Mark Scheme

Q1.

Question Number	Scheme		Marks
	(a) $\frac{2}{x+2}+\frac{4}{x^2+5}-\frac{18}{(x+2)(x^2+5)}=\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$		M1A1
	$=\frac{2x(x+2)}{(x+2)(x^2+5)}$		M1
	$=\frac{2x}{(x^2+5)}$		A1*
			(4)
	(b) $h'(x)=\frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$		M1A1
	$h'(x)=\frac{10-2x^2}{(x^2+5)^2}$	cso	A1
			(3)
	(c) Maximum occurs when $h'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$		M1
	$\Rightarrow x=\sqrt{5}$		A1
	(c) Maximum occurs when $h'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$		M1
	$\Rightarrow x=\sqrt{5}$		A1
	When $x=\sqrt{5}\Rightarrow h(x)=\frac{\sqrt{5}}{5}$		M1,A1
	Range of h(x) is $0\le h(x)\le\frac{\sqrt{5}}{5}$		A1ft
			(5)
			(12 marks)

(a) M1 Combines the three fractions to form a single fraction with a common denominator.
Allow errors on the numerator but at least one must have been adapted.
Condone 'invisible' brackets for this mark.
Accept three separate fractions with the same denominator.
Amongst possible options allowed for this method are

$\frac{2x^2+5+4x+2-18}{(x+2)(x^2+5)}$ Eg 1 An example of 'invisible' brackets

$\frac{2(x^2+5)}{(x+2)(x^2+5)}+\frac{4}{(x+2)(x^2+5)}-\frac{18}{(x+2)(x^2+5)}$ Eg 2An example of an error (on middle term), 1st term has been adapted

$\frac{2(x^2+5)^2(x+2)+4(x+2)^2(x^2+5)-18(x^2+5)(x+2)}{(x+2)^2(x^2+5)^2}$ Eg 3 An example of a correct fraction with a different denominator

A1 Award for a correct un simplified fraction with the correct (lowest) common denominator.
$\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$

Accept if there are three separate fractions with the correct (lowest) common denominator.
Eg $\frac{2(x^2+5)}{(x+2)(x^2+5)}+\frac{4(x+2)}{(x+2)(x^2+5)}-\frac{18}{(x+2)(x^2+5)}$

Note, Example 3 would score M1A0 as it does not have the correct lowest common denominator

M1 There must be a single denominator. Terms must be collected on the numerator.
A factor of (x+2) must be taken out of the numerator and then cancelled with one in the denominator. The cancelling may be assumed if the term 'disappears'

A1* Cso $\frac{2x}{(x^2+5)}$ This is a given solution and this mark should be withheld if there are any errors

(b) M1 Applies the quotient rule to $\frac{2x}{(x^2+5)}$, a form of which appears in the formula book.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{(x^2+5)\times A-2x\times Bx}{(x^2+5)^2} \quad \text{where } A, B>0$$

A1 Correct unsimplified answer $h'(x)=\frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$

A1 $h'(x)=\frac{10-2x^2}{(x^2+5)^2}$ The correct simplified answer. Accept $\frac{2(5-x^2)}{(x^2+5)^2}$ $\frac{-2(x^2-5)}{(x^2+5)^2}$, $\frac{10-2x^2}{(x^4+10x^2+25)}$

DO NOT ISW FOR PART (b). INCORRECT SIMPLIFICATION IS A0

(c) M1 Sets their h'(x)=0 and proceeds with a correct method to find x. There must have been an attempt to differentiate. Allow numerical errors but do not allow solutions from 'unsolvable' equations.

A1 Finds the correct x value of the maximum point $x=\sqrt{5}$.
Ignore the solution $x=-\sqrt{5}$ but withhold this mark if other positive values found.

M1 Substitutes their answer into their h'(x)=0 in h(x) to determine the maximum value

A1 Cso-the maximum value of h(x) = $\frac{\sqrt{5}}{5}$. Accept equivalents such as $\frac{2\sqrt{5}}{10}$ **but not** 0.447

A1ft Range of h(x) is $0\le h(x)\le\frac{\sqrt{5}}{5}$. Follow through on their maximum value if the M's have been scored. Allow $0\le y\le\frac{\sqrt{5}}{5}$, $0\le Range\le\frac{\sqrt{5}}{5}$, $\left[0,\frac{\sqrt{5}}{5}\right]$ but not $0\le x\le\frac{\sqrt{5}}{5}$, $\left(0,\frac{\sqrt{5}}{5}\right)$

If a candidate attempts to work out $h^{-1}(x)$ in (b) and does all that is required for (b) in (c), then allow. Do not allow $h^{-1}(x)$ to be used for h'(x) in part (c). For this question (b) and (c) can be scored together.

Alternative to (b) using the product rule

M1 Sets $h(x)=2x(x^2+5)^{-1}$ and applies the product rule vu'+uv' with terms being $2x$ and $(x^2+5)^{-1}$
If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$(x^2+5)^{-1}\times A+2x\times\pm Bx(x^2+5)^{-2}$$

A1 Correct un simplified answer $(x^2+5)^{-1}\times 2+2x\times -2x(x^2+5)^{-2}$

A1 The question asks for h'(x) to be put in its simplest form. Hence in this method the terms need to be combined to form a single correct expression.

