $\frac{27}{32}$ or 0.84375 cao	A1
		[3]
	$\left\{ t = \frac{x-3}{4} \Rightarrow \right\} y = 4\left(\frac{x-3}{4}\right) + 8 + \frac{5}{2\left(\frac{x-3}{4}\right)}$ Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \frac{10}{x-3}$	
	$y = \frac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ or $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ See notes	dM1
	or $y = \frac{(x+5)(x-3) + 10}{x-3}$ or $y = \frac{(x+5)(x-3)}{x-3} + \frac{10}{x-3}$	
	$\Rightarrow y = \frac{x^2 + 2x - 5}{x-3}, \{a = 2 \text{ and } b = -5\}$ Correct algebra leading to $y = \frac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$	A1 cso
		[3] 6

Question Number	Scheme	Marks
(b)	<p>Alternative Method 1 of Equating Coefficients</p> $y = \frac{x^2 + ax + b}{x - 3} \Rightarrow y(x - 3) = x^2 + ax + b$ $y(x - 3) = (4t + 3)^2 + 2(4t + 3) - 5 = 16t^2 + 32t + 10$ $x^2 + ax + b = (4t + 3)^2 + a(4t + 3) + b$ <hr/> $(4t + 3)^2 + a(4t + 3) + b = 16t^2 + 32t + 10$ <p>Correct method of obtaining an equation in only t, a and b</p> <p>t: $24 + 4a = 32 \Rightarrow a = 2$ Equates their coefficients in t and finds both $a = \dots$ and $b = \dots$ constant: $9 + 3a + b = 10 \Rightarrow b = -5$ $a = 2$ and $b = -5$</p>	<p>M1</p> <p>dM1</p> <p>A1</p> <p>[3]</p>
(b)	<p>Alternative Method 2 of Equating Coefficients</p> $\left\{ t = \frac{x - 3}{4} \Rightarrow \right\} y = 4 \left(\frac{x - 3}{4} \right) + 8 + \frac{5}{2 \left(\frac{x - 3}{4} \right)}$ <p>Eliminates t to achieve an equation in only x and y</p> <hr/> $y = x - 3 + 8 + \frac{10}{x - 3} \Rightarrow y = x + 5 + \frac{10}{x - 3}$ $y(x - 3) = (x + 5)(x - 3) + 10 \Rightarrow x^2 + ax + b = (x + 5)(x - 3) + 10$ <hr/> $\Rightarrow y = \frac{x^2 + 2x - 5}{x - 3}$ <p>Correct algebra leading to give $a = 2$ and $b = -5$ or $y = \frac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$</p>	<p>M1</p> <p>dM1</p> <p>A1 eso</p> <p>[3]</p>

Question Notes		
(a)	<p>B1 $\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$ or $\frac{dy}{dt} = \frac{8t^2 - 5}{2t^2}$ or $\frac{dy}{dt} = 4 - 5(2t)^{-2}(2)$, etc.</p> <p>Note $\frac{dy}{dt}$ can be simplified or un-simplified.</p> <p>Note You can imply the B1 mark by later working.</p> <p>M1 Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$ or $\frac{dy}{dt}$ multiplied by a candidate's $\frac{dt}{dx}$</p> <p>Note M1 can be also be obtained by substituting $t = 2$ into both their $\frac{dy}{dt}$ and their $\frac{dx}{dt}$ and then dividing their values the correct way round.</p> <p>A1 $\frac{27}{32}$ or 0.84375 cao</p>	
(b)	<p>M1 Eliminates t to achieve an equation in only x and y.</p> <p>dM1 dependent on the first method mark being awarded. Either: (ignoring sign slips or constant slips, noting that k can be 1)</p> <ul style="list-style-type: none"> Combining all three parts of their $x - 3 + \bar{8} + \left(\frac{10}{x - 3} \right)$ to form a single fraction with a common denominator of $\pm k(x - 3)$. Accept three separate fractions with the same denominator. Combining both parts of their $x + 5 + \left(\frac{10}{x - 3} \right)$, (where $x + 5$ is their $4 \left(\frac{x - 3}{4} \right) + 8$), to form a single fraction with a common denominator of $\pm k(x - 3)$. Accept two separate fractions with the same denominator. Multiplies both sides of their $y = x - 3 + \bar{8} + \left(\frac{10}{x - 3} \right)$ or their $y = x + 5 + \left(\frac{10}{x - 3} \right)$ by $\pm k(x - 3)$. Note that all terms in their equation must be multiplied by $\pm k(x - 3)$. <p>Note Condone "invisible" brackets for dM1.</p> <p>A1 Correct algebra with no incorrect working leading to $y = \frac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$</p> <p>Note Some examples for the award of dM1 in (b):</p> <p>dM0 for $y = x - 3 + 8 + \frac{10}{x - 3} \rightarrow y = \frac{(x - 3)(x - 3) + 8 + 10}{x - 3}$. Should be $\dots + 8(x - 3) + \dots$</p> <p>dM0 for $y = x - 3 + \frac{10}{x - 3} \rightarrow y = \frac{(x - 3)(x - 3) + 10}{x - 3}$. The "8" part has been omitted.</p> <p>dM0 for $y = x + 5 + \frac{10}{x - 3} \rightarrow y = \frac{x(x - 3) + 5 + 10}{x - 3}$. Should be $\dots + 5(x - 3) + \dots$</p> <p>dM0 for $y = x + 5 + \frac{10}{x - 3} \rightarrow y(x - 3) = x(x - 3) + 5(x - 3) + 10(x - 3)$. Should be just 10.</p> <p>Note $y = x + 5 + \frac{10}{x - 3} \rightarrow y = \frac{x^2 + 2x - 5}{x - 3}$ with no intermediate working is dM1A1.</p>	

