Mark Scheme

Q1.

Question Number	Scheme	Notes	Marks
	$\mathbf{A}=\begin{pmatrix}-2 & 1 & -3\\ k & 1 & 3\\ 2 & -1 & k\end{pmatrix}$		
	$\det \mathbf{A}=-2(k+3)-(k^2-6)-3(-k-2)\ row1$ or e.g. $\det \mathbf{A}=-k(k-3)+(-2k+6)-3(2-2)\ row2$ $\det \mathbf{A}=2(3+3)+(-6+3k)+k(-2-k)\ row3$ $\det \mathbf{A}=-2(k+3)-k(k-3)+2(3+3)\ col1$ $\det \mathbf{A}=-(k^2-6)+(-2k+6)+(-6+3k)\ col2$ $\det \mathbf{A}=-3(-2-k)-3(2-2)+k(-2-k)\ col3$	M1: Correct attempt at determinant (3 'elements' (may be implied if one is zero) with at least two elements correct). **Note that there are various alternatives depending on the choice of row or column.** A1: Correct determinant **in any form**	M1A1
	Note that e.g. $\det \mathbf{A}=-2\begin{vmatrix}1 & 3\\ -1 & k\end{vmatrix}-\begin{vmatrix}k & 3\\ 2 & k\end{vmatrix}-3\begin{vmatrix}k & 1\\ 2 & -1\end{vmatrix}$ scores no marks until the determinants are 'extracted'.		
	$-2(k+3)-(k^2-6)-3(-k-2)=0\Rightarrow k=\ldots$	Sets their $\det\mathbf{A}=0$ (= 0 may be implied) and attempts to solve a 3 term quadratic (see general guidance) as far as $k=\ldots$ NB Correct quadratic is $k^2-k-6=0$	M1
	$(k+2)(k-3)=0\Rightarrow k=-2,\ 3$	Both values correct	A1
			(4)
			Total 4

(**Q07 6669/01, June 2016**)

Q2.

Question Number	Scheme		Marks
	$\mathbf{A}=\begin{pmatrix}2&-1\\4&3\end{pmatrix}$, $\mathbf{P}=\begin{pmatrix}3&6\\11&-8\end{pmatrix}$		
(a)	$\mathbf{A}^{-1}=\frac{1}{10}\begin{pmatrix}3&1\\-4&2\end{pmatrix}$	Either $\frac{1}{10}$ or $\begin{pmatrix}3&1\\-4&2\end{pmatrix}$	M1
		Correct matrix seen.	A1
			[2]
(b) **Way 1**	$\mathbf{P}=\mathbf{AB}$ $\Rightarrow\ \mathbf{A}^{-1}\mathbf{P}=\mathbf{A}^{-1}\mathbf{AB}\ \Rightarrow\ \mathbf{B}=\mathbf{A}^{-1}\mathbf{P}$		
	$\mathbf{B}\quad=\frac{1}{10}\begin{pmatrix}3&1\\-4&2\end{pmatrix}\begin{pmatrix}3&6\\11&-8\end{pmatrix}$	Multiplies **their** $\mathbf{A}^{-1}$ by **P** in correct order. This substituted statement is sufficient.	M1
	$=\begin{pmatrix}2&1\\1&-4\end{pmatrix}$	At least 2 elements correct or $k\begin{pmatrix}20&10\\10&-40\end{pmatrix}$ oe. May be unsimplified	A1
		Correct simplified matrix.	A1
			[3]
(b) **Way 2**	$\{\mathbf{P}=\mathbf{AB}\Rightarrow\}$ $\begin{pmatrix}3&6\\11&-8\end{pmatrix}=\begin{pmatrix}2&-1\\4&3\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}$		
	$\begin{pmatrix}3&6\\11&-8\end{pmatrix}=\begin{pmatrix}2a-c&2b-d\\4a+3c&4b+3d\end{pmatrix}$	Attempt to multiply **A** by **B** in the correct order and puts equal to **P**	M1
	$\Rightarrow a=2,\ c=1,\ b=1,\ d=-4$		
	So, $\mathbf{B}=\begin{pmatrix}2&1\\1&-4\end{pmatrix}$	At least 2 elements are correct.	A1
		Correct matrix.	A1
			[3] **5**

(**Q03 6667/01, June 2017**)

Q3.

Question Number	Scheme		Marks
(a)	$\mathbf{A}+\mathbf{B}=\begin{pmatrix}1 & 2\\-1 & 1\end{pmatrix}$	M1: Correct attempt at matrix addition with 3 elements correct A1: Correct matrix	M1A1
	$2\mathbf{A}-\mathbf{B}=\begin{pmatrix}5 & 1\\-2 & -1\end{pmatrix}$	M1: Correct attempt to double **A** and subtract **B** 3 elements correct A1: Correct matrix	M1A1
	$(\mathbf{A}+\mathbf{B})(2\mathbf{A}-\mathbf{B})=\begin{pmatrix}1 & 2\\-1 & 1\end{pmatrix}\begin{pmatrix}5 & 1\\-2 & -1\end{pmatrix}$		
	$\begin{pmatrix}1 & 2\\-1 & 1\end{pmatrix}\begin{pmatrix}5 & 1\\-2 & -1\end{pmatrix}=\begin{pmatrix}1 & -1\\-7 & -2\end{pmatrix}$	M1: Correct method to multiply A1: cao	M1A1
			(6)
(a) Way 2	$(\mathbf{A}+\mathbf{B})(2\mathbf{A}-\mathbf{B})=2\mathbf{A}^2+2\mathbf{BA}-\mathbf{AB}-\mathbf{B}^2$	M1: Expands brackets with at least 3 correct terms A1: Correct expansion	M1A1
	$\mathbf{A}^2=\begin{pmatrix}3 & 2\\-2 & -1\end{pmatrix}, \mathbf{BA}=\begin{pmatrix}-3 & -1\\-1 & 0\end{pmatrix},$ $\mathbf{AB}=\begin{pmatrix}-2 & 3\\1 & -1\end{pmatrix}, \mathbf{B}^2=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$	M1: Attempts $\mathbf{A}^2$, $\mathbf{B}^2$ and **AB** or **BA** A1: Correct matrices	M1A1
	$2\mathbf{A}^2+2\mathbf{BA}-\mathbf{AB}-\mathbf{B}^2=\begin{pmatrix}1 & -1\\-7 & -2\end{pmatrix}$	M1: Substitutes into their expansion A1: Correct matrix	M1A1
(b)	$\mathbf{MC}=\mathbf{A}\Rightarrow\mathbf{C}=\mathbf{M}^{-1}\mathbf{A}$	May be implied by later work	B1
	$\mathbf{M}^{-1}=\frac{1}{-2-7}\begin{pmatrix}-2 & 1\\7 & 1\end{pmatrix}$	An attempt at their $\frac{1}{\det \mathbf{M}}\begin{pmatrix}-2 & 1\\7 & 1\end{pmatrix}$	M1
	$\mathbf{C}=\frac{1}{-2-7}\begin{pmatrix}-2 & 1\\7 & 1\end{pmatrix}\begin{pmatrix}2 & 1\\-1 & 0\end{pmatrix}$	Correct order required and an attempt to multiply	dM1
	$\mathbf{C}=-\frac{1}{9}\begin{pmatrix}-5 & -2\\13 & 7\end{pmatrix}$	oe	A1
			(4)
			Total 10
(b) Way 2	$\begin{pmatrix}1 & -1\\-7 & -2\end{pmatrix}\begin{pmatrix}a & b\\c & d\end{pmatrix}=\begin{pmatrix}2 & 1\\-1 & 0\end{pmatrix}$	Correct statement	B1
	$a-c=2, b-d=1$ $-7a-2c=-1, -7b-2d=0$	Multiplies correctly to obtain 4 equations	M1
	$a=\frac{5}{9}, b=\frac{2}{9}, c=-\frac{13}{9}, d=-\frac{7}{9}$	M1: Solves to obtain values for a, b, c and d A1: Correct values	M1A1

(**Q06 6667/01/R, June 2014**)

Q4.

