Questions

Q1.

A publisher plans to produce three versions of the same book: a paperback, a hardcover, and a deluxe edition.

- Each paperback takes 4 minutes to print and 1 minute to bind
- Each hardcover takes 8 minutes to print and 5 minutes to bind
- Each deluxe edition takes 15 minutes to print and 12 minutes to bind

The printing machine is available for at most 150 hours and the binding machine must be used for at least 60 hours.

The publisher decides to produce

- at least 1600 books in total
- at least three times as many paperbacks as hardcovers

The profit on each paperback sold is £8, the profit on each hardcover sold is £20 and the profit on each deluxe edition sold is £40

Let x, y and z represent the number of paperbacks, hardcovers and deluxe editions produced.

(a) Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

(5)

The publisher decides to solve this linear programming problem by using the two-stage simplex method.

(b) Set up an initial tableau for solving this problem using the two-stage simplex method.

As part of your solution, you must show how

- the constraints have been made into equations by using slack variables, exactly two surplus variables and exactly two artificial variables
- the rows for the two objective functions are formed

(6)

The following tableau is obtained after two iterations of the first stage of the two-stage simplex method.

b.v.	x	y	z	s_1	s_2	s_3	s_4	a_1	a_2	Value
s_1	0	0	0	1	1	3	0	−1	−3	600
z	0	$\frac{4}{11}$	1	0	$-\frac{1}{11}$	$\frac{1}{11}$	0	$\frac{1}{11}$	$-\frac{1}{11}$	$\frac{2000}{11}$
x	1	$\frac{7}{11}$	0	0	$\frac{1}{11}$	$-\frac{12}{11}$	0	$-\frac{1}{11}$	$\frac{12}{11}$	$\frac{15600}{11}$
s_4	0	$\frac{40}{11}$	0	0	$\frac{1}{11}$	$-\frac{12}{11}$	1	$-\frac{1}{11}$	$\frac{12}{11}$	$\frac{15600}{11}$
P	0	$-\frac{4}{11}$	0	0	$-\frac{32}{11}$	$-\frac{56}{11}$	0	$\frac{32}{11}$	$\frac{56}{11}$	$\frac{204800}{11}$
I	0	0	0	0	0	0	0	1	1	0

(c) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the second stage of the two-stage simplex method to obtain a new tableau. Make your method clear by stating the row operations you use.

(5)

After three iterations of the second stage of the two-stage simplex method, the following tableau is obtained.

b.v.	x	y	z	s_1	s_2	s_3	s_4	Value
s_2	0	0	0	1	1	3	0	600
z	0	0	1	$\frac{1}{10}$	0	$\frac{1}{2}$	$-\frac{1}{10}$	100
x	1	0	0	$-\frac{3}{40}$	0	$-\frac{9}{8}$	$-\frac{7}{40}$	1125
y	0	1	0	$-\frac{1}{40}$	0	$-\frac{3}{8}$	$\frac{11}{40}$	375
P	0	0	0	$\frac{29}{10}$	0	$\frac{7}{2}$	$\frac{1}{10}$	20500

Given that the publisher produces the optimal number of each version of the book,

(d) (i) state the maximum profit the publisher can earn,

(ii) find the number of hours the binding machine will be used.

(2)

(e) Give a reason why the publisher may not earn the profit stated in (d)(i).

(1)

(Total for question = 19 marks)

Q2.

A linear programming problem in *x*, *y* and *z* is to be solved using the big-M method.
The initial tableau is shown below.

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	2	3	4	1	0	0	0	0	13
a_1	1	–2	2	0	–1	0	1	0	8
a_2	3	0	–4	0	0	–1	0	1	12
P	$2 - 4M$	$-3 + 2M$	$-1 + 2M$	0	M	M	0	0	$-20M$

(a) Using the information in the above tableau, formulate the linear programming problem. You should

- list each of the constraints as an inequality
- state the two possible objectives

(4)

(b) Obtain the most efficient pivot for a first iteration of the big-M method. You must give reasons for your answer.

(2)

(Total for question = 6 marks)

Q3.

A garden centre makes hanging baskets to sell to its customers. Three types of hanging basket are made, *Sunshine*, *Drama* and *Peaceful*. The plants used are categorised as *Impact*, *Flowering* or *Trailing*.

Each *Sunshine* basket contains 2 *Impact* plants, 4 *Flowering* plants and 3 *Trailing* plants.
Each *Drama* basket contains 3 *Impact* plants, 2 *Flowering* plants and 4 *Trailing* plants.
Each *Peaceful* basket contains 1 *Impact* plant, 3 *Flowering* plants and 2 *Trailing* plants.

The garden centre can use at most 80 *Impact* plants, at most 140 *Flowering* plants and at most 96 *Trailing* plants each day.