For a correct simplified answer accept
$h'(x)=\frac{10-2x^2}{(x^2+5)^2}=\frac{2(5-x^2)}{(x^2+5)^2}=\frac{-2(x^2-5)}{(x^2+5)^2}=(10-2x^2)(x^2+5)^{-2}$

Q2.

Question	Scheme	Marks
(a)	$x^2+x-6\overline{)x^4+x^3-3x^2+7x-6}$ with quotient x^2+3	
	$\underline{x^4+x^3-6x^2}$	
	$3x^2+7x-6$	M1 A1
	$\underline{3x^2+3x-18}$	
	$4x+12$	
	$\dfrac{x^4+x^3-3x^2+7x-6}{x^2+x-6} \equiv x^2+3+\dfrac{4(x+3)}{(x+3)(x-2)}$	M1
	$\equiv x^2+3+\dfrac{4}{(x-2)}$	A1
		(4)
(b)	$\text{f}'(x)=2x-\dfrac{4}{(x-2)^2}$	M1A1ft
	Subs $x=3$ into $\text{f}'(x=3)=2\times3-\dfrac{4}{(3-2)^2}=(2)$	M1
	Uses $m=-\dfrac{1}{\text{f}'(3)}=\left(-\dfrac{1}{2}\right)$ with $(3,\text{f}(3))=(3,16)$ to form eqn of normal	
	$y-16=-\dfrac{1}{2}(x-3)$ or equivalent cso	M1A1
		(5)
		(9 marks)

(a)

M1 Divides $x^4+x^3-3x^2+7x-6$ by x^2+x-6 to get a quadratic quotient and a linear or constant remainder. To award this look for a minimum of the following

$$\begin{array}{r} x^2(+..x)+A \\ x^2+x-6\overline{)x^4+x^3-3x^2+7x-6} \\ \underline{x^4+x^3-6x^2} \\ \\ \overline{(Cx)+D} \end{array}$$

If they divide by $(x+3)$ first they must then divide their by result by $(x-2)$ before they score this method mark. Look for a cubic quotient with a constant remainder followed by a quadratic quotient and a constant remainder

Note: FYI Dividing by $(x+3)$ gives x^3-2x^2+3x-2 and $(x^3-2x^2+3x-2)\div(x-2)=x^2+3$ with a remainder of 4.

Division by $(x-2)$ first is possible but difficult.....please send to review any you feel deserves credit.

A1 Quotient = x^2+3 and Remainder = $4x+12$

M1 Factorises x^2+x-6 and writes their expression in the appropriate form.

$$\left(\frac{x^4+x^3-3x^2+7x-6}{x^2+x-6}\right) \equiv \text{Their Quadratic Quotient} + \frac{\text{Their Linear Remainder}}{(x+3)(x-2)}$$

It is possible to do this part by partial fractions. To score M1 under this method the terms must be correct and it must be a full method to find both "numerators"

A1 $x^2+3+\frac{4}{(x-2)}$ or $A=3, B=4$ but don't penalise after a correct statement.

(b)

M1 $x^2+A+\frac{B}{x-2} \rightarrow 2x \pm \frac{B}{(x-2)^2}$

If they fail in part (a) to get a function in the form $x^2+A+\frac{B}{x-2}$ allow candidates to pick up this method mark for differentiating a function of the form $x^2+Px+Q+\frac{Rx+S}{x \pm T}$ using the quotient rule oe.

A1ft $x^2+A+\frac{B}{x-2} \rightarrow 2x-\frac{B}{(x-2)^2}$ oe. FT on their numerical A, B for for $x^2+A+\frac{B}{x-2}$ only

M1 Subs $x=3$ into their $f'(x)$ in an attempt to find a numerical gradient

M1 For the correct method of finding an equation of a normal. The gradient must be $-\frac{1}{\text{their } f'(3)}$ and the point must be $(3, f(3))$. Don't be overly concerned about how they found their f(3), ie accept x=3 y =.

Look for $y-f(3)=-\frac{1}{f'(3)}(x-3)$ or $(y-f(3)) \times -f'(3)=(x-3)$

If the form $y=mx+c$ is used they must proceed as far as $c=$

A1 cso $y-16=-\frac{1}{2}(x-3)$ oe such as $2y+x-35=0$ but remember to isw after a correct answer.

Alt (a) attempted by equating terms.

Alt (a)	$x^4+x^3-3x^2+7x-6 \equiv (x^2+A)(x^2+x-6)+B(x+3)$ Compare 2 terms (or substitute 2 values) AND solve simultaneously ie $x^2 \Rightarrow A-6=-3, \quad x \Rightarrow A+B=7, \quad$ const $\Rightarrow -6A+3B=-6$ $A=3, B=4$	M1 M1 A1,A1

1st Mark M1 Scored for multiplying by (x^2+x-6) and cancelling/dividing to achieve $x^4+x^3-3x^2+7x-6 \equiv (x^2+A)(x^2+x-6)+B(x \pm 3)$

3rd Mark M1 Scored for comparing two terms (or for substituting two values) **AND** solving simultaneously to get values of A and B.

2nd Mark A1 Either $A=3$ or $B=4$. One value may be correct by substitution of say $x=-3$

4th Mark A1 Both $A=3$ and $B=4$

Alt (b) is attempted by the quotient (or product rule)

ALT (b) 1st 3 marks	$f'(x)=\frac{(x^2+x-6)(4x^3+3x^2-6x+7)-(x^4+x^3-3x^2+7x-6)(2x+1)}{(x^2+x-6)^2}$ Subs $x=3$ into	M1A1 M1

M1 Attempt to use the **quotient rule** $\frac{vu'-uv'}{v^2}$ with $u=x^4+x^3-3x^2+7x-6$ and $v=x^2+x-6$ and achieves an expression of the form $f'(x)=\frac{(x^2+x-6)(..x^3.......)-(x^4+x^3-3x^2+7x-6)(..x..)}{(x^2+x-6)^2}$.

