Questions

Q1.

In this question you must show detailed reasoning.

$f(x) = x^3 - 21x^2 + Ax - 91$ where A is a real constant

The roots of $f(x) = 0$ are

α, $\alpha + 3\beta$ and $\alpha + 6\beta$

where α and β are real constants.

Use algebra to determine the value of each of these roots.

(7)

(Total for question = 7 marks)

Q2.

The cubic equation

$z^3 - 3z^2 + z + 5 = 0$

has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(2\alpha + 1)$, $(2\beta + 1)$ and $(2\gamma + 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p, q and r are integers to be found.

(5)

(Total for question = 5 marks)

Q3.

f(z) = $z^3 + z^2 + pz + q$

where p and q are real constants.

The equation f(z) = 0 has roots z_1, z_2 and z_3
When plotted on an Argand diagram, the points representing z_1, z_2 and z_3 form the vertices of a triangle of area 35

Given that $z_1 = 3$, find the values of p and q.

(7)

(Total for question = 7 marks)

Q4.

The cubic equation

$x^3 + 3x^2 - 8x + 6 = 0$

has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(\alpha - 1)$, $(\beta - 1)$ and $(\gamma - 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p, q and r are integers to be found.

(5)

(Total for question = 5 marks)

Q5.

$f(z) = z^3 - 8z^2 + pz - 24$

where p is a real constant.

Given that the equation $f(z) = 0$ has distinct roots

$$\alpha, \beta \text{ and } \left(\alpha + \frac{12}{\alpha} - \beta\right)$$

(a) solve completely the equation $f(z) = 0$

(6)

(b) Hence find the value of p.

(2)

(Total for question = 8 marks)

Q6.

The roots of the equation

$x^3 - 2x^2 + 4x - 5 = 0$

are p, q and r.

Without solving the equation, find the value of

(i) $\frac{2}{p} + \frac{2}{q} + \frac{2}{r}$

(ii) $(p - 4)(q - 4)(r - 4)$

(iii) $p^3 + q^3 + r^3$

(Total for question = 8 marks)

Q7.

The roots of the equation

$x^3 - 8x^2 + 28x - 32 = 0$

are α, β and γ

Without solving the equation, find the value of

(i) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

(ii) $(\alpha + 2)(\beta + 2)(\gamma + 2)$

(iii) $\alpha^2 + \beta^2 + \gamma^2$

(8)

(Total for question = 8 marks)

Q8.

The cubic equation

$3x^3 + x^2 - 4x + 1 = 0$

has roots α, β, and γ.

Without solving the cubic equation,

(a) determine the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

(3)

(b) find a cubic equation that has roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$, giving your answer in the form

$x^3 + ax^2 + bx + c = 0$, where a, b and c are integers to be determined.

(3)

(Total for question = 6 marks)

Q9.

The cubic equation

$$9x^3 - 5x^2 + 4x + 7 = 0$$

has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(3\alpha - 2)$, $(3\beta - 2)$ and $(3\gamma - 2)$, giving your answer in the form $aw^3 + bw^2 + cw + d = 0$, where a, b, c and d are integers to be determined.

(Total for question = 5 marks)

Q10.

The roots of the quartic equation

$3x^4 + 5x^3 - 7x + 6 = 0$

are α, β, γ and δ

Making your method clear and without solving the equation, determine the exact value of

(i) $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$

(3)

(ii) $\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma} + \frac{2}{\delta}$

(3)

(iii) $(3 - \alpha)(3 - \beta)(3 - \gamma)(3 - \delta)$

(3)

(Total for question = 9 marks)

Q11.

The cubic equation

$4x^3 + px^2 - 14x + q = 0$

where p and q are real positive constants, has roots α, β and γ

Given that $\alpha^2 + \beta^2 + \gamma^2 = 16$

(a) show that $p = 12$

(3)

Given that $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{14}{3}$

(b) determine the value of q

(3)

Without solving the cubic equation,

(c) determine the value of $(\alpha - 1)(\beta - 1)(\gamma - 1)$

(4)

(Total for question = 10 marks)

Q12.

The cubic equation

$2x^3 - 3x^2 + 5x + 7 = 0$

has roots α, β and γ.

Without solving the equation, determine the exact value of

(i) $\alpha^2 + \beta^2 + \gamma^2$

(3)

(ii) $\frac{3}{\alpha} + \frac{3}{\beta} + \frac{3}{\gamma}$

(3)

(iii) $(5 - \alpha)(5 - \beta)(5 - \gamma)$

(3)

(Total for question = 9 marks)

Q13.

The quartic equation

$2x^4 + Ax^3 - Ax^2 - 5x + 6 = 0$

where A is a real constant, has roots α, β, γ and δ

(a) Determine the value of

$$\frac{3}{\alpha} + \frac{3}{\beta} + \frac{3}{\gamma} + \frac{3}{\delta}$$

(3)

Given that $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = -\frac{3}{4}$

(b) determine the possible values of A

(5)

(Total for question = 8 marks)

Q14.

$f(z) = z^3 + pz^2 + qz - 15$

where p and q are real constants.

Given that the equation $f(z) = 0$ has roots

$$\alpha, \frac{5}{\alpha} \text{ and } \left(\alpha + \frac{5}{\alpha} - 1\right)$$

(a) solve completely the equation $f(z) = 0$

(5)

(b) Hence find the value of p.

(2)

(Total for question = 7 marks)