Q5.

Question	Scheme	Marks	AOs
(a)(i)	$y \times \frac{dx}{dt} = 5 \sin 2t \times 6 \cos t$ or $5 \times 2 \sin t \cos t \times 6 \cos t$	M1	1.2
	$(\text{Area} =) \int 5 \sin 2t \times 6 \cos t \, dt = \int 5 \times 2 \sin t \cos t \times 6 \cos t \, dt$ <p style="text-align: center;">or</p> $\int 5 \sin 2t \times 6 \cos t \, dt = \int 60 \sin t \cos^2 t \, dt$	dM1	1.1b
	$(\text{Area} =) \int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt *$	A1*	2.1*
		(3)	
(a)(ii)	$\int 60 \sin t \cos^2 t \, dt = -20 \cos^3 t$	M1 A1	1.1b 1.1b
	$\text{Area} = \left[-20 \cos^3 t \right]_0^{\frac{\pi}{2}} = 0 - (-20) = 20 *$	A1*	2.1
		(3)	
(b)	$5 \sin 2t = 4.2 \Rightarrow \sin 2t = \frac{4.2}{5}$	M1	3.4
	$t = 0.4986..., 1.072...$	A1	1.1b
	Attempts to find the x values at both t values	dM1	3.4
	$t = 0.4986... \Rightarrow x = 2.869...$ $t = 1.072 \Rightarrow x = 5.269...$	A1	1.1b
	Width of path = 2.40 metres	A1	3.2a
		(5)	
(11 marks)			

Notes:

(a)(i)

M1: Attempts to multiply y by $\frac{dx}{dt}$ to obtain $A \sin 2t \cos t$ but may apply $\sin 2t = 2 \sin t \cos t$ here

dM1: Attempts to use $\sin 2t = 2 \sin t \cos t$ within an integral which may be implied by

$$\text{e.g. } A \int \sin 2t \times \cos t \, dt = \int k \sin t \cos^2 t \, dt$$

A1*: Fully correct work leading to the given answer.

This must include $\sin 2t = 2 \sin t \cos t$ or e.g. $5 \sin 2t = 10 \sin t \cos t$ seen explicitly in their proof and a correct intermediate line that includes an integral sign and the “dt”

Allow the limits to just “appear” in the final answer e.g. working need not be shown for the limits.

(a)(ii)

M1: Obtains $\int 60 \sin t \cos^2 t \, dt = k \cos^3 t$. This may be attempted via a substitution of $u = \cos t$ to obtain

$$\int 60 \sin t \cos^2 t \, dt = ku^3$$

A1: Correct integration $-20 \cos^3 t$ or equivalent e.g. $-20u^3$

A1*: Rigorous proof with all aspects correct including the correct limits and the $0 - (-20)$ and

$$\text{not just: } -20 \cos^3 \frac{\pi}{2} - (-20 \cos^3 0) = 20$$

(b)

M1: Uses the given model and attempts to find value(s) of t when $\sin 2t = \frac{4.2}{5}$. Look for $2t = \sin^{-1} \frac{4.2}{5} \Rightarrow t = \dots$

A1: At least one correct value for t , correct to 2 dp. FYI $t = 0.4986\dots, 1.072\dots$ or in degrees $t = 28.57\dots, 61.42\dots$

dM1: Attempts to find **TWO** distinct values of x when $\sin 2t = \frac{4.2}{5}$. Condone poor trig work and allow this mark if 2

values of x are attempted from 2 values of t .