Use a similar approach to the product rule with $u=x^4+x^3-3x^2+7x-6$ and $v=(x^2+x-6)^{-1}$

Note that this can score full marks from a partially solved part (a) where $f(x) \equiv x^2+3+\frac{4x+12}{x^2+x-6}$

Q3.

Question	Scheme	Marks	AOs
(a)	Correct method used in attempting to differentiate $y=\frac{5x^2+10x}{(x+1)^2}$	M1	3.1a
	$\frac{dy}{dx}=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times2(x+1)}{(x+1)^4}$ oe	A1	1.1b
	Factorises/Cancels term in $(x+1)$ and attempts to simplify $\frac{dy}{dx}=\frac{(x+1)\times(10x+10)-(5x^2+10x)\times2}{(x+1)^3}=\frac{A}{(x+1)^3}$	M1	2.1
	$\frac{dy}{dx}=\frac{10}{(x+1)^3}$	A1	1.1b
		(4)	
(b)	For $x<-1$ Follow through on their $\frac{dy}{dx}=\frac{A}{(x+1)^n}, n=1,3$	B1ft	2.2a
		(1)	
			(5 marks)

(a)

M1: Attempts to use a correct rule to differentiate Eg: Use of quotient (& chain) rules on $y=\frac{5x^2+10x}{(x+1)^2}$

Alternatively uses the product (and chain) rules on $y=(5x^2+10x)(x+1)^{-2}$

Condone slips but expect $\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{(x+1)^4}$ $(A,B,C,D>0)$ or

$\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{((x+1)^2)^2}$ $(A,B,C,D>0)$ using the quotient rule

or $\left(\frac{dy}{dx}\right)=(x+1)^{-2}\times(Ax+B)+(5x^2+10x)\times C(x+1)^{-3}$ $(A,B,C\neq 0)$ using the product rule.

Condone missing brackets and slips for the M mark. For instance if they quote $u=5x^2+10$, $v=(x+1)^2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow where they quote the correct formula, give values of u and v, but only have v rather than v^2 the denominator.

A1: A correct (unsimplified) answer

Eg. $\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$ or equivalent via the quotient rule.

OR $\left(\frac{dy}{dx}\right)=(x+1)^{-2}\times(10x+10)+(5x^2+10x)\times -2(x+1)^{-3}$ or equivalent via the product rule

M1: A valid attempt to proceed to the given form of the answer.

It is dependent upon having a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v^2}$ and proceeding to $\frac{A}{(x+1)^3}$

It can also be scored on a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v}$ and proceeding to $\frac{A}{(x+1)}$

You may see candidates expanding terms in the numerator. FYI $10x^3+30x^2+30x+10-10x^3-30x^2-20x$ but under this method they must reach the same expression as required by the main method.

Using the product rule expect to see a common denominator being used correctly before the above

A1: $\frac{dy}{dx}=\frac{10}{(x+1)^3}$ There is no requirement to see $\frac{dy}{dx}=$ and they can recover from missing brackets/slips.

(b)

B1ft: Score for deducing the correct answer of $x<-1$ This can be scored independent of their answer to part (a). Alternatively score for a correct **ft** answer for their $\frac{dy}{dx}=\frac{A}{(x+1)^n}$ where $A<0$ and $n=1,3$ award for $x>-1$. So for example if $A>0$ and $n=1,3\Rightarrow x<-1$

Question	Scheme	Marks	AOs
Alt via division	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in form $y=A\pm\frac{B}{(x+1)^2}$ $A, B \neq 0$	M1	3.1a
	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in the form $y=5-\frac{5}{(x+1)^2}$	A1	1.1b
	Uses the chain rule $\Rightarrow \frac{dy}{dx}=\frac{C}{(x+1)^3}$ (May be scored from $A=0$)	M1	2.1
	$\frac{dy}{dx}=\frac{10}{(x+1)^3}$ which cannot be awarded from incorrect value of A	A1	1.1b
		(4)	
(b)	For $x<-1$ or correct follow through	B1ft	2.2a
		(1)	
			(5 marks)

Q4.

Question Number	Scheme	Marks	
	(a) $x = 1 - 2y^3 \Rightarrow y = \left(\frac{1-x}{2}\right)^{1/3}$ *or* $\sqrt[3]{\frac{1-x}{2}}$	M1 A1	(2)
	$f^{-1} : x \mapsto \left(\frac{1-x}{2}\right)^{1/3}$ Ignore domain		
	(b) $gf(x) = \frac{3}{1-2x^3} - 4$	M1 A1	
	$= \frac{3 - 4(1-2x^3)}{1-2x^3}$	M1	
	$= \frac{8x^3 - 1}{1-2x^3}$ ∗ cso	A1	(4)
	$gf : x \mapsto \frac{8x^3-1}{1-2x^3}$ Ignore domain		
	(c) $8x^3 - 1 = 0$ Attempting solution of numerator = 0	M1	
	$x = \frac{1}{2}$ Correct answer and no additional answers	A1	(2)
	(d) $\frac{dy}{dx} = \frac{(1-2x^3) \times 24x^2 + (8x^3-1) \times 6x^2}{(1-2x^3)^2}$	M1 A1	
	$= \frac{18x^2}{(1-2x^3)^2}$	A1	
	Solving their numerator = 0 and substituting to find y.	M1	
	$x = 0, y = -1$	A1	(5) [13]

Q5.

