1.Some Basic Formulae:

i. $(a + b)(a - b) = (a^2 - b^2)$
ii. $(a + b)^2 = (a^2 + b^2 + 2ab)$
iii. $(a - b)^2 = (a^2 + b^2 - 2ab)$
iv. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
v. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
vi. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
vii. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
viii. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Problems on Numbers Questions and Answers

1. If one-third of one-fourth of a number is 15, then three-tenth of that number is:

A.35 B.36
C.45 D.54

EXPLANATION

Let the number be x.

Then, $\frac{1}{3}$ of $\frac{1}{4}$ of $x = 15 \Leftrightarrow x = 15 \times 12 = 180$.

So, required number $= \left(\frac{3}{10} \times 180\right) = 54$.

2. Three times the first of three consecutive odd integers is 3 more than twice the third. The third integer is:

A.9 B.11
C.13 D.15

EXPLANATION

Let the three integers be x, $x + 2$ and $x + 4$.

Then, $3x = 2(x + 4) + 3 \Leftrightarrow x = 11$.

$\therefore$ Third integer $= x + 4 = 15$.

3. The difference between a two-digit number and the number obtained by interchanging the positions of its digits is 36. What is the difference between the two digits of that number?

A.3 B.4
C.9 D.Cannot be determined
E. None of these

EXPLANATION

Let the ten's digit be x and unit's digit be y.

Then, $(10x + y) - (10y + x) = 36$

$\Rightarrow 9(x - y) = 36$

$\Rightarrow x - y = 4.$

4. The difference between a two-digit number and the number obtained by interchanging the digits is 36. What is the difference between the sum and the difference of the digits of the number if the ratio between the digits of the number is 1 : 2 ?

A.4 B.8
C.16 D.None of these

EXPLANATION

Since the number is greater than the number obtained on reversing the digits, so the ten's digit is greater than the unit's digit.

Let ten's and unit's digits be $2x$ and x respectively.

Then, $(10 \times 2x + x) - (10x + 2x) = 36$

$\Rightarrow 9x = 36$

$\Rightarrow x = 4.$

$\therefore$ Required difference $= (2x + x) - (2x - x) = 2x = 8.$

5. A two-digit number is such that the product of the digits is 8. When 18 is added to the number, then the digits are reversed. The number is:

A.18 B.24
C.42 D.81

EXPLANATION

Let the ten's and unit digit be x and $\frac{8}{x}$ respectively.

Then, $\left(10x + \frac{8}{x}\right) + 18 = 10 \times \frac{8}{x} + x$

$\Rightarrow 10x^2 + 8 + 18x = 80 + x^2$

$\Rightarrow 9x^2 + 18x - 72 = 0$

$\Rightarrow x^2 + 2x - 8 = 0$

$\Rightarrow (x + 4)(x - 2) = 0$

$\Rightarrow x = 2.$

6. The sum of the digits of a two-digit number is 15 and the difference between the digits is 3. What is the two-digit number?

A.69 B.78
C.96 D.Cannot be determined
E. None of these

EXPLANATION

Let the ten's digit be x and unit's digit be y.

Then, $x + y = 15$ and $x - y = 3$ or $y - x = 3$.

Solving $x + y = 15$ and $x - y = 3$, we get: $x = 9, y = 6$.

Solving $x + y = 15$ and $y - x = 3$, we get: $x = 6, y = 9$.

So, the number is either 96 or 69.

Hence, the number cannot be determined.

7. The sum of the squares of three numbers is 138, while the sum of their products taken two at a time is 131. Their sum is:

A.20 B.30
C.40 D.None of these

EXPLANATION

Let the numbers be a, b and c.

Then, $a^2 + b^2 + c^2 = 138$ and $(ab + bc + ca) = 131$.

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = 138 + 2 \times 131 = 400$.

$\Rightarrow (a + b + c) = \sqrt{400} = 20$.

