Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$\mathbf{r}\bullet\begin{pmatrix}3\\-1\\2\end{pmatrix}=\begin{pmatrix}5\\-3\\-4\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\2\end{pmatrix}$	M1	1.1b
	$3x-y+2z=10$	A1	2.5
		(2)	
(b)	$\begin{pmatrix}3\\-1\\2\end{pmatrix}\bullet\begin{pmatrix}-1\\-5\\3\end{pmatrix}=8$	B1	1.1b
	$\sqrt{(3)^2+(-1)^2+(2)^2}\,.\sqrt{(-1)^2+(-5)^2+(3)^2}\,\cos\alpha=\text{"}-3+5+6\text{"}$	M1	1.1b
	$\theta=90^\circ-\arccos\left(\frac{8}{\sqrt{14}.\sqrt{35}}\right)$ or $\sin\theta=\frac{8}{\sqrt{14}.\sqrt{35}}$	M1	2.1
	$\theta=21.2°$ (1 dp) * cso	A1*	1.1b
		(4)	
(c)	$3(7-\lambda)-(3-5\lambda)+2(-2+3\lambda)=10\Rightarrow\lambda=\ldots$	M1	3.1a
	$\lambda=-\frac{1}{2}$	A1	1.1b
	$\overrightarrow{OX}=\begin{pmatrix}7\\3\\-2\end{pmatrix}-\frac{1}{2}\begin{pmatrix}-1\\-5\\3\end{pmatrix}=\begin{pmatrix}\ldots\\\ldots\\\ldots\end{pmatrix}$	M1	1.1b
	$X(7.5, 5.5, -3.5)$	A1ft	1.1b
		(4)	
		(10 marks)	

		Question Notes
(a)	M1	Attempts to apply the formula $\mathbf{r.n}=\mathbf{a.n}$
	A1	Correct Cartesian notation. e.g. $3x-y+2z=10$ or $-3x+y-2z=-10$
	Note	Do not allow final answer given as $\mathbf{r}\bullet(3\mathbf{i}-\mathbf{j}+2\mathbf{k})=10$, o.e.
(b)	B1	$\overrightarrow{OA}\bullet\mathbf{n}=8$
	M1	An attempt to apply the correct dot product formula between **n** and **d**.
	M1	Depends on previous M mark. Applies the dot product formula to find the angle between Π and l.
	A1*	21.2° cso
(c)	M1	Substitutes l into Π and solves the resulting equation to give $\lambda=\ldots$.
	A1	$\lambda=-\frac{1}{2}$ o.e.
	M1	Depends on previous M mark. Substitutes their λ into l and finds at least one of the coordinates.
	A1ft	$(7.5, 5.5, -3.5)$ but follow through on their value of λ.

Q2.

Question	Scheme	Marks	AOs
(a)	Need **k** component to be zero at ground, so $0.84+0.8\lambda-\lambda^2=0 \Rightarrow \lambda=\ldots$	M1	1.1b
	$\lambda=-\frac{3}{5},\frac{7}{5}$, but $\lambda \geqslant 0$ so $\lambda=\frac{7}{5}$	A1	1.1b
		(2)	
(b)	Direction is $(9-4.6\times1.4)\mathbf{i}+15\mathbf{j}+(0.8-2\times1.4)$ $=2.56\mathbf{i}+15\mathbf{j}-2\mathbf{k}$ or $\frac{64}{25}\mathbf{i}+15\mathbf{j}-2\mathbf{k}$	B1ft	2.2a
		(1)	
(c)	Direction perpendicular to ground is $a\mathbf{k}$, so angle to perpendicular is given by $(\cos\theta)=\frac{a\mathbf{k}.(2.56\mathbf{i}+15\mathbf{j}-2\mathbf{k})}{a\times\lvert 2.56\mathbf{i}+15\mathbf{j}-2\mathbf{k}\rvert}$ or $\frac{\begin{pmatrix}2.56\\15\\-2\end{pmatrix}\bullet\begin{pmatrix}0\\0\\a\end{pmatrix}}{\begin{vmatrix}2.56\\15\\-2\end{vmatrix}\begin{vmatrix}0\\0\\a\end{vmatrix}}$ or angle between $\begin{pmatrix}2.56\\15\\-2\end{pmatrix}$ and $\begin{pmatrix}2.56\\15\\0\end{pmatrix}$ is given by $(\cos\theta)=\frac{\begin{pmatrix}2.56\\15\\-2\end{pmatrix}\bullet\begin{pmatrix}2.56\\15\\0\end{pmatrix}}{\begin{vmatrix}2.56\\15\\-2\end{vmatrix}\begin{vmatrix}2.56\\15\\0\end{vmatrix}}$	M1	1.1b
	$=\frac{-2}{\sqrt{2.56^2+15^2+(-2)^2}}(=-0.130\ldots)$ Or $=\frac{231.5536}{\sqrt{2.56^2+15^2+(-2)^2}\sqrt{2.56^2+15^2+(0)^2}}=0.991\ldots$	M1	1.1b
	$90°-\arccos('-0.130...')=-7.48...$ or $\arccos(0.991...)$	ddM1	3.1b
	So the tennis ball hits ground at angle of $7.5°$ (1d.p.) cao	A1	3.2a
	Alternative Finds the length of the vector in the ij plane $=\sqrt{2.56^2+15^2}$	M1	1.1b
	$\tan\theta=\frac{2}{\sqrt{2.56^2+15^2}}$	M1	1.1b
	$\theta=\arctan\left(\frac{2}{\sqrt{2.56^2+15^2}}\right)$ or $\theta=90-\arctan\left(\frac{\sqrt{2.56^2+15^2}}{2}\right)$	ddM1	3.1b

	So the tennis ball hits ground at angle of $7.5°$ (1d.p.)	A1	3.2a
		(4)	
(d)	In same plane as net when $\mathbf{r}.\mathbf{j}=0$, $\begin{pmatrix}-4.1+9\lambda-2.3\lambda^2\\-10.25+15\lambda\\0.84+0.8\lambda-\lambda^2\end{pmatrix}\bullet\begin{pmatrix}0\\1\\0\end{pmatrix}$ leading to $-10.25+15\lambda=0\Rightarrow\lambda=\ldots$ $\left(=\frac{41}{60}=0.683333\ldots\right)$	M1	3.1b
	So is at position $\left(-4.1+9\times\frac{41}{60}-2.3\left(\frac{41}{60}\right)^2\right)\mathbf{i}+0\mathbf{j}+\left(0.84+0.8\times\frac{41}{60}-\left(\frac{41}{60}\right)^2\right)\mathbf{k}$	M1	1.1b
	$=$ awrt $0.976\mathbf{i}+$ awrt $0.920\mathbf{k}$ or $=$ awrt $0.976\mathbf{i}+0.92\mathbf{k}$ (to 3 s.f.) or $=$ awrt $0.976\mathbf{i}+\frac{3311}{3600}\mathbf{k}$	A1	1.1b
		(3)	
(e)	Modelling as a line, height of net is 0.9m along its length so as **0.92 > 0.9** the ball **will** pass over the net according to the model.	B1ft	3.2a
		(1)	
(f)	Identifies a suitable feature of the model that affects the outcome And uses it to draw a compatible conclusion. For example • The ball is **not a particle** and will have **diameter/radius**, therefore it will hit the net and not pass over. • As above, but so the **ball will clip the net** but it's momentum will take it over as it is mostly above the net. • The **model** says that the ball will clear the net by 2cm which may be smaller than the balls **diameter** • The **net will not be a straight line/taut** so will not be 0.9m high, so the ball **will** have enough clearance to **pass** over the net.	M1 A1	3.2b 2.2b
		(2)	
			(13 marks)