Question Number	Scheme	Marks
(a)	$p = 4\pi^2$ or $(2\pi)^2$	B1
		(1)
(b)	$x=(4y-\sin 2y)^2 \Rightarrow \frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	M1A1
	Sub $y=\frac{\pi}{2}$ into $\frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	
	$\Rightarrow \frac{dx}{dy}=24\pi \quad (=75.4)$ / $\frac{dy}{dx}=\frac{1}{24\pi}(=0.013)$	M1
	Equation of tangent $y-\frac{\pi}{2}=\frac{1}{24\pi}(x-4\pi^2)$	M1
	Using $y-\frac{\pi}{2}=\frac{1}{24\pi}(x-4\pi^2)$ with $x=0 \Rightarrow y=\frac{\pi}{3}$ cso	M1, A1
		(6)
		(7 marks)
Alt (b) I	$x=(4y-\sin 2y)^2 \Rightarrow x^{0.5}=4y-\sin 2y$ $\Rightarrow 0.5x^{-0.5}\frac{dx}{dy}=4-2\cos 2y$	M1A1
Alt (b) II	$x=\left(16y^2-8y\sin 2y+\sin^2 2y\right)$ $\Rightarrow 1=32y\frac{dy}{dx}-8\sin 2y\frac{dy}{dx}-16y\cos 2y\frac{dy}{dx}+4\sin 2y\cos 2y\frac{dy}{dx}$ Or $1dx=32y\,dy-8\sin 2y\,dy-16y\cos 2y\,dy+4\sin 2y\cos 2y\,dy$	M1A1

(a)

B1 $p = 4\pi^2$ or exact equivalent $(2\pi)^2$

Also allow $x = 4\pi^2$

(b)

M1 Uses the chain rule of differentiation to get a form

$A(4y-\sin 2y)(B \pm C\cos 2y), \quad A, B, C \neq 0$ on the right hand side

Alternatively attempts to expand and then differentiate using product rule and chain rule to a form $x=\left(16y^2-8y\sin 2y+\sin^2 2y\right) \Rightarrow \frac{dx}{dy}=Py \pm Q\sin 2y \pm Ry\cos 2y \pm S\sin 2y\cos 2y \quad P, Q, R, S \neq 0$

A second method is to take the square root first. To score the method look for a differentiated expression of the form $Px^{-0.5}... = 4-Q\cos 2y$

A third method is to multiply out and use implicit differentiation. Look for the correct terms, condoning errors on just the constants.

A1 $\frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$ or $\frac{dy}{dx}=\frac{1}{2(4y-\sin 2y)(4-2\cos 2y)}$ with both sides correct. The lhs may be seen elsewhere if clearly linked to the rhs.

In the alternative $\frac{dx}{dy}=32y-8\sin 2y-16y\cos 2y+4\sin 2y\cos 2y$

M1 Sub $y=\frac{\pi}{2}$ into their $\frac{dx}{dy}$ or inverted $\frac{dx}{dy}$. Evidence could be minimal, eg $y=\frac{\pi}{2} \Rightarrow \frac{dx}{dy}=...$

It is not dependent upon the previous M1 but it must be a changed $x=(4y-\sin 2y)^2$

M1 Score for a correct method for finding the equation of the tangent at $\left('4\pi^2', \frac{\pi}{2}\right)$.

Allow for $y-\frac{\pi}{2}=\frac{1}{\text{their numerical } \left(\frac{dx}{dy}\right)}\left(x-\text{their } 4\pi^2\right)$

Allow for $\left(y-\frac{\pi}{2}\right) \times \text{their numerical } \left(\frac{dx}{dy}\right)=\left(x-\text{their } 4\pi^2\right)$

Even allow for $y-\frac{\pi}{2}=\frac{1}{\text{their numerical } \left(\frac{dx}{dy}\right)}(x-p)$

It is possible to score this by stating the equation $y=\frac{1}{24\pi}x+c$ as long as $\left('4\pi^2', \frac{\pi}{2}\right)$ is used in a subsequent line.

M1 Score for writing their equation in the form $y=mx+c$ and stating the value of 'c'

Or setting $x=0$ in their $y-\frac{\pi}{2}=\frac{1}{24\pi}\left(x-4\pi^2\right)$ and solving for y.

Alternatively using the gradient of the line segment AP = gradient of tangent.

Look for $\frac{\frac{\pi}{2}-y}{4\pi^2}=\frac{1}{24\pi} \Rightarrow y=..$ Such a method scores the previous M mark as well.

At this stage all of the constants must be numerical. It is not dependent and it is possible to score this using the "incorrect" gradient.

A1 cso $y=\frac{\pi}{3}$. You do not have to see $\left(0, \frac{\pi}{3}\right)$

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Q6.