Question Number	Scheme	Notes	Marks
	$\mathbf{A}=\begin{pmatrix}1&2\\3&-1\\4&5\end{pmatrix}, \mathbf{B}=\begin{pmatrix}2&-1&4\\1&3&1\end{pmatrix}$		
(i)(a)	$\begin{pmatrix}1&2\\3&-1\\4&5\end{pmatrix}\begin{pmatrix}2&-1&4\\1&3&1\end{pmatrix}=\begin{pmatrix}4&5&6\\5&-6&11\\13&11&21\end{pmatrix}$	M1: 3x3 matrix with a number or numerical expression for each element A2:cao (-1 each error) Only 1 error award A1A0	M1A2
(b)	$\mathbf{BA}=\begin{pmatrix}2&-1&4\\1&3&1\end{pmatrix}\begin{pmatrix}1&2\\3&-1\\4&5\end{pmatrix}=\begin{pmatrix}15&25\\14&4\end{pmatrix}$	Allow any convincing argument. E.g.s BA is a 2x2 matrix (so $AB \neq BA$) or dimensionally different. Attempt to evaluate product not required. NB 'Not commutative' only is B0	B1
			(4)
(ii)	$(\det \mathbf{C} =)2k\times k - 3\times(-2)$	Correct attempt at determinant	M1
	$\mathbf{C}^{-1}=\frac{1}{2k^2+6}\begin{pmatrix}k&2\\-3&2k\end{pmatrix}$	M1: $\frac{1}{\text{their}\det\mathbf{C}}\begin{pmatrix}k&2\\-3&2k\end{pmatrix}$ A1:cao oe	M1A1
			(3)
			Total 7

(**Q04 6667/01, June 2014**)

Q5.

Question Number	Scheme		Marks
	$\mathbf{A} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix}$, $\mathbf{B} = \mathbf{A} + 3\mathbf{I}$		
(i)(a)	$\mathbf{B} = \mathbf{A} + 3\mathbf{I} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix} + 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	For applying $\mathbf{A} + 3\mathbf{I}$. Can be implied by three out of four correct elements in candidate's final answer. Solution must come from addition.	M1
	$= \begin{pmatrix} 2k+4 & k \\ -3 & -2 \end{pmatrix}$	Correct answer.	A1
			[2]
(b)	**B** is singular $\Rightarrow$ $\det \mathbf{B} = 0$.		
	$-2(2k+4) - (-3k) = 0$	Applies "$ad - bc$" to **B** and equates to 0	M1
	$-4k - 8 + 3k = 0$		
	$k = -8$	$k = -8$	A1cao
			[2]
(ii)	$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 2 & -1 & 5 \end{pmatrix}$, $\mathbf{E} = \mathbf{CD}$		
	$\mathbf{E} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}\begin{pmatrix} 2 & -1 & 5 \end{pmatrix} = \begin{pmatrix} 4 & -2 & 10 \\ -6 & 3 & -15 \\ 8 & -4 & 20 \end{pmatrix}$	Candidate writes down a 3×3 matrix.	M1
		Correct answer.	A1
			[2] 6

(**Q03 6667/01/R, June 2013**)

Q6.

Question Number	Scheme	Notes	Marks
(a)	$\mathbf{A}^2 = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} 44 & -14 \\ -28 & 9 \end{pmatrix}$	M1:Attempt both $\mathbf{A}^2$ and $7\mathbf{A} + 2\mathbf{I}$	M1A1
	$7\mathbf{A} + 2\mathbf{I} = \begin{pmatrix} 42 & -14 \\ -28 & 7 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 44 & -14 \\ -28 & 9 \end{pmatrix}$	A1: Both matrices correct	
	OR $\mathbf{A}^2 - 7\mathbf{A} = \mathbf{A}(\mathbf{A} - 7\mathbf{I})$	M1 for expression and attempt to substitute and multiply (2x2)(2x2)=2x2	
	$\mathbf{A}(\mathbf{A} - 7\mathbf{I}) = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2\mathbf{I}$	A1 cso	
			(2)
(b)	$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I} \Rightarrow \mathbf{A} = 7\mathbf{I} + 2\mathbf{A}^{-1}$	Require one correct line using accurate expressions involving $\mathbf{A}^{-1}$ and identity matrix to be clearly stated as $\mathbf{I}$.	M1
	$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})^*$		A1* cso
	Numerical approach award 0/2.		
			(2)
(c)	$\mathbf{A}^{-1} = \frac{1}{2}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix}$	Correct inverse matrix or equivalent	B1
	$\frac{1}{2}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix}\begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -2k-8+4k+10 \\ -8k-32+12k+30 \end{pmatrix}$	Matrix multiplication involving their inverse and k: (2x2)(2x1)=2x1. N.B. $\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix}$ is M0	M1
	$\begin{pmatrix} k+1 \\ 2k-1 \end{pmatrix}$ or $(k+1,\ 2k-1)$	$(k+1)$ first A1, $(2k-1)$ second A1	A1,A1
	Or:		
	$\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix}$	Correct matrix equation.	B1
	$6x - 2y = 2k + 8$ $-4x + y = -2k - 5 \Rightarrow x = ...or\ y = ...$	Multiply out and attempt to solve simultaneous equations for x or y in terms of k.	M1
	$\begin{pmatrix} k+1 \\ 2k-1 \end{pmatrix}$ or $(k+1,\ 2k-1)$	$(k+1)$ first A1, $(2k-1)$ second A1	A1,A1
			(4)
			Total 8

(**Q06 6667/01, June 2013**)

Q7.

Question Number	Scheme	Notes	Marks
(a)	$\det(\mathbf{A}) = 3\times 0 - 2\times 1 (= -2)$	Correct attempt at the determinant	M1
	$\det(\mathbf{A}) \neq 0$ (so **A** is non singular)	det(A) = -2 **and** some reference to zero	A1
	$\frac{1}{\det(\mathbf{A})}$ **scores M0**		**(2)**
(b)	$\mathbf{BA}^2 = \mathbf{A} \Rightarrow \mathbf{BA} = \mathbf{I} \Rightarrow \mathbf{B} = \mathbf{A}^{-1}$	Recognising that $\mathbf{A}^{-1}$ is required	M1
	$\mathbf{B} = -\frac{1}{2}\begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix}$	At least 3 correct terms in $\begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix}$	M1
		$\frac{1}{\text{their det(A)}}\begin{pmatrix} * & * \\ * & * \end{pmatrix}$	B1ft
		Fully correct answer	A1
	Correct answer only score 4/4		**(4)**
	Ignore poor matrix algebra notation if the intention is clear		**Total 6**
(b) Way 2	$\mathbf{A}^2 = \begin{pmatrix} 2 & 3 \\ 6 & 11 \end{pmatrix}$	Correct matrix	B1
	$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 6 & 11 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \Rightarrow \begin{matrix} 2a+6b=0 \\ 3a+11b=1 \end{matrix}$ *or* $\begin{matrix} 2c+6d=2 \\ 3c+11d=3 \end{matrix}$	2 equations in a and b **or** 2 equations in c and d	M1
	$a = -\frac{3}{2}, b = \frac{1}{2}, c = 1, d = 0$	M1 Solves for a and b or c and d A1 All 4 values correct	M1A1
(b) Way 3	$\mathbf{A}^2 = \begin{pmatrix} 2 & 3 \\ 6 & 11 \end{pmatrix}$	Correct matrix	B1
	$\left(\mathbf{A}^2\right)^{-1} = \frac{1}{\text{"2"}\times\text{"11"} - \text{"3"}\times\text{"6"}}\begin{pmatrix} \text{"11"} & \text{"}-3\text{"} \\ \text{"}-6\text{"} & \text{"2"} \end{pmatrix}$ **see note**	Attempt inverse of $\mathbf{A}^2$	M1
	$\mathbf{A}\left(\mathbf{A}^2\right)^{-1} = \frac{1}{4}\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 11 & -3 \\ -6 & 2 \end{pmatrix}$ *or* $\frac{1}{4}\begin{pmatrix} 11 & -3 \\ -6 & 2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$	Attempts $\mathbf{A}\left(\mathbf{A}^2\right)^{-1}$ *or* $\left(\mathbf{A}^2\right)^{-1}\mathbf{A}$	M1
	$\mathbf{B} = -\frac{1}{2}\begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix}$	Fully correct answer	A1
(b) Way 4	$\mathbf{BA} = \mathbf{I}$	Recognising that $\mathbf{BA} = \mathbf{I}$	B1
	$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{matrix} 2b=1 \\ a+3b=0 \end{matrix}$ *or* $\begin{matrix} 2d=0 \\ c+3d=1 \end{matrix}$	2 equations in a and b **or** 2 equations in c and d	M1
	$a = -\frac{3}{2}, b = \frac{1}{2}, c = 1, d = 0$	M1 Solves for a and b or c and d A1 All 4 values correct	M1A1

Extra Notes

(b) Way 3

Attempting inverse of $\mathbf{A}^2$ needs to be recognisable as an attempt at an inverse

E.g. $(\mathbf{A}^2)^{-1} = \dfrac{1}{Their\ Det(\mathbf{A}^2)}$ (A changed $\mathbf{A}^2$)

(**Q06 6667/01, Jan 2012**)

Online Maths Teaching

Q8.