The profit on *Sunshine*, *Drama* and *Peaceful* baskets are £12, £20 and £16 respectively.
The garden centre wishes to maximise its profit.

Let x, y and z be the number of *Sunshine*, *Drama* and *Peaceful* baskets respectively, produced each day.

(a) Formulate this situation as a linear programming problem, giving your constraints as inequalities.

(5)

(b) State the further restriction that applies to the values of x, y and z in this context.

(1)

The Simplex algorithm is used to solve this problem. After one iteration, the tableau is

b.v.	x	y	z	r	s	t	Value
r	$-\frac{1}{4}$	0	$-\frac{1}{2}$	1	0	$-\frac{3}{4}$	8
s	$\frac{5}{2}$	0	2	0	1	$-\frac{1}{2}$	92
y	$\frac{3}{4}$	1	$\frac{1}{2}$	0	0	$\frac{1}{4}$	24
P	3	0	-6	0	0	5	480

(c) State the variable that was increased in the first iteration. Justify your answer.

(2)

(d) Determine how many plants in total are being used after only one iteration of the Simplex algorithm

.

(1)

(e) Explain why for a second iteration of the Simplex algorithm the 2 in the *z* column is the pivot value.

(2)

After a second iteration, the tableau is

b.v.	x	y	z	r	s	t	Value
r	$\frac{3}{8}$	0	0	1	$\frac{1}{4}$	$-\frac{7}{8}$	31
z	$\frac{5}{4}$	0	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	46
y	$\frac{1}{8}$	1	0	0	$-\frac{1}{4}$	$\frac{3}{8}$	1
P	$\frac{21}{2}$	0	0	0	3	$\frac{7}{2}$	756

(f) Use algebra to explain why this tableau is optimal.

(1)

(g) State the optimal number of each type of basket that should be made.

(1)

The manager of the garden centre is able to increase the number of *Impact* plants available each day from 80 to 100. She wants to know if this would increase her profit.

(h) Use your final tableau to determine the effect of this increase. (You should not carry out any further calculations.)

(2)

(Total for question = 15 marks)

Q4.

A linear programming problem in x, y and z is described as follows.

Maximise $P = 3x + 2y + 2z$

subject to

$2x + 2y + z \leq 25$
$x + 4y \leq 15$
$x \geq 3$

(a) Explain why the Simplex algorithm cannot be used to solve this linear programming problem.

(1)

The big-M method is to be used to solve this linear programming problem.

(b) Define, in this context, what M represents. You must use correct mathematical language in your answer.

(1)

The initial tableau for a big-M solution to the problem is shown below.

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value
s_1	2	2	1	1	0	0	0	25
s_2	1	4	0	0	1	0	0	15
t_1	1	0	0	0	0	−1	1	3
P	$-(3+M)$	−2	−2	0	0	M	0	$-3M$

(c) Explain clearly how the equation represented in the b.v. t_1 row was derived.

(1)

(d) Show how the equation represented in the b.v. P row was derived.

(2)

The tableau obtained from the first iteration of the big-M method is shown below.

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value
s_1	0	2	1	1	0	2	-2	19
s_2	0	4	0	0	1	1	-1	12
x	1	0	0	0	0	-1	1	3
P	0	-2	-2	0	0	-3	$3+M$	9

(e) Solve the linear programming problem, starting from this second tableau. You must

- give a detailed explanation of your method by clearly stating the row operations you use and
- state the solution by deducing the final values of *P*, *x*, *y* and *z*.

(7)

(Total for question = 12 marks)

Q5.

The tableau below is the initial tableau for a linear programming problem in x, y and z. The objective is to maximise the profit, P.

basic variable	x	y	z	r	s	t	Value
r	12	4	5	1	0	0	246
s	9	6	3	0	1	0	153
t	5	2	-2	0	0	1	171
P	-2	-4	-3	0	0	0	0

Using the information in the tableau, write down

(a) the objective function,

(2)

(b) the three constraints as inequalities with integer coefficients.

(3)

Taking the most negative number in the profit row to indicate the pivot column at each stage,

(c) solve this linear programming problem. Make your method clear by stating the row operations you use.

basic variable	x	y	z	r	s	t	Value
r	12	4	5	1	0	0	246
s	9	6	3	0	1	0	153
t	5	2	-2	0	0	1	171
P	-2	-4	-3	0	0	0	0

(9)

(d) State the final values of the objective function and each variable.

(3)

One of the constraints is not at capacity.

(e) Explain how it can be identified.

(1)

(Total 18 marks)

Q6.

The tableau below is the initial tableau for a three-variable linear programming problem in *x*, *y* and *z*. The objective is to maximise the profit, *P*.