Question Number	Scheme	Marks
	(a) $-32=(2w-3)^5 \Rightarrow w=\frac{1}{2}$ oe	M1A1
		(2)
	(b) $\frac{dy}{dx}=5\times(2x-3)^4\times 2$ or $10(2x-3)^4$	M1A1
	When $x=\frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is $'160'=\frac{y-(-32)}{x-'\frac{1}{2}'}$ oe	dM1
	$y=160x-112$ cso	A1
		(5)
		(7 marks)

(a) M1 Substitute y=-32 into $y=(2w-3)^5$ **and** proceed to w=….. [Accept positive sign used of y, ie y=+32]

A1 Obtains w or $x=\frac{1}{2}$ oe with no incorrect working seen. Accept alternatives such as 0.5.

Sight of just the answer would score both marks as long as no incorrect working is seen.

(b) M1 Attempts to differentiate $y=(2x-3)^5$ using the chain rule.

Sight of $\pm A(2x-3)^4$ where A is a non- zero constant is sufficient for the method mark.

A1 A correct (un simplified) form of the differential.

Accept $\frac{dy}{dx}=5\times(2x-3)^4\times 2$ or $\frac{dy}{dx}=10(2x-3)^4$

M1 This is awarded for an attempt to find the gradient of the tangent to the curve at P
Award for substituting their numerical value to part (a) into their differential to find the numerical gradient of the tangent

dM1 Award for a correct method to find an equation of the tangent to the curve at P. It is dependent upon the previous M mark being awarded.

Award for $'\textit{their } 160'=\frac{y-(-32)}{x-\textit{their }'\frac{1}{2}'}$

If they use $y=mx+c$ it must be a full method, using m= 'their 160', their '$\frac{1}{2}$' and -32.

An attempt must be seen to find c=…

A1 cso $y=160x-112$. The question is specific and requires the answer in this form.
You may isw in this question after a correct answer.

Q7.

Question Number	Scheme	Marks
	(i)(a) $\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$	M1A1A1
	$= 3x^2 \ln 2x + x^2$	
		(3)
	(i)(b) $\frac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2\cos 2x)$	**B1**M1A1
		(3)
	(ii) $\frac{dx}{dy} = -\text{cosec}^2 y$	M1A1
	$\frac{dy}{dx} = -\frac{1}{\text{cosec}^2 y}$	M1
	Uses $\text{cosec}^2 y = 1 + \cot^2 y$ and $x = \cot y$ in $\frac{dy}{dx}$ or $\frac{dx}{dy}$ to get an expression in x	
	$\frac{dy}{dx} = -\frac{1}{\text{cosec}^2 y} = -\frac{1}{1+\cot^2 y} = -\frac{1}{1+x^2}$ cso	M1, A1*
		(5)
		(11 marks)

(i)(a) M1 Applies the product rule vu'+uv' to $x^3 \ln 2x$.
If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$Ax^2 \times \ln 2x + x^3 \times \frac{B}{x} \qquad \text{where } A, B \text{ are constants} \neq 0$$

A1 One term correct, either $3x^2 \times \ln 2x$ or $x^3 \times \frac{1}{2x} \times 2$

A1 Cao. $\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$. The answer does not need to be simplified.

For reference the simplified answer is $\frac{dy}{dx} = 3x^2 \ln 2x + x^2 = x^2(3\ln 2x + 1)$

(i)(b) B1 Sight of $(x + \sin 2x)^2$

M1 For applying the chain rule to $(x + \sin 2x)^3$. If the rule is quoted it must be correct. If it is not quoted possible forms of evidence could be sight of $C(x + \sin 2x)^2 \times (1 \pm D\cos 2x)$ where C and D are non- zero constants.
Alternatively accept $u = x + \sin 2x$, u'= followed by $Cu^2 \times$ their u'
Do not accept $C(x + \sin 2x)^2 \times 2\cos 2x$ unless you have evidence that this is their u'
Allow 'invisible' brackets for this mark, ie. $C(x + \sin 2x)^2 \times 1 \pm D\cos 2x$

A1 Cao $\frac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2\cos 2x)$. There is no requirement to simplify this.

You may ignore subsequent working (isw) after a correct answer in part (i)(a) and (b)

(ii) M1 Writing the derivative of cot$y$ as -cosec^2y. It must be in terms of y

A1 $\frac{dx}{dy} = -\text{cosec}^2 y$ or $1 = -\text{cosec}^2 y \frac{dy}{dx}$. Both lhs and rhs must be correct.

M1 Using $\frac{dy}{dx} = \frac{1}{dx/dy}$

M1 Using $\text{cosec}^2 y = 1 + \cot^2 y$ **and** $x = \cot y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ just in terms of x.

A1 cso $\frac{dy}{dx} = -\frac{1}{1+x^2}$

Alternative to (a)(i) when ln(2x) is written lnx+ln2

M1 Writes $x^3 \ln 2x$ as $x^3 \ln 2 + x^3 \ln x$.
Achieves Ax^2 for differential of $x^3 \ln 2$ and applies the product rule vu'+uv' to $x^3 \ln x$.