Notes:
Accept any alternative vector notations throughout. (a) **M1**: Attempts to solve the quadratic from equating the **k** component to zero. **A1**: Correct value, must select positive root, so accept 1.4 oe. Correct answer only M1 A1 (b) **B1ft**: For $(2.56, 15, -2)$ o.e or follow through $(9 - 4.6\times'\lambda', 15, 0.8 - 2\times'\lambda')$ for their λ. (c) **M1**: Recognises the angle between the perpendicular and direction vector is needed, and identifies the perpendicular as a**k** for any non-zero a (including 1), and attempts dot product

Alternatively recognises the dot product of $(2.56, 15, -2)$ and $(2.56, 15, 0)$

M1: Applies the dot product formula $\frac{a \bullet b}{|a||b|}$ correctly between *any* two vectors, but must have dot product and modulus evaluated.

ddM1: Dependent on both previous marks. A correct method to proceed to the required angle, usually $90° - \arccos('-0.130...')$ as shown in scheme but may e.g. use $\sin\theta$ instead of $\cos\theta$ in formula.

Alternatively is using dot product of $(2.56, 15, -2)$ and $(2.56, 15, 0)$ finds arccos(0.991…)

A1: For 7.5° cao

Alternative

M1: Finds the length of the vector in the **ij** plane.

M1: Finds the tan of any angle the

ddM1: Dependent on both previous marks. Finds the required angle

A1: For 7.5° cao

(d)

M1: Attempts to find value of λ that gives zero **j** component.

M1: Uses their value of λ in the equation of the path to find position.

A1: Correct position.

(e)

B1ft: States that 0.920 > 0.9 so according to the model the ball will pass over the net. Follow through on their **k** component and draws an appropriate conclusion. May stay the value of k > 0.92

(f)

M1: There must be some reference to the model to score this mark. See scheme for examples. It is likely to be either the ball is not a particle, or the top of the net is not a straight line. Accept references to the ball crossing a long way from the middle.

Do not accept reasons such as "there may be wind/air resistance" as these are not referencing the given model.

A1: For a reasonable conclusion based on their reference to the model.

For example

The ball is not a particle; therefore, it will not go over the net is M1A0 as not explained why – needs reference to radius/diameter

Q3.

Question	Scheme	Marks	AOs
(a)	Any two of: $\begin{cases} \pm k\overrightarrow{AB} = \pm k(5\mathbf{i}+25\mathbf{j}+5\mathbf{k}), \\ \pm k\overrightarrow{AC} = \pm k(-15\mathbf{i}+15\mathbf{j}-10\mathbf{k}), \\ \pm k\overrightarrow{BC} = \pm k(-20\mathbf{i}-10\mathbf{j}-15\mathbf{k}) \end{cases}$	M1	3.3
	Let normal vector be $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ $(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\bullet(\mathbf{i}+5\mathbf{j}+\mathbf{k})=0,\ (a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\bullet(-3\mathbf{i}+3\mathbf{j}-2\mathbf{k})=0$ $\Rightarrow a+5b+c=0,\ -3a+3b-2c=0 \Rightarrow a=...,\ b=...,\ c=...$ Alternative: cross product $\begin{vmatrix} 1 & 5 & 1 \\ -3 & 3 & -2 \end{vmatrix} = (-10-3)\mathbf{i}-(-2+3)\mathbf{j}+(3+15)\mathbf{k}$	M1	1.1b
	$\mathbf{n} = k(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})$	A1	1.1b
	$(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})\bullet(10\mathbf{i}+5\mathbf{j}-50\mathbf{k})=...$	M1	1.1b
	$\mathbf{r}\bullet(13\mathbf{i}+\mathbf{j}-18\mathbf{k})=1035$ o.e. $\mathbf{r}\bullet(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})=-1035$ $\mathbf{r}\bullet(325\mathbf{i}+25\mathbf{j}-450\mathbf{k})=25875$	A1	2.5
		(5)	
(b)	Attempts the scalar product between their normal vector and the vector **k** and uses trigonometry to find an angle	M1	3.1b
	$(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})\bullet\mathbf{k} = -18 = \sqrt{13^2+1^2+18^2}\cos\alpha$	M1	1.1b
	$\cos\alpha = \frac{-18}{\sqrt{494}} \Rightarrow \alpha = 144.08... \Rightarrow \theta = 36°$	A1	3.2a
		(3)	
(c)	Distance required is $\lvert\lambda\rvert$ where $\begin{pmatrix} 13 \\ 1 \\ -18 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ 12 \\ \lambda \end{pmatrix} = 1035$	M1	3.4
	$\lvert\lambda\rvert = 53.2\text{m}$	A1	1.1b
		(2)	

(d)	E.g. • The mineral layer will **not** be perfectly **flat/smooth** and will not form a plane • The mineral layer will have a **depth** and this should be taken into account	B1	3.5b
		(1)	
			(11 marks)

Notes

(a)
M1: Attempts to find at least 2 vectors in the plane that can be used to set up the model. Two correct value implies the correct method if not explicitly seen.
M1: Attempts a normal vector using an appropriate method. E.g. as in main scheme or may use vector product
A1: A correct normal vector
M1: Applies **r.n** = *d* with their normal vector and a point in the plane to find a value for *d*
A1: Correct equation (allow any multiple)
(b)
M1: Realises the scalar product between their from part (a) and a vector parallel to **k** and so applies it and uses trigonometry to find an angle
M1: Forms the scalar product between their from part (a) and a vector parallel to **k**
A1: Correct angle
(c)
M1: Uses the model and a correct strategy to establish the distance from (5, 12, 0) to the plane vertically downwards
A1: Correct distance
(d)
B1: Any reasonable limitation – see scheme

Q4.

Question	Scheme	Marks	AOs
(a)	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\left\{ \overrightarrow{OF} \cdot \overrightarrow{AB} = 0 \Rightarrow \right\} \begin{pmatrix} -3+12\lambda \\ 1+3\lambda \\ -7+18\lambda \end{pmatrix} \cdot \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = 0$	M1	1.1b
	$\Rightarrow -36 + 144\lambda + 3 + 9\lambda - 126 + 324\lambda = 0 \Rightarrow 477\lambda - 159 = 0$		
	$\Rightarrow \lambda = \frac{1}{3}$	A1	1.1b
	$\left\{ \overrightarrow{OF} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and minimum distance $= \sqrt{(1)^2 + (2)^2 + (-1)^2}$	M1	3.1a
	$= \sqrt{6}$ or $2.449\ldots$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F.	A1ft	3.2a
		(7)	
(b)	E.g. Fish F may not swim in an exact straight line from A to B. Fish F may hit an obstacle whilst swimming from A to B. Fish F may deviate his path to avoid being caught by the octopus.	B1	3.5b
		(1)	
(c)	E.g. Octopus is effectively modelled as a particle – so we may need to look at where the octopus's mass is distributed. Octopus may during the fish F's motion move away from its fixed location at O.	B1	3.5b
		(1)	
			(9 marks)