8. A number consists of two digits. If the digits interchange places and the new number is added to the original number, then the resulting number will be divisible by:

A.3 B.5
C.9 D.11

EXPLANATION

Let the ten's digit be x and unit's digit be y.

Then, number = $10x + y$.

Number obtained by interchanging the digits = $10y + x$.

$\therefore (10x + y) + (10y + x) = 11(x + y)$, which is divisible by 11.

9. In a two-digit, if it is known that its unit's digit exceeds its ten's digit by 2 and that the product of the given number and the sum of its digits is equal to 144, then the number is:

A.24 B.26
C.42 D.46

EXPLANATION

Let the ten's digit be x.

Then, unit's digit = $x + 2$.

Number = $10x + (x + 2) = 11x + 2$.

Sum of digits = $x + (x + 2) = 2x + 2$.

$\therefore (11x + 2)(2x + 2) = 144$

$\Rightarrow 22x^2 + 26x - 140 = 0$

$\Rightarrow 11x^2 + 13x - 70 = 0$

$\Rightarrow (x - 2)(11x + 35) = 0$

$\Rightarrow x = 2.$

Hence, required number = $11x + 2 = 24$.

10. Find a positive number which when increased by 17 is equal to 60 times the reciprocal of the number.

A.3 B.10
C.17 D.20

EXPLANATION

Let the number be x.

Then, $x + 17 = \frac{60}{x}$

$\Rightarrow x^2 + 17x - 60 = 0$

$\Rightarrow (x + 20)(x - 3) = 0$

$\Rightarrow x = 3.$

11. The product of two numbers is 9375 and the quotient, when the larger one is divided by the smaller, is 15. The sum of the numbers is:

A.380 B.395
C.400 D.425

EXPLANATION

Let the numbers be x and y.

Then, $xy = 9375$ and $\frac{x}{y} = 15$.

$$\frac{xy}{(x/y)} = \frac{9375}{15}$$

$\Rightarrow y^2 = 625.$

$\Rightarrow y = 25.$

$\Rightarrow x = 15y = (15 \times 25) = 375.$

$\therefore$ Sum of the numbers $= x + y = 375 + 25 = 400.$

12. The product of two numbers is 120 and the sum of their squares is 289. The sum of the number is:

A.20 B.23

C.169 D.None of these

EXPLANATION

Let the numbers be x and y.

Then, $xy = 120$ and $x^2 + y^2 = 289$.

$\therefore (x + y)^2 = x^2 + y^2 + 2xy = 289 + (2 \times 120) = 529$

$\therefore x + y = \sqrt{529} = 23.$

13. A number consists of 3 digits whose sum is 10. The middle digit is equal to the sum of the other two and the number will be increased by 99 if its digits are reversed. The number is:

A.145 B.253

C.370 D.352

EXPLANATION

Let the middle digit be x.

Then, $2x = 10$ or $x = 5$. So, the number is either 253 or 352.

Since the number increases on reversing the digits, so the hundred's digits is smaller than the unit's digit.

Hence, required number = 253.

14. The sum of two number is 25 and their difference is 13. Find their product.

A.104 B.114

C.315 D.325

EXPLANATION

Let the numbers be x and y.

Then, $x + y = 25$ and $x - y = 13$.

$4xy = (x + y)^2 - (x - y)^2$

$= (25)^2 - (13)^2$

$= (625 - 169)$

$= 456$

$\therefore xy = 114$.

15. What is the sum of two consecutive even numbers, the difference of whose squares is 84?

A.34 B.38
C.42 D.46

EXPLANATION

Let the numbers be x and $x + 2$.

Then, $(x + 2)^2 - x^2 = 84$

$\Rightarrow 4x + 4 = 84$

$\Rightarrow 4x = 80$

$\Rightarrow x = 20$.

$\therefore$ The required sum $= x + (x + 2) = 2x + 2 = 42$.