Question	Scheme	Marks	AOs
(a)	$\begin{vmatrix} k & 3 & -1 \\ 3 & -1 & 1 \\ -16 & -k & -k \end{vmatrix} = k(k+k)-3(-3k+16)-1(-3k-16)$	M1	2.1
	Solves det = 0 $\Rightarrow 2k^2+12k-32=0$ or $k^2+6k-16=0$ To achieve $k=2$ ($k=-8$ must be rejected)	A1	1.1b
		(2)	
	Special case $\begin{vmatrix} 2 & 3 & -1 \\ 3 & -1 & 1 \\ -16 & -2 & -2 \end{vmatrix} = 2(2+2)-3(-3\times2+16)-1(-3\times2-16)$ Shows det = 0, therefore when $k=2$ there is no unique solution	M1 A0	2.1 1.1b

(b)	Eliminates z to achieve **two** equations in x and y e.g. $5x+2y=1$ $-10x-4y=-2$ $20x+8y=4$	Eliminates x to achieve **two** equations in y and z e.g. $11y-5z=13$ $22y-10z=26$ $-22y-10z=-26$	Eliminates y to achieve **two** equations in x and z e.g. $11x+2z=-3$ $22x+4z=-6$ $-44x-8z=12$	M1 A1	3.1a 1.1b
	Must give a **reason**: e.g. Two equations are a linear multiple of each other e.g. shows they are the same equation therefore the equations are **consistent**.			A1	2.4
	Alternative Eliminates two different variables to form two equations, should be one equation from two of the three sections in the main scheme. e.g $5x+2y=1$ and $11y-5z=13$ rearranges and substitutes in to one of the original equations in three variables. e.g. $2x+3\left(\frac{1-5x}{2}\right)-\left(\frac{-3-11x}{2}\right)=3$			M1	3.1a
	Correct equations e.g $5x+2y=1$ and $11y-5z=13$			A1	1.1b
	Shows that the equations are a solution e.g. 3 = 3 therefore **consistent**			A1	2.4
(c)	The three **planes** form a **sheaf.**			B1	2.2a
				(1)	
					(6 marks)

Notes:
(a) **M1**: Finds the determinant of the matrix corresponding to the system of equations. **A1**: Sets determinant = 0 and solves their 3TQ to achieve $k = 2$ ($k = -8$ must be rejected)
(a) Special case **M1A0**: Uses $k = 2$ and finds the determinant of the matrix corresponding to the system of equations Shows that determinant = 0 and concludes that when $k = 2$ there is no unique solution
(b) **M1**: A complete method eliminating one variable from the equations using two different pairs of equations. Condone if a different value of k is used **A1**: Achieves two equations in the same two variables **A1**: **Must give a reason,** shows that the equations are a linear multiple of each other therefore they are **consistent**.
(b) Alternative **M1**: A complete method eliminating one variable from the equations using two different pairs of equations. Substitutes these equations into one of the original equations in three variables. **A1**: Achieves two correct equations in two different variables **A1**: Shows that the equation works therefore they are **consistent**.
(c) **B1**: The three **planes** form a **sheaf.** They must have full marks in (b) to award this mark.

(**Q01 8FM0/01, Oct 2020**)

Online Maths Teaching

Q9.

Question	Scheme	Marks	AOs
(a)	Rotation	B1	1.1b
	120 degrees (anticlockwise) or $\frac{2\pi}{3}$ radians (anticlockwise) Or 240 degrees clockwise or $\frac{4\pi}{3}$ radians clockwise	B1	2.5
	About (from) the origin. Allow (0, 0) or O for origin.	B1	1.2
		(3)	
(b)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1	1.1b
		(1)	
(c)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \ldots & \ldots \\ \ldots & \ldots \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$	A1ft	1.1b
		(2)	

(d)	$\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix} = \ldots$ or $\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \ldots$ Note: $\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2}+\frac{1}{2}k \\ \frac{1}{2}+\frac{\sqrt{3}}{2}k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix}$ can score M1 (for the matrix equation) but needs an equation to be "extracted" to score the next A1	M1	3.1a
	$-\frac{\sqrt{3}}{2}+\frac{1}{2}k=1$ or $\frac{1}{2}+\frac{\sqrt{3}}{2}k=k$ or $x=-\frac{\sqrt{3}}{2}x+\frac{1}{2}y$ or $y=\frac{1}{2}x+\frac{\sqrt{3}}{2}y$ (Note that candidates may then substitute x = 1 which is acceptable)	A1ft	1.1b
	$-\frac{\sqrt{3}}{2}+\frac{1}{2}k=1$ or $x=-\frac{\sqrt{3}}{2}x+\frac{1}{2}y \Rightarrow k=2+\sqrt{3}\left(\text{or } \frac{1}{2-\sqrt{3}}\right)$	A1	1.1b
	$\frac{1}{2}+\frac{\sqrt{3}}{2}k=k$ or $y=\frac{1}{2}x+\frac{\sqrt{3}}{2}y \Rightarrow k=2+\sqrt{3}\left(\text{or } \frac{1}{2-\sqrt{3}}\right)$	B1	1.1b
		(4)	
			(10 marks)

Notes
(a) B1: Identifies the transformation as a rotation B1: Correct angle. Allow equivalents in degrees or radians. B1: Identifies the origin as the centre of rotation These marks can only be awarded as the elements of a **single transformation** (b) B1: Shows the correct matrix in the correct form (c) M1: Multiplies the matrices in the correct order (evidence of multiplication can be taken from 3 correct or 3 correct ft elements) A1ft: Correct matrix (follow through their matrix from part (b)) A correct matrix or a correct follow through matrix implies both marks. (d) M1: Translates the problem into a matrix multiplication to obtain at least one equation in k or in x and y A1ft: Obtains one correct equation (follow through their matrix from part (c)) A1: Correct value for k in any form B1: Checks their answer by independently solving both equations **correctly** to obtain $2+\sqrt{3}$ both times or substitutes $2+\sqrt{3}$ into the other equation to confirm its validity

(**Q05 8FM0/01, June 2018**)

Q10.

Question	Scheme	Marks	AOs
(a)	$\mathbf{M}^{-1} = \frac{1}{69}\begin{pmatrix} 1 & 13 & 5 \\ -11 & -5 & 14 \\ -26 & 7 & 8 \end{pmatrix}$	B1 B1	1.1b 1.1b
		(2)	
(b)	$\frac{1}{69}\begin{pmatrix} 1 & 13 & 5 \\ -11 & -5 & 14 \\ -26 & 7 & 8 \end{pmatrix}\begin{pmatrix} -4 \\ 9 \\ 5 \end{pmatrix} = \ldots$	M1	1.1b
	$x = 2, y = 1, z = 3$ or $(2,1,3)$ or $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$	A1	1.1b
		(2)	
(c)	The **point** where three **planes** meet	B1ft	2.2a
		(1)	
		(5 marks)	

Notes
(a) B1: Evidence that the determinant is ± 69 (may be implied by their matrix e.g. where entries are not in exact form: $\pm\begin{pmatrix} 0.014 & 0.188 & 0.072 \\ -0.159 & -0.072 & 0.203 \\ -0.377 & 0.101 & 0.116 \end{pmatrix}$)(Should be mostly correct) **Must be seen in part (a).** B1: Fully correct inverse with all elements in **exact** form (b) M1: Any complete method to find the values of x, y and z (Must be using **their inverse** if using the method in the main scheme) A1: Correct coordinates A solution not using the inverse requires a complete method to find values for x, y and z for the method mark. Correct coordinates only scores both marks. (c) B1: Describes the correct geometrical configuration. Must include the two ideas of **planes** and **meet in a point** with no contradictory statements. This is dependent on having obtained a unique point in part (b)

(**Q01 8FM0/01, June 2018**)

Q11.