Question	Scheme	Marks	AOs
(a) ALT 1	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OA} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \Rightarrow \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\cos\theta \left\{ = \dfrac{\overrightarrow{OA} \bullet \overrightarrow{AB}}{\lvert\overrightarrow{OA}\rvert . \lvert\overrightarrow{AB}\rvert} \right\} = \dfrac{\pm\left(\begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right)}{\sqrt{(-3)^2 + (1)^2 + (-7)^2} . \sqrt{(12)^2 + (3)^2 + (18)^2}}$	dM1	1.1b
	$\left\{ \cos\theta = \dfrac{-36 + 3 - 126}{\sqrt{59}.\sqrt{477}} = \dfrac{-159}{\sqrt{59}.\sqrt{477}} \right\}$		
	$\theta = 161.4038029...$ or $18.59619709...$ or $\sin\theta = 0.3188964021...$	A1	1.1b
	minimum distance $= \sqrt{(-3)^2 + (1)^2 + (-7)^2} \sin(18.59619709...)$	dM1	3.1a
	$= \sqrt{6}$ or $2.449...$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F.	A1ft	3.2a
		(7)	

Question	Scheme	Marks	AOs
(a) ALT 2	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\lvert\overrightarrow{OF}\rvert^2 = (-3 + 12\lambda)^2 + (1 + 3\lambda)^2 + (-7 + 18\lambda)^2$	dM1	1.1b
	$= 9 - 72\lambda + 144\lambda^2 + 1 + 6\lambda + 9\lambda^2 + 49 - 252\lambda + 324\lambda^2$		
	$= 477\lambda^2 - 318\lambda + 59$	A1	1.1b
	$= 53(3\lambda - 1)^2 + 6$	dM1	3.1a
	minimum distance $= \sqrt{6}$ or $2.449...$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F.	A1ft	3.2a
		(7)	

Online Maths

		Question Notes
(a)	M1	Attempts to find $\overrightarrow{OB}-\overrightarrow{OA}$ or $\overrightarrow{OA}-\overrightarrow{OB}$ or the direction vector **d**.
	M1	Applies $\overrightarrow{OA}+\lambda(\text{their } \overrightarrow{AB} \text{ or their } \overrightarrow{BA} \text{ or their } \mathbf{d})$ or equivalent.
	M1	Depends on previous M mark. Writes down (their $\overrightarrow{OF}$ which is in terms of λ)$\bullet$(their $\overrightarrow{AB}$) $=0$. Can be implied.
	A1	Lambda is correct. e.g. $\lambda=\frac{1}{3}$ for $\overrightarrow{AB}=\begin{pmatrix}12\\3\\18\end{pmatrix}$ or $\lambda=1$ for $\mathbf{d}=\begin{pmatrix}4\\1\\6\end{pmatrix}$
	M1	Depends on previous M mark. Complete method for finding $\left\lvert\overrightarrow{OF}\right\rvert$.
	A1	$\sqrt{6}$ or awrt 2.4
	A1ft	Correct follow through conclusion, which is in context with the question.
(a) ALT 1	M1	Attempts to find $\overrightarrow{OB}-\overrightarrow{OA}$ or $\overrightarrow{OA}-\overrightarrow{OB}$ or the direction vector **d**.
	M1	Realisation that the dot product is required between $\overrightarrow{OA}$ and their $\overrightarrow{AB}$. (o.e.)
	M1	Depends on previous M mark. Applies dot product formula between $\overrightarrow{OA}$ and their $\overrightarrow{AB}$. (o.e.)
	A1	$\theta=$ awrt 161.4 or awrt 18.6 or $\sin\theta=$ awrt 0.319
	M1	Depends on previous M mark. (their OA)$\sin$(their θ)
	A1	$\sqrt{6}$ or awrt 2.4
	A1ft	Correct follow through conclusion, which is in context with the question.
(a) ALT 2	M1	Attempts to find $\overrightarrow{OB}-\overrightarrow{OA}$ or $\overrightarrow{OA}-\overrightarrow{OB}$ or the direction vector **d**.
	M1	Applies $\overrightarrow{OA}+\lambda(\text{their } \overrightarrow{AB} \text{ or their } \overrightarrow{BA} \text{ or their } \mathbf{d})$ or equivalent.
	M1	Depends on previous M mark. Applies Pythagoras by finding $\left\lvert\overrightarrow{OF}\right\rvert^2$, o.e.
	A1	$\left\lvert\overrightarrow{OF}\right\rvert^2=477\lambda^2-318\lambda+59$
	M1	Depends on previous M mark. Method of completing the square or differentiating their $\left\lvert\overrightarrow{OF}\right\rvert^2$ w.r.t. λ.
	A1	$\sqrt{6}$ or awrt 2.4
	A1ft	Correct follow through conclusion, which is in context with the question.
(b)	B1	An acceptable criticism for fish F, which is in context with the question.
(c)	B1	An acceptable criticism for the octopus, which is in context with the question.

Q5.

Question	Scheme	Marks	AOs
(a)	Attempts the scalar product between the direction of W and the normal to the road and uses trigonometry to find an angle.	M1	3.1a
	$\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=-9$ or $\left(\begin{pmatrix}-1\\-1\\-3\end{pmatrix}-\begin{pmatrix}1\\2\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=9$	M1 A1	1.1b 1.1b
	$\sqrt{(2)^2+(3)^3+(0)^2}\sqrt{(3)^2+(-5)^3+(-18)^2}\cos\alpha="-9"$ $\theta=90-\arccos\left(\frac{9}{\sqrt{13}\sqrt{358}}\right)$ or $\theta=\arcsin\left(\frac{9}{\sqrt{13}\sqrt{358}}\right)$ Angle between pipe and road $=7.58°$ (3sf) or 0.132 radians (3sf) (Allow $-7.58°$ or -0.132 radians)	M1 A1	1.1b 3.2a
		(5)	

(b)	$W:\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}$ or $\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}$	B1ft	1.1b
	$C \text{ to } W:\left\{\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$ or $\left\{\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$	M1	3.4
	$\begin{pmatrix}2t\\3t+1\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow t=\ldots$ or $\begin{pmatrix}2+2\lambda\\4+3\lambda\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow\lambda=\ldots$ or $(2t)^2+(3t+1)^2+(-3)^2=\ldots$ or $(2+2t)^2+(4+3t)^2+(-3)^2=\ldots$	M1	3.1b
	$t=-\frac{3}{13}$ or $\lambda=-\frac{16}{13}\Rightarrow(C\text{ to }W)_{\min}$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$ or $(2t)^2+(3t+1)^2+(-3)^2=13\left(t+\frac{3}{13}\right)^2+\frac{121}{13}$ or $(2+2t)^2+(4+3t)^2+(-3)^2=13\left(\lambda+\frac{16}{13}\right)^2+\frac{121}{13}$ or $\frac{d\left((2t)^2+(3t+1)^2+(-3)^2\right)}{dt}=0\Rightarrow t=-\frac{3}{13}\Rightarrow C\text{ to }W$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$ Or $\frac{d\left((2+2t)^2+(4+3t)^2+(-3)^2\right)}{dt}=0\Rightarrow t=-\frac{16}{13}\Rightarrow(C\text{ to }W)_{\min}$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$	A1	1.1b
	$d=\sqrt{\left(-\frac{6}{13}\right)^2+\left(\frac{4}{13}\right)^2+(-3)^2}$ or $d=\sqrt{\frac{121}{13}}$	ddM1	1.1b

	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	
			(11 marks)

Notes

(a)

M1: Realises the scalar product between the direction of W and the normal to the road is needed and so applies it and uses trigonometry to find an angle

M1: Calculates the scalar product between $\pm\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)$ and $\pm\begin{pmatrix}3\\-5\\-18\end{pmatrix}$ (Allow sign slips as long as the intention is clear)

A1: $\begin{pmatrix}2\\3\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=-9$ or $\begin{pmatrix}-2\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=9$ or $\begin{pmatrix}2\\3\\0\end{pmatrix}\bullet\begin{pmatrix}-3\\5\\18\end{pmatrix}=9$ or $\begin{pmatrix}-2\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}-3\\5\\18\end{pmatrix}=-9$

M1: A fully complete and correct method for obtaining the acute angle

A1: Awrt 7.58° or awrt 0.132 radians **(must see units)**. Do not isw and withhold this mark if extra answers are given.