A1: Both values correct to 2 dp. NB $x = 2.869\dots, 5.269\dots$

Or may take Cartesian approach

$$5 \sin 2t = 4.2 \Rightarrow 10 \sin t \cos t = 4.2 \Rightarrow 10 \frac{x}{6} \sqrt{1 - \frac{x^2}{36}} = 4.2 \Rightarrow x^4 - 36x^2 + 228.6144 = 0 \Rightarrow x = 2.869\dots, 5.269\dots$$

M1: For converting to Cartesian form **A1:** Correct quartic **M1:** Solves quartic **A1:** Correct values

A1: 2.40 metres or 240 cm

Allow awrt 2.40 m or allow 2.4m (not awrt 2.4 m) and allow awrt 240 cm. **Units are required.**

Q6.

Question Number	Scheme	Marks
(a)	$x = 4 \sin \left(t + \frac{\pi}{6} \right), \quad y = 3 \cos 2t, \quad 0 \leq t < 2\pi$ $\frac{dx}{dt} = 4 \cos \left(t + \frac{\pi}{6} \right), \quad \frac{dy}{dt} = -6 \sin 2t$ $\text{So, } \frac{dy}{dx} = \frac{-6 \sin 2t}{4 \cos \left(t + \frac{\pi}{6} \right)}$	B1 B1 B1 $\sqrt{\quad}$ oe [3]
(b)	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} -6 \sin 2t = 0$ $\text{@ } t = 0, \quad x = 4 \sin \left(\frac{\pi}{6} \right) = 2, \quad y = 3 \cos 0 = 3 \rightarrow (2, 3)$ $\text{@ } t = \frac{\pi}{2}, \quad x = 4 \sin \left(\frac{2\pi}{3} \right) = \frac{4\sqrt{3}}{2}, \quad y = 3 \cos \pi = -3 \rightarrow (2\sqrt{3}, -3)$ $\text{@ } t = \pi, \quad x = 4 \sin \left(\frac{7\pi}{6} \right) = -2, \quad y = 3 \cos 2\pi = 3 \rightarrow (-2, 3)$ $\text{@ } t = \frac{3\pi}{2}, \quad x = 4 \sin \left(\frac{5\pi}{3} \right) = \frac{4(-\sqrt{3})}{2}, \quad y = 3 \cos 3\pi = -3 \rightarrow (-2\sqrt{3}, -3)$	M1 oe M1 A1A1A1 [5] 8