Question	Scheme	Marks	AOs
	x = value of savings account, y = value of property bond account, z = value of share dealing account.	M1	3.1b
	$x+y+z=5000$ $x+400=y$ $0.015x+0.035y-0.025z=79$ **or** $1.015x+1.035y+0.975z=5079$	A1	1.1b
	Let $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.015 & 0.035 & -0.025 \end{pmatrix}$ or $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1.015 & 1.035 & 0.975 \end{pmatrix}$		
	e.g. $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.015 & 0.035 & -0.025 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ -400 \\ 79 \end{pmatrix}$	M1	3.1a
		A1	1.1b
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.015 & 0.035 & -0.025 \end{pmatrix}^{-1}\begin{pmatrix} 5000 \\ -400 \\ 79 \end{pmatrix} = \begin{pmatrix} \ldots \\ \ldots \\ \ldots \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1800 \\ 2200 \\ 1000 \end{pmatrix}$	A1	1.1b
	Tyler invested £1800 in the savings account, £2200 in the property bond account and £1000 in the share dealing account.	A1ft	3.2a
		(7)	
			(7 marks)

	Question Notes	
	M1	Attempts to set up 3 equations with 3 unknowns.
	A1	At least 2 equations are correct with the appropriate variables defined.
	M1	Sets up a matrix equation of the form, e.g. $\begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}$, where "…" are numerical values.
	A1	Correct matrix equation (or equivalent).
	M1	Depends on previous M mark. Applies $(\text{their } \mathbf{A})^{-1}\begin{pmatrix} 5000 \\ \text{their "}-400\text{"} \\ \text{their "79"} \end{pmatrix}$ and obtains at least one value of x, y or z.
	A1	Correct answer.
	A1ft	Correct follow through answer in context.
	Note	$\mathbf{A}^{-1} = \begin{pmatrix} 0.25 & 0.6 & 10 \\ 0.25 & -0.4 & 10 \\ 0.5 & -0.2 & -20 \end{pmatrix}$ or $\begin{pmatrix} -9.75 & 0.6 & 10 \\ -9.75 & -0.4 & 10 \\ 20.5 & -0.2 & -20 \end{pmatrix}$

(**Q03 8FM0/01, Specimen papers**)

Online Maths Teaching

Q12.

Question	Scheme		Marks	AOs
(i) (a)	Multiplies the matrix **A** by itself and sets equal to **I** to form one equation in a only and another equation involving both a and b. $\begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}\begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\Rightarrow 4+a(a-4)=1$ and either $2a+ab=0$ or $2(a-4)+b(a-4)=0$ or $a(a-4)+b^2=1$		M1	3.1a
	Solves a 3TQ involving only the constant a. This could come after a value of b is found and this value substituted into an equation involving both a and b $a^2-4a+3=0\Rightarrow(a-3)(a-1)=0\Rightarrow a=...$		dM1	1.1b
	$a=1, a=3$		A1	11b
	Substitutes a value for a into an equation involving both a and b and solves for b. e.g. $2(1)+(1)b\Rightarrow b=...$ $2(1-4)b+(1-4)=0\Rightarrow b=...$ $(1)(1-4)+b^2=1\Rightarrow b=...$	Alternatively uses $2a+ab=0$ $a(2+b)=0$ As $a\neq 0$ $2+b=0\Rightarrow b=...$	dM1	1.1b
	$b=-2$		A1	1.1b
			(5)	

<table>
<tr><td rowspan="3"></td><td colspan="2">Alternative (i) (a)
Finds $\mathbf{A}^{-1}$ in terms of a and b, sets equal to $\mathbf{A}$ and attempts to find at least two different equations. Allow a single sign slip
$\frac{1}{2b-a(a-4)}\begin{pmatrix} b & -a \\ -(a-4) & 2 \end{pmatrix} = \begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}$
One equation from $\frac{b}{2b-a(a-4)}=2$, $\frac{2}{2b-a(a-4)}=b$
One equation from $\frac{-a}{2b-a(a-4)}=a$, $\frac{-(a-4)}{2b-a(a-4)}=a-4$</td><td>M1</td><td>3.1a</td></tr>
<tr><td>Uses their value of b and their value of the determinant to form and solve a 3TQ involving only the constant a
$a^2-4a+3=0$
$\Rightarrow (a-3)(a-1)=0$
$\Rightarrow a=\ldots$</td><td>Eliminates b from their equations and solve a 3TQ involving only the constant a
$a^2-4a+3=0$
$\Rightarrow (a-3)(a-1)=0$
$\Rightarrow a=\ldots$</td><td>dM1</td><td>1.1b</td></tr>
<tr><td colspan="2">$a=1, a=3$</td><td>A1</td><td>1.1b</td></tr>
<tr><td rowspan="3"></td><td>$\frac{-a}{2b-a(a-4)}=a$
$\Rightarrow 2b-a(a-4)=-1 \Rightarrow \frac{b}{-1}=2$
Or
$\frac{-(a-4)}{2b-a(a-4)}=a-4$
$\Rightarrow 2b-a(a-4)=-1$
$\Rightarrow \frac{2}{-1}=b$</td><td>Substitutes a value for a into an equation to find a value for b</td><td>dM1</td><td>1.1b</td></tr>
<tr><td colspan="2">$b=-2$</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2"></td><td></td><td></td></tr>
</table>

(b)	Uses their smallest value of a and their value for b to form two equations $\begin{pmatrix} 2 & 'a' \\ 'a-4' & 'b' \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow 2x + ay = x$ and $(a-4)x + by = y$ $\begin{pmatrix} 2 & 1 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow 2x + y = x$ and $-3x - 2y = y$	M1	3.1a
	$2x + y = x \Rightarrow x + y = 0$ o.e. and $-3x - 2y = y \Rightarrow x + y = 0$ o.e.	M1	1.1b
	$x + y = 0$ o.e.	A1	2.1
		(3)	
(ii)(a)	Area of the triangle $T = 3$	B1	1.1b
	Complete method to find a value for p. Need to see an attempt at the determinant and setting equal to 15 divided by their area of T. The resulting 3TQ needs to be solved to find a value of p. Determinant $3p \times p - (-1) \times 2p = \frac{15}{\text{'their area'}} \Rightarrow p = \dots$	M1	3.1a
	$3p^2 + 2p - 5 (= 0)$	A1	1.1b
	$p = 1$ must reject $p = -\frac{5}{3}$	A1	1.1b
		(4)	
(b)	$\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$	B1 B1	1.1b 1.1b
		(2)	
(c)	(their matrix found in part (b)) $\begin{pmatrix} 'p' & 2'p' \\ -1 & 3'p' \end{pmatrix} = \begin{pmatrix} \dots & \dots \\ \dots & \dots \end{pmatrix}$ $\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} \dots & \dots \\ \dots & \dots \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} 3 & 6 \\ 2 & -6 \end{pmatrix}$	A1ft	1.1b
		(2)	
			(16 marks)