(b)

B1ft: Forms the correct parametric form for the pipe W. Follow through their direction vector for W from part (a).

M1: Identifies the need to and forms a vector connecting C to W using a parametric form for W

M1: Uses the model to form the scalar product of C to W and the direction of W to find the value of their parameter **or** finds the distance C to W or $(C \text{ to } W)^2$ in terms of their parameter

A1: Correct vector or correct completion of the square

ddM1: Correct use of Pythagoras on their vector CW or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.**

A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m

Alternatives for part (b):

(b) Way 2	$\mathbf{AC}=\begin{pmatrix}0\\1\\-3\end{pmatrix},\quad \mathbf{AB}=\begin{pmatrix}2\\3\\0\end{pmatrix}$	B1ft	1.1b
	$\mathbf{AC.AB}=\begin{pmatrix}0\\1\\-3\end{pmatrix}.\begin{pmatrix}2\\3\\0\end{pmatrix}=3$	M1	3.4
	$\Rightarrow \cos CAB=\frac{3}{\sqrt{10}\sqrt{13}}\Rightarrow CAB=\ldots$	M1	3.1b
	$CAB=74.74\ldots°$	A1	1.1b
	$d=\sqrt{10}\sin 74.74\ldots°$	ddM1	1.1b
	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	

	Notes		
	(b) B1ft: Forms the correct vectors. Follow through their direction vector for W from part (a). M1: Identifies the need to and forms the scalar product between **AC** and **AB** M1: Uses the model to form the scalar product and uses this to find the angle CAB A1: Correct angle ddM1: Correct method using their values or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.** A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m		

(b) Way 3	$\mathbf{AC} = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad \mathbf{AB} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$	B1ft	1.1b
	$\mathbf{AC} \times \mathbf{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 3 \\ 2 & 3 & 0 \end{vmatrix} = \begin{pmatrix} -9 \\ 6 \\ 2 \end{pmatrix}$	M1	3.4
	$\lvert \mathbf{AC} \times \mathbf{AB} \rvert = \sqrt{9^2 + 6^2 + 2^2} = \ldots$	M1	3.1b
	$= 11$	A1	1.1b
	$d = \dfrac{11}{\lvert \mathbf{AB} \rvert} = \dfrac{11}{\sqrt{2^2 + 3^2}} = \ldots$	ddM1	1.1b
	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	
	Notes		
	(b) B1ft: Forms the correct vectors. Follow through their direction vector for W from part (a). M1: Identifies the need to and forms the vector product between **AC** and **AB** M1: Uses the model to find the magnitude of their vector product A1: Correct value ddM1: Correct method using their values or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.** A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m		

Q6.

Question	Scheme	Marks	AOs
(a)	$\begin{pmatrix}3\\0\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=3\{+0\}-3$	M1	1.1b
	$=0$ therefore the lines are **perpendicular.**	A1	2.4
		(2)	
(b)	$\mathbf{r}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=\begin{pmatrix}-2\\2\\0\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=\ldots\{2\}$	M1	1.1b
	$x+2y-3z=2$ o.e.	A1	2.5
		(2)	
(c)	$3+2(1)-3(1)=2$ (therefore lies on the plane)	B1	1.1b
		(1)	

(d)	$\begin{pmatrix}p\\q\\r\end{pmatrix}=\begin{pmatrix}2\\3\\2\end{pmatrix}+\mu\begin{pmatrix}1\\2\\-3\end{pmatrix}$ **or** $\begin{pmatrix}p\\q\\r\end{pmatrix}+\mu\begin{pmatrix}1\\2\\-3\end{pmatrix}=\begin{pmatrix}2\\3\\2\end{pmatrix}$ leading to $p=2-\mu$, $q=3-2\mu$, $r=2+3\mu$	M1	3.1a
	$(p-3)^2+(q-1)^2+(r-1)^2=(2\sqrt{5})^2$ $((2+\mu)-3)^2+((3+2\mu)-1)^2+((2-3\mu)-1)^2=(2\sqrt{5})^2$ $(-1+\mu)^2+(2+2\mu)^2+(1-3\mu)^2=20$ **or** $(p-3)^2+(q-1)^2+(r-1)^2=(2\sqrt{5})^2$ $((2-\mu)-3)^2+((3-2\mu)-1)^2+((2+3\mu)-1)^2=(2\sqrt{5})^2$ $(-1-\mu)^2+(2-2\mu)^2+(1+3\mu)^2=20$	M1	3.1a
	$14\mu^2-14=0$ o.e	A1	1.1b
	Solves their quadratic $\{\mu=-1$ or $\mu=1\}$	M1	1.1b
	Uses $\mu=-1$ Using $\mu=1$	ddM1	1.1b

	$p=2+(-1)=...$ or $p=2-(1)=...$ $q=3+2(-1)=...$ or $q=3-2(1)=...$ $r=2-3(-1)=...$ or $r=2+3(1)=...$		
	$(1,1,5)$ only	A1	3.2a
		(6)	
	Alternative $\lvert AX\rvert=\sqrt{(3-2)^2+(1-3)^2+(1-2)^2}=\sqrt{6}$	M1	3.1a
	Correctly uses Pythagoras to find the length of XB $\lvert XB\rvert=\sqrt{(2\sqrt{5})^2-6}=\sqrt{14}$	M1	3.1a
	Find the magnitude of the direction vector and compares to the length of XB to find a value for μ	M1	1.1b
	$\mu=-1$ or $\mu=1$	A1	1.1b
	Uses $\mu=-1$ / Using $\mu=1$ $p=2+(-1)=...$ or $p=2-(1)=...$ $q=3+2(-1)=...$ or $q=3-2(1)=...$ $r=2-3(-1)=...$ or $r=2+3(1)=...$	ddM1	1.1b
	$(1,1,5)$ only	A1	3.2a
		(6)	
			(11 marks)

Notes:
(a) **M1:** Applies the dot product to the direction vectors. Minimum requirement is 3 – 3 **A1:** Shows that the dot product = 0 and concludes that the lines are **perpendicular**.
(b) **M1:** Applies $\mathbf{r}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=\begin{pmatrix}-2\\2\\0\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=...$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=\begin{pmatrix}-2\\2\\0\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=...$ **A1:** Correct Cartesian equation $x+2y-3z=2$ o.e. Note: $\mathbf{i}+2\mathbf{j}-3\mathbf{k}=2$ is M1A0
(c) **B1:** See scheme, no conclusion required
(d) **M1:** Uses the point of intersection to find the coordinates of B as functions of a parameter **M1:** Uses the distance between the point A and the point B to form an equation for their parameter only. **A1:** Correct simplified quadratic equation

M1: Solves their quadratic equation to find a value for μ

ddM1: Dependent on the first two method marks. Uses any one of their values for their parameter to find the coordinates of *B*, it need not be the correct one.