A1 Either $3x^2 \times \ln 2 + 3x^2 \ln x$ or $x^3 \times \frac{1}{x}$

A1 A correct (un simplified) answer. Eg $3x^2 \times \ln 2 + 3x^2 \ln x + x^3 \times \frac{1}{x}$

Alternative to (ii) using quotient rule

M1 Writes cot y as $\frac{\cos y}{\sin y}$ and applies the quotient rule, a form of which appears in the formula book. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$)
only accept answers of the form $\frac{\sin y \times \pm \sin y - \cos y \times \pm \cos y}{(\sin y)^2}$

A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{dx}{dy} = \frac{\sin y \times -\sin y - \cos y \times \cos y}{(\sin y)^2} = \{-1 - \cot^2 y\}$$

M1 Using $\frac{dy}{dx} = \frac{1}{dx/dy}$

M1 Using $\sin^2 y + \cos^2 y = 1$, $\frac{1}{\sin^2 y} = \text{cosec}^2 y$ and $\text{cosec}^2 y = 1 + \cot^2 y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in x

A1 cso $\frac{dy}{dx} = -\frac{1}{1+x^2}$

Alternative to (ii) using the chain rule, first two marks

M1 Writes cot y as $(\tan y)^{-1}$ and applies the chain rule (or quotient rule).
Accept answers of the form $-(\tan y)^{-2} \times \sec^2 y$

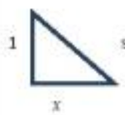
A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{dx}{dy} = -(\tan y)^{-2} \times \sec^2 y$$

Alternative to (ii) using a triangle – last M1

M1 Uses triangle with $\tan y = \frac{1}{x}$ to find sin$y$ and get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ just in terms of x

$x = \cot y \Rightarrow \tan y = \frac{1}{x}$



$\sin y = \frac{1}{\sqrt{1+x^2}}$

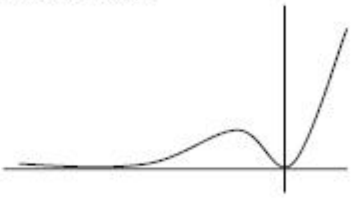
Q8.

Question Number	Scheme	Marks
(a)	$\frac{1}{(x^2+3x+5)} \times \ldots\ , = \frac{2x+3}{(x^2+3x+5)}$	M1,A1
		(2)
(b)	Applying $\frac{vu'-uv'}{v^2}$	M1,
	$\frac{x^2\times -\sin x - \cos x \times 2x}{(x^2)^2} = \frac{-x^2\sin x - 2x\cos x}{x^4} = \frac{-x\sin x - 2\cos x}{x^3}$ oe	A2,1,0
		(3)
		5 Marks

Q9.

Question Number	Scheme		Marks
(a)	$y = \frac{3+\sin 2x}{2+\cos 2x}$		
	Apply quotient rule: $\begin{Bmatrix} u = 3+\sin 2x & v = 2+\cos 2x \\ \frac{du}{dx} = 2\cos 2x & \frac{dv}{dx} = -2\sin 2x \end{Bmatrix}$		
	$\frac{dy}{dx} = \frac{2\cos 2x(2+\cos 2x) - -2\sin 2x(3+\sin 2x)}{(2+\cos 2x)^2}$	Applying $\frac{vu'-uv'}{v^2}$	M1
		Any one term correct on the numerator	A1
		Fully correct (unsimplified).	A1
	$= \frac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2+\cos 2x)^2}$		
	$= \frac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2+\cos 2x)^2}$		
	$= \frac{4\cos 2x + 6\sin 2x + 2}{(2+\cos 2x)^2}$ (as required)	For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$. No errors seen in working.	A1*
			(4)
(b)	When $x = \frac{\pi}{2}$, $y = \frac{3+\sin\pi}{2+\cos\pi} = \frac{3}{1} = 3$	$y = 3$	B1
	At $(\frac{\pi}{2}, 3)$, $m(\mathbf{T}) = \frac{6\sin\pi + 4\cos\pi + 2}{(2+\cos\pi)^2} = \frac{-4+2}{1^2} = -2$	$m(\mathbf{T}) = -2$	B1
	Either **T**: $y - 3 = -2(x - \frac{\pi}{2})$ or $y = -2x + c$ and $3 = -2(\frac{\pi}{2}) + c \Rightarrow c = 3 + \pi$;	$y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient';	M1
	T: $y = -2x + (\pi + 3)$	$y = -2x + \pi + 3$	A1
			(4) [8]

Q10.

Question Number	Scheme	Marks
(*a*)	$\frac{dy}{dx} = x^2e^x + 2xe^x$	M1,A1,A1 (3)
(*b*)	If $\frac{dy}{dx} = 0$, $e^x(x^2 + 2x) = 0$ setting $(a) = 0$ $[e^x \neq 0]$ $x(x+2) = 0$ $(x = 0)$ $x = -2$ $x = 0, y = 0$ **and** $x = -2, y = 4e^{-2}$ $(= 0.54\ldots)$	M1 A1 A1 √ (3)
(*c*)	$\frac{d^2y}{dx^2} = x^2e^x + 2xe^x + 2xe^x + 2e^x$ $\left[= (x^2 + 4x + 2)e^x\right]$	M1, A1 (2)
(*d*)	$x = 0$, $\frac{d^2y}{dx^2} > 0$ $(=2)$ $x = -2$, $\frac{d^2y}{dx^2} < 0$ $[= -2e^{-2}$ $(= -0.270\ldots)]$ M1: Evaluate, or state sign of, candidate's (c) for at least one of candidate's x value(s) from (b) ∴minimum ∴maximum	M1 A1 (cso) (2)
Alt.(*d*)	For M1: Evaluate, or state sign of, $\frac{dy}{dx}$ at two appropriate values – on either side of at least one of their answers from (b) or Evaluate y at two appropriate values – on either side of at least one of their answers from (b) or Sketch curve 	**(10 marks)**

Notes: (a) M for attempt at $f(x)g'(x) + f'(x)g(x)$

1st A1 for one correct, 2nd A1 for the other correct.