Basic variable	x	y	z	r	s	t	Value
r	15	−2	3	1	0	0	180
s	10	1	1	0	1	0	80
t	1	6	−2	0	0	1	100
P	−1	−2	−5	0	0	0	0

(a) Using the information in the tableau, write down

(i) the objective function,
(ii) the three constraints as inequalities.

(3)

(b) Taking the most negative number in the profit row to indicate the pivot column at each stage, solve this linear programming problem. Make your method clear by stating the row operations you use.

(8)

(c) State the final values of the objective function and each variable.

(2)

(Total for question = 13 marks)

Q7.

A three-variable linear programming problem in *x*, *y* and *z* is to be solved. The objective is to maximise the profit, *P*. The following tableau is obtained after the first iteration.

Basic Variable	x	y	z	r	s	t	Value
r	0	5	2	1	–3	0	10
x	1	2	3	0	1	0	18
t	0	1	–1	0	4	1	3
P	0	3	–4	0	1	0	7

(a) State which variable was increased first, giving a reason for your answer.

(1)

(b) Perform **one** complete iteration of the simplex algorithm, to obtain a new tableau, T. Make your method clear by stating the row operations you use.

(5)

(c) Write down the profit equation given by T.

(1)

(d) State whether T is optimal. You must use your answer to (c) to justify your answer.

(2)

(Total for question = 9 marks)

Q8.

The tableau below is the initial tableau for a linear programming problem in *x*, *y* and *z*. The objective is to maximise the profit, *P*.

Basic variable	x	y	z	r	s	t	Value
r	2	−4	1	1	0	0	15
s	4	2	−8	0	1	0	20
t	1	−1	4	0	0	1	8
P	−3	2	7	0	0	0	0

(a) Perform **one** iteration of the Simplex algorithm to obtain a new tableau, *T*. State the row operations you use.

(5)

(b) Write down the profit equation given by *T* and state the current values of the slack variables.

(2)

(Total for question = 7 marks)

Q9.

The tableau below is the initial tableau for a three-variable linear programming problem in *x*, *y* and *z*. The objective is to maximise the profit, *P*.

Basic Variable	x	y	z	r	s	t	Value
r	4	3	$\frac{5}{2}$	1	0	0	50
s	1	2	1	0	1	0	30
t	0	5	1	0	0	1	80
P	–25	–40	–35	0	0	0	0

(a) Taking the most negative number in the profit row to indicate the pivot column at each stage, perform **two** complete iterations of the simplex algorithm to obtain tableau T. Make your method clear by stating the row operations you use.

(9)

(b) Write down the profit equation given by T.

(1)

(c) Use your answer to (b) to determine whether T is optimal, justifying your answer.

(2)

(Total 12 marks)

Q10.