Notes:
(i)(a) **M1**: Forming two equations, one involving a only and one involving a and b **dM1**: Dependent on previous mark, solves a 3TQ involving a **A1**: Correct values for a **dM1**: Dependent on first method mark Substitutes one of their values of a into an equation involving a and b and solve to find a value for b. Alternatively factorises either $2a+ab=0$ and uses $a \neq 0$ to find a value for b. **A1**: Correct value for b
Alternative(i)(a) **M1**: Finds $\mathbf{A}^{-1}$ and sets equal to **A** and forms two different equations **dM1**: Dependent on previous mark. Eliminates b from their equations and solves a 3TQ involving only the constant a. Alternatively if the value of b is found first substitutes their value for b into their determinant $= -1$ to form and solve a 3TQ for a **A1**: Correct value for a **dM1**: Dependent on first method mark. Substitutes a value for a into an equation to find a value for b. Alternatively uses one equation to find the determinant $= -1$ and uses this to find a value of b. **A1**: Correct values for b
(b) **M1**: Extracts simultaneous equations using their matrix **A** with their smaller value of a. **M1**: Gathers terms from their two equations. **A1**: Achieves the correct equations and deduces the correct line. Accept equivalent equations as long as both have been shown to be the same.
(ii)(a) **B1**: Area of the triangle $T = 3$ **M1**: Full method. Finds the determinant, sets equal to 15/their area and solves the resulting 3TQ **A1**: Correct quadratic **A1**: $p=1$ only
(b) **B1** One correct row or column **B1**: All correct
(c) **M1**: Multiplies the matrices **QP** in the correct order (if answer only then evidence can be taken from 3 correct or 3 correct ft elements) **A1ft**: Correct matrix following through on their answer to part (b) and their value of p as long as it is a positive constant

(**Q06 8FM0/01, Oct 2020**)

Online Maths Teaching

Q13.

Question	Scheme	Marks	AOs
(a)	(det(**M**) =) $(4)(-7)-(2)(-5)$	M1	1.1a
	M is non-singular because $\det(\mathbf{M}) = -18$ and so $\det(\mathbf{M}) \neq 0$	A1	2.4
		(2)	
(b)	Area $R = \dfrac{\text{Area } S}{(\pm)\lvert\det \mathbf{M}\rvert} = \ldots$	M1	1.2
	$\text{Area}(R) = \dfrac{63}{\lvert -18 \rvert} = \dfrac{7}{2}$ oe	A1ft	1.1b
		(2)	
(c)	$\begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}\begin{pmatrix} x \\ 2x \end{pmatrix} = \begin{pmatrix} 4x-10x \\ 2x-14x \end{pmatrix}$	M1	1.1b
	$= \begin{pmatrix} -6x \\ -12x \end{pmatrix}$ and so all points on $y = 2x$ map to points on $y = 2x$, hence the line is invariant. OR $= -6\begin{pmatrix} x \\ 2x \end{pmatrix}$ hence $y = 2x$ is invariant.	A1	2.1
		(2)	
	(6 marks)		

Notes		
(a)	M1	An attempt to find $\det(\mathbf{M})$. Just the calculation is sufficient. Site of −18 implies this mark, which may be embedded in an attempt at the inverse..
	A1	$\det(\mathbf{M}) = -18$ **and** reference to zero, e.g. $-18 \neq 0$ **and** conclusion. The conclusion may precede finding the determinant (e.g. "Non-singular if $\det(\mathbf{M}) \neq 0$, $\det(\mathbf{M}) = -18 \neq 0$" is sufficient or accept "Non-singular if $\det(\mathbf{M}) \neq 0$, $\det(\mathbf{M}) = -18$, therefore non-singular" or some other indication of conclusion.) Need not mention "det(**M**)" to gain both marks here, a correct calculation, statement $-18 \neq 0$, and conclusion hence **M** is non-singular can gain M1A1.
(b)	M1	Recalls determinant is needed for area scale factor by dividing 63 by ±their determinant.
	A1ft	$\frac{7}{2}$ or follow through $\frac{63}{\lvert \text{their det} \rvert}$. Must be positive and should be simplified to single fraction or exact decimal. (Allow if made positive following division by a negative determinant.)
(c)	M1	Attempts the matrix multiplication shown or with equivalent, e.g $\begin{pmatrix} \frac{1}{2}y \\ y \end{pmatrix}$. May use $\begin{pmatrix} x \\ y \end{pmatrix}$ **and** substitute $y = 2x$ later and this is fine for the method.
	A1	Correct multiplication and working leading to conclusion that the line is invariant. If the −6 is not extracted, they must make reference to image points being on line $y = 2x$. If the −6 is extracted to show it is a multiple of $\begin{pmatrix} x \\ 2x \end{pmatrix}$ followed by a conclusion "invariant" as minimum.

Alt for (c)	$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{-18}\begin{pmatrix} -7 & 5 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ 2x \end{pmatrix} = \frac{-1}{18}\begin{pmatrix} -7x+10x \\ -2x+8x \end{pmatrix}$	M1	1.1b
	$= \frac{-1}{18}\begin{pmatrix} 3x \\ 6x \end{pmatrix}\left(= \frac{-1}{6}\begin{pmatrix} x \\ 2x \end{pmatrix}\right) \Rightarrow b = 2a$ so points on line $y = 2x$ map to points on $y = 2x$, hence it is invariant.	A1	2.1
	Marks as per main scheme,		
Alt 2	(Since linear transformations map straight lines to straight lines…) E.g. $(1, 2)$ is on line $y = 2x$, and $\begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4-10 \\ 2-14 \end{pmatrix}$	M1	1.1b
	$= \begin{pmatrix} -6 \\ -12 \end{pmatrix}$, which is also on the line y=2x, hence as (0,0) and (1,2) both map to points on y = 2x (and transformation is linear) then y =2x is invariant.	A1	2.1
	Notes		
	M1 Identifies a point on the line $y = 2x$ and finds its image under T. If (0,0) is used there must be a clear statement it is because this is on the line, but for other points accept with any line on $y = 2x$ without statement.		
	A1 Shows the image and another point, which may be (0,0), on y=2x both map to points on y = 2x concludes line is invariant. Need not reference transformation being linear for either mark here.		
Alt 3	$\begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}\begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} X \\ mX+c \end{pmatrix} \Rightarrow \begin{matrix} 4x-5(mx+c)=X \\ 2x-7(mx+c)=mX+c \end{matrix}$ $\Rightarrow 2x-7(mx+c) = m(4x-5(mx+c))+c$ $\Rightarrow (5m^2-11m+2)x+(5m-8)c=0$ $\Rightarrow (5m-1)(m-2)=0 \Rightarrow m = \ldots$ Or similar work with $c = 0$ throughout.	M1	2.1
	$(5m-8 \neq 0 \Rightarrow c = 0)$ Hence $m = 2$ gives an invariant line (with $c = 0$), so $y = 2x$ is invariant.	A1	1.1b
	Notes		
	M1 Attempts to find the equation of a general invariant line, or general invariant line through the origin (so may have $c = 0$ throughout). To gain the method mark they must progress from finding the simultaneous equations to forming a quadratic in m and solving to a value of m.		
	A1 Correct quadratic in m found, with $m = 2$ as solution (ignore the other) and deduction that hence $y = 2x$ is an invariant line. Ignore errors in the $(5m - 8)$ here as $c = 0$ is always a possible solution. No need to see $c = 0$ derived.		

(**Q01 8FM0/01, June 2019**)

Q14.