A1: Correct coordinates for *B*, condone as a vector, if seen (3, 5, -1) must be disregarded

Alternative

M1: Finds the length *AX*

M1: Uses Pythagoras to find the length of *XB*

NOTE the change in order of the M1 and A1

M1: Find the length of the direction vector and compares to find a value for μ

A1: A correct values for μ

ddM1: Dependent on the first two method marks. Uses any one of their values for their parameter to find the coordinates of *B*, it need not be the correct one.

A1: Correct coordinates for *B*, condone as a vector, if seen (3, 5, -1) must be disregarded

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Q7.

Question	Scheme	Marks	AOs
	$(\mathbf{r}=)\begin{pmatrix}-2+\lambda\\5-\lambda\\4-3\lambda\end{pmatrix}$ or $\begin{pmatrix}-2\\5\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\-3\end{pmatrix}$ (oe)	M1	1.1b
	So meet if $\begin{pmatrix}-2+\lambda\\5-\lambda\\4-3\lambda\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\1\end{pmatrix}=-7\Rightarrow(-2+\lambda)\times1+(5-\lambda)\times-2+(4-3\lambda)\times1=-7$	M1 A1	3.1a 1.1b
	$\Rightarrow 0\lambda-8=-7\Rightarrow -8=-7$ a contradiction so no intersection	A1ft	2.3
	Hence l is parallel to Π but not in it.	A1cso	3.2a
		(5)	
	(5 marks)		

Notes				
	M1	Forms a parametric form for the line. Allow one slip.		
	M1	Substitutes into the equation of the plane to an equation in λ. May use Cartesian form of plane to substitute into.		
	A1	Correct equation in λ		
	A1ft	Simplifies and derives a contradiction and deduces line and plane do not meet. Follow through in their initial equation in λ so - contradiction so no intersection if λ disappears and constants unequal - line lies in plane if a tautology is arrived at - meet in a point if a solution for λ is found. **But do not allow for incorrect simplification from a correct initial equation in λ** Note that a miscopy/misread of 7 instead of -7 can therefore score a maximum of M1M1A0A1A0.		
	A1cso	Correct deduction from correct working. This may be seen two separate statements in their working. You may see attempts at showing the line is parallel before/after deducing there is no intersection.		
Alt 1	Note that some may attempt a mix of the main scheme and Alt 1. Mark under main scheme unless Alt 1 would score higher.			
	$\begin{pmatrix}1\\-1\\-3\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\1\end{pmatrix}=1\times1+(-1)\times(-2)+(-3)\times1=0$		M1	3.1a
	Hence l is parallel to Π		A1	1.1b
	$(-2,5,4)$ on l, but $(1)(-2)+(-2)(5)+1(4)=-8$		M1	1.1b
	$-8\neq-7$ so $(-2,5,4)$ is not on the plane.		A1ft	2.3
	Hence l is (parallel to Π but) not in the plane.		A1cso	3.2a
			(5)	
	(5 marks)			

Alt 1 Notes		
	M1	Attempts the dot product between the two direction vectors.
	A1	Shows dot product is zero and makes the correct deduction that line is parallel to plane.
	M1	Finds a point on l and substitutes into the equation of Π (vector or Cartesian)
	A1ft	Simplifies and derives a contradiction – follow through their equation, so if arrive at a tautology, they should deduce the line is in the plane.
	A1cso	Correct deduction from correct working but may be split across working.

Question	Scheme	Marks	AOs
Alt 2	Attempts to solve $\frac{x+2}{1}=\frac{y-5}{-1}=\frac{z-4}{-3}$ and $x-2y+z=-7$ simultaneously – eliminates one variable for M mark.	M1	3.1a
	e.g. $y=-(x+2)+5=-x+3 \Rightarrow x-2(-x+3)+z=-7 \Rightarrow 3x+z=-1$ (oe)	A1	1.1b
	Solves reduced equations, e.g. $-3(x+2)=z-4 \Rightarrow 3x+z=-2$ and $3x+z=-1 \Rightarrow (3x+z)-(3x+z)=-2-(-1)$	M1	1.1b
	$\Rightarrow 0=-1$ a contradiction so no intersection	A1ft	2.3
	Hence l is parallel to Π but not in it.	A1cso	3.2a
		(5)	
	(5 marks)		

Alt 2 notes		
	M1	Attempts to solve the Cartesian equation of the line and plane, using the plane equation to eliminate one variable for the M.
	A1	Correct elimination of their chosen variable. (E.g may see $3-3y+z=-7$ or $-2x-2y-2=-7$ etc)
	M1	Solves the reduced equations in two variables…
	A1ft	… and derives a contradiction/line and plane do not meet. Follow through their result, so may reach a tautology and deduce lies in plane, or find single solution and deduce meet in a point.
	A1cso	Correct deduction from correct working.

Q8.

Question	Scheme	Marks	AOs
(a)	Finds any two vectors $\pm\overrightarrow{LM}, \pm\overrightarrow{LN}$ or $\pm\overrightarrow{MN}$ $\pm\begin{pmatrix}8\\1\\1\end{pmatrix}$ or $\pm\begin{pmatrix}4\\3\\1\end{pmatrix}$ or $\pm\begin{pmatrix}-4\\2\\0\end{pmatrix}$ two out of three values correct is sufficient to imply the correct method	M1	3.3
	Applies the vector equation of the plane formula $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ Where **a** is any coordinate from L, M & N and vectors **b** and **c** come from an attempt at finding any two vectors that lie on the plane.	M1	1.1b
	A correct equation for the plane $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ $\mathbf{a} = \begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ or $\begin{pmatrix}6\\-2\\0\end{pmatrix}$ or $\begin{pmatrix}2\\0\\0\end{pmatrix}$ **b** and **c** are any two vectors from $\pm\begin{pmatrix}8\\1\\1\end{pmatrix}$ or $\pm\begin{pmatrix}4\\3\\1\end{pmatrix}$ or $\pm\begin{pmatrix}-4\\2\\0\end{pmatrix}$	A1	1.1b
		(3)	