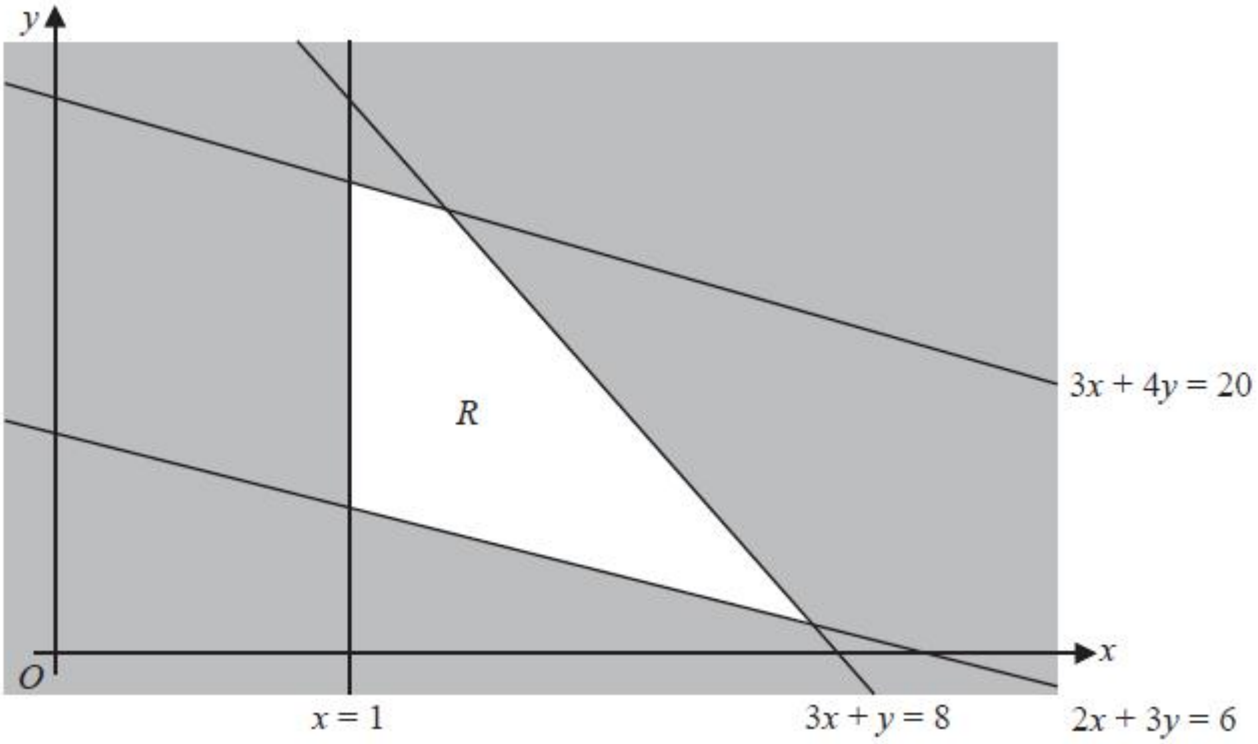

Figure 5

Figure 5 shows the constraints of a linear programming problem in x and y, where R is the feasible region.

The objective is to maximise $P = 11x + ky$, where k is a positive constant.

The optimal value of P is to be found using the big-M method.

(a) Set up an initial tableau for solving this linear programming problem using the big-M method. You should use exactly 2 slack variables, 2 surplus variables and 2 artificial variables.

(7)

After a third iteration of the big-M method, a possible tableau is

b.v.	x	y	s_1	s_2	s_3	s_4	a_1	a_2	Value
s_1	0	0	1	0	$\frac{9}{7}$	$-\frac{1}{7}$	0	$-\frac{9}{7}$	$\frac{78}{7}$
x	1	0	0	0	$\frac{1}{7}$	$\frac{3}{7}$	0	$-\frac{1}{7}$	$\frac{18}{7}$
y	0	1	0	0	$-\frac{3}{7}$	$-\frac{2}{7}$	0	$\frac{3}{7}$	$\frac{2}{7}$
s_2	0	0	0	1	$\frac{1}{7}$	$\frac{3}{7}$	-1	$-\frac{1}{7}$	$\frac{11}{7}$
P	0	0	0	0	$\frac{11}{7}-\frac{3}{7}k$	$\frac{33}{7}-\frac{2}{7}k$	M	$M-\frac{11}{7}+\frac{3}{7}k$	$\frac{198}{7}+\frac{2}{7}k$

(b) Given that the third iteration gives the optimal value for *P*, determine this value.

(5)

(Total for question = 12 marks)

(**Q07 9FM0/03D, June 2025**)

Q11.

Two friends, Anaira and Tommi, play a game involving two positive numbers x and y

Anaira gives Tommi the following clues to see if he can correctly determine the value of x and the value of y

- x is greater than y and the difference between the two is at least 100
- x is at most 5 times as large as y
- the sum of $2x$ and $3y$ is at least 350
- the sum of x and y is as small as possible

Tommi decides to solve this problem by using the big-M method.

(a) Set up an initial tableau for solving this problem using the big-M method.

As part of your solution, you must show

- how the constraints were made into equations using one slack variable, exactly two surplus variables and exactly two artificial variables
- how the objective function was formed

(6)

The big-M method is applied until the tableau containing the optimal solution to the problem is found. One row of this final tableau is as follows.

b.v.	x	y	s_1	s_2	s_3	a_1	a_2	Value
x	1	0	$-\frac{3}{5}$	0	$-\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$	130

(b) (i) State the value of *x*

(ii) Hence deduce the value of *y*, making your reasoning clear.

(3)

(Total for question = 9 marks)

(**Q05 9FM0/03D, June 2024**)

Q12.

A linear programming problem in x, y and z is described as follows.

Maximise $P = 2x + 2y - z$

subject to $3x + y + 2z \leq 30$

$x - y + z \geq 8$

$4y + 2z \geq 15$

$x, y, z \geq 0$

(a) Explain why the Simplex algorithm cannot be used to solve this linear programming problem.

(1)

(b) Set up the initial tableau for solving this linear programming problem using the big-M method.

(7)

After a first iteration of the big-M method, the tableau is

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	3	0	1.5	1	0	0.25	0	−0.25	26.25
a_1	1	0	1.5	0	−1	−0.25	1	0.25	11.75
y	0	1	0.5	0	0	−0.25	0	0.25	3.75
P	$-(2+M)$	0	$2-1.5M$	0	M	$-0.5+0.25M$	0	$0.5+0.75M$	$7.5-11.75M$

(c) State the value of each variable after the first iteration.

(1)

(d) Explain why the solution given by the first iteration is not feasible.

(1)

Taking the most negative entry in the profit row to indicate the pivot column,

(e) obtain the most efficient pivot for a second iteration. You must give reasons for your answer.

(2)

(Total for question = 12 marks)