Question	Scheme	Marks	AOs
(a)	a represents the proportion of juvenile chimpanzees that (survive and) **remain** juvenile chimpanzees the next year.	B1	3.4
		(1)	
(b)(i)	Determinant $= 0.82a - 0.08\times0.15$	M1	1.1b
	$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1} = \ldots\begin{pmatrix} 0.82 & -0.15 \\ -0.08 & a \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1} = \dfrac{1}{0.82a-0.012}\begin{pmatrix} 0.82 & -0.15 \\ -0.08 & a \end{pmatrix}$	A1	1.1b
		(3)	
(ii)	$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1}\begin{pmatrix} 15360 \\ 43008 \end{pmatrix} = \dfrac{1}{0.82a-0.012}\begin{pmatrix} 0.82\times15360-0.15\times43008 \\ (-0.08)\times15360+43008a \end{pmatrix}$ OR forms equations $15360 = aJ_0 + 0.15\times A_0$ $43008 = 0.08\times J_0 + 0.82\times A_0$	M1	3.1a
	$\dfrac{1}{0.82a-0.012}\left[6144+(43008a-1228.8)\right] = 64000$ $\Rightarrow 4915.2+43008a = 64000(0.82a-0.012) \Rightarrow a = \ldots$ OR $A_0 = 64000 - J_0 \Rightarrow 43008 = 0.08\times J_0 + 0.82\times(64000-J_0) = J_0 = \ldots$ $\Rightarrow a = \dfrac{15360-(64000-J_0)}{J_0} = \ldots$	M1	3.1a
	$a = \dfrac{5683.2}{9472} = 0.60$	A1	1.1b
		(3)	
(iii)	Initial juvenile population $= \dfrac{\text{"6144"}}{\text{"0.48"}} = 12800$	M1	3.4
	So change of 2560 juvenile chimpanzees	A1	1.1b
		(2)	

(c)	As the number of juveniles has increased, the model is not initially predicting a decline, so is not suitable in the short term. (Follow through their answer to (b) – but they must have made an attempt at it to find at least a value for J_0)	B1ft	3.5a
		(1)	
(d)	Third category needs to be introduced for chimpanzees aged 40 and above, mature chimpanzees M_n, and a matrix multiplication of increased dimension set up. Accept $3\times3, 3\times2$ or 2×3 matrices including all three categories in the column vector.	M1	3.5c
	The corresponding matrix model will have the form $\begin{pmatrix} J_{n+1} \\ A_{n+1} \\ M_{n+1} \end{pmatrix} = \begin{pmatrix} a & b & \underline{0} \\ 0.08 & c & 0 \\ 0 & d & e \end{pmatrix}\begin{pmatrix} J_n \\ A_n \\ M_n \end{pmatrix}$ (The underlined zero must be correct but do not be concerned about any values used in the other entries.)	A1	3.3
		(2)	
	(12 marks)		

Notes		
(a)	B1	Correct interpretation. Need not mention survival but must be clear it is the (proportion of) **juveniles that remain as juveniles** the next year (ie those that survive but don't progress to adulthood). E.g. accept "(number of) juveniles who do not become adults" but do not accept "surviving juveniles".
		Mark part (b) as a whole.
(b)(i)	M1	Attempts the determinant in terms of a Allow miscopies for the attempt. Allow $0.82a - 0.12$ as a slip.
	M1	Attempts the form of the inverse, swapped leading diagonals and sign changed on both off diagonals. Allow miscopies of the numbers but the signs must be correct.
	A1	Correct inverse matrix
(ii)	M1	Use the inverse matrix and attempts to find the initial juvenile and adult populations. . (May have determinant 1 for this mark.) Alternatively, sets up simultaneous equations from the original system, $15360 = aJ_0 + 0.15 \times A_0$ and $43008 = 0.08 \times J_0 + 0.82 \times A_0$ Accept with J_n and A_n or other appropriate variables.
	M1	Uses the sum of initial populations equals 64000 in an attempt to find a. (May have determinant 1 for this mark.) If using alternative, use of e.g. $A_0 = 64000 - J_0$ in second equation to find J_0, followed by attempt to find a. Award for an attempt to solve the equations, but don't be too concerned with the algebraic process as long as they are attempting to use all three equations.
	A1	Correct value, $a = 0.6$ (or 0.60 or $\frac{3}{5}$).
(iii)	M1	Uses their a to find the value of J_0. This mark may be gained for work done in (ii) if the alternative has been used but must have come from a correct method.
	A1	Correct difference found, as long as there is no contradictory statement – so "decrease of 2560" is A0.
(c)	B1ft	Comments that the change is an increase so does not fit the model. Follow through their answer to (b) as long as at least a value for J_0 has been found. If a decrease has been found allow for commenting the model is suitable. If an answer is given to (b)(iii), follow through on whatever their answer is. If no answer has been given, but an initial population found, a comparison should be made between this value and 153600 with conclusion must be consistent with their answer for J_0
(d)	M1	Introduces a third category (may be *M*ature, *E*lderly or any suitable letter used) and sets up a matrix multiplication (the left hand side may be missing for this mark) with all three categories in the column vector. The dimension of the matrix should be 3 in at least either row or column, and there should be a 3×1 vector.
	A1	Sets up the new matrix equation, including both sides and making clear the zero (underlined) so that the correct progression that no new juveniles arise from the mature chimpanzees is clear. Overlook other values, though ideally the other two zeroes are shown too, to indicate mature chimpanzees do not regress to adulthood, and juveniles cannot proceed directly to mature chimpanzees.

Appendix: Alternatives to (b)

Note that variations may occur with the line equation chosen in part (a), but mark as follows:

Question	Scheme	Marks	AOs
Alt 1 (b)(i)	As per main scheme.	B1 M1	2.2a 3.1b
	$d^2 = (-400+600\lambda)^2 + (200-100\lambda)^2 + (-250+100\lambda)^2$ $= 380000\lambda^2 - 570000\lambda + 262500$ $=380000\left(\lambda - \frac{3}{4}\right)^2 + 48750 \Rightarrow \lambda = ...$	dM1	1.1b
	As per main scheme.	M1 A1	3.4 1.1b
		(5)	
(ii)	Length of tunnel is $\sqrt{"48750"} = ...$	M1	1.1b
	Awrt 221m from correct working, so completion of square must have been correct. (Must include units)	A1	1.1b
		(2)	

	Notes	
(i)	B1M1	As per main scheme.
	M1	Realises the need to find the distance from the point on the mountain to a general point on the line.
	dM1	Attempts the distance or distance squared of $\overrightarrow{MX}$, expands and completes the square to find the value of λ for which distance is minimum. May obtain other forms for the completed square. Look for $A(B\lambda - C)^2 - D + "262500"$ where $A, B, C, D \neq 0$ but B may be 1.
	M1A1	As per main scheme.
(ii)	M1	Correct method for the distance. May be as per main scheme, or via extracting from the completed square constant term.
	A1	Correct distance, including units. Accept awrt 221 m or $25\sqrt{78}$ m

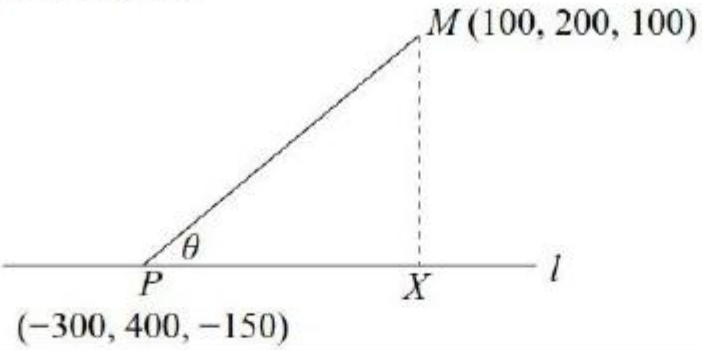
Alt 2 (b)(i)	As per main scheme.	B1 M1	2.2a 3.1b
	$d^2 = (-400+600\lambda)^2 + (200-100\lambda)^2 + (-250+100\lambda)^2$ $= 380000\lambda^2 - 570000\lambda + 262500$ $\frac{d}{dx}(d^2) = 0 \Rightarrow 760000\lambda - 570000 = 0 \Rightarrow \lambda = ...$	dM1	1.1b
	As per main scheme.	M1 A1	3.4 1.1b
		(5)	
(ii)	Length of tunnel is $\sqrt{(150-100)^2 + (325-200)^2 + (-75-100)^2} = ...$	M1	1.1b
	Awrt 221m from correct working, differentiation etc must have been correct. (Must include units)	A1	1.1b
		(2)	

Notes		
		As per main scheme except for:
(i)	**dM1**	Attempts the distance or distance squared of $\overrightarrow{MX}$, differentiates and set to zero to find λ for minimum distance.
(ii)	**M1**	May substitute λ into the distance squared formula to find distance.