Note that x^2e^x on its own scores no marks

(b) 1st A1 $(x = 0)$ may be omitted, but for

2nd A1 both sets of coordinates needed ; f.t only on candidate's $x = -2$

(c) M1 requires complete method for candidate's (a), result may be unsimplified for A1

(d) A1 is cso; $x = 0$, min, and $x = -2$, max and no incorrect working seen,

or (in alternative) sign of $\frac{dy}{dx}$ either side correct, or values of y appropriate to t.p.

Need only consider the quadratic, as may assume $e^x > 0$.

If all marks gained in (a) and (c), and correct x values, give M1A1 for correct statements with no working

Q11

Question	Scheme	Marks	AOs
15(a)	$...xe^x + ...e^x$	M1	1.1b
	$k\left(xe^x + e^x\right)$	A1	1.1b
	$\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right) = \frac{1}{2}\times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$	B1	1.1b
	$\left(f'(x) =\right) \dfrac{(e^{3x}-2)^{\frac{1}{2}}("7"xe^x + "7"e^x) - "\frac{3}{2}"e^{3x}(e^{3x}-2)^{-\frac{1}{2}} \times "7"xe^x}{e^{3x}-2}$	dM1	2.1
	$f'(x) = \dfrac{7e^x\left(e^{3x}(2-x) - 4x - 4\right)}{2(e^{3x}-2)^{\frac{3}{2}}}$	A1	1.1b
		(5)	
(b)	$e^{3x}(2-x) - 4x - 4 = 0 \Rightarrow x(...e^{3x} \pm ...) = ...e^{3x} \pm ...$	M1	1.1b
	$\Rightarrow x = \dfrac{2e^{3x}-4}{e^{3x}+4}$ *	A1*	2.1
		(2)	
(c)	Draws a vertical line $x = 1$ up to the curve then across to the line $y = x$ then up to the curve finishing at the root (need to see a minimum of 2 vertical and horizontal lines tending to the root)	B1	2.1
		(1)	
(d)(i)	$x_2 = \dfrac{2e^3 - 4}{e^3 + 4} = 1.5017756...$	M1	1.1b
	$x_2 = \text{awrt } 1.502$	A1	1.1b
(ii)	$\beta = 1.968$	dB1	2.2b
		(3)	
(e)	$h(x) = \dfrac{2e^{3x}-4}{e^{3x}+4} - x$ $h(0.4315) = -0.000297...$ $h(0.4325) = 0.000947...$	M1	3.1a
	Both calculations correct and e.g. states: • There is a change of sign • e.g $f'(x)$ is continuous • $\alpha = 0.432$ (to 3dp)	A1cao	2.4
		(2)	
		(13 marks)	

Notes

(a)

M1: Attempts the product rule on xe^x (or may be $7xe^x$) achieving an expression of the form $...xe^x \pm ...e^x$. If it is clear that the quotient rule has been applied instead which may be quoted then M0.

A1: $k\left(xe^x + e^x\right)$ (e.g. $7(xe^x + e^x)$) or equivalent which may be unsimplified (may be implied by further work)

B1: $\left(\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right) =\right) \frac{1}{2}\times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$ (simplified or unsimplified)

M1: Attempts the product rule on $xe^x \to ...xe^x \pm ...e^x$ which may be seen within the expression $...e^x(e^{3x}-2)^{-\frac{1}{2}} \pm ...xe^x(e^{3x}-2)^{-\frac{1}{2}} + ...$ simplified or unsimplified.

A1: $k\left(xe^x + e^x\right)$ which may be seen within the expression $k\left(e^x(e^{3x}-2)^{-\frac{1}{2}} + xe^x(e^{3x}-2)^{-\frac{1}{2}}\right) + ...$ simplified or unsimplified.

B1: $\left(-\frac{1}{2}\right) \times 3e^{3x}(e^{3x}-2)^{-\frac{3}{2}}$ which may be seen within the expression $.... + k\left(xe^x \times \left(-\frac{1}{2}\right) \times 3e^{3x}(e^{3x}-2)^{-\frac{3}{2}}\right)$ simplified or unsimplified.

dM1: A complete method using all three products (which may appear all on one line). Do not condone invisible brackets.

A1: As above in main scheme notes.

(b) **Note that if they do not have values $A = -4,\ B = -4$ in (a) (which may be seen later) then maximum score is M1A0***

M1: Sets their $e^{3x}(2-x) - 4x - 4$ equal to zero, collects terms in x on one side of the equation and non x terms on the other and attempts to factorise the side with x as a common factor. Condone sign slips only for this mark. Allow A and B to be used instead of "-4" and "-4"

dM1: Attempts to use the quotient rule. It is dependent on the previous method mark. Score for achieving an expression of the form

$$(f'(x) =)\ \frac{(e^{3x}-2)^{\frac{1}{2}}("7"xe^x + "7"e^x) - "\frac{3}{2}"e^{3x}(e^{3x}-2)^{-\frac{1}{2}} \times "7"xe^x}{e^{3x}-2}$$

or equivalent (do not be concerned by the constants for their "7" or their "$\frac{3}{2}$" which may be both 1)

If it is clear that the quotient rule has been applied the wrong way round then score M0.
Alternatively, applies the product rule. Score for achieving an expression of the form

$$(f'(x) =)\ (e^{3x}-2)^{-\frac{1}{2}}("7"xe^x + "7"e^x) - "\frac{3}{2}"e^{3x}(e^{3x}-2)^{-\frac{3}{2}} \times "7"xe^x$$

or equivalent (do not be concerned by the constants for their "7" or their "$\frac{3}{2}$" which may be both 1)

Do not condone invisible brackets.