(a)	<p>B1: Either one of $\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right)$ or $\frac{dy}{dt} = -6\sin 2t$. They do not have to be simplified.</p> <p>B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. They do not have to be simplified.</p> <p>Any or both of the first two marks can be implied.</p> <p>Don't worry too much about their notation for the first two B1 marks.</p> <p>B1: Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. Note: This is a follow through mark.</p> <p><u>Alternative differentiation in part (a)</u></p> $x = 2\sqrt{3}\sin t + 2\cos t \Rightarrow \frac{dx}{dt} = 2\sqrt{3}\cos t - 2\sin t$ $y = 3(2\cos^2 t - 1) \Rightarrow \frac{dy}{dt} = 3(-4\cos t \sin t)$ $\text{or } y = 3\cos^2 t - 3\sin^2 t \Rightarrow \frac{dy}{dt} = -6\cos t \sin t - 6\sin t \cos t$ $\text{or } y = 3(1 - 2\sin^2 t) \Rightarrow \frac{dy}{dt} = 3(-4\cos t \sin t)$
(b)	<p>M1: Candidate sets their numerator from part (a) or their $\frac{dy}{dt}$ equal to 0.</p> <p>Note that their numerator must be a trig function. Ignore $\frac{dx}{dt}$ equal to 0 at this stage.</p> <p>M1: Candidate substitutes a found value of t, to attempt to find either one of x or y.</p> <p>The first two method marks can be implied by ONE correct set of coordinates for (x, y) or (y, x) interchanged.</p> <p>A correct point coming from NO WORKING can be awarded M1M1.</p> <p>A1: At least TWO sets of coordinates.</p> <p>A1: At least THREE sets of coordinates.</p> <p>A1: ONLY FOUR correct sets of coordinates. If there are more than 4 sets of coordinates then award A0.</p> <p>Note: Candidate can use the diagram's symmetry to write down some of their coordinates.</p> <p>Note: When $x = 4\sin\left(\frac{\pi}{6}\right) = 2$, $y = 3\cos 0 = 3$ is acceptable for a pair of coordinates.</p> <p>Also it is fine for candidates to display their coordinates on a table of values.</p> <p>Note: The coordinates must be exact for the accuracy marks. I.e $(3.46..., -3)$ or $(-3.46..., -3)$ is A0.</p> <p>Note: $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0$ ONLY is fine for the first M1, and potentially the following M1A1A0A0.</p> <p>Note: $\frac{dy}{dx} = 0 \Rightarrow \cos t = 0$ ONLY is fine for the first M1 and potentially the following M1A1A0A0.</p> <p>Note: $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0 \& \cos t = 0$ has the potential to achieve all five marks.</p> <p>Note: It is possible for a candidate to gain full marks in part (b) if they make sign errors in part (a).</p>
	<p><u>(b) An alternative method for finding the coordinates of the two maximum points.</u></p> <p>Some candidates may use $y = 3\cos 2t$ to write down that the y-coordinate of a maximum point is 3.</p> <p>They will then deduce that $t = 0$ or π and proceed to find the x-coordinate of their maximum point. These candidates will receive no credit until they attempt to find one of the x-coordinates for the maximum point.</p> <p>M1M1: Candidate states $y = 3$ and attempts to substitute $t = 0$ or π into $x = 4\sin\left(t + \frac{\pi}{6}\right)$.</p> <p>M1M1 can be implied by candidate stating either $(2, 3)$ or $(2, -3)$.</p> <p>Note: these marks can only be awarded together for a candidate using this method.</p> <p>A1: For both $(2, 3)$ or $(-2, 3)$.</p> <p>A0A0: Candidate cannot achieve the final two marks by using this method. They can, however, achieve these marks by subsequently solving their numerator equal to 0.</p>

Q7.

Question Number	Scheme	Marks
(a)	At A, $x = -1 + 8 = 7$ & $y = (-1)^2 = 1 \Rightarrow A(7,1)$	B1
(b)	$x = t^3 - 8t, \quad y = t^2,$ $\frac{dx}{dt} = 3t^2 - 8, \quad \frac{dy}{dt} = 2t$ $\therefore \frac{dy}{dx} = \frac{2t}{3t^2 - 8}$ At A, $m(T) = \frac{2(-1)}{3(-1)^2 - 8} = \frac{-2}{3 - 8} = \frac{-2}{-5} = \frac{2}{5}$ T: $y - (\text{their } 1) = m_T(x - (\text{their } 7))$ or $1 = \frac{2}{5}(7) + c \Rightarrow c = 1 - \frac{14}{5} = -\frac{9}{5}$ Hence T: $y = \frac{2}{5}x - \frac{9}{5}$ gives T: $2x - 5y - 9 = 0$ AG	<p>$A(7,1)$</p> <p>Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$</p> <p>Correct $\frac{dy}{dx}$</p> <p>Substitutes for t to give any of the four underlined oe:</p> <p>Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$.</p> <p>$2x - 5y - 9 = 0$</p>
(c)	$2(t^3 - 8t) - 5t^2 - 9 = 0$ $2t^3 - 5t^2 - 16t - 9 = 0$ $(t+1)\{(2t^2 - 7t - 9) = 0\}$ $(t+1)\{(t+1)(2t-9) = 0\}$ $\{t = -1 \text{ (at A)}\} \quad t = \frac{9}{2} \text{ at B}$ $x = (\frac{9}{2})^3 - 8(\frac{9}{2}) = \frac{729}{8} - 36 = \frac{441}{8} = 55.125$ or awrt 55.1 $y = (\frac{9}{2})^2 = \frac{81}{4} = 20.25$ or awrt 20.3 Hence B($\frac{441}{8}, \frac{81}{4}$)	<p>Substitution of both $x = t^3 - 8t$ and $y = t^2$ into T</p> <p>A realisation that $(t+1)$ is a factor.</p> <p>$t = \frac{9}{2}$</p> <p>Candidate uses their value of t to find either the x or y coordinate</p> <p>One of either x or y correct.</p> <p>Both x and y correct.</p> <p>awrt</p>