(b)(i) (ii)	Applies 'their' $\mathbf{b}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$ **AND** 'their' $\mathbf{c}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$	**Alternative 1** Finds 'their **b**' – 'their **c**' or vice versa and applies the dot product with $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ **AND** one of their **b** or **c**	**Alternative 2** Applies 'their' $\mathbf{b}.\begin{pmatrix}x\\y\\z\end{pmatrix}$ **AND** 'their' $\mathbf{c}.\begin{pmatrix}x\\y\\z\end{pmatrix}$ and solves to find values of x, y and z	**Alternative 3** Applies the dot product between their answer to part (a) and the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$	M1	1.1b
	Show that both dot product(s) = 0 therefore the lawn is **perpendicular**		**Alternative 1** Shows results is **parallel** to $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ therefore the lawn is **perpendicular**	**Alternative 2** Achieves the value 2 and concludes as **a constant** therefore the lawn is **perpendicular**	A1	2.4
	Outside Specification for this paper – using the cross product Finds the cross product between 'their **b**' and 'their **c**' and either				M1	1.1b

	compares with the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ to show parallel or applies the dot product formula with the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ to show parallel		
	Concludes **parallel** therefore the lawn is **perpendicular**	A1	2.4
	Attempts $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}1\\2\\-10\end{pmatrix}=\mathbf{a}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$ where $\mathbf{a}=\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ or $\begin{pmatrix}6\\-3\\0\end{pmatrix}$ or $\begin{pmatrix}2\\0\\0\end{pmatrix}$ Allow $\mathbf{r}.\begin{pmatrix}1\\2\\-10\end{pmatrix}=\mathbf{a}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$ for this mark	M1	1.1b
	$x+2y-10z=2$ or $x+2y-10z-2=0$	A1	1.1b
		(4)	

(c)	Finds the vector $\overrightarrow{PQ}$ or $\overrightarrow{QP}$ and uses it as the direction vector in the formula $\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$ Two out three values correct is sufficient to imply the correct method	M1	3.3
	$\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$ where $\mathbf{a}=\begin{pmatrix}-10\\8\\2\end{pmatrix}$ or $\begin{pmatrix}6\\4\\3\end{pmatrix}$ and $\mathbf{d}=\pm\begin{pmatrix}16\\-4\\1\end{pmatrix}$	A1	1.1b
		(2)	
(d)	For example: The lawn will not be flat The washing line will not be straight	B1	3.5b
		(1)	
(e)	Applies the distance formula $\frac{\lvert(2\times1)+5\times2+(2.75\times-10)-2\rvert}{\sqrt{1^2+2^2+(-10)^2}}$	M1	3.4
	= 1.71 m or 171 cm	A1	2.2b
		(2)	
(f)	Must have an answer to part (e). Compares their answer to part (e) with 1.5 m and makes an appropriate comment about the model that is consistent with their answer to part (e). If their answer to part (e) is close to 1.5 (e.g. 1.4 to 1.6) they must compare and conclude that the model therefore is good If their answer to part (e) is significantly different to 1.5 they must compare and conclude that the model therefore it is not a good model.	B1ft	3.5a
		(1)	
			(13 marks)

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Notes:

(a)

M1: Finds any two vectors $\pm\overrightarrow{LM}, \pm\overrightarrow{LN}$ or $\pm\overrightarrow{MN}$ by subtracting relevant vectors. Two out three values correct is sufficient to imply the correct method

M1: Applies the vector equation of the plane formula $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ where **a** is any point on the plane and the vectors **b** and **c** are any two from their $\pm\overrightarrow{LM}, \pm\overrightarrow{LN}$ or $\pm\overrightarrow{MN}$

A1: Any correct equation for the plane. Must start with **r** =…

(b)(i)

M1: Applies the dot product between their vectors **b AND c** with the vector $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$

A1: Shows both dot products = 0 and concludes that the lawn is **perpendicular** to the vector $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$

(b)(i) Alternative 1

M1: Applies the dot product between their vector **b** – **c AND** one of their vectors **b** or **c** with the vector $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$

A1: Shows both dot products = 0 and concludes that the lawn is **perpendicular** to the vector $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$

(b)(i) Alternative 2

M1: Applies the dot product between their vectors **b** and **c** $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and attempts to find values of x, y and z

A1: Shows results is **parallel** to $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$ therefore the lawn is **perpendicular**

(b)(i) Alternative 3

M1: Applies the dot product between their answer to part (a) and the vector $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$

A1: Achieves the value 2 and concludes as a constant therefore the lawn is **perpendicular**

(b)(i) Outside Specification for this paper – using the cross product

M1: Finds the cross product between 'their **b**' and 'their **c**' and shows parallel to $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$

A1: Concludes **parallel** therefore the **lawn** is **perpendicular**

Online Maths

(b)(ii) **M1**: Applies the formula $\mathbf{r.n}=\mathbf{a.n}$ where $\mathbf{n}=\begin{pmatrix}1\\2\\-10\end{pmatrix}$ and $\mathbf{a}=\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ or $\begin{pmatrix}6\\-2\\0\end{pmatrix}$ or $\begin{pmatrix}2\\0\\0\end{pmatrix}$ **A1**: Correct Cartesian equation of the plane **Note**: If no method is shown then it must be correct to score **M1 A1**, if incorrect scores **M0 A0. Look** at part (i) to see if there is any method as long as it if used in (ii)
(c) **M1**: Finds the vector $\overrightarrow{PQ}$ or $\overrightarrow{QP}$ and uses it as the direction vector in the formula. $\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$. Two out three values correct is sufficient to imply the correct method **A1**: A correct equation including $\mathbf{r}=\ldots$
(d) **B1**: States an acceptable limitation of the model for the lawn or washing line
(e) **M1**: Applies the distance formula using the point (2, 5, 2.75) and the normal vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ **A1**: 1.71 m or 171 cm
(f) **B1ft**: Compares their answer to part (e) with 1.5 and makes an assessment of the model with a reason with no contradictory statements.

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Q9.

Question	Scheme	Marks	AOs
(a)	Direction: $\pm(3\mathbf{i}+4\mathbf{j}-2\mathbf{k}-(-2\mathbf{i}-8\mathbf{j}-3\mathbf{k}))$	M1	1.1b
	e.g. $\mathbf{r}=3\mathbf{i}+4\mathbf{j}-2\mathbf{k}+\lambda(5\mathbf{i}+12\mathbf{j}+\mathbf{k})$ $\mathbf{r}=-2\mathbf{i}-8\mathbf{j}-3\mathbf{k}+\lambda(5\mathbf{i}+12\mathbf{j}+\mathbf{k})$	A1	2.5
		(2)	
(b)	$z=0\Rightarrow -2+\lambda=0\Rightarrow\lambda=2\Rightarrow C=...$	M1	1.1b
	$\lambda=2\Rightarrow C$ is $(13,\ 28,\ 0)$	A1	1.1b
		(2)	
(c)	$(5\mathbf{i}+12\mathbf{j}+\mathbf{k})\cdot(2\mathbf{i}+4\mathbf{j}-2\mathbf{k})=10+48-2$	M1	3.1b
	$56=\sqrt{5^2+12^2+1^2}\sqrt{2^2+4^2+2^2}\cos\theta\Rightarrow\cos\theta=\frac{56}{\sqrt{170}\sqrt{24}}$	M1	1.1b
	$\Rightarrow\theta=$ awrt $28.8°$	A1	1.1b
		(3)	