Alt 3 (b)(i)	$k = 200$	B1	2.2a
	If M is the point on mountain, then e.g (may use Q rather than P) $\overrightarrow{MP} = \begin{pmatrix} -400 \\ 200 \\ -250 \end{pmatrix} \Rightarrow \cos\theta = \frac{\begin{pmatrix} -400 \\ 200 \\ -250 \end{pmatrix} . \begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix}}{\sqrt{(-400)^2 + 200^2 + (-250)^2}\sqrt{600^2 + (-100)^2 + 100^2}}$ $\Rightarrow \cos\theta = ...$ or $\theta = ...$ (where θ is the angle between the line and $\overrightarrow{MP}$)	M1	3.1b
	$\Rightarrow \lvert\overrightarrow{PX}\rvert = \lvert\overrightarrow{MP}\rvert \cos\theta = ...$	dM1	1.1b
	So e.g. $\overrightarrow{OX} = \begin{pmatrix} -300 \\ 400 \\ -150 \end{pmatrix} + \frac{\lvert\overrightarrow{PX}\rvert}{\left\lvert\begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix}\right\rvert}\begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix} = \begin{pmatrix} -300 \\ 400 \\ -150 \end{pmatrix} + \frac{"75\sqrt{8}"}{100\sqrt{38}}\begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix} = ...$	M1	3.4
	So coordinates of X are $(150, 325, -75)$ Accept as $\begin{pmatrix} 150 \\ 325 \\ -75 \end{pmatrix}$	A1	1.1b
		(5)	
(ii)	Length of tunnel is $\lvert\overrightarrow{MP}\rvert \sin\theta = ...$ (oe)	M1	1.1b
	Awrt 221m from correct working. (Must include units)	A1	1.1b
		(2)	

Notes		
(i)	**B1**	Correct value of k deduced.
	M1	Finds $\overrightarrow{MP}$ (or $\overrightarrow{MQ}$) and attempts scalar product formula with this and the direction of the line to find the angle or cosine of the angle between line and $\overrightarrow{MP}$ (or $\overrightarrow{MQ}$)
	dM1	Uses their angle with the cosine to find the length of $\overrightarrow{PX}$ (or $\overrightarrow{QX}$). Accept equivalent trigonometric methods (e.g. finding opposite side first and using tangent or Pythagoras.
	M1	Uses the length of and $\overrightarrow{PX}$ (or $\overrightarrow{QX}$) to find the coordinates of the point on the line at shortest distance from M.
	A1	Correct point.
(ii)	**M1**	Correct method for the distance. May be as per main scheme, or use of sine ratio with their angle between the line and and $\overrightarrow{MP}$ (or $\overrightarrow{MQ}$). Accept equivalent trigonometric methods.
	A1	Correct distance, including units. Accept awrt 221 m or $25\sqrt{78}$ m

Useful diagram:



Note for P, $\cos\theta = \pm\frac{57}{\sqrt{38}\sqrt{105}}$,

$\theta = 25.5...°$ and $\left|\overrightarrow{PX}\right| = 75\sqrt{38}$

For Q $\cos\theta = \pm\frac{19}{\sqrt{38}\sqrt{29}}$,

$\theta = 55.08...°$, $\left|\overrightarrow{QX}\right| = 25\sqrt{38}$

(**Q10 8FM0/01, June 2019**)

Q15.

Question	Scheme	Marks	AOs
(a)	$\mathbf{MN} = \begin{pmatrix} 2k-24 & 0 & 0 \\ k^2-7k+10 & 6k-44 & -10k+50 \\ 4k-20 & 0 & -14 \end{pmatrix}$	B1 B1	1.1b 1.1b
		(2)	
(b)(i)	$\mathbf{MN} = \begin{pmatrix} -14 & 0 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & -14 \end{pmatrix}$	B1ft	1.1b
(ii)	$\mathbf{M}^{-1} = -\frac{1}{14}\begin{pmatrix} -2 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}$	B1	1.1b
		(2)	
(c)	$\mathbf{M}^{-1} = -\frac{1}{14}\begin{pmatrix} -2 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \ldots$	M1	1.1b
	$\left(-\frac{12}{7}, \frac{40}{7}, -\frac{1}{14}\right)$	A1	1.1b
		(2)	
(d)	The coordinates of the only point at which the **planes** represented by the equations in (c) meet.	B1	2.2a
		(1)	
			(7 marks)

Notes
(a) B1: For 2 correct rows or 2 correct columns (allow unsimplified) B1: Fully correct simplified matrix (b)(i) B1ft: Correct matrix (follow through from part (a)). If an error with part (a) allow the correct matrix stated, restart use of calculator. (ii) B1: Deduces the correct inverse matrix, may use calculator (c) M1: Any complete method to find the values of x, y and z (Must be using **their inverse** if using the method in the main scheme) Allow use of a calculator A1: Correct exact coordinates (allow as a vector or $x = \ldots$, $y = \ldots$, $z = \ldots$) (d) B1: Describes the correct geometrical configuration of the **planes**

(**Q04 8FM0/01, Oct 2021**)

Q16.

Question Number	Scheme	Notes	Marks
(i)(a)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$		B1
(b)	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$		B1
(c)	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	M1: Multiplies their (b) x their (a) in the correct order A1: Correct matrix Correct matrix seen M1A1	M1A1
			(4)
(ii)	Area triangle T $= \frac{1}{2}\times(11-3)\times k = 4k$	M1: Correct method for area for T A1: 4k	M1A1
	$\det\begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix} = 6\times 2 - 1\times(-2)(=14)$	M1: Correct method for determinant A1: 14	M1A1
	Area triangle $T = \frac{364}{"14"}(=26) \Rightarrow 4k = 26$	Uses 364 and their determinant correctly to form an equation in k.	M1
	$k = \frac{26}{4}\left(=\frac{13}{2}\right)$	Accept $k = \pm\frac{13}{2}$ or $k = -\frac{13}{2}$	A1
			(6)
			Total 10

(**Q06 6667/01, June 2014**)

Q17.

Question Number	Scheme	Marks
	(a) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1 **(1)**
	(b) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1 **(1)**
	(c) **R** = **QP**	B1 **(1)**
	(d) R = $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	M1 A1 cao **(2)**
	(e) Reflection in the y axis	B1 B1 **(2)** **[7]**
Notes	(a) and (b) Signs must be clear for B marks. (c) Accept **QP** or their 2x2 matrices in the correct order only for B1. (d) M for their **QP** where answer involves ±1 and 0 in a 2x2 matrix. A for correct answer only. (e) First B for Reflection, Second B for 'y axis' or 'x=0'. Must be single transformation. Ignore any superfluous information.	

(**Q04 6667/01, Jan 2013**)

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Q18.

Question Number	Scheme	Notes	Marks
(a)	$\det \mathbf{M} = 3(-5) - (4)(2) = -15 - 8 = \underline{-23}$	$\underline{-23}$	B1
			[1]
(b)	Therefore, $\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$	Using the information in the question to form the matrix equation. Can be implied by any of the correct equations below.	M1
	Either, $3(2a-7) + 4(a-1) = 25$ or $2(2a-7) - 5(a-1) = -14$ or $\begin{pmatrix} 3(2a-7)+4(a-1) \\ 2(2a-7)-5(a-1) \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$	Any one correct equation (unsimplified) inside or outside matrices	A1
	giving $a = 5$	$a = 5$	A1
			[3]
(c)	$\text{Area}(ORS) = \frac{1}{2}(6)(4); = \underline{12}$ (units)^2	M1: $\frac{1}{2}(6)(\text{Their } a - 1)$ A1: 12 cao and cso	M1A1
	Note A(6, 0) is sometimes misinterpreted as (0, 6) – this is the wrong triangle and scores M0 e.g.1/2x6x3 = 9		
			[2]
(d)	$\text{Area}(OR'S') = \pm 23 \times (12)$	$\pm \det \mathbf{M} \times$ (their part (c) answer)	M1
		$\underline{276}$ (follow through provided area > 0)	A1 $\sqrt{}$
	A method not involving the determinant requires the coordinates of R′ to be calculated ((18, 12)) and then a <u>correct</u> method for the area e.g. (26x25 – 7x13 – 9x12 – 7x25) M1 = 276 A1		
			[2]
(e)	Rotation; 90° anti-clockwise (or 270° clockwise) about $(0, 0)$.	B1: Rotation, Rotates, Rotate, Rotating (not turn) B1: 90° anti-clockwise (or 270° clockwise) about (around/from etc.) $(0, 0)$	B1;B1
			[2]
(f)	$\mathbf{M} = \mathbf{BA}$	$\mathbf{M} = \mathbf{BA}$, seen or implied.	M1
	$\mathbf{A}^{-1} = \frac{1}{(0)(0) - (1)(-1)}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$\mathbf{A}^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	A1
	$\mathbf{B} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	Applies $\mathbf{M}$(their $\mathbf{A}^{-1}$)	M1
	$\mathbf{B} = \begin{pmatrix} -4 & 3 \\ 5 & 2 \end{pmatrix}$		A1
	NB some candidates state $\mathbf{M} = \mathbf{AB}$ and then calculate $\mathbf{MA}^{-1}$ or state $\mathbf{M} = \mathbf{BA}$ and then calculate $\mathbf{A}^{-1}\mathbf{M}$. These could score M0A0 M1A1ft and M1A1M0A0 respectively.		[4]
			14 marks
	Special case		
(f)	$\mathbf{M} = \mathbf{AB}$	$\mathbf{M} = \mathbf{AB}$, seen or implied.	M0
		$\mathbf{A}^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	A0
	$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ -3 & -4 \end{pmatrix}$	Applies (their $\mathbf{A}^{-1}$)$\mathbf{M}$	M1A1ft