A1: $(f'(x) =)\ \dfrac{7e^x\left(e^{3x}(2-x) - 4x - 4\right)}{2(e^{3x}-2)^{\frac{3}{2}}}$ following a fully correct differentiated expression.

You may need to check to see if (a) is continued after other parts for evidence of this.

Condone the lack of $f'(x) =$ on the left hand side or allow the use of $\frac{dy}{dx}$ or y' instead.

Alternative (a) attempt using the triple product rule

e.g. $\frac{d}{dx}\left(7xe^x(e^{3x}-2)^{-\frac{1}{2}}\right) = 7e^x(e^{3x}-2)^{-\frac{1}{2}} + 7xe^x(e^{3x}-2)^{-\frac{1}{2}} + 7xe^x \times \left(-\frac{1}{2}\right) \times 3e^{3x}(e^{3x}-2)^{-\frac{3}{2}}$

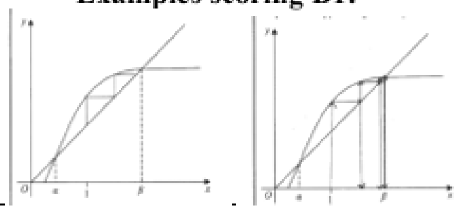
$$\Rightarrow \frac{\left(7e^x + 7xe^x\right)(e^{3x}-2) + 7xe^x \times \left(-\frac{1}{2}\right) \times 3e^{3x}}{(e^{3x}-2)^{\frac{3}{2}}} = \frac{7e^x\left(e^{3x} - 2 + xe^{3x} - 2x - \frac{3}{2}xe^{3x}\right)}{(e^{3x}-2)^{\frac{3}{2}}} \Rightarrow \frac{7e^x\left(e^{3x}(2-x) - 4x - 4\right)}{2(e^{3x}-2)^{\frac{3}{2}}}$$

A1*: Achieves the given answer with no errors including invisible brackets. If they do not reach the printed answer then it is A0. If they subsequently write $x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$ then isw

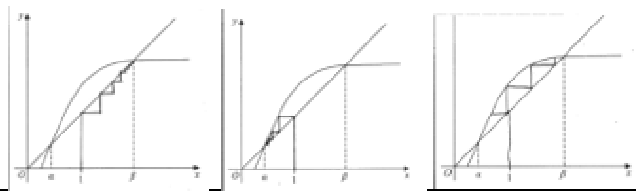
(c)

B1: Starting at $x_1 = 1$ look for at least 2 sets of vertical and horizontal lines drawn (may be dashes) tending to β. Condone a lack of arrows on the lines but the sequence of lines should finish at the point of intersection where the root is. Condone the initial vertical line not starting from the *x*-axis. Mark the intention to draw horizontal and vertical lines. If they have any lines to the left of $x = 1$ this is B0. If they use both diagrams and do not indicate which one they want marking, then the "copy of Diagram 1" should be marked.

Examples scoring B1:



Examples scoring B0:



(d)(i)

M1: Substitutes 1 into the iterative formula. The values embedded in the formula is sufficient for this mark. May be implied by awrt 1.50

A1: awrt 1.502 isw

(d)(ii)

dB1: 1.968 cao (which can only be scored if M1 is scored in (d)(i))

SC: If (d)(i) is rounded to 1.50 then allow 1.97 in (d)(ii) to score M1A0dB1 for (d)

(e)

M1: Attempts to substitute $x = 0.4315$ and 0.4325 into a suitable function and gets one value correct (rounded or truncated to 1sf). It is allowable to use a tighter interval that contains the root 0.4317388728…

If no function is stated then may be implied by their answers to e.g. $f'(0.4315)$, $f'(0.4325)$

You will need to check their calculation is correct.

Other possible functions include:

- $h(x) = x - \frac{2e^{3x} - 4}{e^{3x} + 4}$ (other way round to MS) $h(0.4315) = 0.0002974...$, $h(0.4325) = -0.0009479...$
- their $f'(x) = \pm\left(\frac{7e^{x}\left(e^{3x}(2-x) - 4x - 4\right)}{2(e^{3x} - 2)^{\frac{3}{2}}}\right)$

  (If correct *A* and *B* then $f'(0.4315) = \mp 0.005789...$, $f'(0.4325) = \pm 0.01831...$)
- their $g(x) = \pm\left(e^{3x}(2-x) - 4x - 4\right)$

  (If correct *A* and *B* then $g(0.4315) = \mp 0.002275...$, $g(0.4325) = \pm 0.007261...$)

A1: Requires

- Both calculations correct (rounded or truncated to 1sf)
- A statement that there is a change in sign and that their **function** is continuous (must refer to the function used for the substitution (which is not $f(x)$)

  Accept equivalent statements for $f'(0.4315) < 0$, $f'(0.4325) > 0$ e.g. $f'(0.4315) \times f'(0.4325) < 0$, "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"
- A minimal conclusion e.g. "hence $\alpha = 0.432$ ", "so rounds to 0.432". Do not allow "hence root"