Question	Scheme	Marks	AOs
(d)	$\mathbf{P}_1-\mathbf{P}_2=3\mathbf{i}+4\mathbf{j}-2\mathbf{k}+\lambda(5\mathbf{i}+12\mathbf{j}+\mathbf{k})-(\mathbf{i}+3\mathbf{j}-\mathbf{k}+\mu(2\mathbf{i}+4\mathbf{j}-2\mathbf{k}))$ or $\mathbf{P}_1-\mathbf{P}_2=-2\mathbf{i}-8\mathbf{j}-3\mathbf{k}+\lambda(5\mathbf{i}+12\mathbf{j}+\mathbf{k})-(\mathbf{i}+3\mathbf{j}-\mathbf{k}+\mu(2\mathbf{i}+4\mathbf{j}-2\mathbf{k}))$	M1	3.4
	$((5\lambda-2\mu+2)\mathbf{i}+(12\lambda-4\mu+1)\mathbf{j}+(\lambda+2\mu-1)\mathbf{k})\cdot(5\mathbf{i}+12\mathbf{j}+\mathbf{k})=0$ $\begin{pmatrix}2+5\lambda-2\mu\\1+12\lambda-4\mu\\-1+\lambda+2\mu\end{pmatrix}\bullet\begin{pmatrix}5\\12\\1\end{pmatrix}$ $\Rightarrow 170\lambda-56\mu=-21$ **AND** $((5\lambda-2\mu+2)\mathbf{i}+(12\lambda-4\mu+1)\mathbf{j}+(\lambda+2\mu-1)\mathbf{k})\cdot(2\mathbf{i}+4\mathbf{j}-2\mathbf{k})=0$ $\begin{pmatrix}2+5\lambda-2\mu\\1+12\lambda-4\mu\\-1+\lambda+2\mu\end{pmatrix}\bullet\begin{pmatrix}2\\4\\-2\end{pmatrix}$ $\Rightarrow 56\lambda-24\mu=-10$	M1	3.1b
	$170\lambda-56\mu=-21,\ 56\lambda-24\mu=-10\Rightarrow\lambda=...\left(\frac{7}{118}\right),\ \mu=...\left(\frac{131}{236}\right)$ If using $\mathbf{r}=-2\mathbf{i}-8\mathbf{j}-3\mathbf{k}+\lambda(5\mathbf{i}+12\mathbf{j}+\mathbf{k})$ this leads to parameters of $\lambda=...\left(\frac{125}{118}\right),\ \mu=...\left(\frac{131}{236}\right)$ Either $\mathbf{P}_1-\mathbf{P}_2=...\left(\frac{70}{59}\mathbf{i}-\frac{30}{59}\mathbf{j}+\frac{10}{59}\mathbf{k}\right)$ or $\lvert\mathbf{P}_1-\mathbf{P}_2\rvert=...$	M1	3.4

	$\lvert \mathbf{P}_1 - \mathbf{P}_2 \rvert = \sqrt{\left(\frac{70}{59}\right)^2 + \left(\frac{30}{59}\right)^2 + \left(\frac{10}{59}\right)^2} = \ldots$	dM1	1.1b
	Awrt 1.3{0} m or $\frac{10\sqrt{59}}{59}$ **m units required**	A1	3.2a
		(5)	

	Alternative 1 $\mathbf{P}_1 - \mathbf{P}_2 = 3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k} + \lambda(5\mathbf{i} + 12\mathbf{j} + \mathbf{k}) - (\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}))$	M1	3.4
	Finds $\begin{pmatrix} 5 \\ 12 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} = \ldots\{-28\mathbf{i} + 12\mathbf{j} - 4\mathbf{k}\}$ $\begin{pmatrix} 2+5\lambda-2\mu \\ 1+12\lambda-4\mu \\ -1+\lambda+2\mu \end{pmatrix} = M\begin{pmatrix} -28 \\ 12 \\ -4 \end{pmatrix}$ Or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} 5 \\ 12 \\ 1 \end{pmatrix} = 5x + 12y + z = 0$ And $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} = 2x + 4y - 2z = 0$ Leding to a vector e.g. $\begin{pmatrix} 7 \\ -3 \\ 1 \end{pmatrix}$	M1	3.1b
	$2+5\lambda-2\mu = -28M$ $1+12\lambda-4\mu = 12M \quad \Rightarrow \lambda = \ldots\left(\frac{7}{118}\right), \ \mu = \ldots\left(\frac{131}{236}\right), \left\{M = -\frac{5}{118}\right\}$ $-1+\lambda+2\mu = -4M$ $\mathbf{P}_1 - \mathbf{P}_2 = \ldots\left(\frac{70}{59}\mathbf{i} - \frac{30}{59}\mathbf{j} + \frac{10}{59}\mathbf{k}\right)$	M1	3.4
	$\lvert \mathbf{P}_1 - \mathbf{P}_2 \rvert = \sqrt{\left(\frac{70}{59}\right)^2 + \left(\frac{30}{59}\right)^2 + \left(\frac{10}{59}\right)^2}$	dM1	1.1b
	Awrt 1.3{0} m or $\frac{10\sqrt{59}}{59}$ **m units required**	A1	3.2a
		(5)	
	Alternative 2 outside the spec	M1	3.4

	$\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 12 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$ $\begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$		
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 12 & 1 \\ 2 & 4 & -2 \end{vmatrix} = \mathbf{i}(-24-4) - \mathbf{j}(-10-2) + \mathbf{k}(20-24)$ $= -28\mathbf{i} + 12\mathbf{j} - 4\mathbf{k}$ And $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} -28 \\ 12 \\ -4 \end{pmatrix} = -56 + 12 + 4 = \ldots\{-40\}$	M1	3.4
	$\lvert -28\mathbf{i} + 12\mathbf{j} - 4\mathbf{k} \rvert = \sqrt{(-28)^2 + 12^2 + (-4)^2} = \ldots\{\sqrt{944}\}$	dM1	3.1b
	$\lvert P_1 - P_2 \rvert = \left\lvert \frac{-40}{\sqrt{944}} \right\rvert$	M1	1.1b
	Awrt 1.3{0} **m** or $\frac{10\sqrt{59}}{59}$ **m units required**	A1	3.2a
		(5)	
			(12 marks)

Notes
(a) M1: Subtracts the given coordinates either way round, 2 correct values implies method, and uses as their direction vector. A1: For a correct equation. Allow any equivalent correct equations but must use the correct notation, starts with **r** = …, can be using column vectors (b) M1: Uses $z = 0$ in their P_1 equation to find a value for their parameter and uses this to find the other coordinates. A1: Deduces the correct coordinates, condone written as a vector. (c) M1: Realises the scalar product between their direction from part (a) and the direction vector extracted from the P_2 equation is required and calculates its value M1: Completes the method and at least reaches a value for cosine of the angle. Must be attempting to use the direction vectors A1: Correct acute angle (d) M1: Uses the model to form a general vector connecting the two pipes, condone sign slips if the intention is clear. Condone use of the same parameter for this mark. They will be unable to score any more marks M1: Recognises that the scalar product between the general vector **and** the directions of the lines = 0 and uses this to form 2 simultaneous equations in terms of their parameters. Follow through on their line from part (a)

M1: Attempts to solve the simultaneous equations, writing a value for each parameter is sufficient.
Then uses the values of their parameters in the model and **either**

- shows the shortest vector connecting the 2 lines
- proceeds to a distance with no incorrect working seen.

dM1: Uses their values of their parameters to find the vector which if not correct must be stated and then finds the modulus of their shortest vector to find the shortest distance. Maybe implied by a correct answer. Dependent on previous method mark.
A1: Awrt 1.3(0) **m**
Alternative 1
M1: Uses the model to form a general vector connecting the two pipes, condone sign slips if the intention is clear.
M1: Finds the normal vector to the direction vectors by any method and sets the general equation of the line equal to a multiple of the normal vector.
M1: Attempts to solve the simultaneous equations, writing a value for each parameter is sufficient.
Then uses the values of their parameters in the model and **either**

- shows the shortest vector connecting the 2 lines
- proceeds to a distance with no incorrect working seen.

dM1: Uses their values of their parameters to find the vector which if not correct must be stated and then finds the modulus of their shortest vector to find the shortest distance. Maybe implied by a correct answer. Dependent on previous method mark.
A1: Awrt 1.3(0) **m**

Alternative 2 outside spec
M1: Using their equations to find the vector between the coordinates $(\mathbf{a}-\mathbf{c})$
M1: Find the cross product of their directions $(\mathbf{b}\times\mathbf{d})$ and finds the dot product of their $(\mathbf{a}-\mathbf{c})\bullet(\mathbf{b}\times\mathbf{d})$
M1: Finds their $|\mathbf{b}\times\mathbf{d}|$
M1: Uses $\left|\frac{(\mathbf{a}-\mathbf{c})\bullet(\mathbf{b}\times\mathbf{d})}{|\mathbf{b}\times\mathbf{d}|}\right|$
A1: Awrt 1.3(0) **m.**

Q10.