(**Q06 6667/01, June 2012**)

Q19.

Question Number	Scheme		Marks
(a)	$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$ $\det \mathbf{A} = 2(3) - (-1)(-2) = 6 - 2 = \underline{4}$	$\underline{4}$	B1 (1)
(b)	$\mathbf{A}^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	$\frac{1}{\det \mathbf{A}}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$ $\frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	M1 A1 (2)
(c)	$\text{Area}(R) = \frac{72}{4} = \underline{18}\ (\text{units})^2$	$\frac{72}{\text{their}\det \mathbf{A}}$ or $72(\text{their}\det \mathbf{A})$ $\underline{18}$ or ft answer.	M1 A1$\sqrt{}$ (2)
(d)	$\mathbf{AR} = \mathbf{S} \Rightarrow \mathbf{A}^{-1}\mathbf{AR} = \mathbf{A}^{-1}\mathbf{S} \Rightarrow \mathbf{R} = \mathbf{A}^{-1}\mathbf{S}$ $\mathbf{R} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 8 & 12 \\ 4 & 16 & 4 \end{pmatrix}$ $= \frac{1}{4}\begin{pmatrix} 8 & 56 & 44 \\ 8 & 40 & 20 \end{pmatrix}$ $= \begin{pmatrix} 2 & 14 & 11 \\ 2 & 10 & 5 \end{pmatrix}$ Vertices are $(2, 2)$, $(14, 10)$ and $(11, 5)$.	At least one attempt to apply $\mathbf{A}^{-1}$ by any of the three vertices in $\mathbf{S}$. At least one correct column o.e. At least two correct columns o.e. All three coordinates correct.	M1 A1$\sqrt{}$ A1 A1 (4) [9]

(**Q07 6667/01, Jan 2011**)

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Q20.

Question	Scheme	Marks	AOs
(i)(a)	Stretch	B1	1.1a
	Scale factor 2 parallel to x-axis	B1	2.5
		(2)	
(i)(b)	Any line parallel to the x-axis or $y = k$ or the y-axis or $x = 0$	B1	1.1b
		(1)	

(ii)	Area triangle $=\frac{1}{2}\times(8-2)(6-3)=9$ Area triangle for example $\frac{1}{2}\begin{vmatrix}2 & 3 & 8 & 2\\ 3 & 6 & 3 & 3\end{vmatrix}=\frac{1}{2}\left[2\times6+3\times3+8\times3-(3\times3+8\times6+2\times3)\right]$ Area triangle $=\frac{1}{2}(6)(\sqrt{10})\sin 71.57$ from a valid attempt to find the angle and sides	**M1**	3.1a
	$\begin{vmatrix}\cos 2\theta & 0\\ 1 & \tan 2\theta\end{vmatrix}=\cos 2\theta\tan 2\theta$	**B1**	1.1b
	their '$\cos 2\theta\tan 2\theta$' $=\frac{4.5}{\text{their '9'}}$	**M1**	1.1b
	$\cos 2\theta\tan 2\theta=\frac{4.5}{\text{their 9}}\Rightarrow\cos 2\theta\times\frac{\sin 2\theta}{\cos 2\theta}=\frac{4.5}{\text{their 9}}\Rightarrow\sin 2\theta=k$ where $-1<k<1$	**dM1**	3.1a
	$\sin 2\theta=\frac{1}{2}\Rightarrow\theta=...$ or $\sin 2\theta=-\frac{1}{2}\Rightarrow\theta=...$	**ddM1**	1.1b
	Any two of **<u>Coming from</u>** $\sin 2\theta=\frac{1}{2}$ 15°, 75°, 195°, 255° or $\frac{\pi}{12},\frac{5\pi}{12},\frac{13\pi}{12},\frac{17\pi}{12}$ or awrt 0.26, 1.31, 3.40, 4.45 **<u>Coming from</u>** $\sin 2\theta=-\frac{1}{2}$ 105°, 165°, 285°, 345° or $\frac{7\pi}{12},\frac{11\pi}{12},\frac{19\pi}{12},\frac{23\pi}{12}$ or awrt , 1.83, 2.88, 4.97, 6.02	**A1**	1.1b
	All eight of 15°, 75°, 195°, 255°, 105°, 165°, 285°, 345°	**A1**	2.3
		(7)	
			(10 marks)

Notes:
(i)(a) **B1**: See scheme, enlargement is B0 **B1**: See scheme, if they mention of y-axis must be correct e.g. scale factor 1, stays the same. Condone defining the stretch as from/about the origin and in the x- axis, along the x-axis **(i)(b)** **B1**: See scheme, any incorrect answer stated B0
(ii) **M1**: Correct method to find the area of the triangle T **B1**: Correct determinant of the matrix **M1**: Sets their determinant = 4.5 divided by their area of the triangle T **dM1**: Dependent on previous method mark. Uses the identity $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$ to achieve $\sin 2\theta = k$ where $-1 < k < 1$ **ddM1**: Dependent on previous method marks. Solves $\sin 2\theta = k$ where $-1 < k < 1$ to find a value of θ **A1**: Any two correct values in degrees or radians **A1**: All eight correct values and no others **must be in degrees**

(**Q04 8FM0/01, June 2025**)

Q21.

Question	Scheme	Marks	AOs
(a)	$\det(\mathbf{M}) = (1)(1) - (\sqrt{3})(-\sqrt{3})$	M1	1.1a
	$\mathbf{M}$ is non-singular because $\det(\mathbf{M}) = 4$ and so $\det(\mathbf{M}) \neq 0$	A1	2.4
		(2)	
(b)	$\text{Area}(S) = 4(5) = 20$	B1ft	1.2
		(1)	
(c)	$k = \sqrt{(1)(1) - (\sqrt{3})(-\sqrt{3})}$	M1	1.1b
	$= 2$	A1ft	1.1b
		(2)	
(d)	$\cos\theta = \frac{1}{2}$ or $\sin\theta = \frac{\sqrt{3}}{2}$ or $\tan\theta = \sqrt{3}$	M1	1.1b
	$\theta = 60°$ or $\frac{\pi}{3}$	A1	1.1b
		(2)	
		(7 marks)	

		Question Notes
(a)	M1	An attempt to find $\det(\mathbf{M})$.
	A1	$\det(\mathbf{M}) = 4$ **and** reference to zero, e.g. $4 \neq 0$ **and** conclusion.
(b)	B1ft	20 or a correct ft based on their answer to part (a).
(c)	M1	$\sqrt{(\text{their } \det\mathbf{M})}$
	A1ft	2
(d)	M1	**Either** $\cos\theta = \frac{1}{(\text{their } k)}$ **or** $\sin\theta = \frac{\sqrt{3}}{(\text{their } k)}$ **or** $\tan\theta = \sqrt{3}$
	A1	$\theta = 60^{\circ}$ or $\frac{\pi}{3}$. Also accept any value satisfying $360n + 60^{\circ}$, $n \in \mathbb{Z}$, o.e.

(**Q05 8FM0/01, Specimen papers**)