Question	Scheme	Marks	AOs
(a)	**Note:** Allow alternative vector forms throughout, e.g row vectors, **i**, **j**, **k** notation $\mathbf{b} = \pm\left[\begin{pmatrix}300\\300\\-50\end{pmatrix} - \begin{pmatrix}-300\\400\\-150\end{pmatrix}\right] = \pm\begin{pmatrix}600\\-100\\100\end{pmatrix}$	M1	1.1b
	So $\mathbf{r} = \begin{pmatrix}-300\\400\\-150\end{pmatrix} + \lambda\begin{pmatrix}600\\-100\\100\end{pmatrix}$ oe $\left(\text{e.g. } \mathbf{r} = \begin{pmatrix}300\\300\\-50\end{pmatrix} + \lambda\begin{pmatrix}6\\-1\\1\end{pmatrix}\right)$	A1	2.5
		(2)	
(b)(i)	$k = 200$	B1	2.2a
	If M is the point on mountain, and X a general point on the line then eg. $\overrightarrow{MX} = \begin{pmatrix}-300\\400\\-150\end{pmatrix} + \lambda\begin{pmatrix}600\\-100\\100\end{pmatrix} - \begin{pmatrix}100\\k\\100\end{pmatrix} = \begin{pmatrix}-400+600\lambda\\400-k-100\lambda\\-250+100\lambda\end{pmatrix} = \begin{pmatrix}-400+600\lambda\\200-100\lambda\\-250+100\lambda\end{pmatrix}$ May be in terms of k or with $k = 200$ used.	M1	3.1b
	e.g. $\begin{pmatrix}-400+600\lambda\\200-100\lambda\\-250+100\lambda\end{pmatrix} \bullet \begin{pmatrix}600\\-100\\100\end{pmatrix} = 0 \Rightarrow \lambda = \ldots$	dM1	1.1b
	So e.g. $\overrightarrow{OX} = \begin{pmatrix}-300\\400\\-150\end{pmatrix} + \frac{3}{4}\begin{pmatrix}600\\-100\\100\end{pmatrix} = \ldots$	M1	3.4
	So coordinates of X are $(150, 325, -75)$ Accept as $\begin{pmatrix}150\\325\\-75\end{pmatrix}$	A1	1.1b
		(5)	
(ii)	Length of tunnel is $\sqrt{(150-100)^2 + (325-200)^2 + (-75-100)^2} = \ldots$	M1	1.1b
	Awrt 221m from correct working, so λ must have been correct. (Must include units)	A1	1.1b
		(2)	

(c)	$\left\|\overrightarrow{OP}\right\| = \sqrt{(-300)^2 + 400^2 + (-150)^2} \approx 522$ $\left\|\overrightarrow{OQ}\right\| = \sqrt{300^2 + 300^2 + 50^2} \approx 427$	M1	1.1b
	New tunnel length is signficantly shorter than these values so it is likely that the company will decide to build the accessway. Reason and conclusion needed.	A1ft	2.2b
		(2)	
(d)	E.g. The mountainside is not likely to be flat so a plane may not be a good model. The tunnel and/or pipeline will not have negligible thickness so modelling as lines may not be appropriate. A shortest length tunnel may not be possible, or most practical, as the strata of the rock in the mountain have not been considered by the model.	B1	3.5b
		(1)	
			(12 marks)

Notes

(a)	**M1**	Attempts the direction between positions P and Q. If no method shown, two correct entries imply the method.
	A1	A correct equation in the correct form. Any point on the line may used, and any non-zero multiple of the direction. Must begin **r** = …
(b)		**Note:** mark part (b) as a whole.
(i)	**B1**	Correct value of k deduced.
	M1	Realises the need to find the distance from the point on the mountain to a general point on the line.
	dM1	Takes the dot product with the direction vector of line and sets to zero and proceeds to find a value of λ. If working with k as well, allow for finding either λ in terms of k or k in terms of λ.
	M1	Substitutes their λ into their line equation. (This may not have come from correct work, but the method is for using the line equation here.) May be implied by two out of three correct coordinates for their λ
		Note: May omit this step and substitute λ into $\overrightarrow{MX}$. This gains M0 here, but can gain M1A1 in (ii) for finding the length of $\overrightarrow{MX}$.
	A1	Correct point.
(b)(ii)	**M1**	Uses the distance formula with their point and M, or with their $\overrightarrow{MX}$ from (i). (May be implied by two out of three correct coordinates for their λ)
	A1	Correct distance, including units. Accept awrt 221 m or $25\sqrt{78}$ m
(c)	**M1**	Calculates the two distances OP and OQ.
	A1ft	Makes an appropriate conclusion for their tunnel length, but distances OP and OQ must be correct. A reason and a conclusion is needed. Accept for reason e.g "significantly shorter" or "tunnel is more than 100m less than either existing accessway", as these act as a comparative judgement. But do not accept just "shorter" or just inequalities given with no comparative evidence.
(d)	**B1**	Any appropriate criticism of the model given. The model must be referred to in some way – e.g. criticise the straightness/thickness of line, flatness of plane or lack of taking strata etc of mountain into account (as e.g this means line may not be straight). Note: reference to measurements not being correct is **NOT** a limitation of the model.

For reference Some of the other common equations/values of λ in (b)(i) are:

$$\overrightarrow{MX} = \begin{pmatrix} -300 \\ 400 \\ -150 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 100 \\ 200 \\ 100 \end{pmatrix} = \begin{pmatrix} -400+6\lambda \\ 200-\lambda \\ -250+\lambda \end{pmatrix} \Rightarrow \lambda = 75$$

$$\overrightarrow{MX} = \begin{pmatrix} 300 \\ 300 \\ -50 \end{pmatrix} + \lambda \begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix} - \begin{pmatrix} 100 \\ 200 \\ 100 \end{pmatrix} = \begin{pmatrix} 200+600\lambda \\ 100-100\lambda \\ -150+100\lambda \end{pmatrix} \Rightarrow \lambda = -\frac{1}{4}$$

$$\overrightarrow{MX} = \begin{pmatrix} 300 \\ 300 \\ -50 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 100 \\ 200 \\ 100 \end{pmatrix} = \begin{pmatrix} 200+6\lambda \\ 100-\lambda \\ -150+\lambda \end{pmatrix} \Rightarrow \lambda = -25$$

(If the negative direction vectors are used in any case, the value of λ is just the negative of the above.)

See Appendix for some alternatives to part (b)